

# Flattening Index of the Solar Corona and the Solar Cycle

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**Abstract** The photometrical flattening index of the solar corona  $a + b$  is defined according to Ludendorff. In this paper we have investigated how the flattening index varies with respect to the phase of solar activity and the sunspot number. We have compiled 170 values of the flattening index using the data on 60 total solar eclipses from 1851 to 2010. We have found that the flattening index takes values from 0 to 0.4, and is anticorrelated with solar activity. The value of the flattening index at the beginning of solar cycle 24 was used as a precursor to forecast the amplitude of the cycle. It was found that the amplitude of solar cycle 24 will be about 95 in terms of the smoothed monthly sunspot numbers.

**Keywords** Solar corona · Solar cycle

## 1. Introduction

It is well known that the structure, shape, and brightness of the solar corona change significantly from eclipse to eclipse. Hansky (1897), after analyzing the material of 20 total solar eclipses from 1842 to 1896, first came to important conclusions about the dependence of the shape and brightness of the corona on yearly sunspot numbers. In a sunspot number maximum, the corona forms an entire radiance around the Sun. In a sunspot number minimum, the corona stretches more along the equator, and its brightness is weaker. Naegamvala (1902) compared drawings of the solar corona during 21 total solar eclipses with sunspot activity and confirmed Hansky's (1897) results. Lockyer (1903, 1922, 1931) in a number of papers attempted to find a relationship between the shape (and structure) of the corona and solar prominences. However, these investigations were rather qualitative.

A quantitative analysis of the shape of the solar corona was first made by Ludendorff (1928). He analyzed isophotes of coronal images and defined the averaged flattening of the

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corona by the formula

$$\varepsilon = \frac{\text{I} + \text{II} + \text{VIII}}{\text{IV} + \text{V} + \text{VI}} - 1, \quad (1)$$

where I, II, ..., VIII designate eight diameters of an isophote separated at angles of  $22.5^\circ$ , I being the diameter in the direction of the equator, and so on.

In other words, the flattening of the corona can be defined as follows:

$$\varepsilon = \frac{d_e - d_p}{d_p} = \frac{d_e}{d_p} - 1, \quad (2)$$

where  $d_e$  is the mean of the equatorial diameter of an isophote and two of its diameters at angles  $\pm 22.5^\circ$  to the equatorial direction;  $d_p$  is an analogous quantity for the polar direction.

Ludendorff (1928) found, for the innermost isophotes, a linear relation between the average flattening  $\varepsilon$  and the average equatorial semidiameter  $r_e$ :  $\varepsilon = a + b(r_e - 1)$ , where  $r_e = (\text{I} + \text{II} + \text{VIII})/6 = d_e/2$  and  $a$  and  $b$  are regression coefficients. If  $r_e = 2R_\odot$  (two solar radii), then  $\varepsilon = a + b$ . This value “ $a + b$ ” was indeed used by Ludendorff as a photometrical measure of the coronal flattening. It is therefore also called the photometrical flattening index (or simply the flattening index) of the solar corona.

As a rule, for a coronal image the average flattening  $\varepsilon$  increases nearly linearly from the limb to some distance  $r_{\max} = 1.5 - 2.5R_\odot$ , then it decreases to some distance  $r_{\min} = 3.5 - 5.5R_\odot$  (where the value of  $\varepsilon$  is minimal), and then it again increases with distance from the image center (Hata and Saito, 1966; Dzyubenko, 1983). Note that the initial linear part of the  $\varepsilon(r_e)$  dependence and its growth after  $r_{\min}$  seem to be related to the K-corona and the zodiacal light, respectively.

Ludendorff (1928) also introduced the phase of solar activity  $\Phi$  using the formula

$$\Phi = \frac{T_{\text{ecl}} - T_{\min}}{|T_{\max} - T_{\min}|}, \quad (3)$$

where  $T_{\text{ecl}}$  is the time of eclipse, and  $T_{\max}$  and  $T_{\min}$  are the times of the solar cycle maximum and minimum adjacent to  $T_{\text{ecl}}$ , respectively.

By the way, as noted by Gulyaev (1997), Ludendorff (1928) at first analyzed the relation of both parameters ( $a$  and  $b$ ) separately to sunspot activity, and later he (Ludendorff, 1934) found that it is more convenient to use the sum “ $a + b$ ”. In total 13 solar eclipses from 1893 to 1927 were analyzed in his first paper (Ludendorff, 1928) and 17 eclipses (eclipses of 1922, 1929, 1930, and 1932 were added) in his second paper (Ludendorff, 1934).

After Ludendorff's studies (1928, 1934), the flattening index “ $a + b$ ” was calculated for nearly each of the eclipses observed, and its dependence on the phase of solar activity was studied by many authors. As the most complete studies, we indicate van de Hulst (1953), Hata and Saito (1966), Waldmeier and Weber (1979), Dzyubenko (1983), Rušin and Rybanský (1985), Loucif and Koutchmy (1989), Gulyaev (1997), and Rušin and Rybanský (1999). Note also that Bergstrand (1935) investigated the relation between the flattening index and the sunspot number by taking into account geometrical corrections for coronal rays, and Bernheimer (1938) also looked for correlations between sunspot numbers and the shape of the solar corona.

Nikolsky (1956) defined another quantitative index for the shape of the solar corona called the geometrical flattening index  $H$ . It is based on geometrical characteristics of polar ray systems and coronal streamers. The  $H$  index was calculated for 38 total solar eclipses by Nesmyanovich (Vsekhsyatsky *et al.*, 1965). But this index was not widely calculated later.

Gulyaev (1997) proposed to calculate a new modified coronal flattening index in which the heliographic frame of reference in Ludendorff's definition is substituted by a heliomagnetic one with the magnetic dipole equator as the principal plane of the heliomagnetic coordinate system.

In the present paper we will study the flattening index, and its dependence on sunspot number and phase of solar activity, using all the available data in the literature. We will also attempt to predict the amplitude of solar cycle 24 on the basis of the flattening index at the beginning of cycle 24.

## 2. Compiled Data of Flattening Index

We have compiled 170 flattening index values for 60 total solar eclipses in 1851–2010 using the data in the literature shown in Table 1. Here we calculated 29 values of the flattening index using published isophotes and some images on the Internet. Note that van de Hulst (1953) used a modified version of Equation (2), where  $d_e$  and  $d_p$  were strictly polar and equatorial diameters.

As one can see from Table 1, the values of the flattening index vary for different eclipses, and, as a rule, they take slightly different values for the same eclipse. The first variance seems to be connected with solar activity. The second one can be explained by the fact that the calculated value of the flattening index for an eclipse is sensitive to many factors, namely film emulsion and its characteristics, developer and time of development, number of innermost isophotes selected for linear approximation, possible errors in the orientation of coronal images, *etc.* These factors are responsible for the large scatter in the calculated values of the index for some eclipses (for example, up to 0.16 for the total solar eclipse in 1941). In our study we will use all the flattening index values from Table 1. In particular, we will impose no weighting on the data and consider them to be of equal weight.

The flattening index values are in the range from 0 to 0.4. The largest values appear for the eclipses of 30 June 1954 and 23 October 1976; both occurred near the minimum of solar activity. The largest number of data in a single eclipse is for the total solar eclipse in 1961 (seven).

The monthly sunspot numbers and the phase of solar activity are also shown in Table 1. The monthly sunspot numbers were taken from the Solar Influences Data Analysis Center (SIDC, <http://sidc.oma.be>), and were then smoothed using 13-point running average and interpolated to the eclipse day. The times of minima and maxima of solar activity were found as the extrema of monthly sunspot numbers that were smoothed twice using 13-point running average, and were used to calculate the phase of solar activity. The phase of solar activity  $\Phi$  was calculated using Equation (3). The phase of solar activity  $P$  was calculated as the fraction of the cycle from the minimum to the eclipse epoch.  $P$  is sometimes called Waldmeier's definition of the phase (Waldmeier, Arber, and Bachmann, 1957). Phase  $P$  changes from 0 to 1 and phase  $\Phi$  from  $-1$  to 1. Phases  $P$  and  $\Phi$  can be interpreted as the phases from the minimum to the minimum and from the maximum to the maximum, respectively. These minima and maxima of solar activity that we obtained are very close to the corresponding minima and maxima presented by the National Geophysical Data Center (NGDC, <http://www.ngdc.noaa.gov>).

## 3. Flattening Index and Sunspot Number

Figure 1 shows the dependences of the flattening index on daily and monthly sunspot numbers. The index value decreases with the growth of daily or monthly sunspot numbers. As

**Table 1** Flattening index  $\varepsilon = a + b$  during total solar eclipses in 1851–2009.

Eclipse	Monthly sunspot number	Phase of solar activity		Flattening index ( $\varepsilon = a + b$ )	Reference
		$\Phi [-1; 1]$	$P [0; 1]$		
28 Jul 1851	62.3	-0.60	0.624	0.22	Waldmeier (1951)
16 Apr 1893	81.6	0.88	0.288	0.00	Ludendorff (1928)
				0.00	Ludendorff (1934)
				0.00	van de Hulst (1953)
08 Aug 1896	41.2	-0.65	0.568	0.24	van de Hulst (1953)
				0.24	Ludendorff (1934)
				0.26	Ludendorff (1928)
21 Jan 1898	26.1	-0.46	0.691	0.18	Ludendorff (1928)
				0.18	Ludendorff (1934)
				0.18	van de Hulst (1953)
28 May 1900	9.2	-0.17	0.890	0.30	Ludendorff (1934)
				0.30	van de Hulst (1953)
				0.32	Ludendorff (1928)
17 May 1901	3.9	-0.04	0.972	0.25	Ludendorff (1934)
				0.25	van de Hulst (1953)
				0.30	Ludendorff (1928)
30 Aug 1905	60.2	0.97	0.339	0.01	Ludendorff (1928)
				0.01	Ludendorff (1934)
				0.03	Waldmeier (1943)
				0.04	van de Hulst (1953)
30 Jan 1908	51.7	-0.71	0.540	0.08	Ludendorff (1928)
				0.08	Ludendorff (1934)
				0.08	van de Hulst (1953)
21 Aug 1914	16.3	0.29	0.128	0.16	Ramberg (1951b)
				0.19	Ludendorff (1934)
				0.19	van de Hulst (1953)
				0.21	Ludendorff (1928)
				0.23	Ramberg (1953)
08 Feb 1918	92.9	-0.90	0.511	0.23	Ludendorff (1928)
				0.23	Ludendorff (1934)
				0.23	van de Hulst (1953)
20 Sep 1922	10.4	-0.10	0.943	0.23	van de Hulst (1953)
				0.26	Ludendorff (1934)

**Table 1** (Continued)

Eclipse	Monthly sunspot number	Phase of solar activity		Flattening index ( $\varepsilon = a + b$ )	Reference
		$\Phi$ [-1; 1]	$P$ [0; 1]		
10 Sep 1923	7.6	0.08	0.038	0.22	Ludendorff (1934)
				0.23	van de Hulst (1953)
				0.24	Ludendorff (1928)
				0.24	von Klüber (1948)
25 Jan 1925	28.9	0.35	0.168	0.13	Ludendorff (1934)
				0.13	van de Hulst (1953)
				0.15	Ludendorff (1928)
14 Jan 1926	59.9	0.53	0.260	0.07	Ludendorff (1928)
				0.07	von Klüber (1932)
				0.07	Ludendorff (1934)
				0.07	van de Hulst (1953)
29 Jun 1927	70.2	0.81	0.397	0.04	Ludendorff (1928)
				0.04	Ludendorff (1934)
				0.04	van de Hulst (1953)
09 May 1929	63.4	-0.83	0.573	0.11	von Klüber (1931b)
				0.12	Ludendorff (1934)
				0.12	van de Hulst (1953)
				0.13	von Klüber (1931a)
28 Apr 1930	44.3	-0.66	0.665	0.25	van de Hulst (1953)
				0.27	Ludendorff (1934)
31 Aug 1932	10.9	-0.22	0.886	0.20	Ludendorff (1934)
				0.27	van de Hulst (1953)
14 Feb 1934	6.4	0.07	0.023	0.23	van de Hulst (1953)
				0.04	This work
				0.06	Loucif and Koutchmy (1989)
19 Jun 1936	78.7	0.69	0.251	0.04	van de Hulst (1953)
				0.06	Loucif and Koutchmy (1989)
				0.07	van de Hulst (1953)
08 Jun 1937	111.6	0.95	0.345	0.09	Loucif and Koutchmy (1989)
				0.09	van de Hulst (1953)
21 Sep 1941	46.9	-0.38	0.760	0.18	Bugoslavskaya (1949, 1950)
				0.30	This work
				0.34	Gulyaev (1997)
04 Feb 1943	20.5	-0.17	0.894	0.23	van de Hulst (1953)
09 Jul 1945	37.8	0.38	0.130	0.32 <sup>a</sup>	van de Hulst (1953)
				0.32	Ramberg (1951a)
				0.32	This work

**Table 1** (Continued)

Eclipse	Monthly sunspot number	Phase of solar activity		Flattening index ( $\varepsilon = a + b$ )	Reference
		$\Phi$ [-1; 1]	$P$ [0; 1]		
25 Feb 1952	43.4	-0.33	0.782	0.23	Loucif and Koutchmy (1989)
				0.26	Mikhelson (1958)
				0.27	This work
				0.29	Waldmeier (1954)
30 Jun 1954	7.0	0.04	0.012	0.27	Waldmeier and Bachmann (1955)
				0.29	Loucif and Koutchmy (1989)
				0.32	This work
				0.35	Wallenquist (1957)
				0.39	Waldmeier (1955)
20 Jun 1955	41.0	0.31	0.105	0.24	Waldmeier, Arber, and Bachmann (1957)
				0.25	Saito (1956)
12 Oct 1958	182.8	-0.88	0.423	0.07	Waldmeier (1959)
				0.13	This work
				0.14	Waldmeier (1959)
				0.21	Saito and Yamashita (1962)
02 Oct 1959	144.6	-0.74	0.517	0.14	Loucif and Koutchmy (1989)
				0.14	Waldmeier (1978)
				0.16	This work
15 Feb 1961	74.7	-0.54	0.651	0.12	Madjarska and Dermendjiev (1996)
				0.17	Tifrea (1996)
				0.17	Waldmeier (1962)
				0.17	Loucif and Koutchmy (1989)
				0.19	Cimino and Croce (1965)
				0.20	This work
0.24	Löchel and Högner (1965)				
05 Feb 1962	44.3	-0.40	0.742	0.26	This work
				0.28	Waldmeier (1963)
20 Jul 1963	26.4	-0.18	0.881	0.26	This work
				0.29	Waldmeier (1964)
30 May 1965	15.4	0.16	0.053	0.23	This work
				0.24	Waldmeier (1966)
				0.24	Loucif and Koutchmy (1989)
12 Nov 1966	66.9	0.52	0.178	0.28	Falciani, Righini, and Rigutti (1967)
				0.30	Cimino <i>et al.</i> (1967)
				0.30	This work
				0.32	Loucif and Koutchmy (1989)
				0.33	Waldmeier (1967)

**Table 1** (Continued)

Eclipse	Monthly sunspot number	Phase of solar activity		Flattening index ( $\varepsilon = a + b$ )	Reference
		$\Phi [-1; 1]$	$P [0; 1]$		
22 Sep 1968	107.7	0.98	0.337	0.06	Waldmeier (1969)
				0.06	Loucif and Koutchmy (1989)
				0.08	Khetsuriani <i>et al.</i> (1972)
				0.14	This work
07 Mar 1970	104.4	-0.81	0.462	0.00	Waldmeier (1970)
				0.00	Bappu, Bhattacharyya, and Sivaraman (1973)
				0.00	Loucif and Koutchmy (1989)
				0.05	Caccin, Donati, and Moschi (1970)
				0.08	This work
10 Jul 1972	63.6	-0.51	0.663	0.21	Khetsuriani (1975)
				0.23	Waldmeier (1973)
				0.23	This work
				0.24	Loucif and Koutchmy (1989)
				0.27	This work
30 Jun 1973	39.4	-0.38	0.747	0.19	Khetsuriani (1975)
				0.25	Waldmeier (1974a)
				0.27	Loucif and Koutchmy (1989)
				0.27	This work
20 Jun 1974	31.7	-0.26	0.830	0.15	This work
				0.16	Waldmeier (1974b)
23 Oct 1976	15.6	0.10	0.036	0.36	Waldmeier and Weber (1977)
				0.41	This work
13 Oct 1977	45.4	0.38	0.133	0.24	Waldmeier and Weber (1978)
				0.31	This work
26 Feb 1979	132.4	0.77	0.271	0.15	Waldmeier and Weber (1979)
16 Feb 1980	157.7	-0.97	0.368	0.03	Rušin and Rybanský (1983)
				0.03	Loucif and Koutchmy (1989)
				0.09	This work
31 Jul 1981	140.6	-0.75	0.513	0.20	Lebecq, Koutchmy, and Stellmacher (1985)
				0.20	Loucif and Koutchmy (1989)
				0.21	This work
11 Jun 1983	76.8	-0.46	0.699	0.26	Koutchmy and Nitschelm (1984)
				0.26	Hiei <i>et al.</i> (1986)
				0.26	Loucif and Koutchmy (1989)
22 Sep 1984	33.7	-0.24	0.844	0.23	Pasachoff, Reardon, and MacKenty (1993)
				0.35	Loucif and Koutchmy (1989)

**Table 1** (Continued)

Eclipse	Monthly sunspot number	Phase of solar activity		Flattening index ( $\varepsilon = a + b$ )	Reference
		$\Phi$ [-1; 1]	$P$ [0; 1]		
22 Jul 1990	146.0	-0.88	0.407	0.04 0.12	Markova <i>et al.</i> (1998) Koutchmy <i>et al.</i> (1992)
11 Jul 1991	142.2	-0.73	0.503	0.00 0.00 0.00	Gulyaev, Vanyarkha, and Vanyarkha (1994) Vanyarkha, Vanyarkha, and Gulyaev (1993) Sýkora <i>et al.</i> (1999)
03 Nov 1994	26.1	-0.25	0.832	0.13 0.14 0.27	Sýkora <i>et al.</i> (1996) Badalyan and Sýkora (2008) Marková and Bělik (1995)
24 Oct 1995	13.4	-0.11	0.928	0.13 0.27 0.28	Sýkora <i>et al.</i> (1996) Marková <i>et al.</i> (1996) Rušin <i>et al.</i> (1996)
09 Mar 1997	16.0	0.17	0.052	0.20 0.27	Pintér <i>et al.</i> (1997) Marková <i>et al.</i> (1999)
26 Feb 1998	49.0	0.41	0.130	0.21 0.22	Dorotovič <i>et al.</i> (1999) Marková <i>et al.</i> (1999)
11 Aug 1999	99.1	0.79	0.247	0.04 0.04 0.19	Badalyan and Sýkora (2008) This work Stoeva and Stoev (2006)
21 Jun 2001	110.9	-0.88	0.397	0.07 0.16	This work Golub and Pasachoff (2009) <sup>b</sup>
04 Dec 2002	85.4	-0.71	0.514	0.09	This work
29 Mar 2006	19.2	-0.32	0.781	0.10 0.17 0.17	Stoeva <i>et al.</i> (2008) Pishkalo and Sadovenko (2008) Golub and Pasachoff (2009) <sup>b</sup>
01 Aug 2008	2.7	-0.04	0.970	0.21 0.29	Pishkalo and Baransky (2009) Rušin <i>et al.</i> (2010)
22 Jul 2009	4.2	0.12	0.053	0.24	This work
11 Jul 2010	14.7	0.31	0.139	0.24	This work

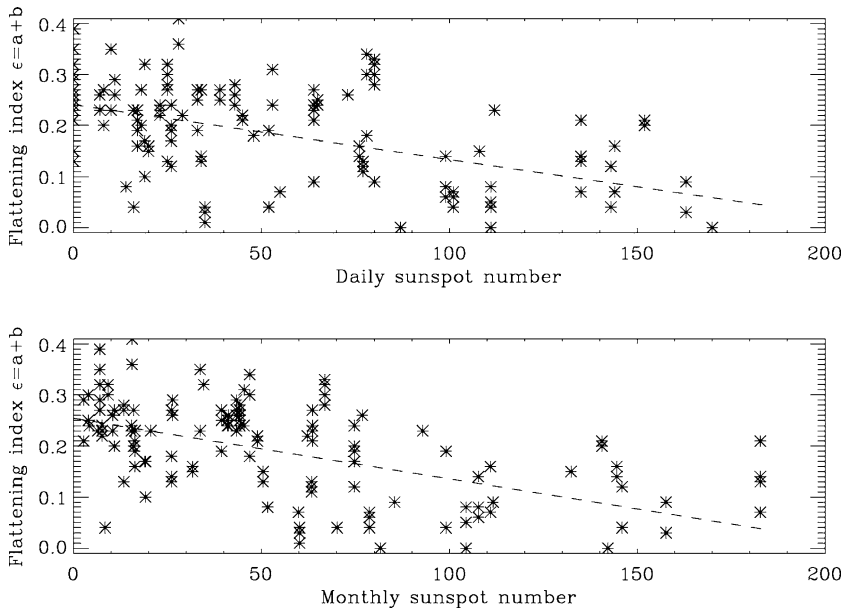
<sup>a</sup>This value was corrected here using Figure 8 of van de Hulst (1953).

<sup>b</sup>These values were obtained by V. Rušin and M. Druckmüller.

Phase values for eclipses in 2009 and 2010 are preliminary. They were calculated using predicted  $T_{\max}$  and duration of solar cycle 24 from Pishkalo (2010b).

we noted above, the sunspot numbers were taken from the SIDC site. The monthly sunspot numbers were also smoothed using a 13-point running average and then interpolated to the





**Figure 1** Flattening index of the solar corona vs. daily (top) and monthly (bottom) sunspot numbers. The best linear fits are shown by dashed straight lines.

time of total solar eclipse. The straight lines in Figure 1 are the best linear fits. One can see that both dependences are similar. They can be described by similar linear equations:

$$a + b = 0.243 - 0.001W_d,$$

$$a + b = 0.255 - 0.001W_m,$$

where  $W_d$  and  $W_m$  are daily and monthly sunspot numbers.

#### 4. Flattening Index and the Solar Cycle

The dependences of the flattening index on the phases of solar activity  $\Phi$  and  $P$  are shown in Figure 2 in the top and bottom panels, respectively. Here the dashed lines are the best quadratic fits to the data, which can be described by the equations

$$a + b = 0.258 - 0.019\Phi - 0.228\Phi^2$$

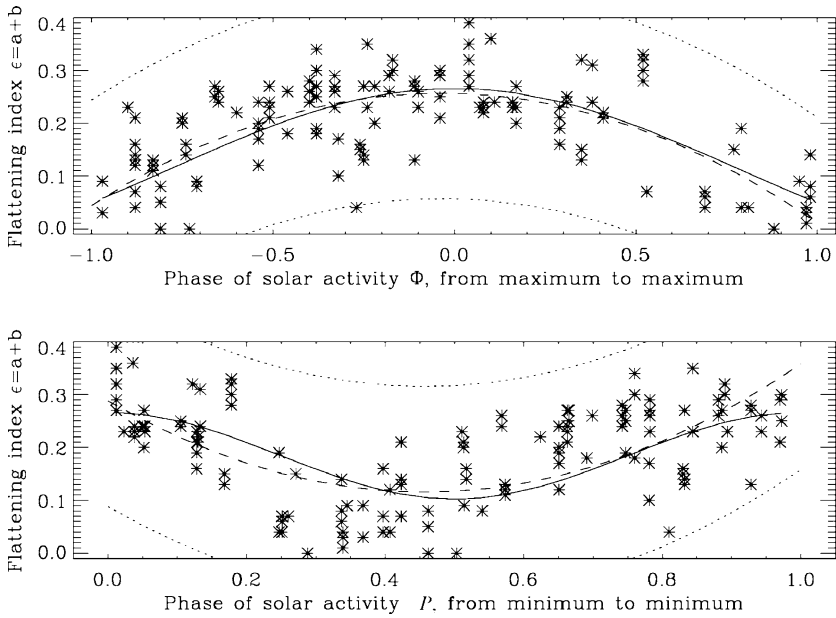
and

$$a + b = 0.288 - 0.748P + 0.821P^2.$$

The dotted lines are drawn at distances of  $\pm 0.2$  from the dashed lines. The solid lines present harmonic (or sinusoidal) functions fitting to the data:

$$a + b = 0.126 + 0.142 \cos(2.102\Phi),$$

$$a + b = 0.186 + 0.081 \cos(2\pi P).$$



**Figure 2** Flattening index of the solar corona vs. the phases of solar activity  $\Phi$  (top) and  $P$  (bottom). The best quadratic and sinusoidal fits are shown by dashed and solid lines, respectively. The dotted lines are drawn at distances of  $\pm 0.2$  from the dashed lines.

Note from Figure 2 that all calculated values of the flattening index are within the limits of 0.2 from the dashed lines. The dispersion is almost uniform and does not exceed 0.4 at different phases of solar activity.

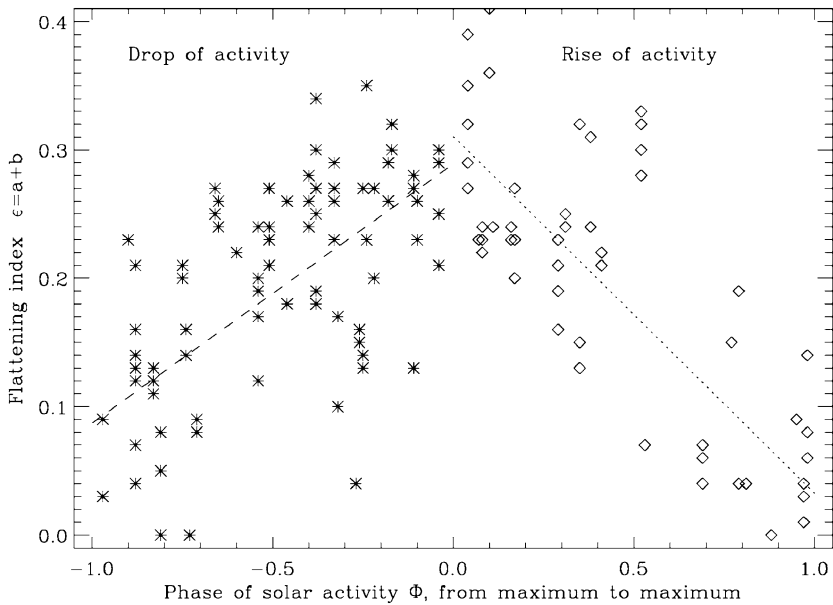
The dependences of the flattening index on the phase of solar activity are shown separately for the ascending and descending parts of the cycle in Figure 3. The best linear fits are shown by straight lines; they can be outlined by the equations

$$\begin{aligned}
 a + b &= 0.292 + 0.202\Phi, & (\Phi < 0), \\
 a + b &= 0.311 - 0.277\Phi, & (\Phi > 0).
 \end{aligned}$$

The dependences are similar; at the rise of solar activity the flattening index drops slightly more sharply than it increases at the decline of activity. Note that faster rise and slower decline are common to all the solar activity phenomena.

After analyzing 17 total solar eclipses from 1893 to 1932, Ludendorff (1934) found that the maximum of flattening index occurred one or two years ( $\Phi = -0.2$ ) before the activity maximum. Later this property was also confirmed by van de Hulst (1953), Vsekhsvyatsky *et al.* (1965), Hata and Saito (1966), Khetsuriani (1975), Dzyubenko (1983), Loucif and Koutchmy (1989), Gulyaev (1997), and Golub and Pasachoff (2009). In these papers, a single value of the flattening index  $a + b$  was derived for each eclipse. However, from Figure 2 one can conclude that when we use all the data compiled from the eclipses listed in Table 1 and plot them as a function of  $\Phi$ , the resulting curve seems to be rather symmetrical.

The photometrical flattening index  $a + b$  is sometimes called the index of ellipticity of the solar corona. This fact reflects the “classical” interpretation of the flattening index, where the solar corona is considered to be spherically symmetrical and ellipsoidal (bulged toward the equator plane) in the activity maximum and minimum, respectively.



**Figure 3** Dependence of the flattening index on the phase of solar activity shown separately for ascending ( $\Phi > 0$ , diamonds) and descending ( $\Phi < 0$ , asterisks) parts of the cycle. The best linear fits are shown by the straight lines.

Now we know that the solar corona is in reality not spherically symmetrical or ellipsoidal. According to modern knowledge, the solar magnetic field is carried out by the solar wind into interplanetary space and forms the interplanetary magnetic field. The global solar magnetic field is mainly dipolar. The time evolution of the global solar magnetic field may be described in an idealized manner as a rotating dipole with a period of about 22 years (Yoshimura, 1977; Starkova and Solov'ev, 1997). The heliospheric current sheet which is represented by the magnetic neutral line at the “source surface” divides the interplanetary space into two parts with oppositely directed open magnetic field lines. Coronal helmet streamers are connected with the heliospheric current sheet and form a closed streamer belt around the Sun with possibly complicated inner structure (see, e.g., Gulyaev and Filippov, 1992; Eselevich and Eselevich, 1999, 2005; and references therein). The surface defining the streamer belt is almost flat near the minima of solar activity. However, it is very complicated, and sometimes looks like a ballerina's skirt, at the maxima of solar activity. Gulyaev (1992) called such an idealized picture “the flat corona.” In this picture, the observed or visible solar corona is mainly defined by the large-scale structure of the streamer belt and its orientation toward the solar equator and the observer. Under such an interpretation, it is natural that the observed structure of the solar corona during the total solar eclipse of 11 July 1991 (near the maximum of solar cycle 22) looks like a minimal corona rotated with respect to the solar equator (Gulyaev, 1992; Sýkora *et al.*, 1999, 2003). It is significant that the shape and structure of the solar corona modeled in the potential-field approach or using magnetohydrodynamic (MHD) simulations (Altschuler and Newkirk, 1969; Riley, Linker, and Mikić, 2001; Sýkora, Badalyan, and Obridko, 2003; Wang *et al.*, 2007; Pishkalo, 2010a; Rušin *et al.*, 2010) are generally consistent with the observed ones for most of the total solar eclipses.

As we note above (see Figure 2), the dispersion of the flattening index at different phases of solar activity is almost uniform and does not exceed  $\pm 0.2$  from the “mean” curve. It is a result of a combination of the structure of the streamer belt, the brightness distribution across coronal rays, and their orientation toward the observer. Furthermore, we can conclude that under some specific conditions at the cycle maximum the flattening index could be negative.

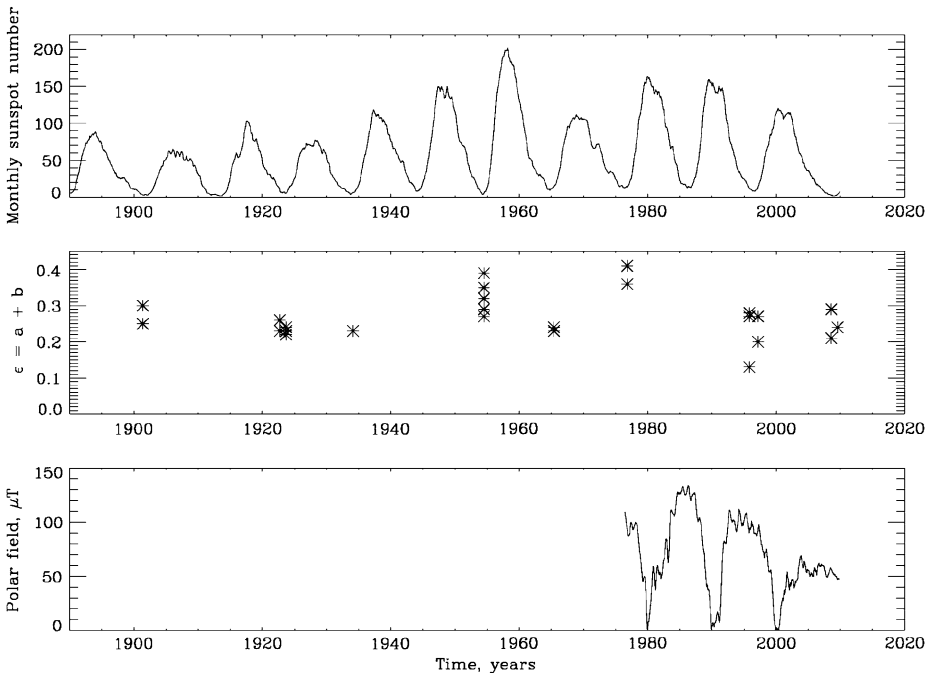
Although the flattening index is not a direct characteristic of spatial sphericity of the solar corona, it was calculated for many eclipses over a long period of time and it is a very important quantity in studying the evolution of the solar corona with solar activity. In particular, the magnitude of the flattening index at a cycle minimum can be used to estimate the maximal amplitude of the cycle.

## 5. Flattening Index and Prediction of Solar Cycle 24

The structure and shape of the solar corona are generally accepted to be related to global and local magnetic fields in the solar atmosphere. The flattening index of the solar corona, which is largest at the activity minimum and vice versa, can be considered as an indirect characteristic of the polar magnetic field of the Sun. This fact can help us to forecast the amplitude of solar cycle 24.

The use of the polar magnetic field as a physical precursor for the next solar cycle is based on the dynamo theory developed by Babcock (1961) and Leighton (1969). According to the dynamo theory, the polar magnetic field near the activity minimum is wrapped up by differential rotation to form toroidal fields which float to the Sun’s surface to form active regions (and sunspots) during the activity maximum. The amplitude of the toroidal magnetic field of the Sun in the activity maximum is therefore expected to be determined by the poloidal magnetic field in the previous minimum. Sunspots (therefore the sunspot number as well) are visual representations of the toroidal magnetic field, whereas the strength of the polar magnetic field characterizes the poloidal field. Therefore, the magnitude of the polar magnetic field of the Sun (and the flattening index  $a + b$ ) in the minimum between solar cycles 23 and 24 can inform us about the amplitude of sunspot numbers in the maximum of the growing solar cycle 24. Note that earlier flattening index values at the minima of solar cycles 21 and 22 were used by Schatten *et al.* (1978) and Layden *et al.* (1991) to predict the amplitude in those cycles.

Figure 4 shows the time evolution of smoothed monthly sunspot number, the flattening index of the solar corona near minima of solar activity, and the magnitude of the Sun’s polar magnetic field. Monthly sunspot numbers were taken from the SIDC home page and then smoothed using a 13-point running average. The values of flattening index  $a + b$  were extracted from Table 1 for the time intervals of  $\pm 1$  year from activity minima. The magnitudes of the polar field were taken from the Wilcox Solar Observatory (WSO, <http://wso.stanford.edu>). Note that the values of the polar field strength observed at WSO and used in this paper are not literally the strength of the magnetic field at the poles. They represent the integrated magnetic flux from about  $\pm 55^\circ$  latitude to the poles. Here for simplicity we call the absolute value of the strength of the polar field, measured from about  $\pm 55^\circ$  latitude to the poles, the magnitude of the polar field. There is a relationship between the value of the flattening index at an activity minimum and the amplitude of the next activity maximum. The higher the  $(a + b)$  value is in a cycle minimum, the higher is the amplitude of the next solar cycle. Figure 5 illustrates this relationship. The correlation between the flattening index near the minimum of solar activity and the smoothed monthly sunspot number at the next maximum is significant (correlation coefficient  $r = 0.59$ , probability of false correlation  $P < 0.01$ ).



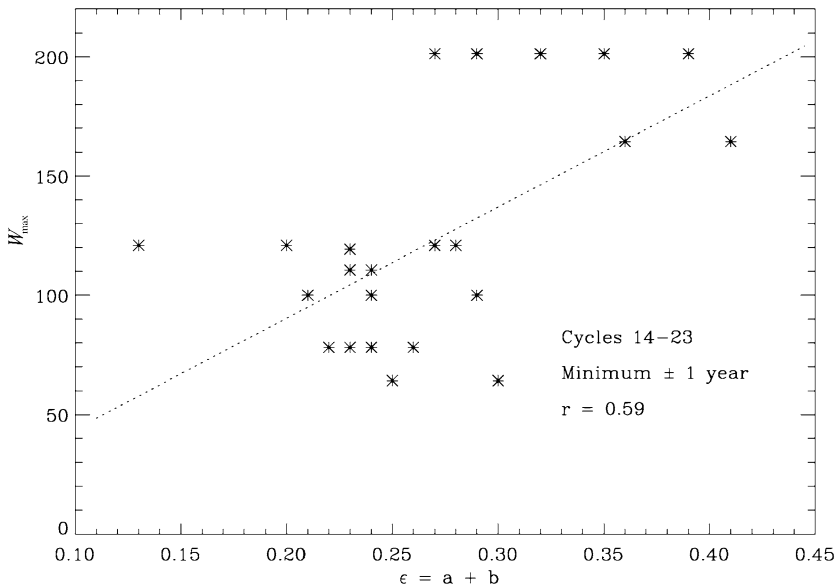
**Figure 4** Smoothed monthly sunspot number (top), the flattening index ( $\epsilon = a + b$ ) near the 11-year cycle minima (middle), and the Sun’s polar magnetic field (bottom) with time.

The best linear fit in Figure 5 is shown by the dotted line; it can be written by the equation

$$W_{\max} = -2.8 + 466.1 \times (a + b).$$

This equation, together with the flattening index value of 0.21 for the total solar eclipse in 2008 obtained in our previous paper (Pishkalo and Baransky, 2009), gives a value of  $95 \pm 65$  for the amplitude of smoothed monthly sunspot number in solar cycle 24. Note that this prediction of the maximum sunspot number in solar cycle 24 has large  $1\sigma$ -uncertainty ( $\pm 65$ ). This uncertainty is due mainly to the large sensitivity of the flattening index value to many factors, as mentioned in Section 2. The uncertainty could be reduced (and  $W_{\max}$  could be slightly changed) by data selection, but we did not do this. (For example, when we exclude from calculation only the value 0.27 for the 1954 total solar eclipse, see Table 1, then the correlation coefficient and the predicted amplitude of solar cycle 24 will be equal to 0.62 and  $92 \pm 60$ , respectively.) The prediction obtained here is slightly higher than the results of Schatten (2005) and Svalgaard, Cliver, and Kamide (2005), both of which were obtained using the Sun’s polar field as the precursor. Their predictions for the amplitude of monthly smoothed sunspot number in solar cycle 24 are equal to  $80 \pm 30$  and  $75 \pm 8$ , respectively. Many other predictions for solar cycle 24 can be found in Pesnell (2008) or at the Janssens panel (<http://users.telenet.be/j.janssens/SC24.html>).

Note that another value of  $(a + b)$  index for the 2008 total solar eclipse, namely 0.29 (Rušin *et al.*, 2010), and our value for the 2009 total solar eclipse ( $a + b = 0.24$ , see Table 1) led to higher amplitudes of solar cycle 24:  $132 \pm 76$  and  $109 \pm 69$  in terms of the smoothed



**Figure 5** Smoothed monthly sunspot number at cycle maxima  $W_{\max}$  vs. the flattening index ( $\epsilon = a + b$ ) near cycle minima. The best linear fit is represented as the dotted line.

monthly sunspot number, respectively. Nevertheless, we prefer the prediction based on our flattening index value for the 2008 total solar eclipse (Pishkalo and Baransky, 2009).

## 6. Conclusions

We have compiled 170 values of the photometrical flattening index (according to Ludendorff's definition) for 60 total solar eclipses from 1851 to 2010 using all the available data in the literature. The values are in the range from 0 to 0.4. The flattening index decreases with the growth of sunspot number. The largest values of the flattening index were found in the eclipses observed near the minima of solar activity and *vice versa*. We also obtained the relationships between the flattening index and the phase of solar activity. The scatter in the values of the flattening index at different phases of solar activity is almost uniform and does not exceed  $\pm 0.2$  from the "mean" curve.

The flattening index, calculated for an eclipse coronal picture, is not a direct characteristic of the spatial sphericity of the solar corona. It is a result of combining the streamer belt structure, the brightness distribution across coronal rays, and their orientation toward the observer.

The flattening index was calculated for many eclipses over a long period of time, and it is a very important quantity in studying the evolution of the solar corona in terms of the activity cycle. The magnitude of the flattening index at a cycle minimum can be used to estimate the amplitude of the cycle. Using the value of the flattening index for the total solar eclipse of 1 August 2008 ( $a + b = 0.21$ , Pishkalo and Baransky, 2009), we have predicted that the amplitude of solar cycle 24 will be  $95 \pm 65$  in terms of the smoothed monthly sunspot number.

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