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The Relationship between Prediction Accuracy and Correlation Coefficient

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Abstract The correlation coefficient (r) between the maximum amplitude (R_{m}) of a sunspot cycle and the preceding minimum *aa* geomagnetic index ($aa_{\min}$), in terms of geomagnetic cycle, can be fitted by a sinusoidal function with a four-cycle periodicity superimposed on a declining trend. The prediction index (χ) of the prediction error relative to its estimated uncertainty based on a geomagnetic precursor method can be fitted by a sinusoidal function with a four-and-half-cycle periodicity. A revised prediction relationship is found between the two quantities: $\chi < 1.2$ if r varies in a rising trend, and $\chi > 1.2$ if r varies in a declining trend. The prediction accuracy of R_{m} depends on the long-term variation in the correlation. These results indicate that the prediction for the next cycle inferred from this method, $R_{\mathrm{m}}(24) = 87 \pm 23$ regarding the 75% level of confidence (1.2-σ), is likely to fail. When using another predictor of sunspot area instead of the geomagnetic index, similar results can be also obtained. Dynamo models will have better predictive powers when having considered the long-term periodicities.

Keywords Solar activity, sunspots, solar cycle · Geomagnetic activity, geomagnetic indices · Prediction

1. Introduction

Various methods have been used to predict the maximum amplitude (R_{m}) of a sunspot cycle, such as those based on climatology, dynamo models, spectral analysis, neural networks, geomagnetic and solar precursor methods (Messerotti *et al.*, 2009). Different methods show a wide range for the prediction of R_{m} for Cycle 24 – from as low as 42 to as high as 190 (Kane, 2007; Pesnell, 2008). However, since the onset of Cycle 24 (December 2008), quite a lot of authors tend to believe that this cycle is a weak one due to the deep, extended

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solar minimum. Most methods can well reproduce past behaviors with very high correlation coefficients, while the future behavior is uncertain.

A prediction of the strength of future solar cycles is of interest in space weather and can be used to test theoretical models of the solar cycle, if the prediction is based on a physical approach (Cameron and Schüssler, 2007; Pesnell, 2008). Recently, dynamo models for the solar cycle (Babcock, 1961; Leighton, 1969) have been attempted to predict R_{m}. Dikpati, de Toma, and Gilman (2006) predicted that Cycle 24 will have a 30% – 50% higher peak than Cycle 23 based on a modified flux-transport dynamo model. In contrast, Choudhuri, Chatterjee, and Jiang (2007) predicted that Cycle 24 will have a 30% – 50% lower peak than Cycle 23 based on a flux-transport dynamo model. The large discrepancy in the dynamo models shows that these models do not as yet possess a predictive capability (Cameron and Schüssler, 2007; Pesnell, 2008).

There are two main kinds of precursor methods: geomagnetic (Ohl, 1966; Thompson, 1993) and solar (Svalgaard, Cliver, and Kamide, 2005; Schatten *et al.*, 1978). Schatten *et al.* (1978) first attempted to make a physical connection between R_{m} and the previous cycle's polar fields. Based on the solar dynamo amplitude index, Schatten (2005) predicted that the R_{m} for Cycle 24 will be low, at about 80. A precursor-based method employs a solar dynamo concept, whereby the poloidal solar magnetic field, which is estimated based on geomagnetic activity during the declining phase of the preceding cycle or at the cycle minimum, provides the seed for future toroidal fields within the Sun that in turn cause solar activity (Schatten *et al.*, 1978).

The geomagnetic precursor method (Ohl, 1966) has been widely applied to predict R_{m} due to the high correlation coefficients between geomagnetic-based parameters and R_{m}, from 0.8 up to 0.99 (Layden *et al.*, 1991; Thompson, 1993; Shastri, 1998; Charvátová, 2009), depending on the parameters and the way in which they are selected (Schüssler, 2007). The parameters can be the *aa* geomagnetic index, the ratio between the variations in the *aa* index and sunspot number (Obridko and Shelting, 2008), the number of geomagnetic disturbed days (Thompson, 1993), and so on. The parameters can be taken from the geomagnetic activity during the descending phase of the sunspot cycle, at sunspot minimum, or at the *aa* maximum of a geomagnetic cycle (Hathaway and Wilson, 2006). When using the *aa* index during the declining phase, one needs to consider the time period in which the *aa* index is selected. Therefore, Wang and Sheeley (2009) recently suggested that a better solar cycle predictor is the lowest value of the *aa* index (aa_{min}) near sunspot minimum.

Geomagnetic precursor method has achieved great success in Cycles 20 – 22 (Ohl, 1966; Wilson, 1990; Hathaway and Wilson, 2006; Kane, 2007). However, when applied to Cycles 23 and 24, the prediction results of this method show great discrepancies (Kane, 2007; Pesnell, 2008; Messerotti *et al.*, 2009). Recently, Du, Li, and Wang (2009) studied this method and found that its predictive power shows a weakening trend and a cyclical behavior that it may succeed in some cycles and then fail in some other ones. One should not be misled by high correlation coefficients for reproducing the past (Cameron and Schüssler, 2007).

The relationship between the accuracy of the prediction of R_{m} and the related correlation is studied using two predictors of aa_{min} and sunspot area. In Section 2, the aa_{min} value is determined from the minimum smoothed monthly *aa* index, in terms of the geomagnetic cycle rather than the minimum value near the sunspot minimum. The predictive power of the geomagnetic precursor method is analyzed in Section 3, by analyzing the prediction index of the prediction error relative to its estimated uncertainty and the variation in the correlation coefficient of R_{m} with the preceding aa_{min}. We analyze in what case a prediction of R_{m} is accurate. Using another predictor, the sunspot area (Javaraiah, 2008), in the application of predicting R_{m} is discussed in Section 4. The results are summarized in Section 5.

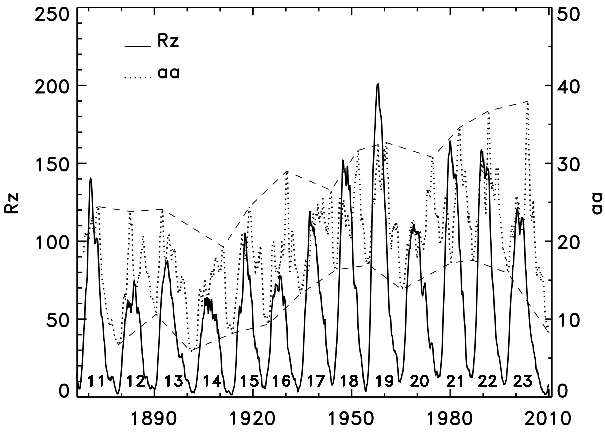

Figure 1 Smoothed R_Z (solid line) since July 1866 and *aa* (dotted) since July 1868. Dashed lines indicate the *aa* maximum (aa_{max}) and minimum (aa_{min}) of the geomagnetic cycle.

2. Data

In a previous paper (Du, Li, and Wang, 2009), we used the parameters of R_m and aa_{min} directly taken from Kane (2007). These parameters are derived from the '12-month running mean' of sunspot number and *aa* indices. Thus, they have small differences from both the annual values and the 'standard' 13-month running means (with half weights at the two ends of data). For Cycles 9 – 11, the aa_{min} values are derived from the equivalent annual *aa* indices (Nevanlinna and Kataja, 1993).

In this study, the data used are the smoothed (13-month running mean of) monthly *aa* index of the reliable values since 1868[1] and the smoothed monthly sunspot number (R_z)[2], as plotted in Figure 1.

As the 11-year solar cycle is very apparent in R_z, the sunspot maximum of the cycle (R_m) can easily be determined. The variation in *aa* is much more complex than that in R_z. Therefore, many ways were used to determine the minimum *aa* index, such as looking during the declining phase of the previous cycle or near the sunspot minimum (R_{min}). In this study, we use the *aa* minimum (aa_{min}) in terms of the geomagnetic cycle as follows. Firstly, we determine the *aa* maximum (aa_{max}, the upper dashed line in Figure 1) and its date between two successive sunspot minima of the cycle. It is seen that aa_{max} is later than R_m by about three years on average and shows a quasi-three-cycle periodicity. (This study does not analyze aa_{max} and its application.) Then, aa_{min} (the lower dashed line in Figure 1) can be determined using the *aa* indices between two successive aa_{max}. The aa_{min} value thus determined has accounted for the effect of time delay of *aa* to R_z and is slightly different from that determined near a sunspot minimum (for Cycle 21).

The parameters of R_m and aa_{min} are listed in Table 1. Because the beginning year (1868) of the *aa* index is later than the onset year (1867) of Cycle 11, an equivalent value from the annual value (16.0) is used for this cycle (Nevanlinna and Kataja, 1993; Kane, 2007). As the onset of Cycle 24 is in December 2008, the current minimum *aa* (8.4) is used to estimate the aa_{min} value for Cycle 24. This value is used only in the prediction of R_m of Cycle 24 and does not affect the conclusion of this study.

[1] ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/RELATED_INDICES/AA_INDEX/.

[2] http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html.

Table 1 Smoothed monthly parameters and results.

n	aa_{min}	R_{m}	A_{N}[b]	r	σ	κ	R_{p}	ΔR_{p}	χ
11	16.0	140.3							
12	6.7	74.6	9.47						
13	10.6	87.9	3.22						
14	5.9	64.2	12.98	0.980	6.7				
15	8.2	105.4	3.74	0.908	12.5	−1	80.3	−25.1	3.75
16	9.4	78.1	33.96	0.883	12.9	−1	93.9	+15.8	1.27
17	13.2	119.2	29.96	0.901	11.8	+1	116.8	−2.4	0.19
18	16.3	151.8	69.35	0.932	11.6	+1	138.5	−13.3	1.12
19	16.9	201.3	15.23	0.905	19.0	−1	148.2	−53.1	4.57
20	13.8	110.6	50.31	0.885	19.5	−1	135.6	+25.0	1.32
21	17.2	164.5	60.05	0.901	18.5	+1	163.3	−1.2	0.06
22	17.5	158.5	29.85	0.907	17.8	+1	166.3	+7.8	0.42
23	15.9	120.8	21.99	0.886	18.8	−1	150.5	+29.7	1.67
24	8.4[a]	?		…	…	?	86.8	…	?

[a]Derived from the monthly values up to May 2010.

[b]From Javaraiah (2008).

3. Result

The linear correlation between R_{m} and aa_{min} is shown in Section 3.1, with a prediction of R_{m} of Cycle 24. The past behavior of the geomagnetic precursor method is checked in Section 3.2, by examining the (post actual) prediction error relative to its estimated uncertainty and the variation in the correlation. In this section, we also analyze whether the prediction of R_{m} of Cycle 24 is reasonable.

3.1. Linear Correlation of R_{m} with aa_{min}

Figure 2a depicts the scatter plot of R_{m} against aa_{min} (triangles) with a least-squares-fit regression equation (dashed line) given by

$$R_{\mathrm{m}} = 11.6 + 8.51 aa_{\mathrm{min}}, \qquad \sigma = 18.8, \tag{1}$$

where σ refers to the standard deviation of the regression equation, which is conventionally used to estimate the uncertainty of a prediction of R_{m}. One can see in this figure that the point of Cycle 19 is far above the regression line and is thus often called an 'outlier' (Kane, 2007).

Using Equation (1) and the aa_{min} value for $n = 24$ (8.4), the peak sunspot number in the next cycle (24) is predicted as (asterisk) $R_{\mathrm{p}}(24) = 86.8 \pm 18.8$ regarding the 66% level of confidence (with 12 degrees of freedom), or $R_{\mathrm{p}}(24) = 87 \pm 23$ regarding the 75% level of confidence (1.2σ). Because the correlation coefficient between R_{m} and aa_{min} is very high, $r = 0.89$ at the 99% level of confidence, most researchers may be satisfied with confidence to infer that the peak sunspot number in the next cycle (24) should be around 87. However, Du, Li, and Wang (2009) pointed out that whether a prediction is accurate depends on the variation of the correlation. We cannot judge whether this prediction is accurate or less accurate using only one correlation coefficient.

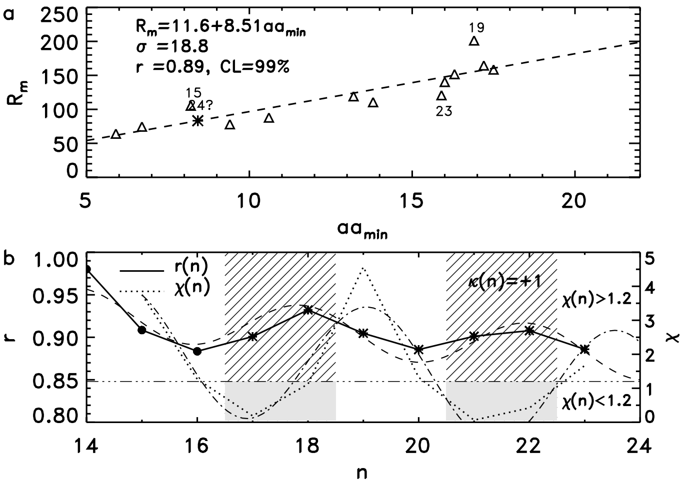

Figure 2 (a) Scatter plot of R_{m} *vs.* $aa_{\min}$ (triangles). The numbers of 15, 19, 23, and 24 refer to the cycle numbers. (b) Correlation coefficient r (solid) with its fitted (dashed) line, and prediction index χ (dotted) with its fitted (dash-dotted) line.

3.2. Past Behavior of the Geomagnetic Method

In this section, we check the predictive power of geomagnetic method in the past cycles (Du, Li, and Wang, 2009). In order to predict R_{m} in cycle n, one uses the values of $R_{\mathrm{m}}(i)$ and $aa_{\min}(i)$ for $i = 11, 12, \ldots, n-1$, to calculate their correlation coefficient $r(n-1)$ and the regression equation similar to Equation (1). Substituting the $aa_{\min}$ value for Cycle n into this equation, the R_{m} value in Cycle n can be predicted, which is denoted by $R_{\mathrm{p}}(n)$. The standard deviation of the regression equation in the previous cycle $\sigma(n-1)$ is used to estimate the uncertainty of $R_{\mathrm{p}}(n)$. The (actual) prediction error

$$\Delta R_{\mathrm{p}}(n) = R_{\mathrm{p}}(n) - R_{\mathrm{m}}(n) \tag{2}$$

is known only when Cycle n is over.

To test the predictive power of this method, we analyze the prediction index,

$$\chi(n) = \frac{|\Delta R_{\mathrm{p}}(n)|}{\sigma(n-1)}, \tag{3}$$

and the prediction parameter (Du, Li, and Wang, 2009),

$$\kappa(n) = \mathbf{Sgn}\left(\frac{r(n)}{r(n-1)} - 1\right), \tag{4}$$

where $y = \mathbf{Sgn}(x)$ is the sign function: $y = 1$ if $x > 0$; $y = -1$ if $x < 0$; and $y = 0$ if $x = 0$.

The $\kappa(n)$ value represents the varying trend of the correlation coefficient $r(n)$: $\kappa(n) = +1$ refers to an increase of $r(n) > r(n-1)$, and $\kappa(n) = -1$ refers to a decrease of $r(n) < r(n-1)$ for the positive correlation $r(n-1) > 0$. The $\chi(n)$ value is a measure to check the degree of a prediction result deviating from the observed value: a smaller $\chi(n)$ refers to a better prediction, and a larger $\chi(n)$ refers to a worse prediction. The results are listed in Table 1.

Figure 2b plots the values of $r(n)$ (solid line) and $\chi(n)$ (dotted). The values for $\kappa(n) = +1$ are indicated by the hatched areas, and those for $\chi(n) < 1.2$ are indicated by the low gray areas. Other (white) areas indicate either $\kappa(n) = -1$ or $\chi(n) > 1.2$. Examining the variations in both $r(n)$ and $\chi(n)$ in Figure 2b, the results can be summarized by the following two cases.

i) If $r(n)$ varies in a rising trend, $\kappa(n) = +1$ (as is the case for $n = 17$, 18, 21, and 22), then $\chi(n) < 1.2$, implying an accurate prediction. This is true for all of the four relevant cases considered here.

ii) On the other hand, if $r(n)$ varies in a declining trend, $\kappa(n) = -1$ (as for $n = 15$, 16, 19, 20, and 23), then $\chi(n) > 1.2$, implying a failed prediction. This is true for all of the five relevant cases considered here.

The average (absolute) prediction error (6.2) under $\kappa(n) = +1$ is much smaller than that (29.7) under $\kappa(n) = -1$. All the prediction results (9/9) satisfy either of the following two cases,

$$\begin{cases} \chi(n) < 1.2 & \text{if } \kappa(n) = +1, \\ \chi(n) > 1.2 & \text{if } \kappa(n) = -1. \end{cases} \tag{5}$$

Here, we used a standard different from Du, Li, and Wang (2009). $\chi(n) < 1.2$ means that there is a probability of about 75% (with 12 degrees of freedom) that the observed R_m lies in the prediction range of $[R_\mathrm{p} - 1.2\sigma, R_\mathrm{p} + 1.2\sigma]$. We select a standard of $\chi_0 = 1.2$ in order that Equation (5) can include all cases of Figure 2b. On using the original standard, $\chi_0 = 1.0$, there will be an exception for $n = 18$, in which $\kappa(18) = +1$, while $\chi(18) = 1.1 > 1$ (see Table 1).

Equation (5) provides a way to improve the prediction of R_m. If $\kappa(n) = +1$ or $r(n) > r(n-1)$, then the observed R_m may likely lie within the prediction range $[R_\mathrm{p} - 1.2\sigma, R_\mathrm{p} + 1.2\sigma]$. In contrast, if $\kappa(n) = -1$ or $r(n) < r(n-1)$, then the observed R_m may likely lie outside the range $[R_\mathrm{p} - 1.2\sigma, R_\mathrm{p} + 1.2\sigma]$, although the relevant correlation coefficient is very high ($r \sim 0.9$; see Table 1). Therefore, an accurate prediction depends on the positive variation in the correlation rather than its high value. If the variation in $r(n)$ can be estimated in advance, one can estimate whether a prediction result is accurate according to Equation (5).

It should be noted from Figure 2b that the $r(n)$ values show a four-cycle periodicity superimposed on a declining trend. For this reason, the function $r = c_0 + c_1 n + A\sin(2\pi n/T + \phi)$ is used to fit them:

$$r(n) = 1.002 - 0.005n + 0.028\sin(2\pi n/4.1 - 47^\mathrm{o}). \tag{6}$$

The correlation coefficient between the fitted (dashed line) and original (solid) values is $r_\mathrm{f} = 0.90$, significant at the 99% level of confidence. It implies that $r(n)$ varies with a periodicity of about $T = 4.1$ (cycles) ~ 44 years. If the four-cycle periodicity continues, the variation in $r(n)$ for the next cycle can be approximately estimated from Equation (6), $r(24) < r(23)$ or $\kappa(24) = -1$. Thus $\chi(24) > 1.2$ according to Equation (5).

In fact, the $\chi(n)$ values also show a periodicity, with their fit equation given by

$$\chi(n) = 4.49 - 0.15n + 1.81\sin(2\pi n/4.5 - 1^\mathrm{o}). \tag{7}$$

The correlation coefficient between the fitted (dash-dotted line) and original (dotted) values is $r_\mathrm{f} = 0.92$, significant at the 99% level of confidence. Implied is that the predictive power of this method varies with a periodicity of about $T = 4.5$ (cycles) ~ 50 years. The value

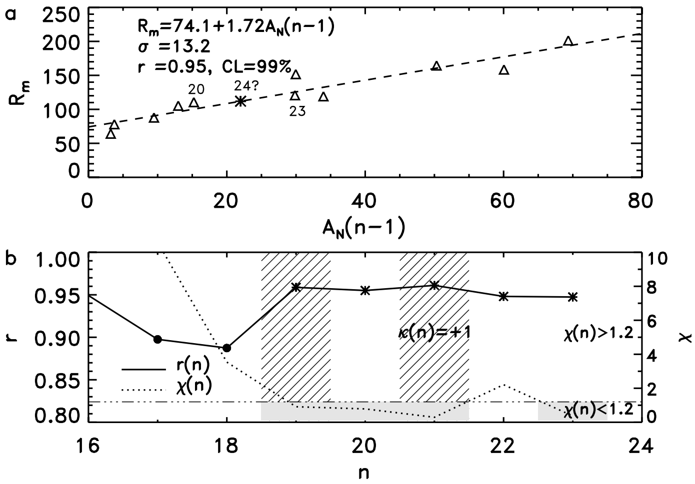

Figure 3 Similar to Figure 2 but using the predictor of A_{N}. (a) Scatter plot of $R_{\mathrm{m}}(n)$ *vs.* $A_{\mathrm{N}}(n-1)$ (triangles). (b) Correlation coefficient r (solid) and prediction index χ (dotted).

for Cycle 24 can be approximately estimated from this equation, $\kappa(24) > \kappa(23) > 1.2$. Both of the above results indicate that the prediction for Cycle 24, $R_{\mathrm{p}}(24) = 87 \pm 23$ regarding the 75% level of confidence (1.2σ uncertainty), would fail, $\kappa(24) > 1.2$.

4. Using Sunspot Area as Another Predictor

The above conclusions are applicable not only to the geomagnetic precursor but also to other predictors. As an example, we employ the sunspot area data used by Javaraiah (2008).

Javaraiah (2008) studied the correlations of R_{m} with several parameters related to the areas of the sunspot groups. One of these parameters is the sum of the areas of the sunspot groups in the $0^{\circ} - 10^{\circ}$ latitude interval of the Sun's northern hemisphere and in the time-interval of -1.35 year to $+2.15$ year from the time of the preceding minimum of a solar cycle, as listed in the fourth column of Table 1 (A_{N}). He found that this parameter in Cycle $n-1$ is well correlated ($r = 0.95$) with R_{m} in Cycle n. Figure 3a depicts the scatter plot of $R_{\mathrm{m}}(n)$ against $A_{\mathrm{N}}(n-1)$ with the regression equation given by

$$R_{\mathrm{m}}(n) = 74.1 + 1.72 A_{\mathrm{N}}(n-1), \qquad \sigma = 13.2. \tag{8}$$

Using this equation, a prediction of R_{m} in Cycle 24 was obtained, 112 ± 13. To analyze whether this prediction succeeds, we test the predictive power of this method using the similar technique in Section 3.2. The results are illustrated in Figure 3b.

One can see from Figure 3b that, in most cases, the results satisfy the condition of Equation (5), with only two exceptions of Cycles 20 and 23. If the R_{m} values in these two cycles are adjusted to be added by a small value of 6 (or the A_{N} values in Cycles 19 and 22 are adjusted to be subtracted by a small value of 3.5), all the results will satisfy the condition (5), as clearly illustrated in Figure 4b.

This adjustment can be viewed as the effect of multiple peaks in the recent cycles, as the traditional 13-month running mean has not perfectly filtered out high-frequency variations (Hathaway, 2010). In addition, the monthly sunspots may have some (25%) systematic

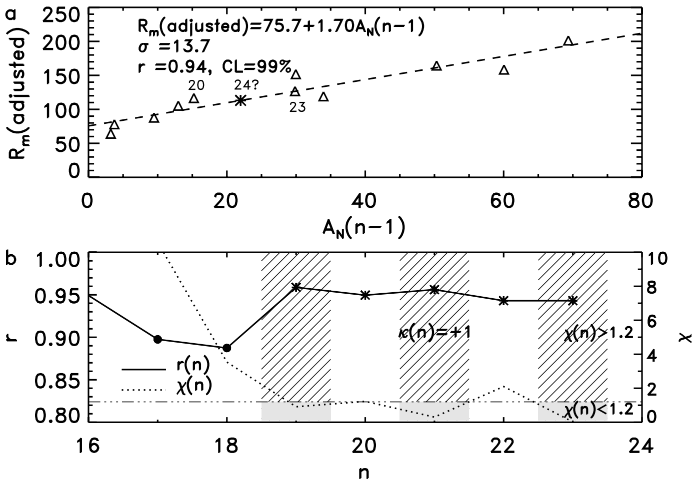

Figure 4 Similar to Figure 3 but R_m is adjusted to be added by 6 in Cycles 20 and 23.

uncertainties (Vitinskij, Kopetskij, and Kuklin, 1986). The regression equation of $R_m(n)$ against $A_N(n-1)$ is now

$$R_m(n) = 75.7 + 1.70A_N(n-1), \qquad \sigma = 13.7. \tag{9}$$

Using this equation, the R_m value in Cycle 24 is predicted as 113 ± 14 (Figure 4a). As the χ value shows a periodicity of two cycles, its next value in Cycle 24 should be greater than 1.2. This suggests that the prediction $R_m(24) = 113 \pm 14$ should fail. It provides another independent estimate of the peak size of Cycle 24, because the A_N value is an independent observational measure to the aa_{min} value.

5. Discussions and Conclusions

Conventionally, one employs all the available data (R_m, aa_{min}) to analyze their correlation and predict R_m with their linear regression equation. Because of the high correlation coefficient ($r \sim 0.9$), this method is widely applied to predict R_m. Since its finding by Ohl (1966), numerous prediction results related to this finding have been published. However, a high r does not always yield an accurate prediction, as shown in Table 1 and Figure 2.

This study shows that both the predictive power (χ) of the geomagnetic precursor method and the correlation (r) of R_m with aa_{min} vary cyclically with a cycle (n) of about 4–4.5 cycles. This periodicity may reflect the double Hale cycle (∼44-year) present in R_m and sunspot area (Javaraiah, 2008). Whether a prediction is accurate depends on the positive variation of the correlation (Du, Li, and Wang, 2009). The reason is that the correlation coefficient (r) will increase when the prediction result is added to the original data pairs, because the prediction is on the regression line (Du, Wang, and Zhang, 2008). Only when the observed and predicted r values have the same trend is it possible that the prediction may succeed.

This study also shows that the prediction relies more on the recent cycle than on the far past ones (Svalgaard, Cliver, and Kamide, 2005; Schatten, 2005; Du, Li, and Wang, 2009).

It is due to the various long-term cycles existing in solar activities. During a particular time period, there is always one that contributes most to the variation of $r(n)$, which is usually a few multiples of the 11-year cycle (the four-cycle periodicity in this study) rather than the very long ones. This is so because a longer cycle varies more slowly and contributes less to $r(n)$ compared with a shorter one. In addition, the effects of the far past data may decrease and even may be canceled by various cycles over time. Therefore, in the prediction of solar cycles (or other similar fields), one need not employ all the data, especially the far past data. Most important is that the data can provide enough useful information about the property of the variations in the data.

An analysis of the correlation between solar and geomagnetic activities is important for understanding the physical mechanism of the 11-year solar cycle. In the current state, dynamo models can reproduce certain features of the 11-year cycle, but they cannot explain the varying amplitudes of maxima and other long-term changes. One reason may be that the actual observational data of poloidal fields are available only from the mid-1970s (Choudhuri, Chatterjee, and Jiang, 2007). Such a short series of data cannot detect long-term periodicities which have not been considered in the current dynamo models. In a long-term view, with the accumulation of observational data of the poloidal field, dynamo models should consider long-term variations in solar activities so as to explain both short- and long-term behaviors of the solar cycle. In that case, dynamo models will possess a better capability to accurately predict the strength of an upcoming cycle.

Studying the variations in the correlations related to solar cycles is useful to understand the evolution of solar activity (Du, Wang, and Zhang, 2009), and to improve the prediction of the activity level (Du, Wang, and Zhang, 2008). The periodicities in r and χ are formed from various long-term periodicities in solar activity. One correlation coefficient cannot provide such information as to its variation. So, there are great discrepancies in the prediction results for Cycle 24 even if similar prediction methods are used (Hathaway and Wilson, 2006; Kane, 2007; Wang and Sheeley, 2009). Analyzing the trend of the variation in the correlation provides information about whether $\kappa = +1$ or $\kappa = -1$ is satisfied, so as to infer whether $\chi < 1.2$ or $\chi > 1.2$ in advance. If $r(n)$ varies in a rising trend, the prediction error will most probably lie within what is expected, where $|\Delta R_{\mathrm{p}}(n)| < 1.2\sigma(n-1)$. On the other hand, if $r(n)$ varies in a declining trend, the prediction error will most probably exceed the estimated uncertainty, $|\Delta R_{\mathrm{p}}(n)| > 1.2\sigma(n-1)$. For the current state, $r(24)$ seems to vary in a declining trend. So, the prediction from this method should fail according to Equation (5). In this case, we should seek other methods to obtain a reasonable prediction (Svalgaard, Cliver, and Kamide, 2005; Schatten and Pesnell, 1993; Schatten, 2005; Kane, 2007; Du, Wang, and Zhang, 2008; Javaraiah, 2008; Pesnell, 2008).

In conclusion, both the predictive power of the geomagnetic precursor method and the relevant correlation coefficient vary cyclically with cycle number. A more accurate or reasonable prediction cannot be obtained only from a high correlation. It depends on the variation of the correlation in the recent cycle. When one obtains a prediction result, its rationality should be analyzed. Otherwise, its success probability will only be 50%, which is no better than tossing a coin (Du and Wang, 2011). If the failure condition $\kappa = -1$ is satisfied, the prediction result should be outside rather than within the range $[R_{\mathrm{p}} - 1.2\sigma,\ R_{\mathrm{p}} + 1.2\sigma]$. Analyzing the temporal variation of the related correlation is important to obtain a more reasonable prediction.

According to the analysis above, the following conclusions are reached.

i) The correlation (r) between the maximum amplitude (R_{m}) of sunspot cycle and the preceding minimum *aa* geomagnetic index (aa_{min}) varies in a sinusoidal function with a four-cycle periodicity superimposed on a declining trend.

ii) The prediction index (χ) of the prediction error relative to its estimated uncertainty varies in a sinusoidal function with a 4.5-cycle periodicity.

iii) A prediction relationship is found between the two quantities: $\chi < 1.2$ if r varies in a rising trend, and $\chi > 1.2$ if r varies in a declining trend.

iv) The geomagnetic precursor method may probably fail in the prediction of R_{m} for the next cycle (24).

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