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Solar Cycle Predictions Based on Extrapolation of Spectral Components: An Update

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Abstract Three series (1876 – 1986, 1886 – 1996, and 1896 – 2006) of 111 annual values of sunspot number R_z in each were subjected to spectral analysis to detect periodicities by the maximum entropy method (MEM), and the periodicities so obtained were used in a multiple regression analysis (MRA) to estimate the amplitudes and phases. All series showed roughly similar spectra with many periodicities (24 or more), but most of these were insignificant. The significant periodicities (far exceeding 2σ) were near 5, 8 – 12, 18, and 37 years. Using the amplitudes and phases of these, we obtained reconstructed series, which showed good correlations (+ 0.7 and more) with the original series. When extrapolated further in time, the reconstructed series indicated R_z(max) in the ranges 80 – 101 (mean 92) for cycle 24 during years 2011 – 2014, 112 – 127 (mean 119) for cycle 25 during years 2022 – 2023, 115 – 120 (mean 118) for cycle 26 during years 2032 – 2034, and 100 – 113 (mean 109) for cycle 27 during 2043 – 2045.

Keywords Sunspots · Periodicities · Predictions · Solar cycle

1. Introduction

For many purposes (*e.g.*, satellite drags), predictions of the strength of a solar (sunspot) cycle are needed. Predictions are made by using different methods. Some are based on sound physical principles, some are precursor methods, and some are statistical, including spectral analysis and predictions by an extrapolation of the significant periodicities. Many earlier workers have reported results of spectral analysis of sunspot numbers (Kimura, 1913; Turner, 1913; Ramanuja Rao, 1973; Cole, 1973; Currie, 1973; Cohen and Lintz, 1974; Radoski, Fougere, and Zawalick, 1975; Kane, 1977). By locating such periodicities and extrapolating them further, predictions have been attempted (*e.g.*, DeMeyer, 1981; Kane and Trivedi, 1985; Rangarajan, 1998). Kane (1999) used the maximum entropy method (MEM), reported results of spectral analysis of annual values of sunspot numbers R_z,

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and used the significant periodicities for predicting $R_z(\max) = 140 \pm 9$ for cycle 23 and $R_z(\max) = 109 \pm 9$ for cycle 24. Since then, data for 10 more years (1996 – 2006) are available and the predictions can be updated. In this communication, updated estimates of $R_z(\max)$ are reported for cycle 24 and estimates are given for further cycles 25, 26, and 27.

2. Data and Methodology of Spectral Analysis

Data used are the conventional sunspot numbers R_z. The sunspot values are available for the past 300 years (from about 1700; Waldmeier, 1961), but the quality of the data is considered as "poor" during 1700 – 1748, "questionable" during 1749 – 1817, "good" during 1818 – 1847, and "reliable" from 1848 onward (McKinnon, 1987). A sunspot cycle (~ 11 years) is defined as from one sunspot minimum to the next. (The interval 1755 – 1765 is designated as Cycle 1.) However, Kane (1999) noticed that when the series for 1748 – 1996 was divided into three intervals of 83 years each, the spectral characteristics differed from one interval to the next, sometimes considerably, notably the high periodicities exceeding 30 years. Later, Kane (2006) showed that the 128-data-point series 1745 – 1872 and 1873 – 2000 for the annual sunspot numbers R_z showed significant periodicities of 8.4 (8.2), 9.9 (9.6), 11.7 (10.6), 15.0 (12.6), and 59.0 (36.1) years (with numbers in parentheses for the latter series 1873 – 2000), some with trivial differences but others with substantial ones. Hence, for prediction purposes, the more recent the data used, the better. However, estimating reliable periodicities needs sufficient data length. As a compromise, it was decided to use 111 values of the most recent data of annual R_z, obtained from the NOAA Web site ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS. Three series were analyzed, namely the annual values of R_z for 1876 – 1986, 1886 – 1996, and 1896 – 2006.

To obtain quantitative estimates of the spectral characteristics, the series were subjected to spectral analysis by the maximum entropy method (Burg, 1967; Ulrych and Bishop, 1975), which locates peaks much more accurately than the conventional BT (Blackman and Tukey, 1958) method. However, the amplitude (power) estimates in MEM are not very reliable (Kane, 1977, 1979; Kane and Trivedi, 1982). Hence, MEM was used only for detecting all the possible peaks T_k ($k = 1$ to n), using the length of the prediction error filter (LPEF) as 60% of the data length. These T_k were then used in the expression

$$f(t) = A_0 + \sum \left[a_k \cos(2\pi t/T_k) + b_k \sin(2\pi t/T_k)\right] + E \tag{1}$$

$$= A_0 + \sum r_k \cos(2\pi t/T_k + \phi_k) + E, \tag{2}$$

where $f(t)$ is the observed series and E is the error factor. A multiple regression analysis (MRA; Bevington, 1969) was then carried out to estimate A_0, a_k, b_k, and their standard errors (by a least-square fit). From these, amplitudes r_k and their standard error σ_k (common for all r_k in this methodology, which assumes white noise) were calculated. Any r_k exceeding 2σ is significant at a 95% (*a priori*) confidence level.

The usefulness of this methodology has been tested earlier by using artificial samples as inputs (Kane, 1977, 1979; Kane and Trivedi, 1982). The amplitudes are revealed with very good accuracy, but phases may not come out very accurate. If all periodicities are considered, earlier (used) data show a good fit (with correlations exceeding +0.95), but future data show unsatisfactory results (with correlations $\sim +0.6$). If only very significant periodicities are chosen, the fits to earlier data are not very good (correlation $\sim +0.8$), but further data show good matching (correlations exceeding $+0.8$). Thus, a compromise is needed while choosing the periodicities to consider for further extrapolation. Proper choice of the LPEF is also a problem. Very small LPEF detects only the most prominent periodicity but it may not be useful. For example, a low LPEF shows only the 11-year periodicity in sunspot numbers, but the prediction will then be the *same value for all cycles*, which is obviously meaningless. If the next prominent periodicities are also considered one by one, the cycle-to-cycle variation is revealed. Here again, a compromise is needed, as very high LPEF gives spurious peaks and/or peak-splitting (Ulrych and Bishop, 1975). We have chosen LPEF = 60% as a compromise. Also, as already mentioned, phases may not come out right (the year of sunspot maximum could be off by 12 months or more), but the amplitudes (as examined for artificial samples) come out with uncertainties of less than 5% (Kane, 1977, 1979; Kane and Trivedi, 1982). In a recent paper (Kane, 2006), several artificial samples are studied and compared with other methods, and MEM turns out to be better than other methods examined by Rigozo *et al.* (2005).

Table 1 lists the periodicities detected by MEM and their amplitudes with standard errors. obtained by MRA. The following may be noted:

(1) There are too many periodicities revealed (~ 25). If year-to-year variations of R_z are large, the spectra show many high harmonics (low periodicities). Some of these may be genuine. Kane (2005) reported periodicities in the QBO (quasi-biennial oscillations, 2 – 3 years) and QTO (quasi-triennial oscillations, 3 – 4 years) in individual sunspot cycles 18 – 23. However, most of these are of small amplitudes (3 – 4 sunspot units), which are insignificant (standard error $\sigma = 2$ sunspot units, $2\sigma = 4$ sunspot units) as compared to the amplitudes of some higher periodicities (40 or more sunspot units) and are not likely to contribute much to the predictions.
(2) However, even in the significant amplitudes (far larger than 2σ, indicated in bold in Table 1), the one near 11 years (strongest as expected) is not unique but shows doublets or triplets in the range 8 – 13 years. This is because, though the average cycle length is ~ 11 years (twelve cycles 12 – 23 in 130 years), the lengths of individual cycles may differ from 11 years by ± 0.5 years or even more. Such splitting has been observed before and commented upon by many workers (Cole, 1973; Currie, 1973; Cohen and Lintz, 1974; Radoski, Fougere, and Zawalick, 1975; Kane, 1977).
(3) There are higher periodicities in the range 17 – 20 years, and one near 37 years (which could be the 33-year periodicity indicated by Ahluwalia, 1998).

To reconstruct the series, Equation (2) can be used. The value of the (constant) A_0 is given in Table 1 for each of the three series, and the amplitudes "r" and phases "ϕ" are also given. First, the reconstruction was made by using all the periodicities. However, the correlation of the reconstructed and original series was not good (less than 0.5). Hence, the periodicities of lowest amplitudes were eliminated one by one and it was noticed that the correlations rose above $+0.7$ only when the highly significant periodicities (marked in bold in Table 1) were considered. The reconstructed series were then extrapolated further and the sunspot maxima were noted for future cycles. Figure 1 shows the plots (for 1876 – 2054, 179 years) of the original series with the reconstructed series superposed, for 1876 – 1986 in (a), for 1886 – 1996 in (b), and for 1896 – 2006 in (c). As can be seen, the phases can be in error by about a year, but the amplitudes should be reliable, as found by using artificial samples as inputs (Kane, 1977, 1979, 2006; Kane and Trivedi, 1982). The results are given in Table 2.

As can be seen from Table 2, the hindsight predictions for cycles 22 and 23 (for which observed values are already available) were largely *underestimates* (with predicted values more than 30% lower than the observed values, *i.e.*, unsatisfactory predictions). Hence, the

Table 1 Periodicities detected by MEM and their amplitudes with standard errors, and phases in radians, obtained by MRA.

1876 – 1986 period (years)	LPEF 60% Amp. r (R_Z units)	Std. error σ_r	Phase ϕ (radians)	1886 – 1996 period (years)	LPEF 60% Amp. r (R_Z units)	Std. error σ_r	Phase ϕ (radians)	1896 – 2006 period (years)	LPEF 60% Amp. r (R_Z units)	Std. error σ_r	Phase ϕ (radians)
Cons. A_0	54.8	1.7		Cons. A_0	59.2	2.9		Cons. A_0	59.7	2.7	
2.02	2.6	2.4	−0.89	2.02	2.0	4.2	−0.78	2.03	1.9	3.9	−0.94
2.12	2.6	2.3	−0.48	2.12	2.7	4.2	0.62	2.11	2.0	4.0	−2.86
2.24	1.7	2.4	0.09	2.3	3.4	4.2	−0.86	2.22	0.9	3.9	1.79
2.30	3.1	2.4	1.01	2.4	1.1	4.2	1.97	2.31	3.0	3.9	2.66
2.46	1.9	2.3	0.80	2.47	2.6	4.3	0.14	2.36	1.4	3.9	−1.93
2.62	1.6	2.4	0.54	2.62	0.9	4.2	2.38	2.49	2.5	3.9	−1.11
2.79	1.9	2.4	0.97	2.79	1.1	4.2	−2.63	2.62	1.3	3.9	−2.06
2.92	0.6	2.4	−2.60	2.96	1.3	4.2	−0.64	2.78	2.0	3.9	−0.23
3.08	1.1	2.4	−2.65	3.15	2.9	4.2	−1.37	2.98	0.9	3.9	0.94
3.23	2.0	2.4	0.90	3.23	3.9	4.2	0.18	3.17	1.9	3.9	2.62
3.44	2.7	2.3	−1.86	3.5	1.3	4.2	2.86	3.26	3.2	3.9	−1.26
3.88	2.7	2.3	−1.42	3.86	1.7	4.2	1.71	3.5	3.0	3.9	−1.94
4.28	0.6	2.3	−0.60	4.2	2.9	4.2	−2.07	3.86	3.4	3.8	−1.45
4.94	4.7	2.4	−2.41	4.56	4.1	4.3	−1.98	4.2	1.7	3.9	0.61
5.26	**7.1**	**2.4**	**−1.59**	4.85	5.2	4.3	−1.62	4.61	1.0	3.9	1.23
5.73	3.0	2.4	1.50	**5.23**	**8.8**	**4.3**	**−0.87**	4.83	5.9	3.9	−1.49
6.47	3.5	2.4	−2.02	5.64	3.8	4.3	−1.09	**5.23**	**10.5**	**4.0**	**−0.25**
7.47	3.7	2.4	−0.03	6.58	3.1	4.2	−0.13	5.57	4.4	3.9	0.80
8.15	**8.8**	**2.4**	**0.21**	**8.29**	**14.6**	**4.2**	**−1.80**	6.55	1.3	3.8	−3.05
9.57	**19.6**	**2.4**	**−2.15**	**9.57**	**16.5**	**4.2**	**−1.79**	**7.96**	**13.2**	**3.9**	**−1.25**
10.62	**44.7**	**2.4**	**−0.72**	**10.5**	**49.3**	**4.2**	**0.00**	**8.83**	**14.4**	**3.9**	**−0.48**
12.69	**13.3**	**2.4**	**3.06**	**12.47**	**11.1**	**4.2**	**−1.47**	**10.44**	**54.7**	**3.9**	**0.59**
17.83	**8.0**	**2.5**	**−1.06**					**11.98**	**11.3**	**3.9**	**1.32**
20.35	**9.3**	**2.5**	**0.21**	19.21	4.5	4.1	−1.27	19.66	5.3	3.9	2.11
37.03	**6.0**	**2.4**	**0.28**	**37.46**	**11.8**	**4.1**	**−1.26**	**37.46**	**11.5**	**3.9**	**−2.84**

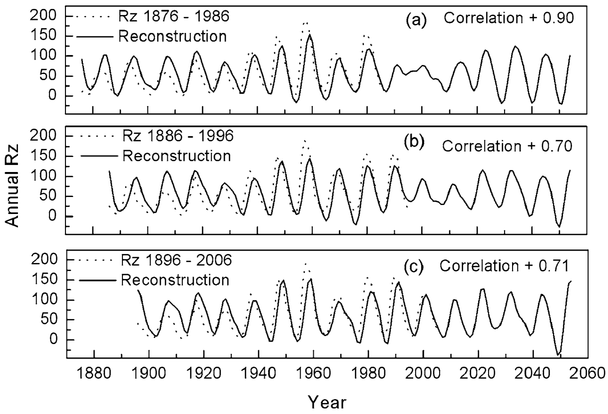

Figure 1 Plot of the annual values of sunspot number R_Z. In (a), the dashed line shows the original R_Z for 1876 – 1986, and the full line shows the reconstructed series. The correlation between original values and the reconstructed values was + 0.90. In (b), similar values of R_Z are shown for 1886 – 1996. In (c), similar values are shown for 1896 – 2006.

Table 2 Results of extrapolation of periodicities detected by MEM – MRA combination.

Cycles			22	23	24	25	26	27
12 – 21	1876 – 1986	Predicted R_Z(max) (P)	69	74	94	112	118	113
		Year	1991	2001	2014	2023	2034	2045
		Observed R_Z(max) (O)	158	120				
		(O – P)/O	56%	38%				
13 – 22	1886 – 1996	Predicted R_Z(max) (P)		94	80	117	115	100
		Year		2000	2011	2022	2033	2044
		Observed R_Z(max) (O)		120				
		(O – P)/O		22%				
14 – 23	1896 – 2006	Predicted R_Z(max) (P)			101	127	120	113
		Year			2011	2022	2032	2043
		Average (P)			92	119	118	109

further predictions for cycles 24, 25, 26, and 27 could also be underestimates. In general, this methodology is expected to yield underestimates for the following reason. The periodicities are *smooth* estimates of the *regular* part of the R_z series and would naturally be conservative, with much lesser weight for the extreme values. This is obvious from the fact that whereas the original series of the annual values of R_z had a range 0 – 190 (0 – 160 if cycle 19, the largest cycle on record, is omitted), the reconstructed series had a range 0 – 155. Thus, maximum values in the reconstructed series were lower by about 5 – 35 units. If this number is added to the predictions of cycles 24 – 27 as given in Table 2, the predictions would increase from the present 90 – 120 to 95 – 155. However, this is all speculation and only indicates the inadequacy of this method, which cannot predict sunspots above ∼ 155. The data length considered (111 values for each of 1876 – 1986, values shifted by 10, 1886 – 1996, and values shifted by 20, 1896 – 2006) does not seem to make much difference, in contrast to the significant differences between the 128-point series 1745 – 1872 and 1873 – 2000 discussed in Kane (2006). The predictions for the future are almost the same: 80 – 101 (mean 92) for cycle 24, 112 – 127 (mean 119) for cycle 25, 115 – 120 (mean 118) for cycle 26, and 100 – 113 (mean 109) for cycle 27, within a standard error of $\sim\pm 10$. The years of occurrence are similar, within ± 1 year.

3. Conclusions and Discussion

The series of annual values of sunspot number R_z were subjected to spectral analysis to detect periodicities by the maximum entropy method (Burg, 1967; Ulrych and Bishop, 1975), and the periodicities so obtained were used in a multiple regression analysis (Bevington, 1969) to estimate the amplitudes and phases. Three series (each of 111 annual values of R_z) were analyzed, namely, 1876 – 1986, 1886 – 1996, and 1896 – 2006. All showed roughly similar spectra with many periodicities (24 or more), but most of these were insignificant. The significant periodicities (far exceeding 2σ) were near 5, 8 – 12, 18, and 37 years. By using the amplitudes and phases of these significant periodicities, reconstructed series were obtained, which showed good correlations (+ 0.7 or more) with the original series. When extrapolated further in time, the reconstructed series indicated R_z(max) in the ranges 80 – 101 (mean 92) for cycle 24 during years 2011 – 2014, 112 – 127 (mean 119) for cycle 25 during years 2022 – 2023, 115 – 120 (mean 118) for cycle 26 during years 2032 – 2034, and 100 – 113 (mean 109) for cycle 27 during years 2043 – 2045.

For cycle 24, many predictions have been made by different methods and these are in a very wide range: 70 – 190 (see the detailed list with references in Kane, 2007). Among these, some based on extrapolation of spectral components are Clilverd *et al.* (2006), $R_z(\max) = 42 \pm 34$; Echer *et al.* (2004), $R_z(\max) = 116 \pm 13$; Gholipour *et al.* (2005), $R_z(\max) = 145$; and Nordemann *et al.* (2007), $R_z(\max) = \sim 100$, in a very wide range of 42 – 145. Ours is $R_z(\max) = (80 – 101) \pm 10$ (*i.e.*, $\sim 92 \pm 10$), which falls in the middle of that range.

For cycle 25, Hathaway and Wilson (2004) predict 70 ± 30, and Du (2006) predicts 103 ± 22. Schatten and Tobiska (2003) claim that solar activity will decrease after cycle 24 and will be heading for a Maunder Minimum in the next few decades. Duhau (2003) suggests that solar activity may be in a declining episode that started about 1993. Clilverd (2005) concludes that after a very small value for cycle 24 ($\sim 45 \pm 27$), a *recovery* is expected during the middle of the century to more typical solar activity cycles with peak sunspot numbers of ~ 120. Our sequence of mean 92 for cycle 24, mean 119 for cycle 25, mean 118 for cycle 26, and mean 109 for cycle 27 does not indicate any unambiguous downward trend.

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