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Synchronization in Sunspot Indices in the Two Hemispheres

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Abstract Historical sunspot records were analyzed by means of nonlinear tools to find synchronization phenomena at different time scales on the Sun. Using cross-recurrence plots it is shown that the north – south sunspot synchronization demonstrates a set of distinct periodic oscillations – 43.29, 18.52, and 7.63 years. Also we have traced the sunspot synchronization on shorter time scales. Very rare and episodic synchronization within half of the Carrington rotation rate was detected. By using the empirical mode decomposition technique the north – south sunspot time series were decomposed into intrinsic oscillatory modes. To determine which modes of the signal are responsible for synchronization we separated them into high- and low-frequency parts. It is shown that phase synchronization is detected only in the low-frequency modes. The high-frequency component demonstrates noisy behavior with amplitude synchronization and strong phase mixing.

1. Introduction

Solar activity presented by the remarkable features at the photosphere such as sunspots exhibits a complex spatial and temporal behavior. Sunspots tend to group in clusters, demonstrating nonrandom distribution over the solar surface and regular evolution during the course of a solar cycle. Active regions (resulting perhaps from their magnetic nature) show strange interconnections, resulting in near-simultaneous eruption of new centers of solar activity in both hemispheres of the Sun.

Synchronization of dynamical processes is one key aspect for understanding the origin and evolution of active regions on the Sun and their various manifestations in the solar corona. According to the modern point of view synchronization is a universal concept in nonlinear sciences (Pikovsky, Rosenblum, and Kurths, 2001).

It is well known that during the Maunder minimum a unique sunspot asymmetry was observed. Sunspots were registered only in the southern hemisphere (Sokoloff and Nesme-Ribes, 1994). However, generally hemispherical sunspot activities are highly synchronized,

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forming the famous "butterfly diagram." If sometimes asynchrony happens, it disappears rather rapidly during a solar cycle. Before offering a possible sunspot synchronization mechanism one needs to describe the main types and features of the phenomenon.

There are numerous papers describing the north – south asymmetry. [Its classical definition is NA $= (N - S)/(N + S)$, where N and S are solar indices for the northern and southern hemispheres.] From one point of view irregular behavior of the asymmetry appears to be random noise rather than deterministic chaos (Watari, 1996). However, in most cases the asymmetry is highly significant and cannot be obtained from the random distribution of sunspot area over the solar surface (Carbonell, Oliver, and Ballester, 1993). Using Fourier and wavelet analysis Knaack, Stenflo, and Berdyugina (2004) reported on significant periodic variations of the magnetic activity between solar hemispheres. In particular, ten meaningful frequencies were obtained by using the north – south asymmetry data sets. However, later Ballester, Oliver, and Carbonell (2005) showed that the use of the discrete Fourier transform to normalized time series leads to misleading results.

We have found recently that the north – south sunspot asymmetry is due to phase asynchrony between the northern and southern hemispherical activities (Zolotova and Ponyavin, 2006a). The classical definition of the asymmetry is inappropriate for determining complex relationships (nonlinear coupling) between sunspot area time series.

Synchronization at the Sun is expected to be imperfect and can display phase, episodic, or delayed behavior. It is not easy to recognize synchronous events by means of linear tools. Strictly speaking, using the linear approach to study nonlinear phenomena is incorrect. The aim of this paper is to find synchronization in sunspot indices at different time scales on the Sun by means of modern nonlinear techniques.

2. Cross-Recurrence Technique

One of the novel techniques of nonlinear data analysis involves the use of recurrence plots (Romano *et al.*, 2005). The method is based on the fundamental feature of dissipative dynamical systems: local divergence of neighboring trajectories in phase space, but recovery close to any former point after a sufficiently long time. Recurrence plot analysis was first proposed by Eckmann, Kamphorst, and Ruelle (1987) as a tool to visualize the recurrence of states of dynamical systems in phase space. Later it was developed by Zbilut, Giuliani, and Webber (1998), Marwan and Kurths (2002, 2005), and others. The method was extended to cross-recurrence plots (CRPs), where two time series are embedded in the same phase space. We have used the CRP Toolbox of Marwan (http://www.agnld.uni-potsdam.de/~marwan/toolbox/), in which the mathematical definition of the CRP is

$$\mathbf{CR}_{i,j}^{m,\varepsilon_i} = \Theta\big(\varepsilon_i - \|\mathbf{x}_i - \mathbf{y}_j\|\big), \quad \mathbf{x}_i, \mathbf{y}_i \in \Re^m, \; i = 1, \ldots, N_x, \; j = 1, \ldots, N_y, \qquad (1)$$

where N is the number of considered states x_i, y_i; ε_i is the threshold distance; $\|\cdot\|$ is the norm (*e.g.,* the Euclidean norm); and Θ is the Heaviside function.

Visually, the CRP is a graphical pattern of matrix $N_x \times N_y$, all elements of which are either zero (white points of the CRP) or one (black points of the CRP); see Figure 2b. The $\mathbf{CR}_{i,j}$ equal one, if point j (belonging to the y trajectory) in phase space is located in the predefined neighborhood ε of point i (belonging to the x trajectory); otherwise the $\mathbf{CR}_{i,j}$ equal zero. If $\mathbf{CR}_{i,j} = 1$ for $\forall i$, then two analyzed processes are fully identical or synchronized. Visually, it means that there is black main diagonal from the lower left to the upper right angle of the cross-recurrence plot. This is the so-called line of synchronization (LOS). However, in common cases the LOS can be broken, curved, or even vanishing, depending on the kind of a relationship between time series. The LOS contains information on temporal and amplitude shifts and time reversals and can trace even small and smooth phase variations (Marwan and Kurths, 2005; Zolotova and Ponyavin, 2006a).

3. Asynchrony and Asymmetry: Significant Periodic Variations

We previously demonstrated that the CRP technique can be applied to nonlinear analysis of sunspot activity (Zolotova and Ponyavin, 2006a). In that paper we carried out the cross-recurrence plot analysis of sunspot activity for the northern and southern hemispheres. We extracted the LOS from the CRP and determined the asymmetry of the northern and southern sunspot activities as an effect of phase asynchronization. Moreover, we mentioned that the LOS exhibits nonrandom quasi-regular behavior.

In this paper we report on significant periodic variations of the LOS. Additionally, we compare our results with findings by Knaack, Stenflo, and Berdyugina (2004).

We use the Royal Greenwich Observatory USAF/NOAA monthly averaged sunspot area data taken separately for northern and southern hemispheres (http://solarscience.msfc.nasa.gov/greenwch.shtml).

Figure 1 displays the LOS and the north – south asymmetry. Wavelet power spectra of the LOS and NA are shown in Figures 1c and 1d. We have applied the wavelet software available at the Web site http://www.pol.ac.uk/home/research/waveletcoherence/. Wavelet power spectra are plotted by using the Morlet mother wavelet.

The lower panels in Figure 1 display the Fourier power spectra of the LOS and NA (black lines). To specify significance levels we do not use white noise as in the paper by Knaack, Stenflo, and Berdyugina (2004). The power spectrum of the LOS is not uniform but decreases on some law. Knaack, Stenflo, and Berdyugina (2004) defined significance levels for NA variations against white noise. We estimated significance levels for the LOS and NA by taking into account a correlation between the present values and those in the distant past. For this reason we calculated the Hurst exponents and found $H_{\mathrm{LOS}} = 0.87 \pm 0.05$ and $H_{\mathrm{NA}} = 0.70 \pm 0.05$. This means that the correlation is positive or the time series are persistent. Interestingly, according to Oliver and Ballester (1998), the value of H for sunspot areas is 0.883, similar to the H_{LOS}. Using the H estimates we generated fractional Gaussian noises for each case. Their Fourier power spectra are shown in Figures 1e and 1f by the gray dashed lines. Significant peaks for the LOS and NA are labeled from “A” to “F” and are listed in Table 1. It should be noticed that significant periodic variations of the LOS are different from the main solar activity periods.

The value of the first “A” peak may be due to a finite-length effect. However, frequencies at other peaks are analogous to findings by Knaack, Stenflo, and Berdyugina (2004) within the range of error. Ballester, Oliver, and Carbonell (2005) with reference to original suggestions made by Yi (1992) showed that the 11-year periodicity comes from the denominator of the normalized sunspot asymmetry index ($N + S$). This period is correct for solar activity coupling of both hemispheres, but it is misleading for their asymmetry or asynchrony. According to the analysis made by Ballester, Oliver, and Carbonell (2005) only three significant peaks appear, with periods of 43.25, 8.65, and 1.44 years. It is interesting to note that peaks for the LOS and NA labeled as “A” and “B”, correspondingly, have approximately the same period, 43.3 ± 7 years.

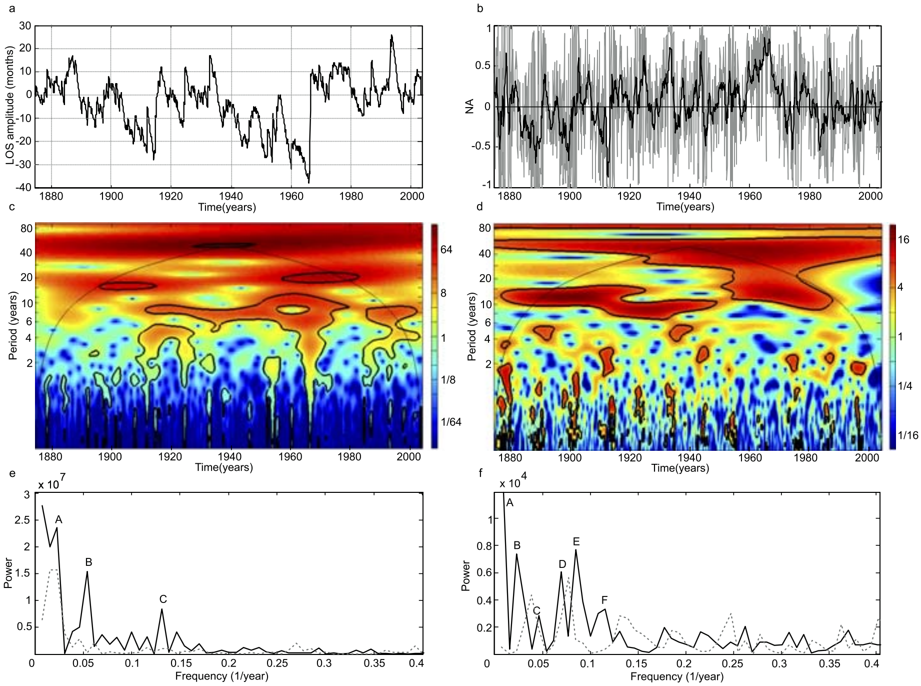

Figure 1 Plots of (a) the line of synchronization (LOS) and (b) the normalized north – south asymmetry index (NA, gray color) versus time. The bold black line is the one-year moving average for NA with a window of one year. (c) and (d) Wavelet power spectra for the LOS and NA, respectively. Bold black contours indicate the 90% significance levels against red noise background; black thin lines are the cone of influence. (e) and (f) Fourier power spectra for the LOS and NA (black lines). Fractional Gaussian noises according to Hurst estimates $H_{\mathrm{LOS}} = 0.87$ and $H_{\mathrm{NA}} = 0.70$ on the base of 130-year realizations give approximate significance levels (gray dashed lines).

Table 1 Main frequencies ν and corresponding periods $T = 1/\nu$ for the LOS and NA. The error is half of the frequency resolution, $\triangle\nu = 0.004$ (1/year)

	LOS			NA	
	Frequency (1/year)	Period (year)		Frequency (1/year)	Period (year)
A	0.023	43.29	A	–	–
B	0.054	18.52	B	0.0231	43.29
C	0.131	7.63	C	0.0463	21.60
			D	0.0694	14.41
			E	0.0848	11.79
			F	0.1080 – 0.1156	9.26 – 8.64

We have shown recently (Zolotova and Ponyavin, 2006a) that the LOS is not a measure of the excess of one hemispheric activity over the other, but it estimates phase asynchronization. We suggest that the present analysis revealing significant periodic variations helps us to understand the origin of the sunspot synchronization.

4. Tracing Synchronization on Shorter Time Scales Using the Hough Transform

It would be inappropriate to suggest that couplings between the two hemispheric magnetic activities are stable and persistent in time. We have proposed a method to identify an onset and loss of synchronization between dynamical systems (Zolotova and Ponyavin, 2006b). The methodology is based on sequential application of two graphical techniques: cross-recurrence analysis and the Hough transform.

According to this approach, the coordinates of a system under consideration (*e.g.,* Cartesian coordinates x and y) are converted into new variables θ and ρ. This transform can be aimed at detecting either straight segments or more complex structures (*e.g.,* circles, ellipses, *etc.*) on the image (Ching, 2001). Since the presence of straight diagonal segments on cross-recurrence plots is responsible for synchronization, in the simplest case, we used the Hough transform, recognizing straight lines $\rho = x\cos(\theta) + y\sin(\theta)$.

We investigate the north – south sunspot synchronization on short time scales using the daily sunspot area data. Figure 2 displays results of the synchronization analysis during solar maximum of the 19th cycle – one of the biggest in solar history. A similar analysis was carried out for other solar cycles. Figure 2a shows sunspot activity, with the black line corresponding to the northern hemisphere and the gray line to the southern hemisphere. The cross-recurrence plot is displayed in Figure 2b and its parametrical Hough space in Figure 2c. Black small squares label intersections of sinusoidal curves, determining the two main directions of straight diagonal segments in the cross-recurrence plot. Figure 2d illustrates a

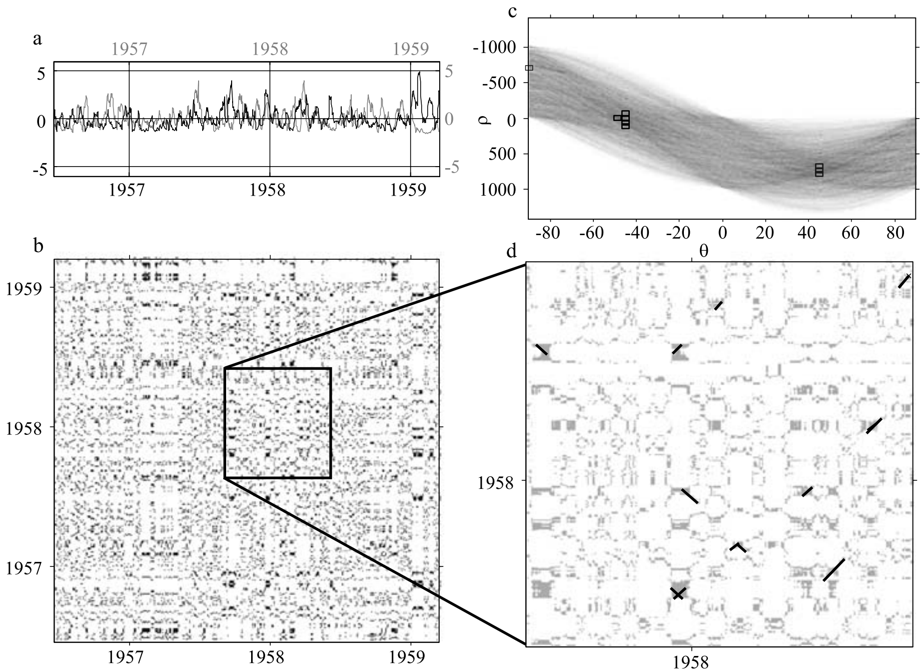

Figure 2 Daily sunspot area (a) during maximum of the 19th solar cycle for the northern (black line) and southern (gray line) hemispheres. Cross-recurrence plot (b) with the following parameters: dimension of 1, delay of 1, and threshold of 10%, with fixed amount of neighbors. Hough parametric space (c) and portion of the CRP (d) with synchronization segments recognized by using the Hough transform (black lines in the gray cross-recurrence background).

zoomed section of the line segments recognized with the Hough transform (black segments on the gray cross-recurrence background). It is seen that synchronization is strongly non-stationary and nonpersistent. The lengths of the segments responsible for synchronization do not exceed 13 days, corresponding to half the Carrington rotation rate. Thus using the daily data as input we cannot trace sunspot synchronization on daily time scales, since a sufficiently significant part of sunspot life passes on the invisible side of the Sun.

5. Tracing Synchronization on Longer Time Scales: Empirical Mode Decomposition

The empirical mode decomposition (EMD) technique was developed to analyze nonlinear and nonstationary data (Huang *et al.*, 1998). The essence of the method is to identify intrinsic oscillatory modes by their characteristic time scales in the data empirically, and then decompose the given signal as a set of intrinsic mode functions (IMFs). The intrinsic mode functions are orthogonal and their number is small and complete. The EMD technique has already been applied to solar and climate variability (Coughlin and Tung, 2004; Ruzmaikin, Feynman, and Yung, 2006) and a host of other applications.

We present the results of application of the EMD to the monthly sunspot area time series from 1874 to 2003.

Figures 3a and 3b display the sunspot area time series relative to the northern and southern hemispheres, respectively. The first set is decomposed into eight intrinsic mode functions and residue (Figure 3c). The second set is decomposed into nine IMFs and residue (Figure 3d).

To determine which modes of signal are responsible for synchronization we sort the IMFs into two parts for each hemisphere. The first is the sum of the first five IMFs ($\sum_{i=1}^{5} \mathrm{IMF}_i$), which compose the high-frequency intrinsic oscillatory modes (Figures 4a and 4c). The top panel represents plots for the northern hemisphere, and bottom panel those for the southern

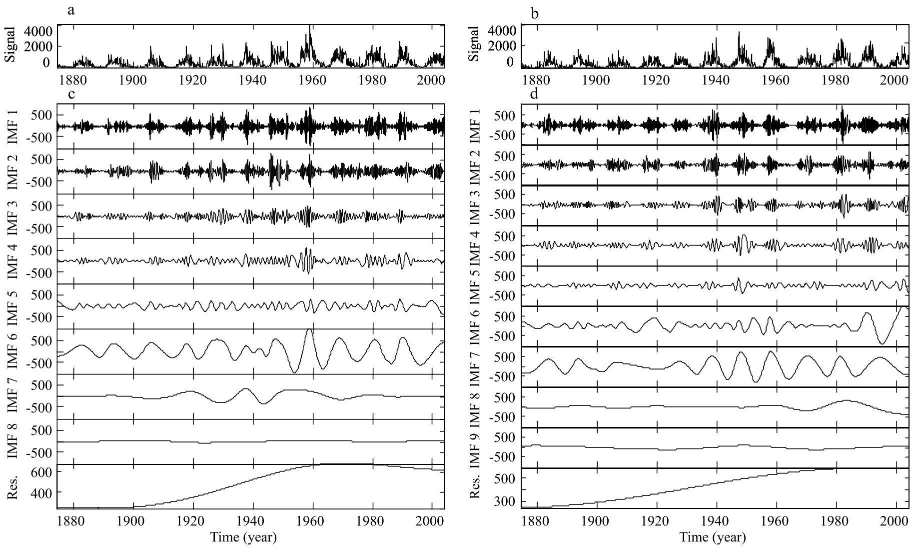

Figure 3 Monthly mean sunspot area for the northern (a) and southern (b) hemispheres. Intrinsic mode functions and residue for the northern (c) and southern (d) hemispheres.

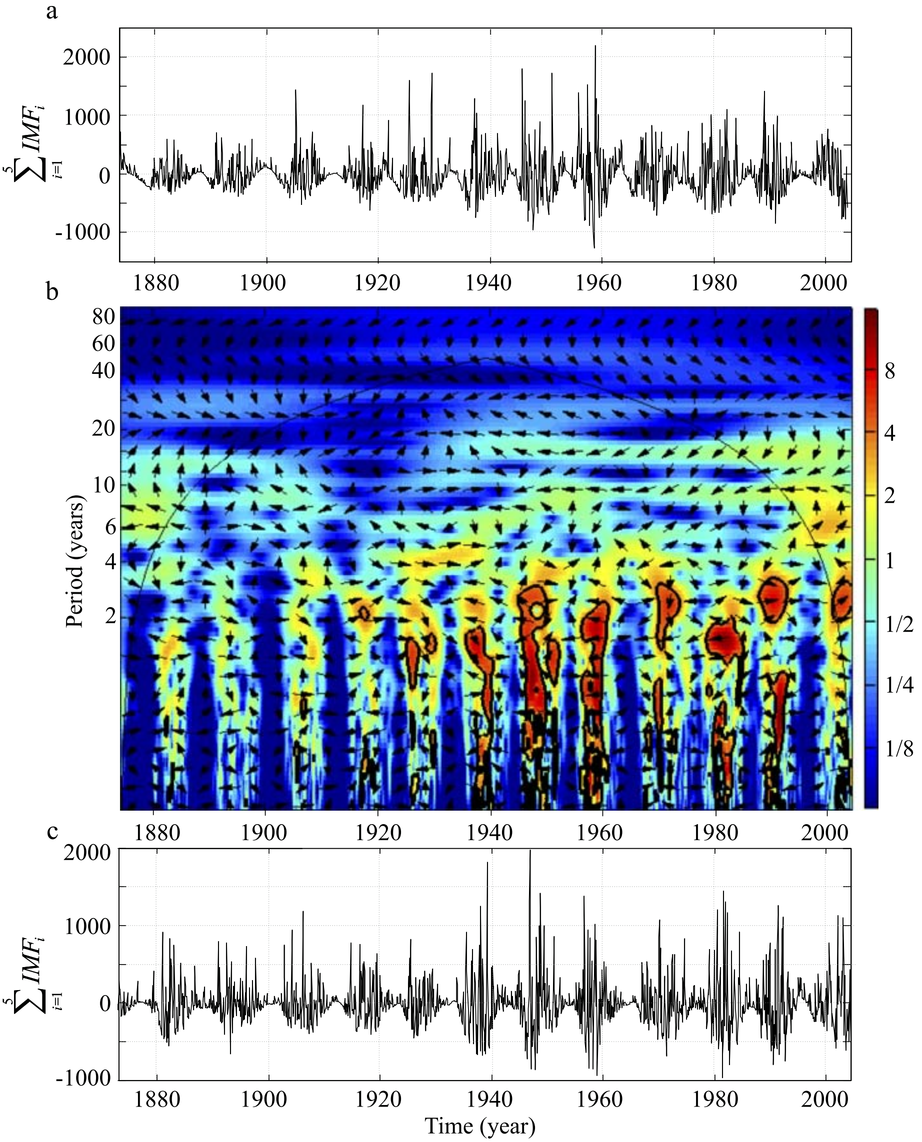

Figure 4 High-frequency modes for the northern (a) and southern (c) hemispheres. (b) Their cross-wavelet spectrum. The bold black contour indicates the 95% significance level against red noise background; the black thin line is the cone of influence.

hemisphere. It is known that the biggest fluctuations of sunspot activity occur during solar maxima. Actually the high-frequency modes are modulated by the 11-year solar cycle and demonstrate noisy behavior with large amplitudes.

To examine relationships between the high-frequency modes we apply the cross-wavelet transform with a significance level against red noise of 0.95% (Figure 4b).

The cross-wavelet transform of two time series X and Y is defined as $W^{XY} = W^X W^{Y*}$, where * denotes complex conjugation and W^X is the continuous wavelet transform (Grinsted, Moore, and Jevrejeva, 2004). The complex argument $\arg(W^{XY})$ can be interpreted as a local relative phase between X and Y in time – frequency space. The relative phase relationship is shown by arrows (Figure 4b). Arrows point to the right when processes are in phase and to the left when they are in antiphase. If an arrow points down, then the first process leads the second one.

The cross-wavelet spectrum displays synchrony in the high-frequency time zone (Figure 4b). Evidently, this is a type of amplitude synchronization (as a result of an 11-year modulation). The phase (arrows) is fairly randomly distributed, suggesting strong phase mixing. Thus the high-frequency modes are not responsible for the strong phase synchronization detected via the cross-recurrence plot (Zolotova and Ponyavin, 2006a). Presumably it corresponds to the noise on the CRP and reveals an existing stochastic component in the sunspot data.

Then we consider the low-frequency part of the sunspot EMD: $\sum_{i=6}^{8} \mathrm{IMF}_i + \mathrm{Residue}$ for the north (Figure 5a) and $\sum_{i=6}^{9} \mathrm{IMF}_i + \mathrm{Residue}$ for the south (Figure 5c). The cross-wavelet spectrum is displayed in Figure 5b. According to the definition, it is constructed from two continuous wavelet transforms. Thus it reveals domains with high common power (Grinsted, Moore, and Jevrejeva, 2004). On Figure 5b such a domain corresponds to the wide frequency band near 11 years. Within this band, arrows point commonly to the right, indicating that the low-frequency IMFs are synchronized in phase.

The most evident phase deviation is observed in the 1960s. Arrows pointing downward indicate that the south sunspot activity leads that in the north in the course of the 19th solar cycle. This result is in accordance with the LOS analysis (Figure 5d). An LOS ≤ 0 means delayed synchronization, when the south leads. Moreover, asynchronization in this period (marked by the gray strip) is the highest – up to 3 years (Zolotova and Ponyavin, 2006a).

However, when applying a cross-wavelet spectral analysis to multivariate nonlinear data one should know about pitfalls. According to Maraun and Kurths (2004), small structures appear on wavelet spectral patterns much more often than larger ones, the area of the peaks plays an important role, and spurious single peaks can be overestimated. Not every structure in a wavelet domain is meaningful, and one might be misled. Thus we have to be careful with conclusions about the significance of single peaks in the cross-wavelet spectrum of the high-frequency modes (Figure 4b). Otherwise it is possible to receive wishful but artificial results. Moreover, Maraun and Kurths (2004) showed that cross-wavelets can demonstrate false peaks even for independent processes, when the continuous wavelet spectrum for one of them exhibits strong peaks. Relative to the low-frequency sunspot modes, they both have 11-year frequency bands in continuous wavelets.

As a final discussion we touch on the problem of solar activity character (Paluš and Novotná, 1999; Kremliovsky, 1994; Letellier *et al.*, 2006). Is it low-dimensional deterministic chaos or stochastic? Lawrence, Cadavid, and Ruzmaikin (1995) showed that a low-dimensional, chaotic behavior of sunspot number operates at time scales longer than about 8 years. But at time scales less than about 2 years the variability can be characterized by a random multiplicative cascade process.

Using the EMD analysis we decomposed the sunspot area time series into a number of IMFs. The low-frequency IMF modes can considered as a long-term trend and the high-frequency modes as a stochastic component that is not random (Carbonell, Oliver, and Ballester, 1993) but amplitude modulated.

Concerning the north – south asymmetry of solar activity we suggest that it is not dominated by a stochastic process (Carbonell, Oliver, and Ballester, 1993) but is a result of imperfect quasi-regular phase and delayed synchronization.

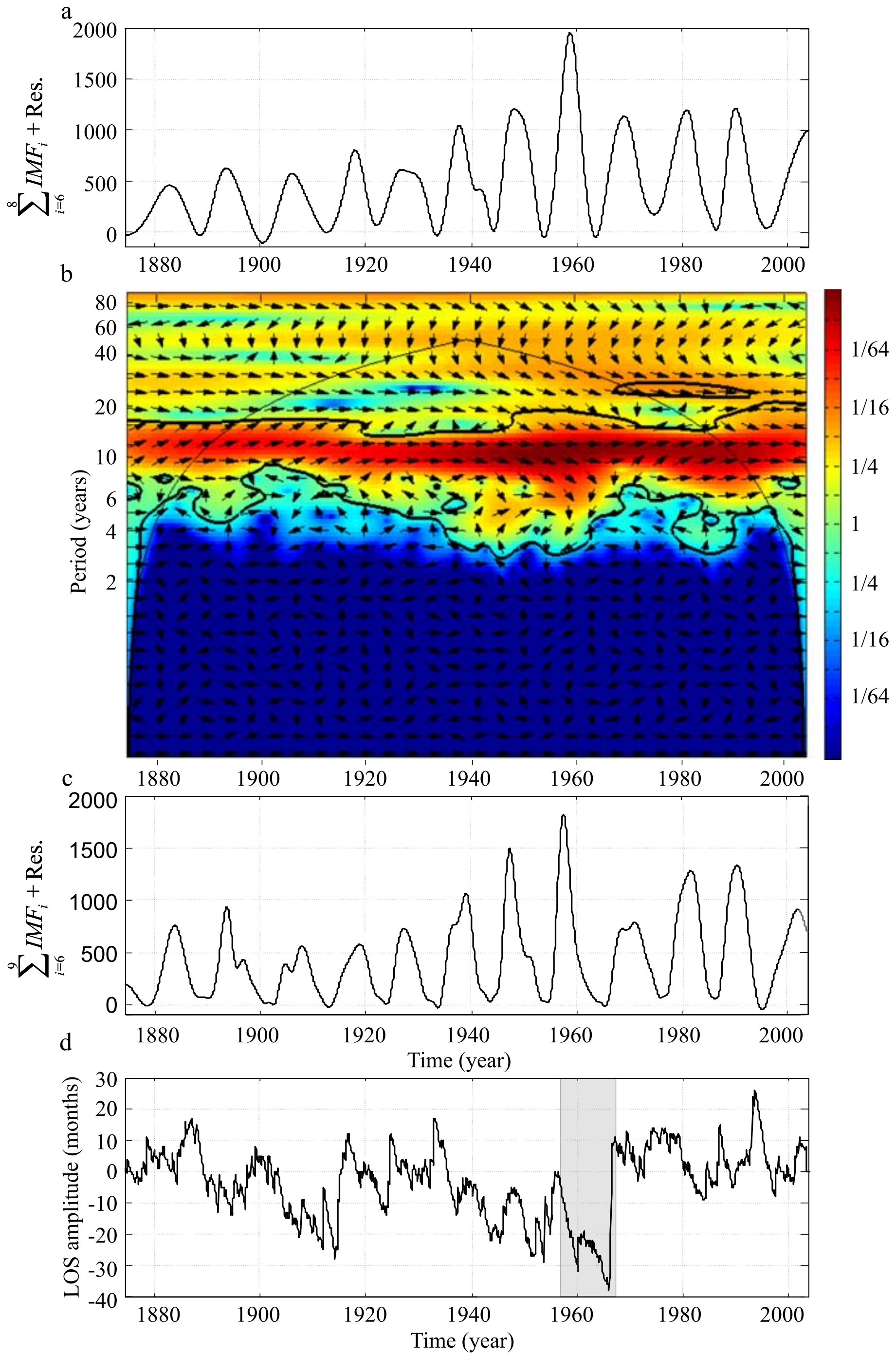

Figure 5 Low-frequency modes for the northern (a) and southern (c) hemispheres. (b) Their cross-wavelet spectrum. The bold black contour indicates the 95% significance level against red noise background; the black thin line is the cone of influence. (d) Line of synchronization for the monthly sunspot area for both hemispheres. The gray strip marks the highest asynchronization.

6. Conclusions

We have examined sunspot activity using monthly and daily sunspot area data from 1874 to 2003 separately for the northern and southern hemispheres. Previously we suggested phase relationships between opposite active latitudes and proposed a measure of phase synchronization, which reveals the origin of sunspot asymmetry.

In the present paper we found distinct periodic oscillations of the LOS (listed in Table 1) and compared these with periods of the normalized asymmetry index (NA). Significant periodic variations of the LOS are different from the main solar activity periods. Of particular interest is the period of 43.3 ± 7 years, which is equal to the first significant period for the NA.

A cross-recurrence plot analysis applied to the daily sunspot area displays a noisy pattern for the analyzed data set. Hough transforms used to identify line structures (*i.e.*, epochs of a similar evolution of segments of phase space trajectories) revealed very rare and episodic synchronization between the north and south sunspot activities within daily time scales. The sunspot area data are only a projection of the real summarized area of all sunspots at the solar surface (since we can take into account only sunspots from the visible side of the Sun). This is why it is difficult to trace synchronization in the sunspot daily observations.

Applying the EMD filtration procedure to the monthly data we separated the north–south sunspot activity into high- and low-frequency parts. Then we performed a cross-wavelet analysis to identify the dominant spatial and time structures in the data set. The high-frequency components demonstrate noisy behavior with an 11-year solar cycle modulation. The type of synchronization corresponds to amplitude synchronization. Phase is randomly distributed, suggesting strong phase mixing. Phase synchronization of sunspot coupling between the hemispheres is detected only in the low-frequency components of solar activity. Local relative phase on the cross-wavelet spectra has established leading and slave processes. The south sunspot activity leads that of the north during the course of the 19th solar cycle. This is confirmed by the highest asynchronization of the LOS behavior observed at the same period.

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References

Ballester, J.L., Oliver, R., Carbonell, M.: 2005, *Astron. Astrophys.* **431**, L5.
Carbonell, M., Oliver, R., Ballester, J.L.: 1993, *Astron. Astrophys.* **274**, 497.
Ching, Y.-T.: 2001, *Pattern Recognit. Lett.* **22**, 421.
Coughlin, K., Tung, K.K.: 2004, *J. Geophys. Res.* **109**, D21105.
Eckmann, J.-P., Kamphorst, S.O., Ruelle, D.: 1987, *Europhys. Lett.* **4**, 973.
Grinsted, A., Moore, J.C., Jevrejeva, S.: 2004, *Nonlinear Process. Geophys.* **11**, 561.
Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.-C., Tung, C.C., Liu, H.H.: 1998, *Proc. Roy. Soc. Lond.* **454**, 903.
Knaack, R., Stenflo, J.O., Berdyugina, S.V.: 2004, *Astron. Astrophys.* **418**, L17.
Kremliovsky, M.N.: 1994, *Solar Phys.* **151**, 351.
Lawrence, J.K., Cadavid, A.C., Ruzmaikin, A.A.: 1995, *Astrophys. J.* **455**, 366.
Letellier, C., Aguirre, L.A., Maquet, J., Gilmore, R.: 2006, *Astron. Astrophys.* **449**, 379.
Maraun, D., Kurths, J.: 2004, *Nonlinear Process. Geophys.* **11**, 505.
Marwan, N., Kurths, J.: 2002, *Phys. Lett. A* **302**, 299.
Marwan, N., Kurths, J.: 2005, *Phys. Lett. A* **336**, 349.
Oliver, R., Ballester, J.L.: 1998, *Phys. Rev. E* **58**, 5650.
Paluš, M., Novotná, D.: 1999, *Phys. Rev. Lett.* **83**, 3406.

Pikovsky, A., Rosenblum, M., Kurths, J.: 2001, *Synchronization: A Universal Concept in Nonlinear Science*, Cambridge University Press, Cambridge.
Romano, M.C., Thiel, M., Kurths, J., Kiss, I.Z., Hudson, J.L.: 2005, *Europhys. Lett.* **71**, 466.
Ruzmaikin, A., Feynman, J., Yung, Y.L.: 2006, *J. Geophys. Res.* **111**, D21114.
Sokoloff, D., Nesme-Ribes, E.: 1994, *Astron. Astrophys.* **288**, 293.
Watari, S.: 1996, *Solar Phys.* **163**, 259.
Yi, W.: 1992, *J. Roy. Astron. Soc. Can.* **86**, 89.
Zbilut, J.P., Giuliani, A., Webber, C.L. Jr.: 1998, *Phys. Lett. A* **246**, 122.
Zolotova, N.V., Ponyavin, D.I.: 2006a, *Astron. Astrophys.* **449**, L1.
Zolotova, N.V., Ponyavin, D.I.: 2006b, *Tech. Phys. Lett.* **32**, 954.