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Multifractality as a Measure of Complexity in Solar Flare Activity

Asok K. Sen

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Abstract In this paper we use the notion of multifractality to describe the complexity in Hα flare activity during the solar cycles 21, 22, and 23. Both northern and southern hemisphere flare indices are analyzed. Multifractal behavior of the flare activity is characterized by calculating the singularity spectrum of the daily flare index time series in terms of the Hölder exponent. The broadness of the singularity spectrum gives a measure of the degree of multifractality or complexity in the flare index data. The broader the spectrum, the richer and more complex is the structure with a higher degree of multifractality. Using this broadness measure, complexity in the flare index data is compared between the northern and southern hemispheres in each of the three cycles, and among the three cycles in each of the two hemispheres. Other parameters of the singularity spectrum can also provide information about the fractal properties of the flare index data. For instance, an asymmetry to the left or right in the singularity spectrum indicates a dominance of high or low fractal exponents, respectively, reflecting a relative abundance of large or small fluctuations in the total energy emitted by the flares. Our results reveal that in the even (22nd) cycle the singularity spectra are very similar for the northern and southern hemispheres, whereas in the odd cycles (21st and 23rd) they differ significantly. In particular, we find that in cycle 21, the northern hemisphere flare index data have higher complexity than its southern counterpart, with an opposite pattern prevailing in cycle 23. Furthermore, small-scale fluctuations in the flare index time series are predominant in the northern hemisphere in the 21st cycle and are predominant in the southern hemisphere in the 23rd cycle. Based on these findings one might suggest that, from cycle to cycle, there exists a smooth switching between the northern and southern hemispheres in the multifractality of the flaring process. This new observational result may bring an insight into the mechanisms of the solar dynamo operation and may also be useful for forecasting solar cycles.

A.K. Sen (✉)
Department of Mathematical Sciences, Indiana University, 402 N. Blackford Street, Indianapolis, IN 46202, USA
e-mail: asen@iupui.edu

1. Introduction

Many physical processes evolve over a multitude of time and/or spatial scales and are governed by complex dynamics. In recent years there has been a great deal of interest in quantifying the complexity of these multiscale processes. However, no clear and unambiguous definition of complexity has been established in the literature. Intuitively, complexity is associated with "meaningful structural richness" (Grassberger, 1991). The notion of entropy has been introduced as a measure of complexity. A higher value of entropy is usually associated with a more irregular, more complex process. Beginning with the information-theoretic definition of Shannon, several other definitions of entropy have been proposed by researchers to describe complexity in a more precise way (see, for example, Pincus, 1991). But, as Costa, Goldberger, and Peng (2002) have pointed out, these traditional entropy measures are postulated on the basis of a single scale, and do not take into account the multiscale features. Based on the idea of coarse-graining the time series, Costa, Goldberger, and Peng (2002) introduced the concept of multiscale entropy (MSE) and used it to describe the nature of complexity in physiological time series such as those associated with cardiac dynamics and gait mechanics. A different form of multiscale entropy, based on wavelet transform has been proposed by Starck, Murtagh, and Gastaud (1998), and is used in astronomical applications (Starck *et al.*, 2001). An alternate approach is to use fractal-based measures of complexity. If the process under consideration is self-similar or scale-invariant, its dynamics can be described by means of a single power-law scaling exponent called the Hurst exponent. Such homogeneous processes are referred to as monofractal and their complexity can be interpreted in terms of a single fractal dimension. However, many complex processes are heterogeneous in the sense that they cannot be described by a single power-law exponent or a fractal dimension, and it is necessary to use more than one scaling exponent or fractal dimension to assess their complexity. Due to the need for using a range of fractal dimensions to describe the scaling properties, these processes are called multifractal, and their complexity can be measured by the degree of multifractality.

By their very nature, solar flares are intermittent, consisting of sudden bursts of large fluctuations separated by intervals of small fluctuations or almost quiescent periods. The large fluctuations (or singularities) in an intermittent process contribute significantly to the statistical moments which lead to multifractality (Frsich, 1995). In fact, intermittency and multifractality are considered to be two different terms for the same phenomenon, with the former typically referring to time series and the latter used for spatial objects (Takayasu, 1990; Frsich, 1995; Cheng, 1999). The purpose of this paper is to characterize the intermittent behavior of solar flares in terms of their multifractality and interpret the degree of multifractality as a measure of their complexity.

A solar flare represents a sudden rise in activity of the Sun accompanied by a large increase in the intensity of radiation emitted around the sunspots. This leads to an intense release of energy which can be as high as 10^{32} ergs. The origin of solar flare occurrences is not clearly understood. It has been suggested that solar flares are the result of rapid conversion of large amounts of energy stored in the magnetically active areas of the Sun and dissipated through magnetic reconnections. A typical flare has a rise time on the order of a few minutes and a decay time on the order of tens of minutes to several hours. Since the first observation of a solar flare by Carrington and Hodgson in 1859, numerous flares have been monitored and studied by researchers (Atac and Ozguc, 2001; Atac, Ozguc, and Rybak, 2005; Bai, 2003; Bazlevskaya *et al.*, 2001; Charbonneau *et al.*, 2001; Joshi and Joshi, 2005; Joshi and Pant, 2005; Li, Schmieder, and Li, 1998; Maris and Popescu, 2004; Ozguc, Atac, and Rybak, 2003; Ozguc *et al.*, 2004; Rybak *et al.*, 2005; Temmer *et al.*, 2001, 2004; Veronig *et al.*, 2002). The flare activity is customarily described by a flare index, Q, which represents the total energy emitted by the flare. It is defined as the product $Q = it$, where i denotes the intensity scale of importance of the flare spectral class and t is the duration of the flare in minutes (Kleczek, 1952). Extensive data on solar flare indices are available at several websites.

In recent years multiscale methods have attracted more and more attention in the analysis of solar phenomena in both spatial (Cadavid *et al.*, 1994; Lepreti *et al.*, 2000; Abramenko *et al.*, 2002; Abramenko, 2005; McAteer, Gallagher, and Ireland, 2005), and temporal domains (Ozguc *et al.*, 2004; Macek, 2005; Rybak *et al.*, 2005). Here we perform a multifractal analysis of the intermittent Hα flare activity and describe their complexity in terms of the degree of multifractality. In particular, we examine the Hα flare activity in both northern and southern hemispheres during the solar cycles 21, 22 and 23. A multifractal analysis of the flare activity consists of constructing a singularity spectrum of the flare index time series in terms of a regularity parameter called the Hölder exponent. The singularity spectrum provides a precise quantitative description of the multifractal measure in terms of interwoven sets with a Hölder exponent α, whose Hausdorff dimension is $f(\alpha)$. The complexity in flare activity can be assessed from the *broadness* of the singularity spectrum. The broadness of the singularity spectrum denotes the range of fractal exponents present in the signal and thus gives a measure of the degree of multifractality of complexity. Using the broadness measure, complexity in the flare index data are compared between the northern and southern hemispheres in each of the three cycles, and also among the three cycles in each of the two hemispheres. Other parameters of the singularity spectrum can also provide information about the fractal properties of the flare index data. For instance, an asymmetry in the spectrum indicates a dominance of high or low fractal exponents, reflecting a relative abundance of large or small fluctuations of energy emitted by the flares.

It can be shown that for a monofractal process, the singularity spectrum reduces to a single point. On the other hand, if the singularity spectrum does not reduce to a single point, it is indicative of multifractal behavior. A multifractal process may be considered to be locally self-similar and the Hölder exponent in a singularity spectrum may be treated as a local Hurst exponent. In other words, a multifractal may be thought of as a superposition of an infinite number of monofractals. Multifractal analysis has been used to describe intermittent processes in a wide variety of applications (see, for example, Meneveau and Sreenivasan, 1991; Davis *et al.*, 1994; Carreras *et al.*, 2000; Schmitt, Schertzer, and Lovejoy, 2000; Fedi, 2003; Zheng *et al.*, 2005). Analysis of solar magnetic fields using fractal and multifractal theories has also been carried out by Abramenko *et al.* (2002), Abramenko (2005), Georgoulis (2005), and McAteer, Gallagher, and Ireland (2005), among others.

All the Hα flare index data analyzed in this paper have been recorded at Kandilli solar observatory from 1976 to 2003. These data were obtained from the archives maintained at the National Geophysical Data Center (NGDC) through the website http://www.ngdc.noaa.gov/stp/SOLAR/ftpsolarflares.html. The data for cycle 21 covers the period from January 1976 to December 1985, cycle 22 data are from January 1986 to October 1996, and the data for cycle 23 range from January 1996 to December 2003. The daily flare index time series for the various cycles are depicted in Figure 1. In this figure, the top two plots are for the northern and southern hemisphere flare indices in cycle 21, the middle two plots are for similar data in cycle 22, and the lower two plots are for the northern and southern hemisphere flare indices during cycle 23. Instead of considering the total flare index in the two hemispheres, it is important to examine the flare index in each hemisphere separately in order to avoid the overlapping effects of the activity of both hemispheres in a full-disk consideration. We will find that the multifractal properties of the flare indices may vary

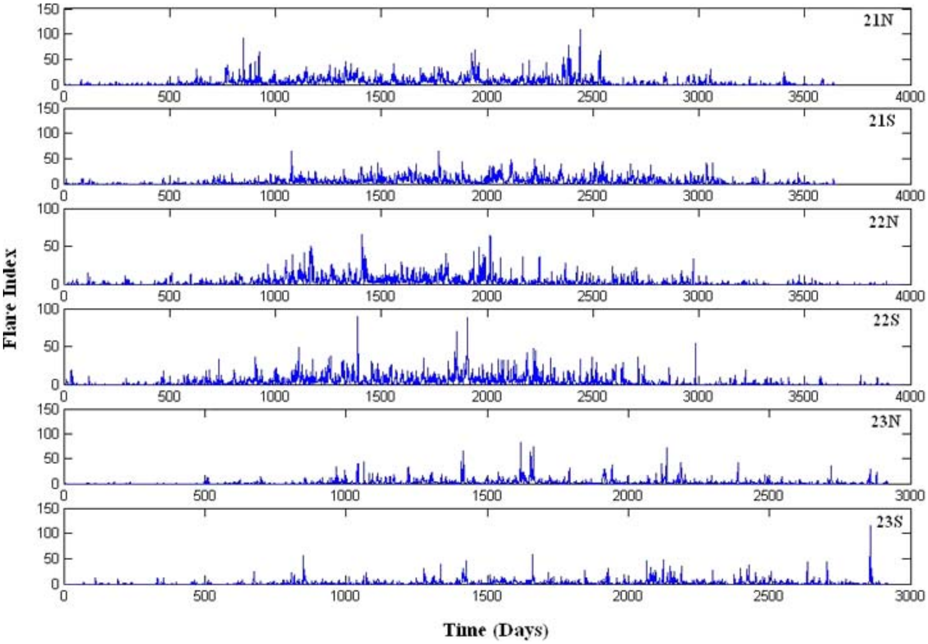

Figure 1 Time series plots of the daily northern and southern hemisphere flare indices. The top two panels (21N and 21S) are for cycle 21, the middle two panels (22N and 22S) are for cycle 22 and the two lower panels (23N and 23S) are for cycle 23. The letters N and S refer to the northern and southern hemispheres, respectively.

between the northern and southern hemispheres during the same cycle and also from one cycle to another in each hemisphere.

The remainder of this paper is organized as follows. In Section 2, we describe the method of multifractal analysis and apply it to the flare index time series. This is followed in Section 3 by a discussion of the results, and in Section 4, some concluding remarks are given.

2. Multifractal Analysis

There are several techniques available for calculating the singularity spectrum of a time series. We have used the method based on a box counting algorithm which was developed by Chhabra and Jensen (1989), and applied by Chhabra *et al.* (1989) and Meneveau and Sreenivasan (1991) to characterize the intermittent structure of turbulent flows. This is a *direct* method for computing the singularity spectrum without the need for calculating the generalized fractal dimension or applying the Legendre transform (Kravchenko, Boast, and Bullock, 1999). Since its inception, this method has been used by researchers in a wide variety of applications (see, for example, Balfas and Dewey, 1995; Pinzon *et al.*, 1995; Ramirez-Rejas *et al.*, 2004; Posadas *et al.*, 2005). It has been found to be a practical and efficient method for direct computation of the singularity spectrum of observed or experimental data.

In a multifractal process, if we divide the support of a measure (*e.g.*, a measured quantity) into boxes of size ℓ, then the probability of the measure in the i-th box is found to vary as:

$$p_i(\ell) \sim \ell^{\alpha_i}, \tag{1}$$

where the exponent α_i may be defined as a singularity strength characterizing the scaling in the i-th box. If we count the number of boxes $N(\alpha)$ where the probability p_i has singularity strength between α and $\alpha + d\alpha$, we can define a function $f(\alpha)$ such that

$$N(\alpha) \sim \ell^{-f(\alpha)}. \tag{2}$$

Consider a signal $x(t)$ given by the positive time series $\{x_n\}, n = 1, 2, 3, \dots, N$. Divide the time series into boxes or segments of length ℓ and introduce the probability measure:

$$p_i(\ell) = \sum_{j=1}^{N(\ell)} x_j \Big/ \sum_{n=1}^{N} x_n. \tag{3}$$

Here the subscript i refers to the i-th box, $x_j$is the magnitude of the signal at location j inside the i-th box, and $N(\ell)$ is the number of boxes of length ℓ. Based on the above measure we introduce a one-parameter family of normalized measures $\mu_i(q, \ell)$ according to the expression:

$$\mu_i(q, \ell) = \frac{[p_i(\ell)]^q}{\sum_k [p_k(\ell)]^q}. \tag{4}$$

The parameter q may be considered to be a microscope for exploring different regions of the singular measure. For values of $q > 1$, the more singular regions are enhanced; for $q < 1$ on the other hand, the less singular regions are accentuated, and for $q = 1$, the original measure is replicated (Chhabra and Jensen, 1989). Using the normalized measure given by (4), Chhabra *et al.* (1989) derived the following expressions for the Hölder exponent and Hausdorff dimension.

$$f(q) = \lim_{\ell \to 0} \frac{\sum_i \mu_i(q, \ell) \ln[\mu_i(q, \ell)]}{\ln \ell}, \tag{5}$$

$$\alpha(q) = \lim_{\ell \to 0} \frac{\sum_i \mu_i(q, \ell) \ln[p_i(\ell)]}{\ln \ell}. \tag{6}$$

(See Chhabra and Jensen, 1989; Chhabra *et al.*, 1989 for details.) These two equations provide a relationship between the Hölder exponent and Hausdorff dimension as implicit functions of the parameter q. By evaluating $f(q)$and $\alpha(q)$for different values of q, the singularity spectrum given by$f(\alpha)$can be easily derived. For a chosen value of q, values of $f(q)$and $\alpha(q)$are estimated from the slopes of the functions given in the numerators of (5) and (6) with respect to $\ln \ell$.

Using the direct method outlined above, we have computed the singularity spectrum of the flare index data in the northern and southern hemispheres for each of the three cycles 21, 22 and 23 as follows. Consider, for example, the flare index data in the northern hemisphere in cycle 21. We start with $q = -10$ and find $f(q)$ and $\alpha(q)$ from (3) and (4), respectively. Next the value of q is increased in steps of 0.1 to $q = 10$, and for each q, $f(q)$and $\alpha(q)$ are found using (5) and (6). The collection of the pairs of points $\alpha(q) - f(q)$ yields the singularity spectrum.

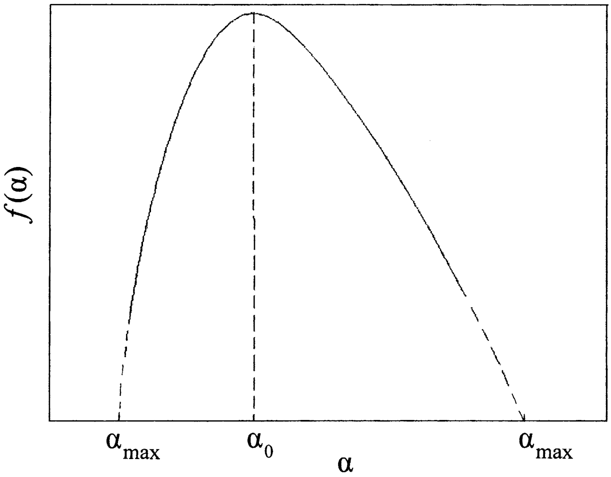

Figure 2 A typical singularity spectrum of a multifractal time series. The broadness of the singularity spectrum is given by $\Delta\alpha = \alpha_{max} - \alpha_{min}$, where α_{min} and α_{max} are the intersections of the (extrapolated) singularity spectrum on the α-axis.

3. Results and Discussion

To facilitate our discussion of the results, we begin by presenting a typical singularity spectrum of a multifractal time series. This is shown by the solid curve in Figure 2. The singularity spectrum has a characteristic unimodal (single-hump) appearance as seen in this figure. The broadness or width of the singularity spectrum may be assessed by the range of values of α where $f(\alpha) > 0$, by extrapolating the spectral curve at each end to meet the abscissa as shown by the dashed lines. The broadness is given by $\Delta\alpha = \alpha_{max} - \alpha_{min}$, where α_{min} and α_{max} are the smaller and larger values of the intersections of the extrapolated curve with the abscissa. The broadness, $\Delta\alpha$, represents the range of possible fractal exponents in the signal with $f(\alpha) > 0$, and thus gives a measure of the degree of multifractality or complexity of the time series. The broader the spectrum, the richer and more complex is the structure with a higher degree of multifractality. A small value of broadness, on the other hand, approaches a homogenous monofractal limit. As mentioned in the introduction, an asymmetry in the shape of the singularity spectrum can also provide information about the fractal properties of the time series. For instance, a left- or right-skewed spectrum implies a dominance of high or low fractal exponents, respectively, and indicates a relative abundance of large or and small fluctuations in the data (Kravchenko, Boast, and Bullock, 1999; Posadas *et al.*, 2005; Telesca, Lapenna, and Macchiato, 2005).

Figures 3, 4 and 5 depict the singularity spectra of the northern and southern hemisphere flare index time series in the cycles 21, 22 and 23, respectively. Table 1 lists the broadness and skewness parameters of the various singularity spectra. These results may be summarized as follows:

(a) As revealed by the broadness of the singularity spectra, the flare index data in cycle 21 have a higher degree of multifractality in the northern hemisphere than in the southern hemisphere (Figure 3). In cycle 22, the degrees of multifractality in the two hemispheres are nearly the same (Figure 4), but in cycle 23, the northern hemisphere flare index data possess a lower degree of multifractality in comparison to the southern hemisphere (Figure 5). In other words, between the two hemispheres, the northern hemisphere flare indices are richer and more complex in structure in cycle 21, less complex in cycle 23, and have almost the same order of complexity as the southern hemisphere in cycle 22.

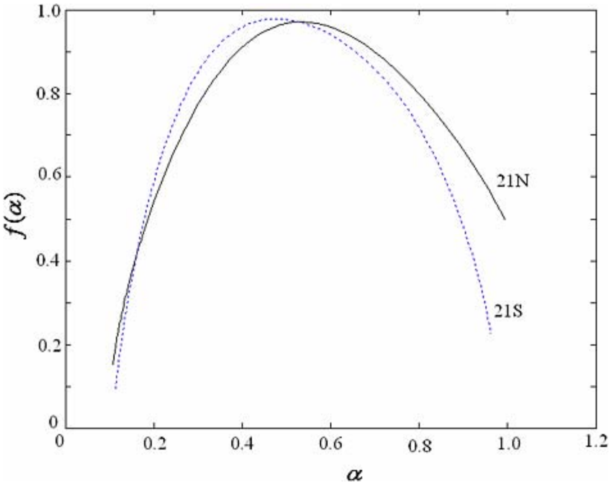

Figure 3 Singularity spectra of the northern and southern hemisphere flare index time series for cycle 21. A broader spectrum for the northern hemisphere indicates higher multifractal complexity in this hemisphere than that in the southern hemisphere. In Figures 3–7, the letters N and S denote, respectively, the northern and southern hemispheres.

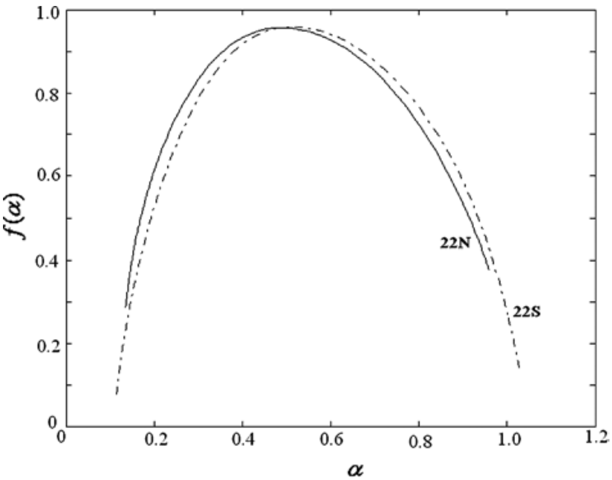

Figure 4 Singularity spectra of the northern and southern hemisphere flare index time series for cycle 22. Because the broadness of the singularity spectra is nearly the same in both hemispheres, there is no significant difference in multifractal complexity between the two hemispheres.

(b) Figures 3 and 4 also reveal that in each of the cycles 21 and 22, the singularity spectra are right-skewed in both hemispheres, indicating a dominance of low fractal exponents and reflecting a relative abundance of small fluctuations in the flare index data. By contrast, in cycle 23, the singularity spectrum for the northern hemisphere is left-skewed denoting a dominance of high fractal exponents, whereas that for the southern hemisphere is slightly right-skewed indicating a prevalence of low fractal exponents (Figure 5). Therefore, in this cycle there is a relative abundance of large fluctuations in the northern hemisphere and a prevalence of small fluctuations in the southern hemisphere.

Next we consider the flare index data in the northern hemisphere and southern hemisphere separately in the three cycles. Figure 6 presents the singularity spectra of the northern hemisphere flare indices for the cycles 21, 22 and 23. We observe that the data in cycle 21 possess the highest degree of multifractality whereas the degrees of multifractality of the data in cycles 22 and 23 are nearly equal (see Table 1). Finally, the singularity spectra of the southern hemisphere flare indices for the three cycles are displayed in Figure 7. In

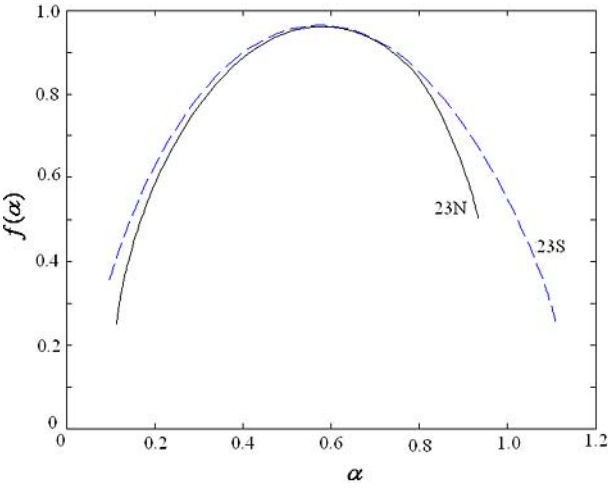

Figure 5 Singularity spectra of the northern and southern hemisphere flare index time series for cycle 23. A broader spectrum for the southern hemisphere indicates higher multifractal complexity in this hemisphere than in the northern hemisphere.

Table 1 Broadness and skewness parameters of the singularity spectrum for the northern (N) and southern (S) hemispheres flare index data in cycles 21, 22 and 23.

Cycle		Broadness ($\Delta\alpha$)	Skewness
Cycle 21	N	1.08	Right-skewed
	S	0.89	Right-skewed
Cycle 22	N	0.93	Right-skewed
	S	0.94	Right-skewed
Cycle 23	N	0.92	Left-skewed
	S	1.15	Slightly Right-skewed

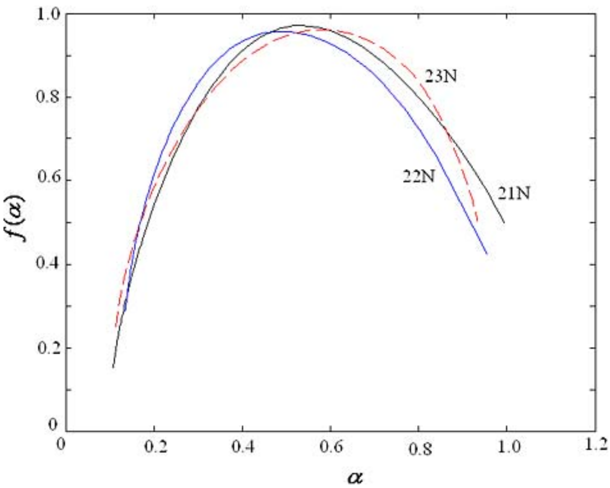

Figure 6 Singularity spectra of the northern hemisphere flare index time series for the cycles 21, 22 and 23. From the broadness of the three spectra it is apparent that the flare index data in cycle 21 are more complex than those in cycles 22 and 23, which have nearly the same order of complexity.

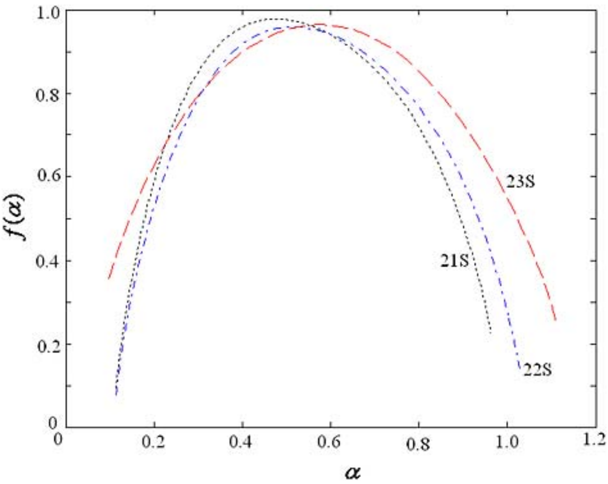

Figure 7 Singularity spectra of the southern hemisphere flare index time series for the cycles 21, 22 and 23. From the broadness of the three spectra it is evident that the flares in cycle 23 have the highest complexity, followed in order by those in cycles 22 and 21.

this hemisphere, the degree of multifractality is seen to be the lowest in cycle 21, higher in cycle 22 and even higher in cycle 23. Effects of asymmetry in the shape of the singularity spectra in Figures 6 and 7 can be interpreted in the manner discussed above.

4. Concluding Remarks

By calculating the singularity spectra of the daily flare index time series in terms of the Hölder exponent, we have presented a multifractal characterization of the complexity in Hα flare activity in the solar cycles 21, 22 and 23. The flare indices in the northern and southern hemispheres have been analyzed separately in order to avoid any overlapping effects of the two hemispheres in a full-disk consideration. The broadness of the spectrum is used for comparing the degree of multifractality or complexity of the intermittent solar flares between the two hemispheres in the same cycle and also among different cycles in the same hemisphere. An asymmetry to the left or right in the singularity spectrum reveals a dominance of high or low fractal exponents and reflects a prevalence of large or small fluctuations in the total energy emitted by the flares.

Our results reveal that in the even (22nd) cycle, the singularity spectra are very similar for the northern and southern hemispheres, whereas in the odd cycles (21st and 23rd) they differ significantly. In particular, we find that in cycle 21, the northern hemisphere flare index data have higher complexity than those for its southern counterpart, with an opposite pattern prevailing in cycle 23. Furthermore, small-scale fluctuations in the flare index time series are predominant in the northern hemisphere in the 21st cycle and are predominant in the southern hemisphere in the 23rd cycle. Based on these findings one might suggest that, from cycle to cycle, there exists a smooth switching between the northern and southern hemispheres in the multifractality of the flaring process. This new observational result may bring an insight into the mechanisms of the solar dynamo operation and may be useful for forecasting of the solar cycles.

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