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A New Extension of Bourguignon and Chakravarty Index to Measure Educational Poverty and Its Application to the OECD Countries

Juan-Francisco Sánchez-García[1] · María-del-Carmen Sánchez-Antón[2] · Rosa Badillo-Amador[3] · María-del-Carmen Marco-Gil[3] · Juan-Vicente LLinares-Ciscar[2] · Susana Álvarez-Díez[4]

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Abstract
The consequences that educational underperformance has on both individuals and society as a whole lead policy makers and planners to focus on how to measure it properly. The aim of this paper is to propose an index to measure educational poverty which, taking as a starting point the economic literature on multidimensional poverty measurement, turns out to be appropriate in the educational context. With this purpose, the following two features are demanded: (1) an individual should be identified as poor whenever they do not reach the basic level of knowledge in at least one of the relevant subjects; (2) the degree of poverty of individuals who present the same level of insufficiency in some subjects but have different scores in others should be different. Based on these premises, we introduce a multidimensional adjusted poverty index, called BC^a index, which is an extension of Bourguignon and Chakravarty index, and we apply it to measure educational poverty in the OECD countries by using data from PISA 2012 and 2015 reports.

Keywords Multidimensional adjusted poverty measurement · Educational poverty · PISA 2012 and 2015

JEL Classification I24 · I32 · D31 · D63

1 Introduction

The consequences that educational underperformance has on both individuals and society as a whole lead policy makers and planners to design politics geared towards reducing it and to focus on how to measure properly the extent of educational underperforming. Research literature related to poor educational performance interprets it as educational poverty and it argues that the indices commonly used in the analysis of inequality and poverty

✉ Juan-Vicente LLinares-Ciscar
llinares@um.es

Extended author information available on the last page of the article

provide useful information (Thomas et al. 2000; Denny 2002; Lohmann and Ferger 2014; Villar 2016; Minzyuk and Russo 2016).

Many studies have considered the poverty as a multidimensional phenomenon in order to define it from monetary and non-monetary deprivations following the axiomatic approach. Alkire et al. (2015) presented an excellent and comprehensive analysis of the research in this field. The contributions introduced by Bourguignon and Chakravarty (2003) and Permanyer (2014) are closely related to our proposal.

Bourguignon and Chakravarty (2003) proposed a multidimensional poverty index family specifying a poverty line for each dimension of poverty and using the union criterion to identify the poor, that is, an individual is poor if they are deprived of any attribute. Lasso de la Vega et al. (2009) characterized this poverty index family. Until Bourguignon and Chakravarty (2003) the way of dealing with the multidimensionality of poverty was to assume that the different attributes of an individual might be added in a single cardinal index of well-being and that poverty might be defined in terms of that index. This approach may be regarded as a single dimensional poverty index, with some generalizations of the definition of income.

Permanyer (2014) introduced a general and novel formulation to measure poverty, since it extends many of the indices existing in the literature but, at the same time, allows to cushion the poverty of the individuals with the values reached in the dimensions in which they do not show insufficiencies.

Based on the multidimensional framework proposed by Bourguignon and Chakravarty (2003), the main purpose of this paper is to extend the class of sub-group consistent poverty indices proposed by Foster et al. (1984) to the multidimensional context adding some kind of adjustment in the line of Permanyer (2014). In particular, we introduce a new multidimensional poverty index called *Adjusted Bourguignon Chakravarty Index,* BC^a index hereinafter, which extends Bourguignon and Chakravarty index and considers, at the same time: (1) deprivation in any attribute means educational poverty and (2) the degree of educational poverty can be cushioned by the achievements in the poor's non-deprived attributes. We also analyse a set of standard properties fulfilled by our proposal which have a natural interpretation in the educational context. Moreover, we apply the BC^a index to measure educational poverty by using data from PISA 2012 and 2015 reports, which provide the richest database for the analysis of the academic achievements of 15 year-old students in mathematics, reading and science.

The work is organised as follows. Section 2 provides the notation and basic definitions to introduce the new educational poverty index. Section 3 presents the BC^a index and describes the main properties it satisfies. Section 4 applies the BC^a index to measure educational poverty of the OECD countries by using data from PISA 2012 and 2015. Section 5 summarises the main conclusions. All the proofs are relegated to the "Appendices".

2 Notation and Definitions

We consider a population of $n \geq 2$ individuals, $N = \{1, 2, \dots, n\}$ and a set of k attributes, $J = \{1, 2, \dots, k\}$, where k is given and fixed, which are relevant to assess poverty. We assume that each attribute is representable by a continuous variable. For all $i \in N, j \in J$, let $x_{ij} \in \mathbb{R}_+$ denote the individual i's achievement of attribute $j \in J$. Let $\mathcal{M}$ denote the set of real $n \times k$ matrices. So, a multidimensional distribution among the population is represented by an $n \times k$ real matrix $\mathbb{X} \in \mathcal{M}$ where $\mathbb{X} = \left(x_{ij}\right)_{\substack{1 \leqq i \leqq n \\ 1 \leqq j \leqq k}}, x_{ij} \geqq 0\ \forall i, j$. The i-th row of $\mathbb{X}$, denoted by $x_i = \left(x_{ij}\right)_{1 \leqq j \leqq k}$, represents the individual i's achievement vector.

To identify the poor, we compare the individual i's achievement with a specific poverty line. For any $j \in J$, let $z_j > 0$ the threshold level of attribute j, that is, z_j is the minimal level considered acceptable for attribute j, the subsistence level. So, we denote by $z = \left(z_j\right)_{1 \leqq j \leqq k} \in \mathbb{R}^{\mathrm{k}}_{+}$ the vector of threshold for all the attributes, the poverty line. Whenever an individual i's achievement x_{ij} for an attribute j is below the corresponding threshold level, we say that this individual i is deprived in that attribute.

Although there are different ways to identify the poor, we follow Bourguignon and Chakravarty (2003) and consider that a person is poor if they are deprived in any attribute. Let $\rho : \mathbb{R}^{\mathrm{k}}_{+} \times \mathbb{R}^{\mathrm{k}}_{+} \to \{0, 1\}$ be the *poverty indicator variable function* (Chakravarty 2009) which is defined by setting

$$\rho\left(x_i, z\right) = \begin{cases} 1 & \textit{if } \exists j \in \{1, 2, \ldots, k\} : x_{ij} < z_j \\ 0 & \textit{otherwise} \end{cases} \tag{1}$$

Therefore, an individual $i \in N$ is poor if and only if $\rho\left(x_i, z\right) = 1$; the number of poor is given by $q = \sum_i^n \rho\left(x_i, z\right)$ and the incidence of the educational poverty by $H = q/n$, that is, the proportion of the poor in the population, named the *Headcount ratio*.

Once the poor have been identified, a multidimensional poverty index P has to be defined to aggregate the attributes into an overall indicator and measure the poverty of the society. A multidimensional poverty index P (Chakravarty 2009) is a non-constant real valued function $\mathrm{P} : \mathcal{M} \times \mathbb{R}^{\mathrm{k}}_{+} \to \mathbb{R}$ where $\mathrm{P}(\mathbb{X}, \mathrm{z})$ determines the poverty level associated with the achievement matrix $\mathbb{X}$ and the threshold vector z.

In the next section, we propose a new multidimensional poverty index based on the idea that the achievements in the non-deprived attributes of the poor can affect the degree of their poverty.

3 The Multidimensional Adjusted Poverty Index

Next, we introduce a new multidimensional poverty index, called *Adjusted Bourguignon Chakravarty index*, BC^a, which allows for *adjustments* by the attributes of the poor which do not fall below the corresponding threshold level, without changing the identification of an individual as poor.

Our proposal is to define an index in terms of deprivations as well as of non-deprivations of the poor. For any $(\mathbb{X}, z) \in \mathcal{M} \times \mathbb{R}^k_+$ we define the deprivation matrix[1] by setting,

$$G : \mathcal{M} \times \mathbb{R}^k_+ \to \mathcal{M}$$

$$G(\mathbb{X}, z) = \left(g_{ij}(\mathbb{X}, z)\right)_{\substack{1 \leqq i \leqq n \\ 1 \leqq j \leqq k}} \tag{2}$$

[1] Once we have constructed the deprivation matrix, $G(\mathbb{X}, \mathrm{z})$, the proportion of people who are poor and deprived in any set of attributes can be easily obtained (see "Appendix 1").

$$g_{ij}(\mathbb{X}, z) = max\left\{0, \frac{z_j - x_{ij}}{z_j}\right\} \tag{3}$$

and the *non-deprivation matrix* by considering,

$$R : \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k \to \mathcal{M}$$

$$R(\mathbb{X}, z, m) = \left(r_{ij}(\mathbb{X}, z, m)\right)_{\substack{1 \leqq i \leqq n \\ 1 \leqq j \leqq k}} \tag{4}$$

$$r_{ij}(\mathbb{X}, z, m) = max\left\{0, \frac{x_{ij} - z_j}{m_j - z_j}\right\} \tag{5}$$

where m_j is the maximum level that an individual could achieve in the j-th attribute and we denote by $m = \left(m_j\right)_{1 \leqq j \leqq k} \in \mathbb{R}_+^k$ the vector of maximum level for all the attributes, the top line.

Note that $g_{ij}(\mathbb{X}, z)$ are the usual normalized poverty gaps considered in the literature (Chakravarty 2009) for the individual's deprived attributes. Symmetrically, we introduce $r_{ij}(\mathbb{X}, z, m)$ as the surplus gaps for the individual's non-deprived attributes.

To present our index, we need to extend the definition of multidimensional poverty index by considering it as a non-constant real valued function $\mathrm{P} : \mathcal{M} \times \mathbb{R}_+^{\mathrm{k}} \times \mathbb{R}_+^{\mathrm{k}} \to \mathbb{R}$ where $P(\mathbb{X}, z, m)$ determines the poverty level associated with the achievement matrix $\mathbb{X}$, the threshold vector z and the top line m.

Next, we define an individual poverty index (adjusted individual poverty index) by making use of the p-norm ($p > 0$). Formally, let $\phi : \mathbb{R}^{n \times k} \to \mathbb{R}^n$ be a non-constant real function defined as follows,

$$\phi(\mathbb{X}) = (\phi_1(\mathbb{X}),\ \phi_2(\mathbb{X}), \dots, \phi_n(\mathbb{X}))$$

where, for each $1 \leqq i \leqq n$,

$$\phi_i(\mathbb{X}) = \left[\frac{1}{k}\left(\sum_{1 \leq j \leq k} x_{ij}^p\right)\right]^{1/p} \tag{6}$$

Then, given $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$, for each $1 \leqq i \leqq n$ and for each $\theta > 0$, we will consider the deprivation level, $\phi_i(G(\mathbb{X}, z))$, and the non-deprivation level, $\phi_i(R(\mathbb{X}, z, m))$, as follows,

$$\phi_i(G(\mathbb{X}, z)) = \left[\frac{1}{k}\left(\sum_{1 \leq j \leq k} g_{ij}^\theta(\mathbb{X}, z)\right)\right]^{1/\theta} \tag{7}$$

$$\phi_i(R(\mathbb{X}, z, m)) = \left[\frac{1}{k}\left(\sum_{1 \leq j \leq k} r_{ij}^\theta(\mathbb{X}, z, m)\right)\right]^{1/\theta} \tag{8}$$

Let us note that when individual i is deprived in no attribute $\phi_i(G(\mathbb{X}, z)) = 0$, whereas when individual i is totally deprived in all attributes $\phi_i(G(\mathbb{X}, z)) = 1$. Analogously, when individual i is deprived in all attributes $\phi_i(R(\mathbb{X},z,m)) = 0$, whereas when individual i gets the maximum achievement in all attributes $\phi_i(R(\mathbb{X},z,m)) = 1$.

The expression $\phi_i(G(\mathbb{X},z))$ is, in fact, the family of individual multidimensional indices proposed by Bourguignon and Chakravarty (2003) (by considering equal weights for all attributes). In order to define our index, we follow Dutta et al.'s approach (2003), and define the adjusted individual poverty index by aggregating information from deprivation and non-deprivation matrices in the line proposed by Permanyer (2014).

Definition 1 The *Adjusted Individual Poverty index, BC_i^a*, is a function,

$$BC_i^a : \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k \rightarrow [0,1]$$

$$BC_i^a(\mathbb{X},z,m) = \phi_i(G(\mathbb{X},z)) \times A_i(\mathbb{X},z,m) \tag{9}$$

where $A_i(\mathbb{X},z,m)$, called *Adjustment Factor*, is defined by

$$A_i(\mathbb{X},z,m) = \left(\frac{\phi_i(G(\mathbb{X},z))}{\phi_i(G(\mathbb{X},z)) + \phi_i(R(\mathbb{X},z,m))}\right). \tag{10}$$

Next proposition presents the main features of the *Adjustment Factor*. On the one hand, it has been constructed to satisfy the basic properties to be considered as such. That is, reduce the deprivation level of an individual whenever they are non-deprived in some attributes. On the other hand, it has some properties that we find to be appropriate in the educational context. First, $A_i(\mathbb{X},z,m)$ depends on both the individual deprivation and non-deprivation levels which are defined in a symmetric way (applying the same function to surplus and poverty gaps) looking for consistency. Second, $A_i(\mathbb{X},z,m)$ is sensitive to the deprivation levels of the individuals since its value depends increasingly on it as a whole. Third, when an individual has a greater deprivation level, an increase in the same is compensated by a greater increase in the non-deprivation level to maintain constant their poverty level.

Proposition 1 *Given $(\mathbb{X},z,m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$ the Adjustment Factor, $A_i(\mathbb{X},z,m)$, satisfies:*

i. *It is non-negative; if for any $i \in N$ and $j,t \in \{1,\dots,k\}$, $j \neq t$, $r_{ij}(\mathbb{X},z,m) > 0$ and $g_{it}(\mathbb{X},z) > 0$, then $A_i(\mathbb{X},z,m) > 0$; and $A_i(\mathbb{X},z,m) = 1$ when the individual is deprived in all the attributes.*
ii. *It is increasing with respect to $\phi_i(G(\mathbb{X},z))$ and decreasing with respect to $\phi_i(R(\mathbb{X},z,m))$;*
iii. *Deprivation and non-deprivation levels considered to construct it are defined in a parallel way;*
iv. *For each $i \in N$, the curves defined by the sets of pairs $\{(\phi_i(G(\mathbb{X},z)), \phi_i(R(\mathbb{X},z,m)) \in [0,1]x[0,1] \,/\, BC_i^a(\mathbb{X},z,m) = C\}$, called BC_i^a-isopoverty curves, are increasing and convex.*

Proof (See "Appendix 2").

Remark 1 Although the main idea of our extension is pretty close to Permanyer's approach, it has some characteristics that clearly differentiate it from the Permanyer's index. Next, we list them.

i. The Permanyer's index is not consistent in the sense that it measures the deprivation and non-deprivation levels by applying different functions to surplus and poverty gaps.
ii. The construction of Permanyer's index requires estimating $[k(k-1)+1]$ parameters more than our index (where k is the number of attributes).
iii. Permanyer's index is not sensitive to the deprivation levels of the individuals because it is not a function of $\phi_i(G(\mathbb{X},z))$.
iv. In general, the isopoverty curves defined from Permanyer's index are not convex.

The previous statements are proved in "Appendix 3".

Note that it is straightforward to prove that an individual $i \in N$ is poor ($\rho\left(x_i, z\right) = 1$) if and only if $BC_i^a(\mathbb{X}, z, m) > 0$.

Moreover, it is important to highlight that if an individual is poor, a high achievement in a non-deprived attribute cannot change their identification as poor but simply adjusts (decreases) their poverty degree. Therefore, two individuals with the same level in all deprived attributes could have different poverty degrees, depending on their achievements in non-deprived attributes, but both will be either poor or non-poor. That is, the identification of poor is independent of the non-deprived attribute achievements they possess. We call this property *Strong Focus Identification*.

Strong Focus Identification *(SFI)* For any $(\mathbb{X}, z, m), (\mathbb{Y}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$, $j \in \{1, \ldots, k\}$, $i \in \{1, \ldots, n\}$, if $x_{ij} \geq z_j$, $y_{ij} = x_{ij} + \delta$, where $\delta > 0, y_{tj} = x_{tj}$ for all $t \neq \mathrm{i}$ and $j \in \{1, \ldots, k\}$, then $P(\mathbb{X}, z, m) > 0 \leftrightarrow P(\mathbb{Y}, z, m) > 0$.

Finally, we apply the well-known Foster–Greer–Thorbecke's approach (1984) to define the *Adjusted Bourguignon Chakravarty index, BC^a*, from the *Adjusted Individual Poverty* indices, BC_{i}^a.

Definition 2 The *Adjusted Bourguignon Chakravarty index*, BC^a, is the real valued function,

$$BC^a : \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k \rightarrow [0, 1]$$

$$BC^a(\mathbb{X}, z, m) = \frac{1}{n} \sum_{i=1}^{n} \left(BC_i^a(\mathbb{X}, z, m)\right)^\alpha \tag{11}$$

where $\alpha > 0$. For $\alpha = 0$, $\left(BC_i^a(\mathbb{X}, z, m)\right)^0$ denotes the *poverty indicator variable function*.

Remark 2 The BC^a index provides different poverty measures depending on the value of the parameter α. In this regard, if $\alpha = 0$ the index indicates the incidence of the poverty since it coincides with the proportion of poor people, $H = {}^{q}/_{n}$. If $\alpha = 1$ the index stands for poverty per capita in the total population and can be expressed by $H \times I$, where $I = \frac{1}{q} \sum_{i=1}^{n} BC_i^a(\mathbb{X}, z, m)$ represents the poverty intensity among the poor. When $\alpha > 1$, a larger value of α involves giving more weight to the poorest of the poor, therefore inequality among the poor is taken into account when measuring the poverty of the population. The most common value of the parameter α in the literature about poverty is $\alpha = 2$. In this case, the index combines the three aspects that, according to Sen (1976), a poverty index should consider: incidence, intensity and inequality of poverty.

Next, a list of properties that the BC^a index satisfies is introduced. They are either standard in the literature or extensions we have defined in our context, which considers the possibility of some kind of adjustment among different attributes.

Weak Focus requires that the index is independent of the attribute levels of the non-poor people.

Weak Focus *(WF)* For any $n \in N$, $\mathbb{X}, \mathbb{Y} \in \mathcal{M}$, $z, m \in \mathbb{R}^k_+$, if for some $i, x_{ik} \geqq z_k \forall k$ and (1) for any $j \in \{1, 2, \dots, k\}$, $y_{ij} = x_{ij} + \delta$, where $\delta > 0$, (2) $y_{it} = x_{it} \forall t \neq j$ and (3) $y_{rs} = x_{rs} \forall r \neq i$ and all s, then $P(\mathbb{Y}, z, m) = P(\mathbb{X}, z, m)$.

Symmetry and *Normalization* are standard assumptions. In particular *Symmetry* demands anonymity. That is, it requires the index to be independent of characteristics of individuals other than the quantities of individual achievements.

Symmetry *(SYM)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}^k_+ \times \mathbb{R}^k_+$, $P(\mathbb{X}, z, m) = P(\Pi\mathbb{X}, z, m)$ where Π is any permutation matrix of appropriate order.

Normalization is the cardinalization of the index. It states that if all the individuals in society are non-poor, then the index value is 0.

Normalization *(NOM) For all* $z, m \in \mathbb{R}^k_+, \mathbb{X} \in \mathcal{M}$, *if* $x_{ij} \geqq z_j$ $\forall i \in \{1, 2, \dots, n\}$, and $\forall j \in \{1, 2, \dots, k\}$, then $P(\mathbb{X}, z, m) = 0$.

With respect to monotonicity we consider two different types (*Monotonicity* and *ND-Monotonicity*), depending either if the position of a poor person improves with respect to an attribute in which they are deprived or not. *Monotonicity* is the usual one in the literature and demands that if the position of person i who is poor and deprived with respect to attribute j improves in this attribute, then overall poverty should not increase. In the same line but from the point of view of non-deprived attributes, *ND-Monotonicity* demands that if the position of a person who is poor but not deprived with respect to attribute j improves in this attribute, then overall poverty should not increase.

Monotonicity *(MON)* For any $n \in N$, $\mathbb{X} \in \mathcal{M}$, $z, m \in \mathbb{R}^k_+$, $j \in \{1, 2, \dots, k\}$, if (1) for any i, $y_{ij} = x_{ij} + \delta$, where $x_{ij} < z_j$, $\delta > 0$, (2) $y_{tj} = x_{tj} \forall t \neq i$, and (3) $y_{is} = x_{is}$ for all i and for all $s \neq j$, then $P(\mathbb{Y}, z, m) \leq P(\mathbb{X}, z, m)$.

ND-Monotonicity *(NDMON)* For any $n \in N$, $\mathbb{X} \in \mathcal{M}$, $z, m \in \mathbb{R}^k_+$, $j \in \{1, 2, \dots, k\}$, if (1) for any i, $y_{ij} = x_{ij} + \delta$, where $x_{ij} > z_j$, $\delta > 0$, (2) $y_{tj} = x_{tj} \forall t \neq i$, and (3) $y_{is} = x_{is}$ for all i and for all $s \neq j$, then $P(\mathbb{Y}, z, m) \leq P(\mathbb{X}, z, m)$.

Continuity establishes that the poverty index varies continuously with the individual achievements. That is, small changes in the attributes, the threshold and the top line imply small changes in the value of the poverty index.

Continuity *(CONT)* For any $z, m \in \mathbb{R}^k_+, \mathbb{X} \in \mathcal{M}, P(\mathbb{X}, z, m)$ is continuous on $\mathcal{M}$.

Scale Invariance requires the poverty index to be invariant under scale transformations of the attributes, the threshold and the top line.

Scale Invariance *(SI)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}^k_+ \times \mathbb{R}^k_+$, $P(\mathbb{X}, z, m) = P\left(\mathbb{X}', z', \mathrm{m}'\right)$ where $\mathbb{X}' = \mathbb{X}\Lambda, z' = z\Lambda, m' = m\Lambda, \Lambda$ being the diagonal matrix $\mathrm{diag}\left(\lambda_1, \lambda_2, \dots, \lambda_k\right), \lambda_j > 0$ for all j.

Principle of Population allows us to compare poverty levels of societies with different population sizes. It requires that if an attribute matrix is replicated several times, then poverty remains unchanged. This property is particularly relevant to analyse inter-temporal and inter-regional poverty comparisons.

Principle of Population *(PP)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k, r \in \mathbb{N}$, finite, $P(\mathbb{X}^r, z, m) = P(\mathbb{X}, z, m)$ where $\mathbb{X}^r$ is the r-fold replication of $\mathbb{X}$.

Subgroup Decomposability establishes that if the population is split into subgroups, according to homogeneous characteristics, say age, gender, region, and so on, then the overall poverty is the population weighted average of the subgroup poverty levels. Therefore, this requirement allows us to calculate percentage contributions of different subgroups to total poverty and to identify the subgroups that are more afflicted by poverty.

Subgroup Decomposability *(SUD)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$ and any partition of the population into s subgroups, $s \geq 2$, $P(\mathbb{X}, z, m) = \sum_{l=1}^{s} \frac{n_l}{n} P\left(\mathbb{X}_l, z, m\right)$, where $\mathbb{X}_l, n_l$ denote the achievement data and the population size of subgroup l, respectively, for all $l = 1, 2, \ldots, s$ and $\sum_{l=1}^{s} n_l = n$.

The particularisation of this property to the extreme case where each group is a single individual is known as *Decomposability*.

The next result shows that the *Adjusted Bourguignon Chakravarty index, BC^a* satisfies all the previous properties.

Proposition 2 *The BC^a index satisfies SFI, WF, SYM, NOM, MON, NDMON, CONT, SI, PP and SUD.*

Proof (See "Appendix 4").

Finally, we consider the *Factor Decomposability* property. It demands a poverty index to be additive across attribute. Then, if a poverty index satisfies this property, the shares of different attributes to total poverty can be determined.

Factor Decomposability *(FD)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$, if $\alpha = \theta$, then $P(\mathbb{X}, z, m) = \sum_{j=1}^{k} b_j P\left(\mathbb{X}_j, z_j, m_j\right)$, where $\mathbb{X}_j$ denotes the j-th column of achievement matrix $\mathbb{X}$, $b_j \in \mathbb{R}^+$ such that $\sum_{j=1}^{k} b_j = 1$.

Proposition 3 *If $\alpha = \theta$, the BC^a index satisfies FD.*

Proof (See "Appendix 5").

Note that, when this property is exhibited by a poverty index in conjunction with *Subgroup Decomposability*, as it is for the BC^a index, the contributions of different subgroups to aggregate poverty with respect to different attributes can be calculated and the subgroup-attribute combinations that are more susceptible to poverty can be identified. This is very important in designing antipoverty policies when a society has limited resources to eliminate poverty for an entire subgroup or for a specific attribute.

4 An Application to Educational Poverty

The OECD Programme for International Student Assessment (PISA) tries to quantify if 15-year-old students, approaching the end of compulsory studying, are well prepared to meet the challenges of the future. PISA surveys take place every 3 years in the OECD countries and a group of partner countries, with together make up close to 90% of the world Economy. Through it, three subjects are evaluated: reading comprehension (reading), mathematics and problem solving (mathematics) and comprehension of scientific texts (science), focusing, in each wave, on one of them in a more exhaustive way. Next, we analyse the level of educational poverty in the OECD countries from the PISA data for 2012 (OECD 2014a) and 2015 (OECD 2016a, b).

In PISA 2012 and PISA 2015 around 510,000 and 514,000 students, respectively, completed the assessment, representing about 28 million and 29 million 15-year-old students in the schools of the participating countries and economies (65 in 2012 and 72 in 2015). The PISA 2012 was focused on mathematics, with reading and science as minor areas of assessment, whereas PISA 2015 was focused on science.

For each student different variables are stored, including ten plausible values in 2015 (five in 2012 and previous years), which represent the uncertainty about the right value to impute in each dimension (mathematics, reading, science) as long as the weights associated to each student (OECD 2014b) which are used to adjust for the probabilities of selection for individual schools and students, for school or student nonresponse, and for errors in estimating the size of the school or the number of 15-year-olds in the school at the time of sampling (Kelly et al. 2013).

Since PISA 2003, student scores are transformed to the PISA scale (0–1000), so that the mean is 500 and the standard deviation 100, and six levels of proficiency are defined. It is generally accepted that the level 2 is the minimum one to ensure that the student will be able to succeed in the labour market in the future. For this reason, we can identify students with scores linked to level 1, and below level 1, in any subject, to be poor in terms of education.

For PISA 2012 and 2015, the cut scores for level 2 for mathematics, reading and science literacy are 420.07, 407.47 and 409.54, respectively. Any score below these amounts will imply that the student is poor in the corresponding competency or dimension. Therefore, the poverty line is $z = (420.07, 407.47, 409.54)$ and, according to our definition of "poor student", any individual will be classified as "educationally poor" if they don't get these values in all the subjects.

In order to apply the BC^a index to determine the educational poverty level in the OECD countries, we present the choice of the parameters α and θ. To set the value of the parameter α, we consider, following Sen (1976), that a poverty index should combine the three essential aspects of poverty: incidence, intensity and inequality. Therefore, α should be strictly greater than 1. Among all the possibilities, we set the most common value, that is, $\alpha = 2$. So, according to Proposition 3, the value of $\theta = \alpha$ ensures that the BC^a index satisfies *Factor Decomposability*, an important property for the design of antipoverty policies, therefore $\theta = 2$. Moreover, this value of θ implies that inequality in scores of the different subjects also affects the level of individual poverty, characteristic that seems to be appropriate in the educational context. Finally, when $\theta = 2$, both the individual deprivation level and the individual non-deprivation level have a clear geometric interpretation, the Euclidean distance. All these reasons have lead us to choose $\alpha = \theta = 2$.

Tables 1 and 2 show the main results of the application of the BC^a index to the OECD countries with data from PISA 2012 and 2015, respectively. In both tables countries are listed in alphabetical order. Each table presents the value of the BC^a index together with the per capita poverty (intensity of poverty) and the *Headcount ratio* (proportion of poor people, or incidence of educational poverty), and their corresponding rankings (1st position corresponds to the lowest value). Although the per capita poverty and the *Headcount ratio* are the BC^a index for $\alpha = 1$ and $\alpha = 0$, respectively, as stated above, we do not think they gather global poverty accurately, but we include them because they provide simple and interesting information related to educational poverty. In both tables, the values with the shaded background correspond to those which are above the OECD average, for the BC^a index, the per capita poverty and the proportion of poor, and to those whose positions are above the median, for the rankings.

Table 1 BC^a index, per capita poverty and proportion of poor. PISA 2012. *Source*: Own elaboration with data from PISA 2012

	Per Capita Poverty		Global Poverty (BC^a)		Proportion of Poor
Country	**Ranking**	**Value**	**Ranking**	**Value**	**Value**
Australia	12	0.0194	14	0.0038	0.2240
Austria	15	0.0217	13	0.0038	0.2496
Belgium	20	0.0257	24	0.0058	0.2351
Canada	6	0.0128	5	0.0022	0.1638
Chile	33	0.0530	31	0.0091	0.5541
Czech Republic	13	0.0203	15	0.0038	0.2528
Denmark	11	0.0189	12	0.0035	0.2176
Estonia	1	0.0066	1	0.0008	0.1288
Finland	4	0.0116	4	0.0021	0.1505
France	25	0.0298	27	0.0068	0.2666
Germany	10	0.0180	10	0.0033	0.2085
Greece	30	0.0396	30	0.0084	0.3948
Hungary	23	0.0280	20	0.0051	0.3083
Iceland	26	0.0317	28	0.0071	0.2910
Ireland	7	0.0135	7	0.0023	0.1811
Israel	32	0.0499	34	0.0128	0.3749
Italy	24	0.0283	23	0.0056	0.2946
Japan	5	0.0116	6	0.0023	0.1310
Korea	2	0.0089	3	0.0017	0.1091
Luxembourg	27	0.0321	26	0.0066	0.3001
Mexico	34	0.0664	33	0.0128	0.6221
Netherlands	9	0.0164	9	0.0030	0.1866
New Zealand	17	0.0236	17	0.0047	0.2540
Norway	21	0.0259	25	0.0058	0.2671
Poland	3	0.0106	2	0.0016	0.1640
Portugal	22	0.0273	21	0.0052	0.2837
Slovak Republic	31	0.0454	32	0.0111	0.3525
Slovenia	14	0.0208	11	0.0035	0.2719
Spain	19	0.0243	18	0.0047	0.2761
Sweden	29	0.0352	29	0.0082	0.3158
Switzerland	8	0.0146	8	0.0025	0.1775
Turkey	28	0.0349	22	0.0054	0.4549
United Kingdom	18	0.0239	19	0.0051	0.2387
United States of America	16	0.0232	16	0.0040	0.2802
OECD		**0.0271**		**0.0051**	**0.2943**

Table 2 BC^a index, per capita poverty and proportion of poor. PISA 2015. *Source*: Own elaboration with data from PISA 2015

	Per Capita Poverty		Global Poverty (BC^a)		Proportion of Poor
Country	**Ranking**	**Value**	**Value**	**Value**	**Value**
Australia	18	0.0235	18	0.0042	0.2501
Austria	24	0.0286	24	0.0053	0.2819
Belgium	19	0.0253	19	0.0046	0.2490
Canada	2	0.0112	2	0.0017	0.1610
Chile	32	0.0540	30	0.0103	0.5167
Czech Republic	21	0.0265	20	0.0047	0.2711
Denmark	6	0.0149	6	0.0023	0.1986
Estonia	1	0.0085	1	0.0011	0.1403
Finland	5	0.0129	5	0.0022	0.1577
France	26	0.0318	27	0.0064	0.2792
Germany	11	0.0190	11	0.0033	0.2125
Greece	29	0.0455	29	0.0090	0.4056
Hungary	28	0.0372	28	0.0069	0.3365
Iceland	25	0.0299	25	0.0055	0.3183
Ireland	4	0.0123	3	0.0019	0.1804
Israel	31	0.0485	31	0.0110	0.3752
Italy	22	0.0279	21	0.0048	0.3033
Japan	3	0.0117	4	0.0019	0.1456
Korea	9	0.0168	9	0.0030	0.1901
Luxembourg	27	0.0338	26	0.0062	0.3193
Mexico	34	0.0675	34	0.0126	0.6177
Netherlands	13	0.0212	14	0.0037	0.2235
New Zealand	15	0.0221	15	0.0038	0.2514
Norway	10	0.0189	10	0.0032	0.2208
Poland	7	0.0160	8	0.0025	0.2142
Portugal	16	0.0223	13	0.0036	0.2674
Slovak Republic	30	0.0483	32	0.0113	0.3750
Slovenia	8	0.0163	7	0.0025	0.2097
Spain	12	0.0212	12	0.0034	0.2530
Sweden	20	0.0260	22	0.0050	0.2641
Switzerland	14	0.0219	17	0.0038	0.2370
Turkey	33	0.0641	33	0.0119	0.5700
United Kingdom	17	0.0226	16	0.0038	0.2661
United States of America	23	0.0283	23	0.0051	0.3084
OECD		**0.0316**		**0.0058**	**0.3213**

Considering the OECD countries as a whole, from 2012 to 2015 not only does the percentage of the poor increase by 9.19% (from 29.43 to 32.13%) but also the per capita and global poverty levels. The per capita poverty rises by 16.75% (from 0.0271 to 0.0316) and the global poverty level by 13.00% (from 0.0051 to 0.0058).[2] Therefore, in 2015 there are more students that do not reach the basic level of knowledge in at least one subject than in 2012. On average, their level of knowledge is lower and the global educational poverty increases. The simultaneous increase of these three indicators occurs for 15 of the considered countries. Among countries that stand out for their level of educational deterioration we find Korea, Switzerland, Poland and Turkey, whose percentage of poor increase, from 2012 to 2015, by 74.31%, 33.56%, 30.57% and 25.30%, respectively, their per capita poverty level by 88.69%, 50.10%, 49.98% and 83.72%, respectively, and their global poverty level by 77.63%, 51.14%, 63.90% and 122.73%, respectively.

Obviously, this trend is not general across all OECD member countries, 9 of them experience a decrease in all the three mentioned indicators, among which we can distinguish Slovenia, Norway, Sweden and Denmark. The proportion of poor of these countries decreases, in the analysed period, by 22.88%, 17.33%, 16.36% and 8.74%, respectively, their per capita poverty level by 21.53%, 27.24%, 26.05% and 20.99%, respectively, and their global poverty level by 26.62%, 44.92%. 38.56% and 34.29%, respectively.

It is important to note that not in all countries these three indicators vary in the same direction. Particularly, in Belgium, Iceland, Israel, Italy and United Kingdom the percentage of the poor increases from 2012 to 2015, by 5.92%, 9.38%, 0.09%, 2.94% and 11.48%, respectively, whereas the per capita poverty decreases, specifically by 1.55%, 5.58%, 2.86%, 1.52% and 5.32%, respectively, and the global poverty level also decreases by 20.23%, 21.64%, 14.41%, 15.55% and 24.49%. On the contrary, only in Chile there is a decrease in the percentage of the poor, by 6.76%, and an increase of both the per capita poverty by 1.93% and the global poverty by 12.46%.

For most OECD countries, the variation, from 2012 to 2015, of both the per capita and global poverty levels presents the same direction. The exceptions are France, Japan, Luxembourg and Mexico. In all of these countries the per capita poverty goes up by 6.62%, 0.11%, 5.31% and 1.63%, respectively, whereas the global poverty level goes down by 6.00%, 18.31%, 6.35% and 1.60% respectively. From this information we can conclude that, although the average level of knowledge of individuals increases, the inequality among the poor decreases in these countries.

Focusing on the relative position of the different OECD countries with respect to their level of educational poverty, the following two facts can be highlighted. Firstly, both the group of countries with the six greatest per capita and global educational poverty levels and the one with the six lowest ones coincide. In 2012 the first group consists of Mexico, Chile, Israel, Slovak Republic, Greece and Sweden, and in the second one Estonia, Korea, Poland, Finland, Japan and Canada are found. In 2015, Turkey joins the first group and Sweden leaves it while Ireland and Denmark enter the second group and Korea and Poland exit. Moreover, Mexico is both in 2012 and 2015 the poorest country globally and with the greatest per capita poverty level and Estonia the richest country globally and with the least per capita poverty level, even though both indicators increase significantly from 2012 to 2015 by 28.01% and 28.12%, respectively. Secondly, there are notable differences in the ranking of some countries between 2012 and 2015. Particularly, for regarding the per capita poverty level, it is worth mentioning the relative improvement of some countries in more than 5 positions: Norway (11 positions), Sweden (9 positions), Spain (7 positions) and Portugal and Slovenia (6 positions for each). As well as the relative worsening

[2] These percentages and the following ones may differ from the values calculated from Tables 1, 2 and 3 because, in terms of simplicity, only four decimal digits are presented.

of countries in more than 5 positions: Austria (9 positions), Czech Republic (8 positions), Korea and United States of America (7 positions for each) and Australia and Switzerland (6 positions for each). These variations are similar for the global poverty level with small changes. Specifically, Denmark joins the first group and Slovenia leaves it while Hungary enter the second group and Slovak Republic exits.

In spite of all the variations across OECD countries in the different educational variables considered, there are two group of countries that, for both 2012 and 2015, are kept either below or above the means of the percentage of poor and the per capita and global poverty levels, and the medians of the rankings. The first group (below the means and medians) includes 13 countries: Canada, Denmark, Estonia, Finland, Germany, Ireland, Japan, Korea, Netherlands, New Zealand, Poland, Slovenia and Switzerland. The following 7 countries belong to the second group (above the means and medians): Chile, Greece, Hungary, Israel, Mexico, Slovak Republic and Turkey.

Regarding the inequality between levels of educational poverty across OECD countries, measured by the Coefficient of Gini (GC) and the Coefficient of Variation (CV), different results are obtained if we consider the BC^a index, the per capita poverty or the percentage of poor in education. As it is shown in Table 3, the inequality measures show an increase in inequality in the OECD countries for both the global and the per capita poverty levels in 2015 compared to 2012. However, when we focus on the proportion of poor across OECD countries, the inequality decreases from 2012 to 2015. Particularly, if the per capita poverty is considered, the CV increases from 2012 to 2015 by 3.41% and the GC by 1.86%. But for the global poverty index, both inequality coefficients grow in a greater proportion (by 4.94% and 2.99%, respectively). On the contrary, when the proportion of educational poor is considered, the CV decreases in 2015 with respect to 2012 by 4.31% and the GC by 5.62%. This shows that, among OECD countries, inequality in the proportion of poor decreases from 2012 to 2015, whereas inequality in the per capita and global poverty levels increases in the same period.

It is important to point out that the analysis of educational poverty in the OECD countries carried out by Villar (2016) according to PISA 2012 data differs considerably from ours. On the one hand, the proportion of the poor in the population is much lower (from 6% in Estonia to 48% in Mexico, with an average of 18%) than our estimation (from 12.88% in Estonia to 62.21% in México, with an average of 29.43%). The reason of this disparity comes from the fact that, according to Villar's analysis, a large part of the students who have not reached the basic knowledge in some subject are not identified as poor since he bases the identification of the poor on the geometric mean of relative scores in all the subjects. On the other hand, the ranking of the OECD countries when applying the two methodologies are different for 13 countries if we consider the per capita poverty level, and for 24 when paying attention to the global poverty level. These differences do not exceed two

Table 3 Measures of inequality. *Source*: Own elaboration with data from PISA 2012 and 2015

	2012			2015		
	Per capita poverty	Global poverty (BC^a)	Proportion of poor	Per capita poverty	Global poverty (BC^a)	Proportion of poor
Coefficient of variation (CV)	0.5192	0.5911	0.4154	0.5369	0.6203	0.3975
Gini coefficient (GC)	0.2748	0.3143	0.2137	0.2798	0.3237	0.2017

positions in the first case but they reach up to four positions in the second one, as it is the case of Norway and Turkey.

5 Conclusion

Based on the multidimensional framework proposed by Bourguignon and Chakravarty (2003) and Permanyer (2014), we have proposed a new multidimensional adjusted poverty measure which extends the Bourguignon and Chakravarty index, called the BC^a index. Our proposal has some characteristics that are really suitable for its application to education and differentiate it from Permanyer's index. The introduction of the *Adjustment Factor* to define the BC^a index (see Definition 1), the characteristics that it has over against the Permanyer's one (see Proposition 1 and Remark 1) and the properties that the BC^a index satisfies (see Propositions 2 and 3) constitute our main theoretical contribution.

The application of the BC^a index to the estimation of the educational poverty across OECD member countries, using PISA 2012 and 2015 data, leads to set an accurate evaluation of school poverty that could be undoubtedly relevant for policy makers and planners. Next, the main results are summarised.

From a global perspective of educational poverty, that is, considering OECD member countries as a whole, there has been an educational deterioration that can be quantified by means of an increase in all the three indicators analysed: the proportion of poor and the per capita and global poverty levels. Therefore, not only the proportion of poor in the population has increased but also, in average, the per capita and global poverty levels. This simultaneous increase of these three indicators occurs for 15 of the OECD countries, being Korea the country that presents the greatest worsening in both the percentage of the poor and the per capita poverty level, and Turkey the country that reaches the greatest increase in the global poverty.

Across all OECD member countries, only 9 of them experience a simultaneous decrease in all the three mentioned indicators, that is, an educational improvement from the different perspectives considered. In this group, the greatest variation in the percentage of the poor correspond to Slovenia, whereas Norway is the country with the greatest improvement of the per capita and global poverty levels.

Focusing on the relative position of the OECD countries with respect to their level of educational poverty, for both the per capita and global poverty levels, as for the years 2012 and 2015, Mexico, Chile, Israel, Slovak Republic and Greece belong to the group of countries with the six greatest educational poverty levels, and Estonia, Finland, Japan and Canada are part of the group of countries with the six lowest educational poverty levels. Moreover, Mexico is both in 2012 and 2015 the poorest country by considering the per capita and global poverty levels, and Estonia the least poor one. In this regard, it should also be noted the most notable changes in the poverty level rankings of the OECD countries from 2012 to 2015. The greatest movement forwards corresponds to Norway, for both, the per capita and global poverty levels, and the largest one backwards is in Austria, when the per capita poverty is considered, and in Turkey, for the global poverty level.

Finally, regarding the inequality of educational poverty across OECD countries, we can conclude that the inequality in the proportion of the poor decreases from 2012 to 2015, whereas inequality in the per capita and global poverty levels increase in the same period.

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Appendix 1: Calculation of the Proportion of the People Who Are Poor and Deprived in Any Set of Attributes

Let $k \in N, J = \{1, \dots, k\}$ and $S \in P(J)\backslash\emptyset$, where $P(J)$ denotes the set of all the subsets of J. We define the *exclusive identification function*, $\mathrm{ID}^E : \mathbb{R}_+^k \times P(J)\backslash\emptyset \to \{0, 1\}$, such that for any $x \in \mathbb{R}_+^k$ and any $S \in P(J)\backslash\emptyset$,

$$ID^E(x, S) = \begin{cases} 1 & \text{if } x_j \neq 0\, \forall j \in S,\ x_t = 0\, \forall t \in J\backslash S \\ 0 & \text{otherwise} \end{cases}$$

Therefore, for any $(\mathbb{X}, z) \in \mathcal{M} \times \mathbb{R}_+^k$, $J = \{1, \dots, k\}$, $S \in P(J)\backslash\emptyset$, and by denoting the i-th row of matrix G $(\mathbb{X}, z)$ by $g_i(\mathbb{X}, z)$, then

$$\sum_{i=1}^{n} ID^E\left(g_i(\mathbb{X}, z), S\right)$$

provides the number of poor who are exclusively deprived in the set of attributes S.

Therefore, for any $(\mathbb{X}, z) \in \mathcal{M} \times \mathbb{R}_+^k$, $j \in J = \{1, \dots, k\}$,

$$\sum_{i=1}^{n} ID^E\left(g_i(\mathbb{X}, z), \{j\}\right)$$

provides the number of poor people who are exclusively deprived in attribute j. Moreover, $\sum_{i=1}^{n} ID^E\left(g_i(\mathbb{X}, z), J\right)$ provides the number of people deprived in all the attributes.

We denote, for all $x \in \mathbb{R}$, by

$$ID(\mathrm{x}) = \begin{cases} 1 & \text{if } x \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Moreover, for any $(\mathbb{X}, z) \in \mathcal{M} \times \mathbb{R}_+^k$, $j \in J = \{1, \dots, k\}$, $i \in N = \{1, \dots, n\}$ and $S \in P(J)\backslash\emptyset$,

$$\sum_{i=1}^{n} ID\left(\sum_{S \in P(J)\backslash\emptyset} ID^E\left(g_i(\mathbb{X}, z), S\right)\right)$$

provides the number of poor in the population;

$$\frac{\sum_{i=1}^{n} ID^E\left(g_i(\mathbb{X}, z), S\right)}{\sum_{i=1}^{n} ID\left(\sum_{S \in P(J)\backslash\emptyset} ID^E\left(g_i(\mathbb{X}, z), S\right)\right)}$$

is the proportion of poor who are exclusively deprived in all the attributes belonging to S;

$$\frac{\sum_{i=1}^{n} ID^E\left(g_i(\mathbb{X}, z), S\right)}{n}$$

is the proportion of the population that is exclusively deprived in all the attributes belonging to S.

Appendix 2: Proof of Proposition 1

Proposition 1 *Given* $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$ *the Adjustment Factor,* $A_i(\mathbb{X}, z, m)$*, satisfies:*

i. *It is non-negative; if for any* $\mathrm{i} \in \mathrm{N}$ *and* $j, t \in \{1, \dots, k\}$, $j \neq t, r_{ij}(\mathbb{X}, z, m) > 0$ *and* $g_{it}(\mathbb{X}, z) > 0$, *then* $A_i(\mathbb{X}, z, m) > 0$; *and* $A_i(\mathbb{X}, z, m) = 1$ *when the individual is deprived in all the attributes.*
ii. *It is increasing with respect to* $\phi_i(G(\mathbb{X}, z))$ *and decreasing with respect to* $\phi_i(R(\mathbb{X}, z, m))$.
iii. *Deprivation and non-deprivation levels considered to construct it are defined in a parallel way.*
iv. *For each* $i \in \mathrm{N}$, *the curves defined by the sets of pairs* $\left\{(\phi_i(G(\mathbb{X}, z)), \phi_i(R(\mathbb{X}, z, m)) \in [0,1] \times [0,1] \,/\, BC_i^a(\mathbb{X}, z, m) = C\right\}$, *called* BC_i^a*-isopoverty curves, are increasing and convex.*

□

Proof

i. It is obviously satisfied by definition of the *Adjustment Factor,* $A_i(\mathbb{X}, z, m)$.
ii. $A_i(\mathbb{X}, z, m)$ is increasing with respect to $\phi_i(G(\mathbb{X}, z))$
 since

$$\partial A_i(\mathbb{X}, z, m) / \partial \phi_i(G(\mathbb{X}, z)) = \frac{\phi_i(R(\mathbb{X}, z, m))}{\left[\phi_i(G(\mathbb{X}, z)) + \phi_i(R(\mathbb{X}, z, m))\right]^2} > 0.$$

 $A_i(\mathbb{X}, z, m)$ is decreasing with respect to $\phi_i(R(\mathbb{X}, z, m)))$ since

$$\partial A_i(\mathbb{X}, z, m) / \partial \phi_i(R(\mathbb{X}, z, m)) = \frac{-\phi_i(G(\mathbb{X}, z))}{\left[\phi_i(G(\mathbb{X}, z)) + \phi_i(R(\mathbb{X}, z, m))\right]^2} < 0.$$

iii. Deprivation and non-deprivation levels considered to construct the *Adjustment Factor* are defined in a parallel way since $A_i(\mathbb{X}, z, m) = \zeta(f_1(G(\mathbb{X}, z)), f_2(R(\mathbb{X}, z, m))$ where $f_1 = f_2 = \phi_i$.
iv. For each $i \in \mathrm{N}$, consider the BC_i^a-isopoverty curve $\left\{(\phi_i(G(\mathbb{X}, z)), \phi_i(R(\mathbb{X}, z, m)) \in [0,1] \times [0,1] \,/\, BC_i^a(\mathbb{X}, z, m) = C\right\}$.
 Since

$$BC_i^a(\mathbb{X}, z, m) = \frac{\left(\phi_i(G(\mathbb{X}, z))\right)^2}{\phi_i(G(\mathbb{X}, z)) + \phi_i(R(\mathbb{X}, z, m))} = C$$

 then

$$\phi_i(R(\mathbb{X}, z, m)) = \frac{\left(\phi_i(G(\mathbb{X}, z))\right)^2}{C} - \phi_i(G(\mathbb{X}, z)).$$

Then,

$$d\phi_i(R(\mathbb{X},z,m))/d\phi_i(G(\mathbb{X},z)) = \big(2\phi_i(G(\mathbb{X},z)) - C\big)/C > 0$$

since
$C = BC_i^a(\mathbb{X},z,m) = \phi_i(G(\mathbb{X},z)) \times A_i(\mathbb{X},z,m) \leq \phi_i(G(\mathbb{X},z)) < 2\phi_i(G(\mathbb{X},z))$, that is, the BC_i^a-isopoverty curve is increasing.

Moreover,

$$d^2\phi_i(R(\mathbb{X},z,m))/d\big(\phi_i(G(\mathbb{X},z))\big)^2 = 2/C > 0,$$

so, the BC_i^a-isopoverty curve is convex.

□

Appendix 3: Proof of Remark 1

i. The Permanyer's index is not consistent in the sense that it measures the deprivation and non-deprivation levels by applying different functions to surplus and poverty gaps.
ii. The construction of Permanyer's index requires estimating $[k(k-1)+1]$ parameters more than our index (where k is the number of attributes).
iii. Permanyer's index is not sensitive to the deprivation levels of the individuals because it is not a function of $\phi_i(G(\mathbb{X},z))$.
iv. In general, the isopoverty curves defined from Permanyer's index are not convex.

Proof

i. See the definition of excess gaps and poverty gaps.
ii. By definition, Permanyer's index (Permanyer 2014) requires estimating γ and the values of λ_{jl} for all $j, l \in \{1,\ldots,k\}$, such that $l \neq j$, that is, $[k(k-1)+1]$ parameters more than our proposal, where k is the number of attributes. In Permanyer's words, his proposal is over-parametrised.

In order to prove (iii) and (iv), we consider the following specification of the general Permanyer's formulation of a multidimensional poverty index (P^P).

$$P^P(\mathbb{X},z,m) = \frac{1}{n}\sum_{i=1}^{n}\psi\left(\left[\sum_{j=1}^{k}\left(g_{ij}(\mathbb{X},z)\prod_{l=1}^{k}\varphi_{jl}\big(r_{il}(\mathbb{X},z,m)\big)\right)^{\theta}\right]^{1/\theta}\right)$$

with $\varphi_{jl}\big(r_{il}((\mathbb{X},z,m)\big) = 1 + \big(\lambda_{jl}-1\big)r_{il}^{\gamma}((\mathbb{X},z,m))$, where $\psi(x) = \big(x/k^{1/\theta}\big)^{\alpha}$, $\lambda_{jl} = \lambda$ for all j, l, $\lambda \in (0,1]$ and $\gamma > 0$. Then,

$$P^P(\mathbb{X},z,m) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{1}{k}\left(\sum_{1\leq j\leq k} g_{ij}^{\theta}(\mathbb{X},z)\right)\right)^{1/\theta} \times \big(A_i^P(\mathbb{X},z,m)\big)\right]^{\alpha} = \frac{1}{n}\sum_{i=1}^{n}\big(P_i^P(\mathbb{X},z,m)\big)^{\alpha}$$

If $1-\lambda=\beta, \beta\in[0,1)$, the Permanyer's correction function for any agent i can be written as $A_i^P=\left(1-\beta r_{i1}^{\gamma}(\mathbb{X},z,m)\right)\left(1-\beta r_{i2}^{\gamma}(\mathbb{X},z,m)\right)\cdots\left(1-\beta r_{ik}^{\gamma}(\mathbb{X},z,m)\right)=\prod_{l=1}^{k}\left(1-\beta r_{il}^{\gamma}(\mathbb{X},z,m)\right)$. Thus, the individual poverty levels of any individual $i\in\{1,\dots,n\}$ according to this index is:

$$P_i^P((\mathbb{X},z,m))=\phi_i(G(\mathbb{X},z))\times A_i^P(\mathbb{X},z,m)=\phi_i(G(\mathbb{X},z))\prod_{l=1}^{k}\left(1-\beta r_{il}^{\gamma}(\mathbb{X},z,m)\right).$$

iii. The previous expression shows that Permanyer's correction function, $A_i^P(\mathbb{X},z,m)$ is not a function depending on $\phi_i(G(\mathbb{X},z))$. That is, it is not sensitive to the level of deprivation of the individual since $\partial A_i^P(\mathbb{X},z,m)/\partial\phi_i(G(\mathbb{X},z))=0$.

iv. Next counter example shows that the isopoverty curves defined from Permanyer's index, $\left\{(\phi_i(G(\mathbb{X},z)),\phi_i(R(\mathbb{X},z,m))\in[0,1]\times[0,1]\ /\ P_i^P(\mathbb{X},z,m)=C\right\}$, are not convex.

Suppose there are two attributes, $j=\{1,2\}, \theta=2, \gamma=1, \beta=1/2, g_i=(g_{i1},0)$ and $r_i=(0,r_{i2})$.

Then, $\phi_i(G(\mathbb{X},z))=g_{i1}/\sqrt{2}$, $A_i^P=1-(r_{i2}/2)$ and $P_i^P=\left(g_{i1}/\sqrt{2}\right)\left[1-(r_{i2}/2)\right]$.

If $g_i=(0.5,0)$ and $r_i=(0,0.71)$, we have that

$$P_i^P=\left(0.5/\sqrt{2}\right)\left[1-(0.71/2)\right]=0.3225/\sqrt{2}.$$

Now, we want to know the non-deprivation level, r_{i2}^*, which combined with the previous deprivation level increased by 0.05, $g_{i1}+0.05$, provides the same poverty level, $P_i^P=0.3225/\sqrt{2}$.

Then, solving the equation

$$0.3225/\sqrt{2}=\left[\left(0.5/\sqrt{2}\right)+0.05\right]\left[1-\left(r_{i2}^*/2\right)\right]$$

we obtain $r_{i2}^*=0.869830152$, so $\Phi_i^R\left[(r_{i1},r_{i2}^*)\right]=0.615062799$. Therefore, the pair $\left(\left(0.5/\sqrt{2}\right)+0.05,0.615062799\right)$ also belongs to the isopoverty curve of level $0.3225/\sqrt{2}$.

Analogously, we want to know the non-deprivation level which combined with the initial deprivation level increased by another 0.1, $g_{i1}+0.1$, provides the same isopoverty level, $P_i^P=0.3225/\sqrt{2}$.

Then, solving the equation

$$0.3225/\sqrt{2}=\left[\left(0.5/\sqrt{2}\right)+0.1\right]\left[1-\left(r_{i2}^{**}/2\right)\right]$$

we obtain $r_{i2}^{**}=0.994420759$, so $\Phi_i^R\left[(r_{i1},r_{i2}^*)\right]=0.703161662$. Therefore, the pair $\left(\left(0.5/\sqrt{2}\right)+0.1,0.703161662\right)$ also belongs to the isopoverty curve of level $0.3225/\sqrt{2}$.

Then, we have that when $\Phi_i^G=0.5/\sqrt{2}$ and it increases by 0.05, it has to be compensated by an increase of the non-deprived level that amounts to $\Phi_i^R\left[(r_{i1},r_{i2}^*)\right]-\Phi_i^R\left[(r_{i1},r_{i2})\right]=0.113016985$ to maintain the poverty level at

$0.3225/\sqrt{2}$. However, when $\Phi_i^G = \left(0.5/\sqrt{2}\right) + 0.05$ and it increases by 0.05, it has to be compensated by a smaller increase of the non-deprived level, which amounts to $\Phi_i^R\left[(r_{i1}, r_{i2}^{**})\right] - \Phi_i^R\left[(r_{i1}, r_{i2}^{*})\right] = 0.088098863$, to maintain the poverty level at $0.3225/\sqrt{2}$, which is completely *unnatural* in the educational context.

□

Appendix 4: Proof of Proposition 2

Proposition 2 *The BC^a index, $BC^a : \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k \to [0, 1]$, satisfies SFI, WF, SYM, NOM, MON, NDMON, CONT, SI, PP and SUD.*

Proof

- *(SFI)* Let us consider $(\mathbb{X}, z, m)$ and $(\mathbb{Y}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$, $j \in \{1, \dots, k\}$, $i \in \{1, \dots, n\}$ if $x_{ij} \geq z_j$, $y_{ij} = x_{ij} + \delta$, where $\delta > 0$, $y_{tj} = x_{tj}$ for all $t \neq i$ and $j \in \{1, \dots, k\}$, then

  $BC_i^a(\mathbb{X}, z, m) > 0 \leftrightarrow \phi_i(G(\mathbb{X}, z)) > 0 \leftrightarrow \phi_i(G(\mathbb{Y}, z)) > 0 \leftrightarrow BC_i^a(\mathbb{Y}, z, m) > 0.$
- *(WF)* If for some i, $x_{ik} \geqq z_k \forall k$, then $g_{ik} = 0\ \forall k$, which implies $\phi_i(G(\mathbb{X}, z)) = 0$, so $BC_i^a(\mathbb{X}, z, m) = BC_i^a(\mathbb{Y}, z, m) = 0$. Moreover, since $\forall r \neq i\ BC_{\mathrm{r}}^a(\mathbb{X}, z, m) = BC_{\mathrm{r}}^a(\mathbb{Y}, z, m)$, then $BC^a(\mathbb{Y}, z, m) = BC^a(\mathbb{X}, z, m)$.
- *(SYM)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$, if Π is any permutation of the rows, then

$$\Pi\mathbb{X} = (\mathrm{x}_{\sigma(i)j})_{\substack{i=1,2,..,n \\ j=1,2,\dots,k}}.$$

Since $g_{ij}(\mathbb{X}, z) = max\left\{0, \frac{z_j - x_{ij}}{z_j}\right\} = max\left\{0, \frac{z_j - x_{\sigma(i)j}}{z_j}\right\} = g_{\sigma(i)j}(\Pi\mathbb{X}, z)$,
it follows that

$$\phi_i(G(\mathbb{X}, z)) = \left[\frac{1}{k}\left(\sum_{1 \leq j \leq k} g_{ij}^{\theta}(\mathbb{X}, z)\right)\right]^{\frac{1}{\theta}} = \left[\frac{1}{k}\left(\sum_{1 \leq j \leq k} g_{\sigma(i)j}^{\theta}(\Pi\mathbb{X}, z)\right)\right]^{\frac{1}{\theta}} = \phi_{\sigma(i)}(G(\Pi\mathbb{X}, z))$$

and

$$\phi_i(R(\mathbb{X}, z, m)) = \left[\frac{1}{k}\left(\sum_{1 \leq j \leq k} r_{ij}^{\theta}(\mathbb{X}, z, m)\right)\right]^{\frac{1}{\theta}} = \phi_{\sigma(i)}(R(\Pi\mathbb{X}, z, m))$$

then,

$$BC_i^a(\mathbb{X}, z, m) = \phi_i(G(\mathbb{X}, z))A_i(\mathbb{X}, z, m) = \phi_{\sigma(i)}(G(\Pi\mathbb{X}, z))A_{\sigma(i)}(\Pi\mathbb{X}, z, m) = BC_{\sigma(i)}^a(\Pi\mathbb{X}, z, m)$$

and $BC^a(\mathbb{X}, z, m) = \frac{1}{n}\sum_{i=1}^{n}\left(BC_i^a(\mathbb{X}, z, m)\right)^{\alpha} = \frac{1}{n}\sum_{i=1}^{n}\left(BC_{\sigma(i)}^a(\Pi\mathbb{X}, z)\right)^{\alpha} = BC^a(\Pi\mathbb{X}, z)$

- *(NOM) For all* $z \in \mathbb{R}_+^k, \mathbb{X} \in \mathcal{M}$, *if* $x_{ij} \geqq z_j \forall i \in \{1, 2, \ldots, n\}$, and $\forall j \in \{1, 2, \ldots, k\}$, then $\phi_i(G(\mathbb{X}, z)) = 0$, so $BC_i^a(\mathbb{X}, z, m) = 0$, therefore $BC^a(\mathbb{X}, z, m) = 0$.
- *(MON)* Since $y_{ij} = x_{ij} + \delta$,

  (a) if $y_{ij} < z_j$, then $g_{ij}(\mathbb{Y}, \mathrm{z}) = g_{ij}(\mathbb{X}, \mathrm{z}) - \frac{\delta}{z_j}$ and $r_{ij}(\mathbb{Y}, \mathrm{z}, \mathrm{m}) = 0$, therefore $g_{ij}(\mathbb{Y}, \mathrm{z}) < g_{ij}(\mathbb{X}, \mathrm{z})$ and $\phi_i(G(\mathbb{Y}, z)) < \phi_i(G(\mathbb{X}, z))$. Moreover $\phi_i(R(\mathbb{Y}, z, m)) = \phi_i(R(\mathbb{X}, z, m))$. By applying (ii) of Proposition 1, $A_i(\mathbb{Y}, z, m) < A_i(\mathbb{X}, z, m)$, so $BC_{\mathrm{i}}^a(\mathbb{Y}, z, m) < BC_{\mathrm{i}}^a(\mathbb{X}, z, m)$, then $BC^a(\mathbb{Y}, z, m) < BC^a(\mathbb{X}, z, m)$.

  (b) If $y_{ij} > z_j, g_{ij}(\mathbb{Y}, \mathrm{z}) = 0, g_{ij}(\mathbb{X}, \mathrm{z}) > 0$ and $r_{ij}(\mathbb{Y}, \mathrm{z}, \mathrm{m}) > 0$, then $\phi_i(G(\mathbb{Y}, z)) < \phi_i(G(\mathbb{X}, z))$ and $\phi_i(R(\mathbb{Y}, z, m)) > \phi_i(R(\mathbb{X}, z, m))$. Analogously by applying (ii) in Proposition 1, $A_i(\mathbb{Y}, z, m) < A_i(\mathbb{X}, z, m)$, so $BC_{\mathrm{i}}^a(\mathbb{Y}, \mathrm{z}, \mathrm{m}) < BC_{\mathrm{i}}^a(\mathbb{X}, \mathrm{z}, \mathrm{m})$, then $BC^a(\mathbb{Y}, z, m) < BC^a(\mathbb{X}, z, m)$.
- *(NDMON)* Since $x_{ij} > z_j$, then $y_{ij} = x_{ij} + \delta > x_{ij} > z_j$ implies that $g_{ij}(\mathbb{Y}, \mathrm{z}) = g_{ij}(\mathbb{X}, \mathrm{z}) = 0$, therefore $\phi_i(G(\mathbb{Y}, z)) = \phi_i(G(\mathbb{X}, z))$. Moreover,

$$r_{ij}(\mathbb{Y}, z, m) = \frac{x_{ij} + \delta - z_j}{m_j - z_j} > \frac{x_{ij} - z_j}{m_j - z_j} = r_{ij}(\mathbb{X}, z, m),$$

  then

$$\phi_i(R(\mathbb{Y}, z, m)) > \phi_i(R(\mathbb{X}, z, m)).$$

  So, by applying Proposition 1, $A_i(\mathbb{Y}, z, m) < A_i(\mathbb{X}, z, m)$, and

  $BC_i^a(\mathbb{Y}, z, m) \leq BC_i^a(\mathbb{X}, z, m) = \phi_i(G(\mathbb{X}, z)) \times A_i(\mathbb{X}, z, m)$. *Therefore* $BC^a(\mathbb{Y}, z, m) \leq BC^a(\mathbb{X}, z, m)$.
- $(CONT)$ $BC^a(\mathbb{X}, z, m)$ is a composition of continuous functions.
- *(SI)* Since $x'_{ij} = \lambda_j x_{ij}$, $z'_j = \lambda_j z_j, m'_j = \lambda_j m_j$, it is obtained that

$$g_{ij}(\mathbb{X}', z') = max\left\{0, \frac{z'_j - x'_{ij}}{z'_j}\right\} = max\left\{0, \frac{\lambda_j z_j - \lambda_j x_{ij}}{\lambda_j z_j}\right\} = g_{ij}(\mathbb{X}, \mathrm{z})$$

  and

$$r_{ij}(\mathbb{X}', z', m') = max\left\{0, \frac{x'_{ij} - z'_j}{m'_j - z'_j}\right\} = r_{ij}(\mathbb{X}, z, m)$$

  Then,
  $\phi_i(G(\mathbb{X}', z')) = \phi_i(G(\mathbb{X}, z))$ and $\phi_i(R(\mathbb{X}', z', m')) = \phi_i(R(\mathbb{X}, z, m))$, therefore

$$BC^a(\mathbb{X}, z, m) = BC^a(\mathbb{X}', z', m').$$

- *(PP)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^k \times \mathbb{R}_+^k$, $r \in \mathbb{N}$, finite, if $\mathbb{X}^r$ is the r-fold of $\mathbb{X}$,

$$BC^a(\mathbb{X}^r, z, m) = \frac{1}{nr} \sum_{i=1}^{nr} \left(BC_i^a(\mathbb{X}^r, z, m)\right)^\alpha = \frac{1}{nr} \sum_{i=1}^{n} \sum_{h=1}^{r} \left(BC_{i_h}^a(\mathbb{X}^r, z, m)\right)^\alpha$$
$$= \frac{1}{nr} \sum_{i=1}^{n} r\left(BC_i^a(\mathbb{X}^r, z, m)\right)^\alpha = BC^a(\mathbb{X}, z, m)$$

- *(SUD)* For any $(\mathbb{X}, z, m) \in \mathcal{M} \times \mathbb{R}_+^{\mathrm{k}} \times \mathbb{R}_+^{\mathrm{k}}$ and any partition of the population into s subgroups, $s \geq 2$:

$$BC^a(\mathbb{X},z,m)=\frac{1}{n}\sum_{i=1}^{n}\left(BC_i^a(\mathbb{X},z,m)\right)^{\alpha}=\frac{n_l}{n}\sum_{l=1}^{S}\frac{1}{n_l}\sum_{i=1}^{n_l}\left(BC_i^a(\mathbb{X},z,m)\right)^{\alpha}=\frac{n_l}{n}\sum_{l=1}^{S}BC^a\left(\mathbb{X}_l,z,m\right)$$

□

Appendix 5: Proof of Proposition 3

Proposition 3 The BC^a index, $BC^a:\mathcal{M}\times\mathbb{R}_+^k\times\mathbb{R}_+^k\rightarrow[0,1]$, satisfies FD if $\alpha=\theta$.

Proof For any $(\mathbb{X},z,m)\in\mathcal{M}\times\mathbb{R}_+^k\times\mathbb{R}_+^k$, let $x_j=\left(x_{ij}\right)_{1\leqq i\leqq n}$ denote the j-th column of $\mathbb{X}$, which represents the achievement vector in attribute j of all the individuals $i=1,..,n$. Then, the poverty level due to attribute or dimension j, according to our poverty index BC^a, denoted by BC_{dj}^a, is defined for $\alpha=\theta$, as:

$$BC_{dj}^a(\mathbb{X},z,m)=\frac{1}{n}\sum_{i=1}^{n}\left[\left(\left(g_{ij}(\mathbb{X},z)\right)^{\theta}\right)^{\frac{1}{\theta}}A_i(\mathbb{X},z,m)\right]^{\theta}.$$

Then,

$$\begin{aligned}
BC^a(\mathbb{X},z,m)&=\frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{1}{k}\sum_{1\leq j\leq k}g_{ij}^{\theta}(\mathbb{X},z)\right)^{\frac{1}{\theta}}A_i(\mathbb{X},z,m)\right]^{\theta}\\
&=\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}g_{ij}^{\theta}(\mathbb{X},z)A_i(\mathbb{X},z,m)^{\theta}\\
&=\frac{1}{k}\sum_{j=1}^{k}\frac{1}{n}\sum_{i=1}^{n}g_{ij}^{\theta}(\mathbb{X},z)A_i(\mathbb{X},z,m)^{\theta}\\
&=\frac{1}{k}\sum_{j=1}^{k}\frac{1}{n}\sum_{i=1}^{n}\left[\left(\left(g_{ij}(\mathbb{X},z)\right)^{\theta}\right)^{\frac{1}{\theta}}A_i(\mathbb{X},z,m)\right]^{\theta}\\
&=\frac{1}{k}\sum_{j=1}^{k}BC_{dj}^a(\mathbb{X},z,m).
\end{aligned}$$

□

References

Alkire, S., Foster, J., Seth, S., Santos, M. E., Roche, J. M., & Ballón, P. (2015). *Multidimensional poverty measurement and analysis*. Oxford: Oxford University Press.

Bourguignon, F., & Chakravarty, S. R. (2003). The measurement of multidimensional poverty. *Journal of Economic Inequality, 1,* 25–49.

Chakravarty, S. R. (2009). *Inequality, polarization and poverty*. Berlin: Springer.

Denny, K. (2002). New methods for comparing literacy across populations: insights from the measurement of poverty. *Jouranl of the Royal Statistical Society A, 165*(Part 3), 481–493.

Dutta, I., Pattanaik, P. K., & Xu, Y. (2003). On measuring deprivation and the standard of living in a multidimensional framework on the basis of aggregate data. *Economica, 70,* 197–221.
Foster, J. E., Greer, J., & Thorbecke, E. (1984). A class of decomposable poverty measures. *Econometrica, 52,* 761–766.
Kelly, D., Nord, C. W., Jenkins, F., Chan, J. Y., & Kastberg, D. (2013). *Performance of US 15-year-old students in mathematics, science, and reading literacy in an international context. first look at PISA 2012*. NCES 2014-024. National Center for Education Statistics.
Lasso de la Vega, C., Urrutia, A., & Diez, H. (2009). *The Bourguignon and Chakravarty multidimensional poverty family: A characterization*. Working Papers No 109, ECINEQ, Society of the Study of Economic Inequality (109).
Lohmann, H., & Ferger, F. (2014). *Educational poverty in a comparative perspective: Theoretical and empirical implications*. SFB 882 Working Paper Series, no 26, DFG Research Center (SFB) from Heterogeneities to Inequalities. http://www.sfb882.uni-bilefeld.de/. Accessed 21 Dec 2018.
Minzyuk, L., & Russo, F. (2016). La misurazione multidimensionale della povertà in istruzione in Italia multidimensional measurement of educational poverty in Italy. *Politica economica, 1,* 65–122.
OECD. (2014a). *PISA 2012 results: What students know and can do—Student performance in mathematics, reading and science* (Revised ed., Vol. I). Paris: PISA, OECD Publishing. https://doi.org/10.1787/9789264208780-en.
OECD. (2014b). *PISA 2012 technical report*. Paris: OECD Publishing.
OECD. (2016a). *PISA 2015 results: Excellence and equity in education*. Paris: PISA, OECD Publishing. https://doi.org/10.1787/9789264266490-en.
OECD. (2016b). *PISA 2015 results in focus*. https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf. Accessed 21 Dec 2018.
Permanyer, I. (2014). Assessing individuals' deprivation in a multidimensional framework. *Journal of Development Economics, 109,* 1–16.
Sen, A. K. (1976). Poverty: an ordinal approach to measurement. *Econometrica, 44,* 219–231.
Thomas, V., Wang, Y., & Fan, X. (2000) *Measuring educational inequality: Gini coefficients of education*. Working Paper 2525. World Bank, Washington DC: World Bank.
Villar, A. (2016). Educational poverty as a welfare loss: Low performance in the OECD according to PISA 2012. *Modern Economy, 7,* 441–449. https://doi.org/10.4236/me.2016.74049.

Affiliations

Juan-Francisco Sánchez-García[1] · María-del-Carmen Sánchez-Antón[2] · Rosa Badillo-Amador[3] · María-del-Carmen Marco-Gil[3] · Juan-Vicente LLinares-Ciscar[2] · Susana Álvarez-Díez[4]

Juan-Francisco Sánchez-García
jf.sanchez@upct.es

María-del-Carmen Sánchez-Antón
csanchez@um.es

Rosa Badillo-Amador
rosa.badillo@upct.es

María-del-Carmen Marco-Gil
carmen.marco@upct.es

Susana Álvarez-Díez
salvarez@um.es

1 Department of Quantitative Methods, Legal Sciences and Modern Languages, Technical University of Cartagena, Cartagena, Murcia, Spain

2 Department of Economic Analysis, University of Murcia, Murcia, Spain

[3] Department of Economics, Accounting and Finance, Technical University of Cartagena, Cartagena, Murcia, Spain

[4] Department of Quantitative Methods for Economics and Business, University of Murcia, Murcia, Spain