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First Order Dominance Techniques and Multidimensional Poverty Indices: An Empirical Comparison of Different Approaches

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Abstract In this empirically driven paper we compare the performance of two techniques in the literature of poverty measurement with ordinal data: multidimensional poverty indices and first order dominance techniques (FOD). Combining multiple scenario simulated data with observed data from 48 Demographic and Health Surveys around the developing world, our empirical findings suggest that the FOD approach can be implemented as a useful robustness check for ordinal poverty indices like the multidimensional poverty index (MPI; the United Nations Development Program's flagship poverty indicator) to distinguish between those country comparisons that are sensitive to alternative specifications of basic measurement assumptions and those which are not. To the extent that the FOD approach is able to uncover the socio-economic gradient that exists between countries, it can be proposed as a viable complement to the MPI with the advantage of not having to rely on many of the normatively binding assumptions that underpin the construction of the index.

Keywords Multidimensional poverty measurement · Poverty index · First order dominance · Robustness · Simulations

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1 Introduction

For a long time, poverty has been analyzed on the basis of income distributions alone (see Chakravarty 2009 for a review of the income poverty measures employed in the economics literature). Nevertheless, it is nowadays widely acknowledged that both monetary *and* non-monetary attributes are essential to conceptualize and measure individuals' welfare levels (see, for instance, Bourguignon and Chakravarty 2003: 26). This paper aims to empirically compare two different approaches to the measurement of multidimensional poverty in the context of ordinal data: poverty indices and the so-called 'First Order Dominance' (henceforth FOD) techniques. Multidimensional poverty indices attempt to generalize well-known income poverty measures to a multiple attribute framework by taking into consideration the joint distribution of several variables (e.g. basic amenities, education and health—see examples in Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty et al. (2008), Alkire and Foster (2011) and Fattore (2016)). The proposal by Alkire and Foster is perhaps the most popular one since the United Nations Development Program (UNDP) adopted it as the multidimensional poverty index (MPI) in 2010. While measures of this kind can potentially give very precise assessments of existing poverty levels, their construction is based on a wide range of debatable assumptions. To illustrate: important decisions have to be made regarding the choice of the functional form of the index, the weights that are applied to each dimension,[1] the ways in which the different indicators are chosen and normalized or the extent to which deprivations are going to be traded-off between dimensions (i.e.: complementarity and substitutability issues across dimensions, which ultimately depend on the underlying social welfare function), each of which having crucial ethical implications.

Given these concerns, another strand of the literature has attempted to develop robust methods for making multidimensional poverty comparisons. Essentially, such methods aim at reaching conclusions that remain valid for a wide range of assumptions concerning the ways in which poverty measures are conceptualized and constructed [see, for instance, Duclos et al. (2006) and Bourguignon and Chakravarty (2008) in the context of cardinal variables, and Duclos et al. (2007) and Yalonetzky (2014) for ordinal variables]. Yet, it turns out that the aforementioned studies still rely—one way or another—on certain assumptions regarding the signs of the second or higher order cross-derivatives of the underlying social welfare function determining, among other things, the complementarity or substitutability between attributes. In parallel, other recent contributions have proposed the FOD techniques to perform simple welfare comparisons in multivariate settings (see Arndt et al. 2012, and Hussain et al. 2015). Basically one says that 'population group *A* FOD dominates population group *B*' if the distribution of *B*'s individuals on all possible outcomes can be obtained from *A*'s by moving population shares from better to worse outcomes in *A*'s distribution. According to Arndt et al. (2012: 2291), such techniques do not make any kind of assumptions regarding "the relative desirability of changes between levels within or between dimensions or the complementarity/substitutability between dimensions", so they might a priori offer an interesting alternative to currently existing multivariate dominance methods in poverty analysis—an issue we want to investigate here.

The main aim of this article is to investigate the empirical suitability and tractability of the FOD techniques following a two-pronged strategy. Firstly, we generate several simulated datasets with which we can study the performance of the FOD approach in multiple

[1] As shown in Cherchye et al. (2008), Permanyer (2011, 2012) and Foster et al. (2013), certain composite indices of well-being can be highly sensitive to the choice of alternative weighting schemes.

settings and compare it with multidimensional poverty indices. Introducing a large diversity of scenarios varying the number of variables, their corresponding distributions within and across countries and the correlation structures between them, we aim at reasonably approximating many 'real-world' situations that could be encountered in practice. Secondly, we complement the previous approach using real data from 48 Demographic and Health Surveys across the developing world. Since the use of FOD techniques has been quite sparse (very often working with a single or a quite reduced number of countries—e.g. Arndt et al. 2012; Hussain et al. 2015) it is entirely unknown whether or not they provide a coherent and consistent picture of multidimensional poverty rankings at the international level when compared to well-known indices like UNDP's MPI.

To the extent that current international cooperation, development and aid programs are guided by the rankings derived from these measures, the issues analyzed in this paper are not a mere academic curiosity but have practical and financial implications for the design of effective poverty eradication strategies. The implications of having one level of association or another between alternative methodologies can be completely different. If the alternative methodologies turn out to be very highly correlated we can safely conclude that our assessments of multidimensional poverty are not highly distorted when using one approach or the other. If this were the case, it would suggest that the ordinal information provided by the FOD approach could be incorporated as a useful robustness check to distinguish between those country comparisons that are overly sensitive to alternative measurement specifications and those which are not. At the other extreme, a lack of significantly positive association between the two approaches would suggest that the cardinal and FOD perspectives might highlight complementary aspects of the same phenomenon: poverty. In addition, such results would raise some red flags that would caution against a thoughtless use of existing multidimensional poverty measures.

The remainder of this paper is organized as follows. In Sect. 2 we present the different methodologies that are being compared and in Sect. 3 we present the results obtained from the simulated data. Section 4 shows the empirical results from the DHS data and Sect. 5 concludes.

2 Two Approaches to the Measurement of Multidimensional Poverty

In this section we present in some detail the definitions that are used in the two approaches to the measurement of multidimensional poverty compared in this paper: the use of indices and the first order dominance perspective.

2.1 Multidimensional Poverty Indices

If one agrees that poverty is a multidimensional phenomenon, it is quite common to introduce the so-called 'multidimensional poverty indices' in order to measure it. Following the seminal contribution of Sen (1976), when constructing poverty indices it is almost universal to divide the procedure in two steps: 'identification' and 'aggregation'. In the first step, one must present a criterion to decide who should be considered as being multidimensionally poor. In the second step, information regarding the poverty levels of the individuals is aggregated into a single number. In order to identify the poor, it is common to define the so-called 'union', 'intersection' and 'intermediate' approaches'—which basically identify an individual as being 'poor' depending on the number of dimensions in which she or he is deprived (the so-called 'poverty cutoff level').[2] Regarding the aggregation step, there is a variety of alternatives that have recently been proposed in the last few years (see Permanyer 2014, Table 1).

Let k and N be the number of dimensions and individuals we are taking into account respectively. For each dimension j, denote the corresponding deprivation cutoff (i.e.: the level of achievement considered to be sufficient in order to be non-deprived in that dimension) as z_j (with $z_j > 0$). An individual i is 'deprived' in dimension j whenever her/his achievement level x_{ij} (with $x_{ij} \geq 0$) is below z_j and 'non-deprived' otherwise. One can define the deprivation gap of individual i in dimension j as $g_{ij} = \max\{0, (z_j - x_{ij})/z_j\}$. Each dimension j is given a weight $w_j > 0$ according to its relative importance. In this paper we will focus on the following class of poverty measures:

$$P_{\alpha,\beta} = \frac{1}{N}\sum_{i\in Q}\left[\sum_{j=1}^{k} w_j \left(g_{ij}^{c}\right)^{\alpha}\right]^{\alpha/\beta} \tag{1}$$

where Q is the set of individuals identified as poor according to the intermediate approach, $\alpha \geq 0$, $\beta \geq 0$ and $c \geq 0$. Choosing the appropriate values for α and β the different variables become complements or substitutes (see Bourguignon and Chakravarty 2003). Since in this paper we focus our attention on binary data, we set $c = 0$. In that case, g_{ij}^{0} takes a value of 1 whenever individual i is deprived in attribute j and 0 otherwise. The functional form of (1) is based on the multidimensional poverty index suggested by Bourguignon and Chakravarty (2003) but incorporating a more general identification method that goes beyond the union approach. Interestingly, when $\alpha = \beta$ it turns out that $P_{\alpha,\beta}$ coincides with the ordinal poverty index M_0 suggested by Alkire and Foster (2011) which is used by UNDP in the construction of the MPI for the Human Development Report since 2010.

2.2 The FOD Approach

Consider a simple model in which the deprivation of individuals is measured with $k \in \mathbb{N}$ ordinal variables. The set of all possible achievements in that space will be denoted as S. The elements of S can be partially ordered[3] by $\leq$, the usual relationship of vector dominance.[4] Hence, $(S, \leq)$ is a 'partially ordered set' (or a *poset* for short). The elements of S are sometimes referred to as 'achievement profiles'. Given $p \in S$, the *down-set* of p (written $\downarrow p$) is the set of elements vector dominated by p (i.e.$\downarrow p := \{t \in S | t \leq p\}$). For a given set $T \subseteq S$, $\downarrow T := \bigcup_{t\in T} \downarrow t$. The distribution of well-being of some population is described by a probability mass function f over S (that is: $\sum_s f(s) = 1$ and $f(s) \geq 0$ for all s in S). If there are two such distributions f and g, we say that 'f first order dominates g' if any of the following equivalent conditions hold:

[2] According to the 'union' approach, an individual should be labeled as 'poor' if s/he is deprived in at least one dimension. At the other extreme, the 'intersection' approach states that an individual is 'poor' if s/he is deprived in all dimensions simultaneously. In between these extreme perspectives, Alkire and Foster (2011) proposed a counting approach based on Atkinson (2003) suggesting that an individual is 'poor' when s/he is deprived in an intermediate number of dimensions that has to be decided by the analyst. These well-known approaches can be seen as particular cases of the more general identification method using partially ordered sets suggested in Fattore (2016).

[3] A partial order relation in a set X is a binary relation satisfying *reflexivity* ($x \leq x$ for all $x \in X$), *antisymmetry* (if $x \leq y$ and $y \leq x$, then $x = y$ for all $x, y \in X$) and *transitivity* (if $x \leq y$ and $y \leq z$, then $x \leq z$ for all $x, y, z \in X$) (see Davey and Priestley 2002).

[4] We say that $x = (x_1, \ldots, x_k)$ vector dominates $y = (y_1, \ldots, y_k)$ if $x_i \geq y_i$ for all $i = 1, \ldots, k$.

1. g can be obtained from f after a finite sequence of bilateral transfers of density to less desirable outcomes.
2. $\sum_s f(s)h(s) \geq \sum_s g(s)h(s)$ for any non-decreasing real function h.
3. $\sum_{s \in T} g(s) \geq \sum_{s \in T} f(s)$ for any set $T \subseteq S$ such that $\downarrow T = T$.

In words, condition (1) says that one distribution FOD another if one could hypothetically move from one distribution to the other by sequentially shifting population mass in the direction from a better outcome to a worse outcome. Therefore, whenever f FOD g, the population represented by f is unambiguously better off than the one represented by g. The FOD approach does not rely on weighting schemes or on assumptions regarding substitutability/complementarity relationships between welfare dimensions. However, the FOD approach is not always able to determine a ranking when two given countries are compared because conditions (1), (2) and (3) might fail to be satisfied. When this happens, the criterion remains inconclusive.

Let X denote the set of countries being compared and let FOD_k denote the FOD relationship when the number of dimensions we take into account is k. Given any two countries A, B, whenever a country A FOD_k dominates another country B, we will write $B \leq_{FOD_k} A$. Since the FOD_k relation is a partial order, the pair $(X, \leq_{FOD_k})$ is also a partially ordered set. We will denote the set of pairs of countries that are *comparable* in terms of $\leq_{FOD_k}$ as C_{FOD_k}, that is:

$$C_{FOD_k} := \left\{(A, B) \in X \times X | A \leq_{FOD_k} B \ or \ B \leq_{FOD_k} A\right\}. \tag{2}$$

The complement of this set—which will be denoted by N_{FOD_k}—is the set of *incomparable* pairs. The sets C_{FOD_k} and N_{FOD_k} are a partition of $X \times X$, that is: $C_{FOD_k} \cup N_{FOD_k} = X \times X$ and $C_{FOD_k} \cap N_{FOD_k} = \emptyset$. As the number of dimensions (k) increases FOD_k becomes increasingly demanding, so the set of comparable pairs (C_{FOD_k}) tends to become smaller—an issue that becomes apparent in Sects. 3 and 4. At the opposite extreme, when $k = 1$, $C_{FOD_1} = X \times X$ and $N_{FOD_1} = \emptyset$. In Appendix 1 we give some details on how to establish the existence of FOD relationships and in Appendix 2 we give further details on how the approach works for the multidimensional binary case that is applied in the empirical sections of the paper.

Even if FOD methods were originally introduced to perform robust welfare comparisons, in this paper we have adapted them to generate multivariate poverty orderings. For that purpose, we have used the dimension-specific poverty thresholds z_j to dichotomize the support of our underlying variables in two groups, i.e.: above and below z_j. From now onwards, “1” will be used to denote the “good” outcome (e.g.: being non-deprived) and “0” the “bad” one (being deprived). As is common in poverty analysis, rather than considering the entire support of the distribution we are just concerned with what happens below the poverty lines.

The techniques analyzed in this paper bear some resemblance with other approaches recently proposed in the literature of multidimensional poverty in ordinal settings. One of these approaches suggests using the theory of partially ordered sets to measure multidimensional deprivation (e.g. Fattore 2016) and the other proposes robust multidimensional poverty comparison techniques applied to ordinal data (see Yalonetzky 2014). For clarification purposes, in Appendix 3 we highlight what they have in common and what are the key differences between them.

3 Simulating Datasets

In order to explore the performance of the FOD approach in different settings and compare it with multidimensional poverty indices we have generated several simulated distributions. These simulations correspond to alternative scenarios where we take into consideration (1) a varying amount of binary variables, with k running from two to ten;[5] (2) different welfare distributions across simulated countries (left skewed, symmetrical and right skewed), and (3) different correlation structures across variables (low, medium and high correlations). This results in 81 different scenarios (9 values of $k\times$ 3 between country distributions $\times$ 3 correlation coefficients). This way, we aim at covering a wide range of hypothetical distributions resembling what practitioners might encounter in practice when attempting to implement the FOD approach.

In our simulations we compare the performance of 200 hypothetical 'countries' (this number is chosen because it approximates the number of countries in the world) and assume that in each country we have 2000 individuals (this is a common sample size in many socio-demographic household surveys). For each individual in each country we randomly generate k binary variables ($2 \leq k \leq 10$) as follows. We start generating k random variables $X_1, \ldots, X_k$ from a multivariate normal distribution $N(0, \Sigma)$ where all diagonal elements equal 1 and, for the sake of simplicity, all off-diagonal elements are equal to a constant ρ. We have considered three different values of ρ (near 0.1, 0.5 and 0.9) so that the correlations between our k binary variables end up being close to those three values in different scenarios. To generate the binary variables $B_1, \ldots, B_k$ from the continuous variables $X_1, \ldots, X_k$ within each country 'i' we have applied the next rule: whenever the value of X_j exceeds a given threshold t_{ij}, then the corresponding binary variable B_j equals 1 (the 'good' outcome); otherwise it equals 0 (the 'bad' outcome). The choice of the thresholds t_{ij} for country 'i' and variable 'j' has been guided by the following principle. If we define the 'welfare' level of a given country as the share of 1 s among its 2000 $\cdot k$ entries, we have chosen the set of thresholds t_{ij} so that the international welfare distribution among our 200 countries has three possible shapes: skewed to the right (with many countries with low welfare levels and fewer ones with high welfare), symmetrical and skewed to the right (with many countries with high welfare levels and fewer ones with low welfare), see Fig. 1. With this approach we aim at simulating three 'development scenarios': low, medium and high international development levels.

3.1 Results for the Simulated Datasets

In Fig. 2 we plot the percent of FOD occurrences among all possible country pairs for the 81 scenarios considered in this paper (since there are 200 simulated countries, the number of country pairs is 200·199/2 = 19,900). As expected, the larger the number of variables the lower the share of FOD occurrences, irrespective of the shape of the 'international' welfare distribution and the correlation between variables. In addition, the extent of FOD relationships is strongly influenced by the correlation level between variables. No matter how many dimensions we take into account or what shape the international welfare distribution has, higher correlation between variables leads to a higher prevalence of FOD occurrences. In addition, this relationship seems to grow stronger as the number of

[5] The time needed to run these simulations increases rapidly with k (for the case $k = 10$ the computation time for anIntel® Xeon® E5-1650 v3 3, 5 GHz, RAM 16 GB is about 24 h). For this reason, in our simulations we have not considered more than 10 variables.

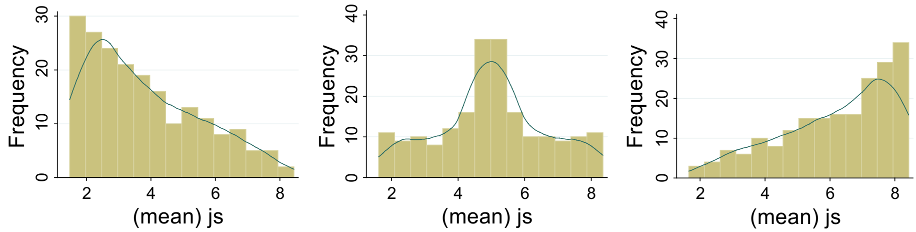

Fig. 1 International distribution of welfare levels for the simulated data in three different scenarios (right skewed, symmetrical and left skewed) for $\rho = 0.5$ and $k = 10$. *Source*: Author's calculations based on simulated data

dimensions increases: when $k = 10$, the drop in FOD occurrences is particularly large when the correlation goes from $\rho = 0.5$ to $\rho = 0.1$. As can be seen such drop is even larger in the right skewed scenario—which is the one that resembles the most our empirical data in Sect. 4 (with a few rich countries and a majority of poor ones).

We now explore how the FOD and poverty index approaches are related to each other. There are good reasons to suspect that the differences in poverty levels for the pairs of countries included in C_{FOD_k} might be significantly bigger than the differences in poverty levels for the pairs of countries included in N_{FOD_k}. That is: we expect the differences in poverty levels to be higher when the corresponding pairs of countries are 'FOD-comparable' than when they are not. In order to investigate this issue, for each simulated scenario discussed before and for each value of k between 2 and 10 we have generated two *scaled* density functions of the values of $\left|P_{1,1}(A) - P_{1,1}(B)\right|$: one for the pairs $(A, B) \in N_{FODk}$ (which is denoted as $f_{N,k}$) and another one for the pairs $(A, B) \in C_{FODk}$ (which is denoted as

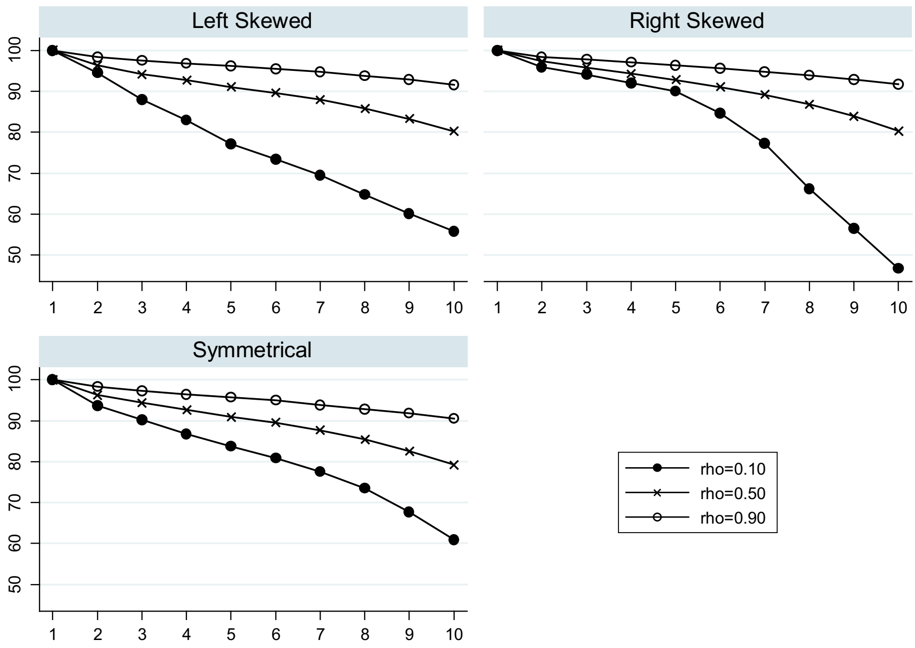

Fig. 2 Percentage of FOD_k occurrences for the different simulated data scenarios when $k = 2,\ldots,10$. *Source*: Author's calculations based on simulated data

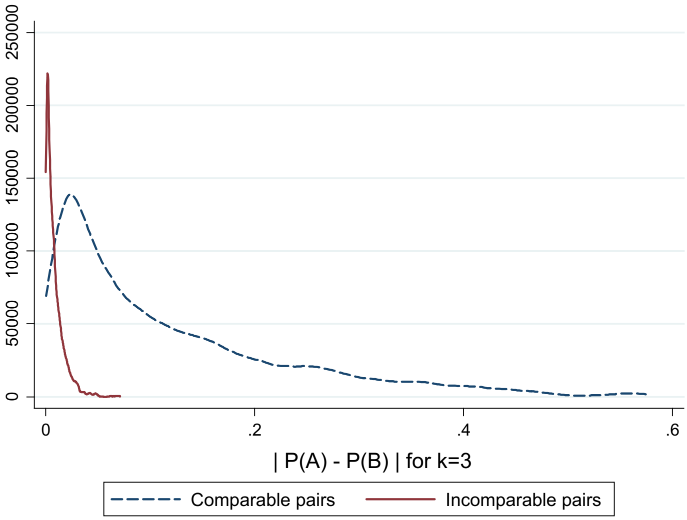

Fig. 3 Weighted density functions comparing $f_{N,3}$ (incomparable pairs) and $f_{C,3}$ (comparable pairs) for the left skewed, $\rho = 0.1$ scenario. *Source*: Author's calculations based on simulated data

$f_{C,k}$).[6] Figure 3 shows the scaled density functions $f_{C,k}$ and $f_{N,k}$ (which are scaled by the number of comparable and incomparable pairs[7] respectively) for the case $k = 3$ in the scenario where the 'international' distribution is right skewed and the correlation coefficient between variables equals 0.1 (this is the scenario that resembles the most our survey-based data analyzed in Sect. 4). As can be seen, the range of values in the differences in poverty levels is much larger when the pairs of countries are FOD comparable than when they are not. When the FOD criterion in inconclusive, the poverty levels of the corresponding countries are highly similar and vice versa. In addition, the large difference in the areas below the two curves confirms what we already observed in Fig. 2: for such low value of k the majority of country pairs are FOD comparable (92%).

Figure 4 shows the analogous results when increasing the number of variables to $k = 6$ and $k = 10$. Several interesting patterns arise. On the one hand, the scaled distribution $f_{C,k}$ moves to the right: the mean of that distribution when $k = 3$, 6 and 10 is 0.16, 0.26 and 0.29 respectively. In addition, the higher the value of k the more difficult it is to find country pairs that are FOD comparable and whose difference in poverty levels approach zero. On the other hand, the scaled distribution $f_{N,k}$ not only moves to the right (its means when $k = 3$, 6 and 10 are 0.01, 0.02 and 0.03) but is considerably wider as well, i.e. when the number of variables increase the FOD criterion is increasingly demanding, so it is more likely to find country pairs that are not FOD comparable but whose difference in poverty levels is relatively high. Yet, the relative position of the $f_{N,k}$ and $f_{C,k}$ curves does not vary with k. In line with the findings shown earlier in this section, one can see that at higher

[6] The choice of different values of α and β for the poverty index does not substantially change our findings. The results are not shown here, but are available upon request.

[7] In this example, the number of comparable and incomparable pairs equal 17,605 and 2295 respectively (hence, the relative shares of comparable and incomparable pairs are 88.5 and 11.5%, see Fig. 2). This means that the areas under the scaled density functions $f_{C,3}$ and $f_{N,3}$ equal 17,605 and 2295, respectively.

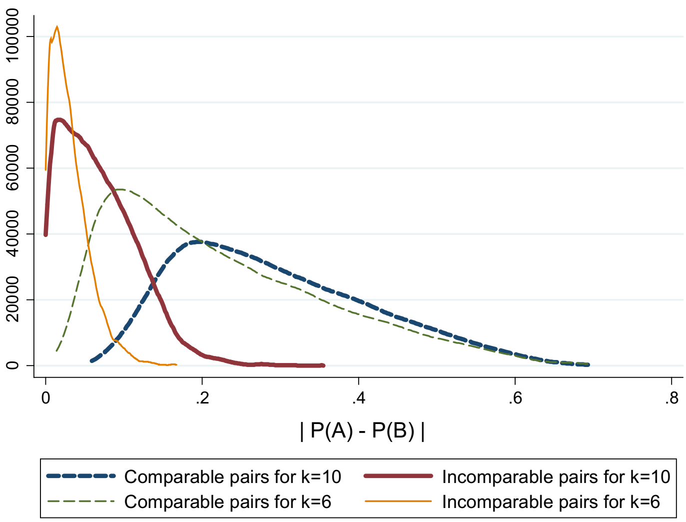

Fig. 4 Scaled density functions $f_{N,k}$ (incomparable pairs) and $f_{C,k}$ (comparable pairs) when $k = 6$ and 10 for the left skewed, $\rho = 0.1$ scenario. *Source*: Author's calculations based on simulated data

values of k the share of incomparable pairs grows in importance: when $k = 6$ and $k = 10$, the share of comparable couples among all country pairs are 83 and 47% respectively.

All in all, these results suggest that the FOD approach can be an effective tool to distinguish those country pairs whose ranking is robust from those pairs in which the ranking is contingent upon the choice of weighting scheme, underlying social welfare function and the like. The patterns we have just described are repeated when using the other simulated scenarios described earlier in this section (the results are not shown here but are available upon request).

4 Case Study: The FOD Approach Across the Developing World

In this section we complement the previous findings presenting some results based on 48 Demographic and Health Surveys across the developing world. After introducing the definitions and basic indicators used in our analysis we show the results for the poverty indices, then for the FOD approach and finally we compare the two approaches.

4.1 Data and Indicators

In order to compare multidimensional poverty measurement approaches across the developing world we have assembled 48 Demographic and Health Surveys (DHS) primarily undertaken between 2006 and 2012 (totaling 761,909 observations among the 48 surveys, see Table 1). We have chosen the DHS surveys that were used in the construction of the official MPI, an issue that ensures comparability with UNPD's well-known measure. Overall, we have obtained an extensive sample including countries from all major regions in the developing world.

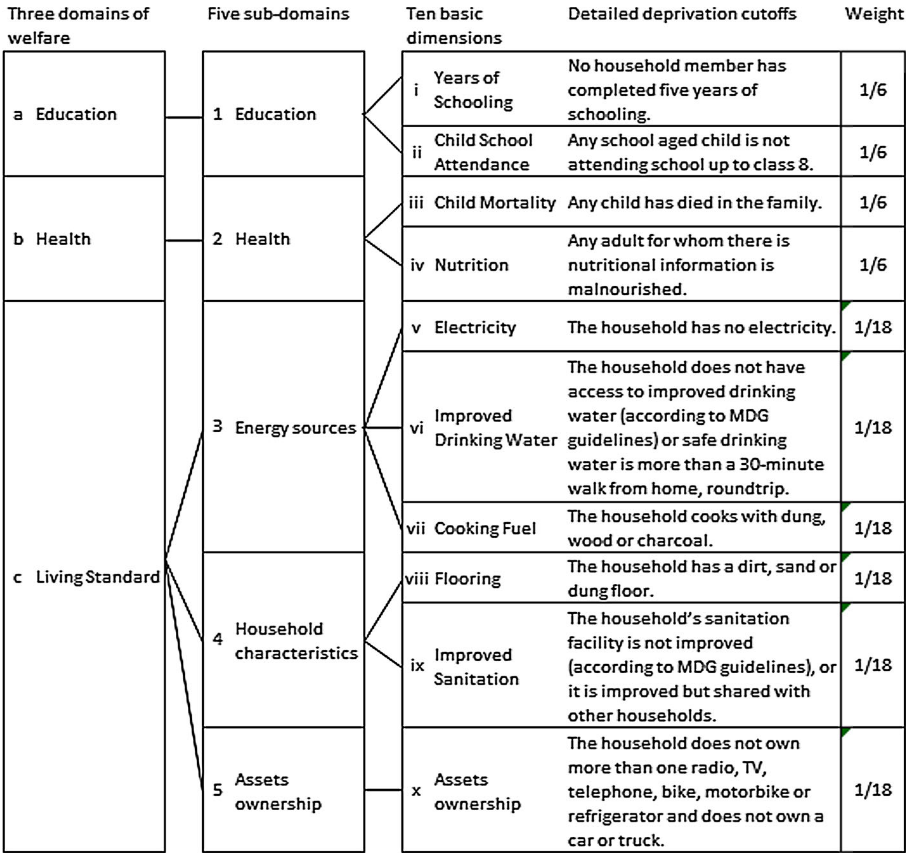

Fig. 5 Welfare dimensions, sub-domains, domains, deprivation cutoffs and weights on the MPI

The information included in the DHS constitutes a formidable database with many variables to compute non-monetary multidimensional poverty indices like $P_{\alpha,\beta}$ and to implement the multivariate FOD methodology. The indices used in this section are hierarchically structured, with ten variables partitioned in three domains ('Health', 'Education' and 'Standard of Living') and five sub-domains[8] ('Health', 'Education', 'Energy sources', 'Household characteristics' and 'Asset ownership')—see the details in Fig. 5. The 'Health' and 'Education' sub-domains are composed of two variables each: one referring to adults and the other to children in the corresponding household. The six variables in the 'Living Standard' domain include several household characteristics. In Fig. 5, we also show the conditions that must be met in order to consider a household deprived in the corresponding variable (that is: they determine the variable-specific poverty thresholds z_j). To create these

[8] In its original definition, the UNDP's MPI does not have the sub-domains we have introduced here. We have introduced them to have a more gradual dimensional refinement that allows exploring in more detail the effects of increasing dimensionality on the occurrence of FOD relationships.

thresholds and to choose the weights that $P_{\alpha,\beta}$ assigns to each variable included in the index we have used the official guidelines provided in UNDP's MPI.

Borrowing techniques from Hussain et al. (2015), we explore the effect of increasing dimensionality on the robustness of the FOD results by gradually refining the well-being dimensions we take into account. For a given set of dimensions, we will require a good outcome in all indicators of a given dimension for an individual to be classified as non-deprived in that dimension—a criterion that mimics the union approach of multidimensional poverty indices (see footnote #2). In the extreme 1-dimensional case, the FOD criterion simply compares the shares of the populations that are not deprived in any of the 10 underlying indicators, thus ensuring the existence of FOD relationships for every pair of countries. The five sub-domains in the FOD_5 case imply an aggregation of the 10 initial dimensions, such that a good outcome in a given sub-domain exists when there are good outcomes in all the corresponding basic welfare dimensions. For instance: a good outcome regarding sub-domain 4 ('Household characteristics') means that both flooring material is of good quality and that there is access to improved sanitation. The three welfare domains similarly are an aggregation of the sub-domains and thereby also implicitly of the ten basic dimensions. A good outcome regarding domain *c* ('Living Standard') in a household thus exists when the outcomes regarding energy source, household characteristics, and assets ownership is good. According to the Lemma shown in Hussain et al. (2015), if a FOD relationship holds between a given pair of countries when using binary indicators, such relationship either holds or vanishes whenever a given dimension is refined into several sub-divisions.

4.2 Results for Multidimensional Poverty Indices

Table 1 shows the values of the poverty index $P_{1,1}$ for the 48 countries included in our study (recall that when $\alpha = \beta, P_{\alpha,\beta}$ corresponds to the Alkire and Foster (2011) M_0 index).[9] Along with the values of the different poverty indices, Table 1 also presents in parentheses the corresponding country rankings (with the countries having lower poverty levels being placed at the better—i.e. smallest in number–positions in the ranking). The range of observed values for $P_{1,1}$ is [0.002, 0.524]. The minimum value is attained by Ukraine and the maximum value is attained by Niger, the poorest country in our sample according to this measure.

As an external consistency check to validate the reasonableness of the measures introduced in this paper, in Table 1 we have also included the values of the official UNDP's MPI. Both our ordinal measure $P_{1,1}$ and the MPI tend to rank countries in the same way: the correlation coefficient between the two measures equals 0.95. Since we are dealing with 48 countries, there are $48 \cdot 47/2 = 1128$ pairs of countries. In 1012 of them (i.e. in 90% of the cases), there is an agreement between both measures when determining which of the corresponding two countries has the highest poverty levels. These results suggest that the indicators and measures proposed in the paper are within reasonable bounds.

[9] Choosing alternative values for α, β leads to results that are highly correlated with the ones presented here, so they will not be reported.

Table 1 Multidimensional poverty measures for 48 countries (the corresponding rankings are indicated in parentheses). *Source*: Authors' calculations using DHS data

Country (year)	$P_{1,1}$	MPI	FOD1
Albania (2009)	0.010 (3)	0.004 (4)	0.619 (3)
Armenia (2010)	0.007 (2)	0.002 (2)	0.542 (2)
Azerbaijan (2006)	0.031 (4)	0.009 (7)	0.897 (10)
Bangladesh (2011)	0.354 (34)	0.236 (26)	0.985 (26)
Benin (2006)	0.384 (40)	0.400 (40)	1.000 (47)
Bolivia (2008)	0.113 (14)	0.096 (16)	1.000 (44)
Burkina Faso (2010)	0.460 (44)	0.507 (44)	0.991 (30)
Burundi (2010)	0.433 (41)	0.441 (42)	0.999 (39)
Cambodia (2010)	0.368 (35)	0.211 (22)	1.000 (46)
Cameroon (2011)	0.195 (20)	0.260 (29)	0.930 (14)
Colombia (2010)	0.039 (8)	0.032 (11)	0.715 (5)
Congo (2012)	0.247 (24)	0.192 (19)	0.986 (27)
Cote Ivoire (2012)	0.293 (29)	0.306 (31)	0.979 (24)
Dominican Rep. (2007)	0.034 (6)	0.026 (9)	0.925 (12)
Egypt (2008)	0.031 (5)	0.035 (12)	1.000 (43)
Ethiopia (2011)	0.513 (47)	0.537 (46)	0.997 (36)
Gabon (2012)	0.085 (12)	0.072 (15)	0.901 (11)
Guinea (2005)	0.435 (42)	0.548 (47)	0.984 (25)
Guyana (2009)	0.035 (7)	0.031 (10)	0.835 (7)
Haiti (2012)	0.335 (32)	0.241 (28)	0.999 (38)
Honduras (2012)	0.164 (18)	0.098 (17)	1.000 (45)
India (2006)	0.294 (30)	0.282 (30)	0.930 (15)
Indonesia (2012)	0.141 (16)	0.024 (8)	1.000 (42)
Jordan (2009)	0.039 (9)	0.003 (3)	0.801 (6)
Kenya (2009)	0.234 (22)	0.226 (24)	0.960 (21)
Lesotho (2009)	0.141 (17)	0.227 (25)	0.936 (17)
Liberia (2007)	0.441 (43)	0.459 (43)	1.000 (48)
Madagascar (2009)	0.377 (39)	0.420 (41)	0.995 (32)
Malawi (2010)	0.269 (25)	0.331 (34)	0.990 (29)
Maldives (2009)	0.064 (11)	0.007 (6)	0.930 (13)
Mali (2006)	0.495 (46)	0.533 (45)	0.998 (37)
Moldova (2005)	0.097 (13)	0.005 (5)	1.000 (41)
Mozambique (2011)	0.488 (45)	0.390 (39)	0.994 (31)
Namibia (2007)	0.197 (21)	0.199 (21)	0.851 (8)
Nepal (2011)	0.277 (27)	0.196 (20)	0.966 (22)
Niger (2012)	0.524 (48)	0.583 (48)	0.997 (35)
Pakistan (2013)	0.240 (23)	0.237 (27)	0.933 (16)
Peru (2012)	0.054 (10)	0.043 (14)	0.684 (4)
Philippines (2008)	0.116 (15)	0.037 (13)	0.940 (18)
Rwanda (2010)	0.376 (36)	0.351 (36)	0.999 (40)
Sao Tome and Prin. (2009)	0.186 (19)	0.216 (23)	0.960 (20)
Senegal (2011)	0.272 (26)	0.389 (38)	0.893 (9)
Tanzania (2010)	0.376 (37)	0.334 (35)	0.996 (33)
Timor-Leste (2010)	0.341 (33)	0.322 (33)	0.986 (28)

Table 1 continued

Country (year)	$P_{1,1}$	MPI	FOD1
Uganda (2011)	0.376 (38)	0.358 (37)	0.996 (34)
Ukraine (2007)	0.002 (1)	0.001 (1)	0.512 (1)
Zambia (2007)	0.325 (31)	0.318 (32)	0.959 (19)
Zimbabwe (2011)	0.283 (28)	0.180 (18)	0.973 (23)

4.3 Results for the FOD Approach

As indicated in Appendix 2, with three binary indicators of deprivation there are $2^3 = 8$ possible welfare indicator combinations. The share of households falling into each of the 8 categories for each country is shown in Table 2.[10] On average (weighted by countries' overall population) 6% of the population experiences the best welfare combination, which means they are not deprived in any of the three dimensions (outcome (1,1,1)). We see a large variation across countries with Ukraine, Armenia and Albania around 40–50%, while countries like Liberia, Benin, Cambodia and a few others are down at 0%. On average some 11% of households experience the worst welfare combination, that is: deprivation with respect to all three dimensions simultaneously (outcome (0,0,0)). Also high variation is seen for this combination since Mozambique, Mali, Ethiopia and Guinea are all above 40%, while Ukraine, Armenia and Albania have no households in this worst off category. The remaining slightly more than 80% of the population experiences intermediate welfare combinations (a mixture of 0/deprivation and 1/non deprivation), which in many cases cannot be internally ranked without assumptions regarding each dimension's importance. The least common combination is deprivation in living standards and non-deprivation in education and health (1,0,1).

The simplest case is where all ten dimensions are collapsed into one binary welfare indicator. In that extreme case, the FOD_1 approach is equivalent to comparing the population shares that are not deprived in any of the 10 welfare indicators. The results are presented under the heading FOD_1 in Table 1. We see that least poverty is seen in Ukraine, which is therefore ranked as no. 1 in this comparison of countries. The second and third best countries are Armenia and Albania. The worst ranked countries are Benin, Bolivia, Cambodia, Egypt, Honduras, Indonesia, Liberia and Moldova (in those countries, *all* households are deprived in at least one indicator).

Despite the many conditions to be fulfilled to obtain FOD_3 relationships (see the 11 inequalities in Appendix 2) there are 531 instances of FOD_3 among the total of 1128 country pairs (i.e.: 47%). The lack of FOD for half of the country comparisons occurs in situations where rankings are dependent on differing evaluation criteria, so the analyst should be careful in making the ranking anyway. In other words, lack of FOD_3 pinpoints those country comparisons where standard index-based rankings should be taken with some caution. Since we are dealing with 48 countries, the maximum number of times a country can dominate another one is 47. As can be seen in Table 3, the highest observed number of dominations is for Ukraine, which dominates 45 countries. Eleven countries do not dominate other countries (Burkina Faso, Benin, Burundi, Ethiopia, India, Liberia, Moldova, Mali, Mozambique, Niger and Rwanda). The highest number of times a country is being dominated is 31 (Mali), while Ethiopia and Liberia each are dominated by 28

[10] For the five and ten dimensional cases the corresponding tables are considerably larger, so they are not shown here (they are available from the authors upon request).

Table 2 Population distribution by welfare indicator combination (in %)

		0/1 indicators by position. 1st: Education. 2nd: Health. 3rd: Living st.								Total
		000 Worst	001 Health	010 Living s	011 LS, He	100 Educ.	101 Ed, He	110 Ed, LS	111 Best	
AL	Albania	0.93	0.32	7.64	5.90	5.05	4.45	37.68	38.02	100
AM	Armenia	0.65	1.22	10.10	17.71	0.71	1.31	22.54	45.75	100
AZ	Azerbaijan	2.79	0.22	8.98	0.95	16.87	1.60	58.35	10.23	100
BD	Bangladesh	32.44	0.97	37.95	2.76	9.83	0.68	13.89	1.48	100
BF	Burkina Faso	35.72	0.27	52.32	1.05	1.26	0.04	8.51	0.84	100
BJ	Benin	26.85	0.00	36.21	0.00	12.28	0.00	24.66	0.00	100
BO	Bolivia	7.19	0.00	24.04	0.00	13.19	0.00	55.58	0.00	100
BU	Burundi	30.76	0.00	51.70	0.02	3.27	0.00	14.22	0.04	100
CG	Congo (Brazzaville)	17.75	0.55	42.60	2.24	5.55	0.08	29.83	1.39	100
CI	Cote Ivoire	18.39	0.41	56.35	4.14	1.78	0.10	16.73	2.09	100
CM	Cameroon	12.36	0.16	20.55	0.63	12.94	1.13	45.29	6.96	100
CO	Colombia	3.15	1.31	14.56	4.92	8.36	8.38	30.83	28.49	100
DR	Dominican Republic	2.14	0.28	20.87	2.38	4.25	0.68	61.99	7.42	100
EG	Egypt	3.58	0.00	24.14	0.00	7.36	0.00	64.92	0.00	100
ET	Ethiopia	40.16	0.20	43.18	0.23	4.60	0.14	11.28	0.21	100
GA	Gabon	6.69	2.55	31.82	15.34	2.35	0.75	30.66	9.84	100
GN	Guinea	40.03	0.82	38.81	0.96	6.56	0.88	10.36	1.59	100
GY	Guyana	2.24	0.33	14.13	3.01	6.73	0.86	56.20	16.49	100
HN	Honduras	7.13	0.00	28.55	0.00	13.18	0.00	51.14	0.00	100
HT	Haiti	18.92	0.01	50.14	0.17	5.14	0.03	25.49	0.09	100
IA	India	22.54	0.88	12.97	0.80	32.18	6.38	17.34	6.91	100
ID	Indonesia	6.79	0.00	53.66	0.00	2.59	0.00	36.96	0.00	100
JO	Jordan	3.84	6.53	18.07	27.35	2.30	3.32	18.73	19.87	100
KE	Kenya	11.84	0.11	24.21	0.37	14.54	0.54	44.45	3.94	100
KH	Cambodia	19.97	0.00	49.93	0.00	6.53	0.00	23.57	0.00	100
LB	Liberia	29.15	0.00	43.97	0.00	7.24	0.00	19.64	0.00	100
LS	Lesotho	3.27	0.06	20.98	0.85	9.32	0.71	58.42	6.39	100
MB	Moldova	5.24	7.35	44.86	42.55	0.00	0.00	0.00	0.00	100
MD	Madagascar	17.61	0.00	40.68	0.02	10.28	0.05	30.91	0.45	100
ML	Mali	44.77	0.01	38.70	0.02	6.63	0.07	9.66	0.14	100
MV	Maldives	9.34	1.61	14.34	2.01	26.57	5.77	33.37	6.98	100
MW	Malawi	13.33	0.00	29.18	0.07	15.40	0.19	40.83	0.99	100
MZ	Mozambique	46.95	0.27	36.67	0.79	5.27	0.10	9.36	0.57	100
NI	Niger	38.58	0.04	47.11	0.09	3.87	0.12	9.89	0.29	100
NM	Namibia	10.37	1.96	19.95	3.32	12.08	5.36	32.08	14.87	100
NP	Nepal	19.07	0.97	47.49	4.71	4.71	0.77	18.91	3.38	100
PE	Peru	1.96	0.44	15.81	5.54	3.98	1.84	38.88	31.56	100
PH	Philippines	6.29	0.54	46.55	6.63	1.36	0.28	32.41	5.95	100
PK	Pakistan	22.90	3.85	37.63	10.91	3.58	1.29	13.17	6.67	100

Table 2 continued

		0/1 indicators by position. 1st: Education. 2nd: Health. 3rd: Living st.								Total
		000 Worst	001 Health	010 Living s	011 LS, He	100 Educ.	101 Ed, He	110 Ed, LS	111 Best	
RW	Rwanda	21.34	0.00	55.87	0.01	2.31	0.00	20.44	0.03	100
SN	Senegal	21.90	1.69	37.15	4.19	5.01	1.35	18.08	10.63	100
ST	Sao Tome and Principe	5.62	0.00	38.88	0.25	7.29	0.30	43.70	3.96	100
TL	Timor-Leste	21.99	0.23	28.67	0.31	20.13	0.68	26.70	1.30	100
TZ	Tanzania	29.75	0.19	44.63	0.26	6.35	0.13	18.36	0.32	100
UA	Ukraine	0.07	0.10	5.44	3.49	0.44	0.70	41.00	48.76	100
UG	Uganda	27.91	0.04	44.09	0.08	5.79	0.07	21.72	0.31	100
ZM	Zambia	20.08	0.79	27.04	0.90	15.83	2.36	28.92	4.08	100
ZW	Zimbabwe	23.93	3.61	32.69	2.93	10.93	2.10	21.14	2.66	100
Average		11.40	1.10	40.39	4.80	4.54	0.78	30.70	6.29	100

countries. Only two countries are not dominated by other countries (Ukraine and Armenia). As expected, there is a clear negative correlation ($r = –0.72$) between the number of dominations and the number of times being dominated—the more a country dominates, the less it is dominated, and vice versa.

We can see from Table 3 that fewer country pairs are comparable when we increase dimensionality from three to five. More specifically we can now rank 345 out of the earlier mentioned 1128 possible comparisons (31%). As indicated in Sect. 3, the reduction in comparability is expected. Although we observe fewer FOD_5 relationships, the relative performance of countries in terms of FOD_3 and FOD_5 is preserved: the correlation coefficient between the number of times a country dominates others in FOD_3 and FOD_5 is $r = 0.93$, and when we compare the number of times countries are dominated by others, $r = 0.87$. If we refine the analysis to the most disaggregated level and conduct the analysis for the ten basic dimensions, we end up with 113 country pairs that can be ranked according to the FOD_{10} methodology (10% of all pairs). Like before, the reduction in FOD comparable pairs is expected as ten dimensions include an even more demanding set of criteria to be fulfilled. The fewer cases where FOD_{10} occurs are likely to reflect very deep differences in the corresponding welfare distributions, an issue that will be further investigated in Sect. 4.4.

How do these findings relate to the simulation results shown in Sect. 3.1? The percentage of FOD_k occurrences across our 48 countries is smaller than the percentages observed in the simulation exercises (see Fig. 2). Inter alia, this can be attributable to two factors. On the one hand, the correlation structure of the ten variables included in the $P_{1,1}$ index—which is shown in Table 4—differs substantially from the uniform correlation structure we adopted, for the sake of simplicity, in our simulation exercises. Even if the average correlation among all variable pairs equals 0.17, some variable pairs are moderately correlated (e.g. $r = 0.453$ for the 'Cooking fuel' and 'Asset index' variables) while others have very small correlation levels (e.g. $r = -0.084$ for the 'Body Mass Index' and 'Water access' variables). On the other hand, the welfare differences across the 48 countries included in our dataset are not as large as the differences across the world

Table 3 Country FOD comparisons. DHS surveys, 2001–2010

		AL	AM	AZ	BD	BF	BJ	BO	BU	CG	CI	CM	CO	DR	EG	ET	GA	GN	GY	HN	HT	IA	ID	JO	KE	Total
AL	Albania				5	10	10	5	10	10	5	5	3		3	10	3	5		10	10	5			10	37
AM	Armenia				5	10	10	5	10	10	5	5				10	3	5		5	10	5	5	3	5	37
AZ	Azerbaijan				3	3	3		5	3		3				10		3			3				3	24
BD	Bangladesh															5										3
BF	Burkina Faso																									0
BJ	Benin																									0
BO	Bolivia						5																			3
BU	Burundi																									0
CG	Congo (Brazzaville)					3			5							5					3					11
CI	Cote Ivoire					3			5							5										6
CM	Cameroon				5	3	3		5							10		5								16
CO	Colombia				5	5	5		5	3		3				10		5			5	5			5	29
DR	Dominican Republic				3	3	3	3	5	3	3	3			3	5		3		5	3		5		3	29
EG	Egypt						5	5												5						5
ET	Ethiopia																									0
GA	Gabon				5	3	5		5	5	5					10		5			10					21
GN	Guinea																									2
GY	Guyana				5	3	3	3	5	3	3	3			3	5		5		5	10	3			3	33
HN	Honduras						3																			3
HT	Haiti								5																	3
IA	India																									0
ID	Indonesia						3																			3
JO	Jordan				5	5	5		5	5	3					10		5			10					22
KE	Kenya					3	3		5							5		3								14

Table 3 continued

		AL	AM	AZ	BD	BF	BJ	BO	BU	CG	CI	CM	CO	DR	EG	ET	GA	GN	GY	HN	HT	IA	ID	JO	KE	Total
KH	Cambodia																									1
LB	Liberia																									0
LS	Lesotho				3	3	3	3	5	3	3					5		3		3	3				3	23
MB	Moldova																									0
MD	Madagascar						3		5																	5
ML	Mali																									0
MV	Maldives				3											5		3								5
MW	Malawi						3		5							5										8
MZ	Mozambique																									0
NI	Niger																									0
NM	Namibia				5	3	3		5							5		5				5				18
NP	Nepal				5	3			5							10		5								10
PE	Peru				5	10	10	3	10	3	3	10			3	10	3	5		10	5	5	5		10	38
PH	Philippines				5	3	3		5	3	3					10		5			5					19
PK	Pakistan					3			5							5		5								7
RW	Rwanda																									0
SN	Senegal				5	3			5							5		5								11
ST	Sao Tome and Principe					3	3		5	3						5		3			5					17
TL	Timor-Leste															5										3
TZ	Tanzania															5										3
UA	Ukraine	3		5	5	10	10	10	10	3	3	10	5	5	3	10	3	5	5	10	5	10	5		10	45
UG	Uganda								5																	3
ZM	Zambia				5		3									5		5								8
ZW	Zimbabwe				5											5		5								6
Total		1	0	1	19	21	23	8	25	13	10	8	2	1	5	28	4	22	1	8	14	7	4	1	9	531

Table 3 continued

		KH	LB	LS	MB	MD	ML	MV	MW	MZ	NI	NM	NP	PE	PH	PK	RW	SN	ST	TL	TZ	UA	UG	ZM	ZW	Total
AL	Albania	10	10	5		10	10	3	5	5	10	5	5			5	5	5	5	10	10		10	10	5	37
AM	Armenia	10	10		5	5	10		5	10	10	5	10		10	10	10	10	5	10	10		10	10	10	37
AZ	Azerbaijan	5	3			3	3		3	3	3		3				5			3	3		3	3	3	24
BD	Bangladesh						3			3																3
BF	Burkina Faso																									0
BJ	Benin																									0
BO	Bolivia	3	5																							3
BU	Burundi																									0
CG	Congo (Brazzaville)	3	5				3			5	5										5		3			11
CI	Cote Ivoire						3			5	10															6
CM	Cameroon		10			5	3		5	5	10									5	5		5	5		16
CO	Colombia	5	5			5	10	3	5	5	5	3	5			3	5	3		10	10		5	5	3	29
DR	Dominican Republic	10	3			3	3		3	5	5		3				5		3	3	3		5	3		29
EG	Egypt	3	10																							5
ET	Ethiopia																									0
GA	Gabon	3	10			5	10			10	10		5			5	5				5		5		5	21
GN	Guinea						3			3																2
GY	Guyana	10	3	3		10	5	3	5	5	5		5				5	3	5	5	5		5	10	3	33
HN	Honduras	3	3																							3
HT	Haiti	3	3																							3
IA	India																									0
ID	Indonesia	3	3																							3
JO	Jordan	10	5			5	5			5	5		5			5	5	3			10		10		3	22
KE	Kenya		3			5	3		5	5	5									5	5		5			14
KH	Cambodia		3																							1
LB	Liberia																									0
LS	Lesotho	3	10			3	3		3	3	10						5			5	3		3			23

Table 3 continued

		KH	LB	LS	MB	MD	ML	MV	MW	MZ	NI	NM	NP	PE	PH	PK	RW	SN	ST	TL	TZ	UA	UG	ZM	ZW	Total
MB	Moldova																									0
MD	Madagascar		3				3																3			5
ML	Mali																									0
MV	Maldives						3			3																5
MW	Malawi		3				3				5										3		3			8
MZ	Mozambique																									0
NI	Niger																									0
NM	Namibia		5				3			5	5					3		3		5	5		5	5	3	18
NP	Nepal		3				3			5	5										5					10
PE	Peru	10	10	3		10	10	3	10	5	10	5	5		5	5	5	3	5	10	10		10	10	3	38
PH	Philippines	3	10				5			5	10		5				5				10		10		3	19
PK	Pakistan						5			5	5															7
RW	Rwanda																									0
SN	Senegal		5				3			5	10										5		5			11
ST	Sao Tome and Principe	3	3			3	3			5	10						5			3	5		5			17
TL	Timor-Leste						3			3																3
TZ	Tanzania						3				3															3
UA	Ukraine	10	10	3	3	10	10	5	10	5	10	10	10	5	5	10	5	5	5	10	10		10	10	3	45
UG	Uganda		3				3																			3
ZM	Zambia						3			5	5									5						8
ZW	Zimbabwe						3			3	5															6
Total		19	28	4	2	14	31	5	11	27	25	5	11	1	3	8	13	8	6	14	20	0	20	10	11	531

The number entry shows the highest dimension in which the row country dominates the column country

Table 4 Correlation between variable pairs in the $P_{1,1}$ index across 48 countries. *Source*: Author's calculations based on DHS data

	yssch	chatt	bmi	chmort	electr	water	toilet	floor	cook	asset
yssch	1.000									
chatt	0.063	1.000								
bmi	−0.023	0.010	1.000							
chmort	0.082	0.194	0.071	1.000						
electr	0.312	0.183	0.033	0.189	1.000					
water	0.066	0.093	−0.084	0.025	0.153	1.000				
toilet	0.179	0.099	0.083	0.092	0.242	0.104	1.000			
floor	0.274	0.134	0.087	0.149	0.479	0.075	0.198	1.000		
cook	0.253	0.189	0.079	0.184	0.538	0.037	0.293	0.413	1.000	
asset	0.304	0.112	0.074	0.117	0.475	0.055	0.302	0.393	0.453	1.000

Yssch years of schooling, *Chatt* child school attendance, *bmi* body mass index, *electr* electricity, *water* access to potable water, *toilet* access to an improved toilet, *floor* good quality floor, *cook* cooking fuel, *asset* asset index (the details of each variable are shown in Fig. 5)

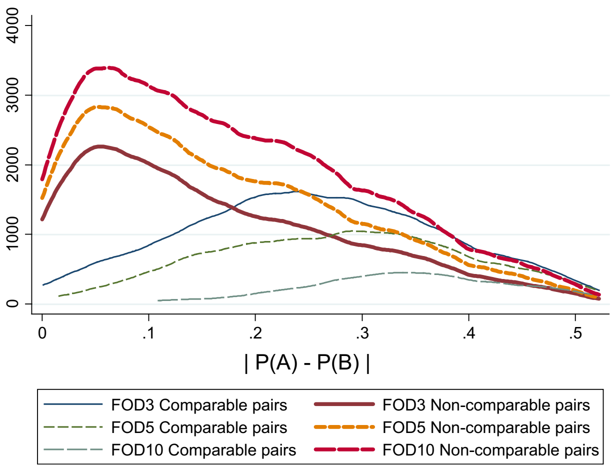

Fig. 6 Weighted density functions for the values of $\left|P_{1,1}(A) - P_{1,1}(B)\right|$ among FOD_k ranked and unranked couples for $k = 3$, 5 and 10. *Source*: Authors' calculations using DHS data

'countries' simulated in Sect. 3. As discussed at the end of that section, the larger the differences in welfare levels across countries the more likely we will observe FOD occurrences. In this regard, we could argue that the differences in poverty levels across our 48 countries might not be large enough to observe as many FOD relationships, an issue to which we now turn.

4.4 Comparison Between Poverty Indices and FOD Approaches

In this section we separately focus our attention on (1) the pairs of countries that *can* be ranked according to the FOD criterion (i.e. the sets C_{FOD3}, C_{FOD5} and C_{FOD10}) and, (2) the pairs of countries that *cannot* be ranked by FOD (i.e. the sets N_{FOD3}, N_{FOD5} and N_{FOD10}). As we did in Sect. 3.1, we compare the distribution of differences in poverty levels $|P_{1,1}(A) - P_{1,1}(B)|$ among country pairs belonging to C_{FOD_k} and to N_{FOD_k} separately. The corresponding scaled density functions (which are denoted as $f_{C,k}$ and $f_{N,k}$ respectively and are scaled by the number of comparable and incomparable country pairs respectively—recall the definitions given in 3.1) are shown in Fig. 6 when $k = 3, 5, 10$. As can be seen, the pairs belonging to N_{FOD_k} tend, on average, to have lower difference in poverty levels $|P_{1,1}(A) - P_{1,1}(B)|$ than those pairs belonging to C_{FOD_k} for all k. The means of $f_{N,k}$ and $f_{C,k}$ in Fig. 6 are 0.137 and 0.247 for $k = 3$, 0.143 and 0.283 for $k = 5$, 0.156 and 0.337 for $k = 10$. As expected, the higher the difference in poverty levels between countries A and B, the higher the probability that these countries can be ordered by FOD$_k$. However, there is a non-negligible number of pairs of countries for which a large difference in poverty levels is not enough to guarantee a concluding poverty assessment in terms of FOD$_k$, and similarly quite a number of country pairs where even small poverty differences results in FOD$_k$. Since higher values of k make the FOD$_k$ relationship more difficult to hold, it is not surprising to find that when they occur the differences in poverty levels tend to be higher as well. This is illustrated by the relative position of the scaled density functions $f_{C,3}$, $f_{C,5}$ and $f_{C,10}$ shown in Fig. 6, which move to the right as k increases and have a smaller area below them.

5 Discussion and Concluding Remarks

In this paper we have contrasted two approaches to the measurement of multidimensional poverty from an international perspective. Combining multiple scenario simulated data with observed data from 48 Demographic and Health Surveys across the developing world, we have investigated the possibility of implementing the so-called first order dominance techniques (FOD) for poverty analysis—a relatively new approach that does not rely on the host of debatable assumptions upon which poverty indices are typically based. Whenever a FOD relationship holds, the domination is robust to any weighting of the dimensions one is taking into account and does not make any assumption regarding the functional form of the underlying social welfare function. This empirical exercise is particularly relevant in a moment where there are intense debates in the international research community on the most appropriate way of conceptualizing and measuring poverty and that are taking place in the post-2015 Sustainable Development Goals framework. This paper attempts to highlight the advantages and disadvantages of currently existing methods to inform and illuminate these debates. To the best of our knowledge, there is currently no assessment of the degree of consistency that might exist between the different approaches in an international context[11]—an issue we have addressed in this work.

A well-known characteristic of dominance techniques in multivariate settings is their dependency on the number of dimensions one is taking into account: the higher it is, the higher the number of conditions that must be satisfied, so the more difficult it becomes for the results to be conclusive. Using simulated datasets we have quantified the prevalence of FOD occurrences as the number of binary variables increases from $k = 2$ to $k = 10$ in a variety of scenarios taking into account the shape of the between country welfare

[11] Iglesias et al. (2016) compare confirmatory factor analysis, the Alkire and Foster counting approach and the posetic approach in the context of contemporary Switzerland.

distribution and the correlation structure across variables. Our simulated data suggest that the correlation structure across variables strongly conditions the occurrence of FOD relationships (the higher the correlation, the higher the occurrence). In addition, we observe that the higher the difference in poverty levels across country pairs, the more likely they will be comparable by the FOD criterion. Interestingly, the general patterns observed in the simulated datasets are similar to the ones found in the DHS data from 48 countries (in the latter case, the occurrence of FOD relationships is lower than observed in the simulated datasets—an issue that is attributable to the complicated correlation structure among the 10 underlying variables and to the relatively small differences in poverty levels among the 48 countries). All in all, our findings suggest that the FOD techniques adapted to the analysis of multidimensional poverty generate a partial ordering between countries that is consistent with the orderings generated by well-known poverty indices such as UNDP's MPI but which might fail to be very informative when the number of dimensions is high (e.g. around $k = 10$).

The findings reported in this paper suggests that the FOD approach can be implemented as a useful robustness check for ordinal poverty indices like UNDP's MPI to distinguish between those country comparisons that are sensitive to alternative specifications of the measurement assumptions and those which are not. To the extent that the FOD approach is able to uncover the socio-economic gradient that exists between countries, it can be postulated as a viable complement to the MPI that has the advantage of not having to rely on many of the normatively binding assumptions that underpin the construction of UNDP's poverty index. Even if the goals of multidimensional poverty indices and the FOD approach are somewhat different—the former aim to measure precisely (i.e. cardinally) how many people are multiply deprived and by how much, while the latter aims at generating robust ordinal information regarding countries' relative standings—the results presented here provide ample support for the inclusion of FOD as a standard technique for researchers involved in distributional analysis. In addition, the FOD approach nicely complements other multidimensional ordinal techniques recently proposed in the literature of poverty measurement: the partially ordered set methods suggested by Fattore (2016) and the dominance techniques suggested by Yalonetzky (2014). Future research may explore the extent to which the FOD approach is also able to uncover territorial variations *within* countries to identify the regions where underdevelopment and social disadvantage are more entrenched. In this regard, it would be interesting to determine if the within-country assessments generated by the FOD approach are consistent with the ones generated by more widely used measures like the multidimensional poverty indices discussed in this paper.

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Appendix 1: Establishing the Existence of FOD

The applied algorithm to establish the existence of FOD relationships is based on a slight rephrasing of Arndt et al. (2015). Let A and B be two populations characterized by probability mass functions f and g respectively. For outcomes s and s′ with $s' \leq s$, let $t_{s,s'}$ be the amount of probability mass transferred from outcome s to s′. Note that the first sub-script denotes the source of the transfer whereas the second denotes the destination. Given the conditions outlined above, population A dominates population B if and only if there exists a feasible solution to the following linear problem

$$f(s) + \sum_{s' \geq s} t_{s',s} - \sum_{s' \leq s} t_{s,s'} = g(s) \forall s \in S, t_{s,s'} \geq 0, t_{s,s} = 0,$$

where S is the space of outcomes—an issue that is determined via the GAMS 23.0 software (see https://www.gams.com).

Appendix 2: The Multidimensional Binary Case

To illustrate the FOD definitions we focus on the case of different binary indicators that is applied in the empirical sections of the paper. While our empirical analysis explores the existence of FOD_k relationships between pairs of countries when k goes from 1 to 10, for simplicity we base our illustration on the 3-dimensional case. In this case, the space of outcomes S is a partially ordered set with $2^3 = 8$ elements ($S = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\}$) and the partial order given by the usual vector dominance $\leq$. Here (0,0,0) denotes the outcome where someone is deprived in all three dimensions simultaneously, (1,1,0) means that someone is only deprived in the third dimension, and so on. We now show the eleven inequalities (denoted as $I_1,\ldots,I_{11}$) derived from condition (3) in the definition of FOD that must be satisfied to conclude that, given two probability mass functions on S (f and g), one of them dominates the other (f FOD$_3$ g).

$$(I_1) g(0,0,0) \geq f(0,0,0)$$
$$(I_2) g(0,0,0) + g(1,0,0) \geq f(0,0,0) + f(1,0,0)$$
$$(I_3) g(0,0,0) + g(0,1,0) \geq f(0,0,0) + f(0,1,0)$$
$$(I_4) g(0,0,0) + g(0,0,1) \geq f(0,0,0) + f(0,0,1)$$
$$(I_5) g(0,0,0) + g(1,0,0) + g(0,1,0) + g(1,1,0) \geq f(0,0,0) + f(1,0,0) + f(0,1,0) + f(1,1,0)$$
$$(I_6) g(0,0,0) + g(1,0,0) + g(0,0,1) + g(1,0,1) \geq f(0,0,0) + f(1,0,0) + f(0,0,1) + f(1,0,1)$$
$$(I_7) g(0,0,0) + g(0,1,0) + g(0,0,1) + g(0,1,1) \geq f(0,0,0) + f(0,1,0) + f(0,0,1) + f(0,1,1)$$
$$(I_8) g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,1,0) + g(1,0,1) \\ \geq f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,1,0) + f(1,0,1)$$
$$(I_9) g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,1,0) + g(0,1,1) \\ \geq f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,1,0) + f(0,1,1)$$
$$(I_{10}) g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,0,1) + g(0,1,1) \\ \geq f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,0,1) + f(0,1,1)$$
$$(I_{11}) g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,1,0) + g(1,0,1) + g(0,1,1) \\ \geq f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,1,0) + f(1,0,1) + f(0,1,1).$$

The FOD concept is illustrated in Fig. 7 using three hypothetical welfare indicators denoted as I, II and III. The entries in the cells represent the probabilities for the joint distribution of all three indicators. We consider three hypothetical distributions: q, r and s. As can be seen, the percentage of individuals that are deprived in all three indicators at the same time in q, r and s are 20, 30 and 10 respectively. We see that q does not FOD r, and vice versa. This is because although $r(0,0,0) > q(0,0,0)$ giving $30 > 20$ (e.g. condition (I_1) is fulfilled), we also see that $r(0,0,0) + r(1,0,0) + r(0,1,0) + r(1,1,0) < q(0,0,0)$

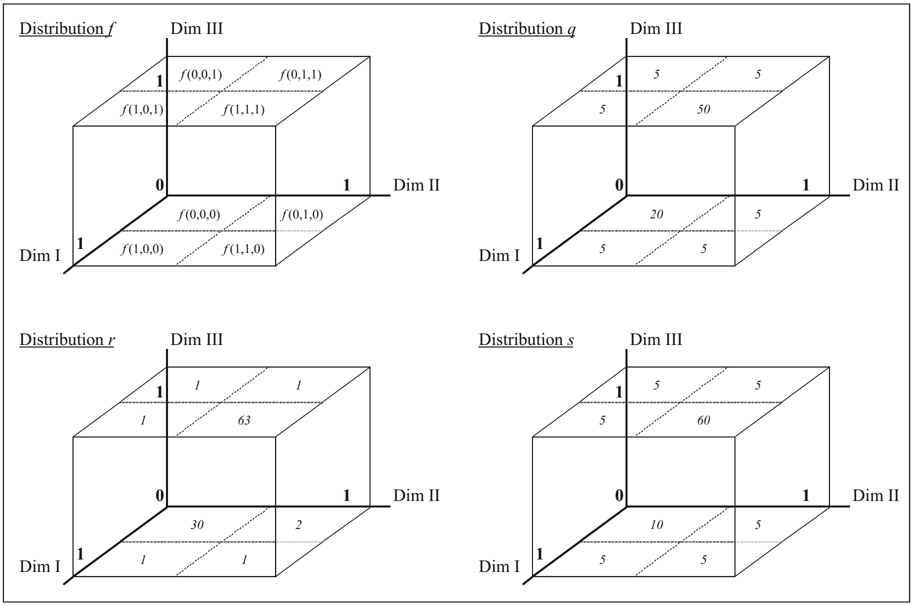

Fig. 7 Hypothetical welfare distributions q, r and s in the three dimensional binary case. *Notes*: Dimensions I, II and III are binary welfare indicators with 0 being a bad outcome and 1 being a good outcome. Numbers in italic are probabilities for the joint distribution of dimensions I–III. The 'floor' and the 'roof' represent bad respectively good outcome with respect to dimension III. The best simultaneous outcome is the lower right quadrant on the 'roof', while the worst is the upper left quadrant on the 'floor'

$+\ q(1,0,0) + q(0,1,0) + q(1,1,0)$ giving $34 < 35$, which is a violation of condition (I_5). The lack of FOD is illustrative of the fact that q and r alter rank depending on evaluation criteria. For instance q is better than r if the criterion is minimization of the group with members who are simultaneously worse off in all dimensions ($q(0,0,0) < r(0,0,0)$ giving $20 < 30$). But r is better than q if the criterion is instead maximization of population shares characterized by good outcomes in the three dimensions separately; shares with good outcomes in dimensions I–III is 66, 67 and 66% for distribution r, compared with 65% for each dimension in distribution q.

Looking next at welfare distributions r and s we also see that none dominates the other, e.g. lack of FOD. Condition (I_1) is fulfilled since $r(0,0,0) > s(0,0,0)$ (giving $30 > 10$), but the last condition (I_{11}) is not fulfilled ($37 < 40$). Again the problem that arises is that domination depends on evaluation criteria. Distribution s is better than q if the criterion is maximization of population shares characterized by good outcomes in the three dimensions separately; shares with good outcomes in dimensions I–III is 75% in each in distribution s, while the population shares are 66, 67 and 66% for distribution r. On the other hand r is better than s if we want to maximize the group that simultaneously does well on all three criteria, e.g. $r(1,1,1) > s(1,1,1)$ giving $63 > 60$.

The remaining comparison is between distributions q and s. If we insert Fig. 7 probabilities in conditions (I_1)–(I_{11}) we see all are met, and we can therefore conclude that s FOD q. To reach that conclusion we can also use the intuitive strategy where we move probability mass from better to worse to see if one distribution (the dominated one) can be generated from the other (the dominating one). In this case we just need to move 10% from the best outcome (1,1,1) to the worst outcome (0,0,0) in distribution s, which will result in distribution q.

Appendix 3: Comparison with Other Related Methods

The techniques analyzed in this paper bear some resemblance with other approaches recently proposed in the literature of multidimensional poverty in ordinal settings. For clarification purposes it will be useful to highlight what they have in common and what are the key differences between them. One of these approaches suggests using the theory of partially ordered sets to measure multidimensional deprivation (e.g. Fattore 2016) and the other proposes robust multidimensional poverty comparison techniques applied to ordinal data (see Yalonetzky 2014).

As regards the former, Annoni et al. (2011), Fattore et al. (2011) or Fattore (2016) among others have made great strides to apply partial order theory to better capture multidimensional poverty in ordinal settings. Like in this paper, individuals' achievements are assessed via $k \in \mathbb{N}$ ordinal variables, so evaluations are also based on the structure of the poset $(S, \leq)$. In a nutshell, the approach can be summarized as follows (for technical details, see Fattore 2016). First, a decision maker must establish deprivation thresholds, that is: identify what combinations of achievements constitute the unambiguously/completely deprived profiles. Second, an identification and severity functions assign the deprivation intensity associated to each achievement profile. These functions satisfy two consistency conditions: (1) if $p \in S$ and $q \in \downarrow p$, then the intensity of poverty in p is lower than the one in q; (2) deprivation thresholds are assigned the maximum deprivation degree. Lastly, a population level deprivation indicator is obtained aggregating individual deprivation levels.

Despite the limited structure of the original poset $(S, \leq)$ where many pairs of achievement profiles are incomparable in terms of vector dominance, the approach successfully generates a *complete* order that—unlike the FOD approach—allows comparing *all* possible pairs of countries (with each country represented as a probability mass function over S). The only assumption that has to be made to reach such completeness—which does not seem to be particularly restrictive—is that all the elements of the set of linear extensions[12] of S (denoted as $\Omega(S)$) are supposed to be equally important. Unfortunately, the huge size of $\Omega(S)$ when the number of achievement profiles in S gets larger severely limits the applicability of the approach: as of now, computations are feasible for posets with a few hundreds of elements[13] (Arcagni and Fattore 2014; Fattore 2016). In this regard, the applicability of the FOD approach is also restricted by computational considerations: in this paper we have dealt with up to $k = 10$ binary variables, thus resulting in posets with $2^{10} = 1024$ achievement profiles; see appendices 1 and 2 and footnote #5).

Another conceptually related approach has been recently suggested by Yalonetzky (2014). In that paper, the author presents the conditions that must be satisfied if one aims to conclude that the poverty levels associated with a multivariate ordinal distribution (A) are unambiguously lower than those of another distribution (B) *for all possible poverty thresholds Z and all possible weighting vectors W*. Yet, the results are only valid (i) when the function to identify the poor corresponds to the extreme union or intersection

[12] A *linear extension of* is a partial order that (i) is complete over (i.e. all pairs of elements are comparable), and (ii) respects the order established by vector dominance.

[13] Yet, imposing some simplifying assumptions (e.g. the k attributes can be completely ordered in terms of relevance) the computational time can be considerably reduced (see Fattore and Arcagni 2017).

approaches; and (ii) when several restrictions on the signs of the cross partial derivatives of the underlying social welfare function—some of which being particularly difficult to interpret—are satisfied. On the other hand, the FOD techniques make no assumption regarding (i) the choice of weights; and (ii) the behavior of the underlying social welfare function. The last point is particularly attractive because in many empirical applications it is unclear whether the different pairs of dimensions should be treated as complements or substitutes.

Summing up, both approaches are interesting on their own right and each of them has its own advantages and disadvantages. While Yalonetzky (2014) method allows dominance analysis to varying deprivation cutoffs and weights but imposes several conditions on the underlying social welfare function, the FOD approach does neither impose conditions on that function nor in the weights but is dependent on the choice of deprivation cutoffs. Since neither of the methods encompasses the other, they have the potential of being complementary tools that can be jointly applied in empirical analyses.

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