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Away from Dissatisfaction, Closer to Well-Being: A Multidimensional Synthetic Measure

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Abstract As part of the international debate on methods for measuring the social progress of a population, there has been increasing interest in individual subjective opinions about different aspects of quality of life (elementary indicators). In the literature, many methods have been introduced for producing measures of subjective well-being based on these opinions. Some of these methods aim to construct synthetic measures that allow us to consider all the aspects simultaneously. This topic often requires subjective methodological choices and/or distributional assumptions and, when the opinions are encoded by means of categorical ordinal values, the eventual quantification of the original variables. Here, starting from the Istat *multipurpose survey on households'—aspects of daily life*, we propose an original method for constructing a global satisfaction index. We introduce a variable based on the joint distribution of all the elementary indicators that is able to express the individual degree of global satisfaction. This approach allows us to maintain the original ordinal data scale and to avoid any aggregation formula. By comparing the observed distribution of the new variable and the theoretical one, which refers to the situation of overall dissatisfaction (all individuals are dissatisfied for every aspect), we propose three indices of global satisfaction. We also implemented two simulation studies that confirms both the efficacy and robustness of our method. We then applied it to measure the global satisfaction degree of the Italian population, using Istat *multipurpose survey* data for the year 2013.

Keywords Global satisfaction index · Individual opinions · Subjective well-being

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1 Introduction

As part of the international debate on methods for measuring the social progress of a population, there has been increasing interest in individual subjective opinions about several aspects of quality of life, especially due to the recommendations of the Final Report by the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz et al. 2009). In fact, also in the past economists had used data on self-reported life satisfaction (see, for example, Easterlin 1974; Di Tella et al. 2001; Alesina et al. 2005).

This topic concerns the more general problem of measuring complex phenomena that are not directly observable and hence approximated by a set of elementary (one-dimensional) indicators. Furthermore, individual subjective feelings have an intrinsic ordinal nature and are often encoded in a Likert scale (Likert 1932), going for example from ‘highly dissatisfied’ to ‘highly satisfied’, or vice-versa. These characteristics imply some methodological issues, for which there exists a wide literature.

In particular, several studies have focused on un-observability of subjective well-being and estimated the latent structure existing under individual opinions by means of latent variable models, item response theory (IRT) models, or combination of discrete uniform and shifted binomial distributions (CUB) models (see, for instance, Gambacorta et al. 2014; Iglesias et al. 2016; Ivaldi et al. 2016; Misajon et al. 2016; Rahman et al. 2004; Trinchera et al. 2008). However, these approaches have shown several drawbacks (see, among the others, Iglesias et al. 2016; Ivaldi et al. 2016; Maggino and Fattore 2011). Firstly, they are not very suitable for addressing a non-academic public (indeed, assessing well-being is often aimed at providing condensed information to policy makers). Secondly, they require both theoretical and methodological assumptions (for instance, local independence of items) that are not always fulfilled, in which case the obtained results may be of dubious reliability. Furthermore, they involve some pragmatic and methodological issues, such as for example: the existence of a not trivial correlation among elementary indicators; the choice of the number of latent factors to be extracted and their relevant relations with observed elementary indicators, hence their interpretability.

Many other studies have been devoted to obtain a synthetic measure of the latent phenomenon’s intensity starting from a set of elementary indicators, in aim to provide a synthetic information useful to analysts or policy makers for communication and for monitoring social changes. A frequent solution is to construct a composite indicator, which usually is a weighted average of the elementary indicators; there is a widely developed statistical literature on this subject (see among the others: OECD 2008; Saltelli 2007; Bird et al. 2005; Cox et al. 1992; Goldstein and Spiegelhalter 1996). Composite indicators show several advantages (for instance, they make easier comparisons among different times, spatial units and social groups; they are easily understandable even by a non-academic audience). However, at the same time they have some critical aspects (see, among the others: Freudenberg and Nardo 2003; Howell et al. 2007; Mazziotta and Pareto 2012; Munda and Nardo 2009; Saisana et al. 2005): they introduce some degree of arbitrariness, requiring subjective choices about normalisation, weighting and aggregation; furthermore, indicators to be synthesized are often heterogeneous and their aggregation could imply an information loss (actually, information loss always occurs when several variables are aggregated).

More recent studies have used original elementary indicators without aggregating them; this is the case, for instance, of the *Better Life Index* (OECD 2013, 2016), the proposals by Fattore and Maggino (2012, 2015) and by Casacci and Pareto (2015), and of the *BES* project (Istat 2014, 2015; specifically, in the 2015 edition, in addition to the analysis of each elementary indicator, there has been proposed some synthetic measures too for some domains).

Both the proposals by Fattore and Maggino and by Casacci and Pareto also dealt with the issue of intrinsic ordinal nature of data concerning subjective opinions. In the literature, a common solution is to quantify ordinal variables, with the idea that quantitative measures can be more precise. There are several quantification methods, such as transformation by expert ratings, estimation from item text and optimal scaling (Băltătescu 2002; Hensler and Stipak 1979; Herzel 1974; Casacci and Pareto 2014). However, several authors consider quantification not consistent with the ordinal nature of opinion data since they are intrinsically ‘characterized by nuances and ambiguities’ (Fattore et al. 2012). Furthermore, even if the attributes of categorical data can be ordered (by ranks), they cannot be interpreted on a numeric scale except for some particular cases (for instance, when it can be assumed that the distances between any two adjacent categories are equal). To avoid quantification, there are different solutions. A simple index of satisfaction can be obtained by dividing the number of people corresponding to the highest levels of satisfaction by the number of respondents. This index, however, underestimates the effective degree of satisfaction, since it does not consider people who declared intermediate levels of satisfaction. Another methodological solution is the aforementioned proposal by Fattore and Maggino (2012, 2015) in the framework of the Partial Order Set (POSET) theory. Specifically, they proposed to assess sequences of individual scores by exploiting the relational structure of data, involving solely the partial ordering of the profiles and no aggregation of the underlying variables. However, this approach seems to be quite cumbersome and requires a threshold identification. Recently, Casacci and Pareto (2015) proposed an interesting method. They considered data from the Istat *multipurpose survey on households—aspects of daily life* (Istat 2006) regarding citizens’ opinions about their level of satisfaction for some aspects of daily life. For each variable, the authors compared the percentage distribution of the Italian population in year t with the analogous distribution referring to a hypothetical population where all people were ‘dissatisfied’. Their idea was that, as the observed distribution was further from that of general dissatisfaction (all individuals are dissatisfied for that aspect), so the level of satisfaction would be higher for the aspect corresponding to that variable. Consequently, the authors proposed measuring the degree of satisfaction by calculating the distance between the two distributions. Specifically, they applied a quadratic index of dissimilarity for ordinal data (Leti 1983), based on the comparison between the two cumulative distribution functions (cdf). Moreover, they applied a normalization procedure to this index to evaluate how high the degree of satisfaction was, thus obtaining the corresponding normalized one with values between 0 and 1 (meaning, respectively, dissatisfaction and maximum satisfaction). They applied this index separately to each variable. This method seems preferable compared to other ones already known in the literature, since it does not need to quantify the values of ordinal variables and it takes into account all the declared different levels of satisfaction without aggregating them into one category (as other methods do). However, the Casacci and Pareto index only considers one aspect (variable) at a time, thus focusing on a one-dimensional perspective of well-being.

Actually, subjective well-being simultaneously concerns all the different aspects that affect the quality of life and that interact with each other. Consequently, an index devoted to measuring subjective well-being in terms of global satisfaction should take into account all the most important aspects of quality of life simultaneously.

Here, following the idea introduced by Guagnano and Sebastiani (2015), we propose an alternative approach. We consider a set of elementary indicators that are observed in a population. Each of these variables represents the individual degree of satisfaction about one of several aspects related to quality of life. In aiming to measure the global satisfaction for the entire population, we firstly define a new ordinal variable based on the joint distribution of all the elementary indicators, which measure the individual degree of global satisfaction. Then, analogously with Casacci and Pareto (2015), we measure the distance between the observed cdf of this new variable and the theoretical one, which refers to the situation of overall dissatisfaction (all individuals are dissatisfied for every aspect). We normalize the distance, thus obtaining an index which takes values between 0 and 1, where 0 and 1 mean, respectively, overall dissatisfaction and maximum overall satisfaction (all individuals show the highest level of satisfaction for every aspect).

Two simulation studies are devoted to verify efficacy and robustness of the proposed method. Finally, we apply the methodology to measure the degree of global satisfaction of the Italian population, using Istat data for the year 2013.

The paper is organized as follows: in Sect. 2 we explain our proposal, in Sect. 3 we discuss the results of the simulation studies, in Sect. 4 we show the outcome obtained from the empirical application on Istat data and, finally, some concluding remarks complete the paper.

2 The Proposed Method

Let us consider a set of elementary indicators, each representing the individual degree of satisfaction about a specific aspect of daily life. Let us suppose that each variable takes a finite number of ordered levels (for instance going from 'very satisfied' to 'dissatisfied') and that it is observed on a population. As a particular case, we can consider the four levels used in the Istat *multipurpose survey on households—aspects of daily life*: (1) 'very satisfied'; (2) 'quite satisfied'; (3) 'not very satisfied'; (4) 'dissatisfied'.

In order to define a global satisfaction index, an intuitive and simple approach could be to calculate the percentage of very or quite satisfied people for all aspects (VQS index, from now on). However, we thus underestimate the real degree of global satisfaction, since we exclude partially satisfied people (i.e. satisfied for at least one aspect but dissatisfied for the remaining ones). Alternatively, we could apply the Casacci and Pareto index (DISS) to each of the elementary indicators and then calculate the average of the obtained values; however, in this way we do not take into account all the information concerning the possible associations among the elementary indicators.

Let us consider the example with two variables illustrated in Table 1. Here, for the first indicator we have DISS = 0.735, while for the second one DISS = 0.624, and their average is equal to 0.680. In this distribution, 65% of people are very or quite satisfied (VQS index) for both indicators, whereas 20% are dissatisfied for at least one of them.

Let us now consider the distribution shown in Table 2. Here, DISS indices and their mean take the same values as before; however in this case the degree of global satisfaction is obviously lower. Indeed, VQS = 40%, whereas 25% people are dissatisfied for at least one indicator.

It is therefore evident that the average of the DISS values does not give correct and complete information about global satisfaction, because it does not consider the joint distribution of the elementary indicators and hence people who are only partially satisfied.

Table 1 Example of the joint distribution of two elementary indicators: 65% of people are very or quite satisfied for both indicators and 20% are dissatisfied for at least one indicator

Indicator 1 Indicator 2	Very satisfied	Quite satisfied	Not very satisfied	Dissatisfied	Total
Very satisfied	3	0	0	0	3
Quite satisfied	7	3	0	0	10
Not very satisfied	0	2	1	1	4
Dissatisfied	0	0	2	1	3
Total	10	5	3	2	20

Table 2 Example of the joint distribution of two elementary indicators: 40% of people are very or quite satisfied for both indicators and 25% are dissatisfied for at least one indicator

Indicator 1 Indicator 2	Very satisfied	Quite satisfied	Not very satisfied	Dissatisfied	Total
Very satisfied	0	0	3	0	3
Quite satisfied	3	5	0	2	10
Not very satisfied	4	0	0	0	4
Dissatisfied	3	0	0	0	3
Total	10	5	3	2	20

In trying to overcome these limitations and to maintain a multidimensional perspective, we could define a new variable that is able to express the individual degree of global satisfaction, since its values depend on the specific degree of satisfaction expressed for each of the elementary indicators simultaneously. We could then compare its observed distribution on a given population with the theoretical one corresponding to the situation of overall dissatisfaction.

To be more general, let K be the number of elementary indicators, M the number of the categories of each indicator, and n the number of individuals. Here we assume that the first and the M-th categories correspond, respectively, to the best and the worst judgements, as for instance in the case of the Istat *multipurpose survey* where $M = 4$. Starting from the K indicators X_1, X_2,…, X_k, let us define a new ordered variable 'individual degree of global satisfaction', named Y, based on the joint distribution of all the elementary indicators. In general, in this distribution there are M^K possible associations among the categories of all the variables; let us call A the whole set of these associations. Each element of A is related to a certain level of individual global satisfaction; at the same time, a certain level of individual global satisfaction could correspond to several associations. There are different ways of defining Y as a function of X_1, X_2,…, X_k; this topic is afforded with more detail in the next sub-section.

To be general, let Y be a variable with L ordered levels. We need to keep separate the case of overall dissatisfaction from all the others, in order to distinguish between the observed distribution of Y and the theoretical one corresponding to overall dissatisfaction. Hence, let us indicate with LS the number of Y levels corresponding to a degree of satisfaction for all aspects, and with LPS the number of Y levels corresponding to partial satisfaction; obviously, $L = LS + LPS + 1$.

Let f_l and F_l be, respectively, the relative frequency and the corresponding cumulative one for the l-th level of Y, $1 \leq l \leq L$. Furthermore, let f_l^* and F_l^* be the analogous quantities for the aforesaid theoretical distribution, where: $f_l^* = F_l^* = 0$ for $l \neq L$, while $f_L^* = F_L^* = F_L = 1$.

To appreciate the contribution given by partially satisfied people to the real degree of global satisfaction, we can proceed as follows. Firstly, we define the index IS_1, that takes into account only people who are satisfied (at any level) for all aspects, as DISS index does. It consequently considers only the first LS levels of Y. Then we integrate information about global satisfaction, taking into account also people who are only partially satisfied (i.e. the further LPS levels of Y), so introducing the IS_2 index.

Following the approach of Casacci and Pareto, IS_1 can be so defined:

$$0 \leq IS_1(h) = \sqrt[h]{\sum_{l=1}^{LS} \left|F_l - F_l^*\right|^h} = \sqrt[h]{\sum_{l=1}^{LS} F_l^h} \leq \sqrt[h]{LS} \tag{1}$$

and the corresponding normalized one:

$$0 \leq is_1(h) = \sqrt[h]{\frac{\sum_{l=1}^{LS} F_l^h}{LS}} \leq 1, \tag{2}$$

where the exponent h is a non-zero real number.

To define IS_2 index, we add the term $\sum_{l=LS+1}^{L-1} F_l^h$ under the h-th root, so obtaining:

$$0 \leq IS_2(h) = \sqrt[h]{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} F_l^h} \leq \sqrt[h]{L-1} \tag{3}$$

and the corresponding normalized index:

$$0 \leq is_2(h) = \sqrt[h]{\frac{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} F_l^h}{L-1}} \leq 1. \tag{4}$$

However, even this last proposal is not completely satisfying, since it is not always possible to discriminate among different situations of partial satisfaction, corresponding to a different number of satisfying aspects. For instance, for $K = 3$, IS_2 and is_2 take the same values in two populations that differ each other in the satisfaction profile of only one individual:

1. in the first population, the individual is dissatisfied for only one aspect and satisfied for the other two;
2. in the second population, the individual is dissatisfied for two aspects and satisfied for the other one;

actually, the degree of global satisfaction should be higher in case 1.

A possible solution is to consider explicitly the number of satisfying aspects into the frequencies f_l corresponding to the Y levels of partial satisfaction ($LS + 1 \leq l \leq L - 1$). So we decompose f_l into the $K - 1$ terms: $f_{l.1}, f_{l.2}, \ldots, f_{l.K-1}$, indicating the relative frequency of people who are satisfied for only 1 aspect, or for 2 aspects, or for $(K - 1)$ aspects, respectively. Such terms should not have the same importance and, for this reason, we introduce a set of corresponding weights. A simple and natural choice consists in defining them equal to the respective proportion of satisfying aspects among the K considered; then we have:

- when $l = 1, 2, \ldots, LS$: $w_l = K/K = 1$;
- when $l = LS + 1, \ldots, L - 1$:
  for $f_{l.1}$, $w_{l.1} = 1/K$; for $f_{l.2}$, $w_{l.2} = 2/K$; …; for $f_{l.K-1}$, $w_{l.\ K-1} = (K-1)/K$.

Then, another index of global satisfaction could be:

$$\begin{aligned} 0 \le IS_3(h) &= \sqrt[h]{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} F_l^h} = \sqrt[h]{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} \left(F_{LS} + \sum_{j=LS+1}^{l} f_j\right)^h} \\ &= \sqrt[h]{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} \left(F_{LS} + \sum_{j=LS+1}^{l} \sum_{i=1}^{K-1} f_{j\cdot i} w_{j\cdot i}\right)^h} \\ &= \sqrt[h]{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} \left(F_{LS} + \sum_{j=LS+1}^{l} \left(f_{j\cdot 1}\frac{1}{K} + f_{j\cdot 2}\frac{2}{K} + \cdots + f_{j\cdot K-1}\frac{K-1}{K}\right)\right)^h} \\ &\le \sqrt[h]{L-1} \end{aligned} \tag{5}$$

and the corresponding normalized one:

$$0 \le is_3(h) = \sqrt[h]{\frac{\sum_{l=1}^{LS} F_l^h + \sum_{l=LS+1}^{L-1} \left(F_{LS} + \sum_{j=LS+1}^{l} \sum_{i=1}^{K-1} f_{j.i} w_{j.i}\right)^h}{L-1}} \le 1. \tag{6}$$

The last proposal allows us to overcome the aforementioned limitations of the previous two indices.

2.1 Definition of the ‘Individual Degree of Global Satisfaction’ Variable

As highlighted in the previous section, there are different ways of defining Y as a function of $X_1, X_2, \ldots, X_K$. Here we cope with the problem of choosing the most suitable definition.

Firstly, in aim to discriminate among different levels of global satisfaction coherently with the elementary indicators, we require that Y takes exactly M ordered categories. Even in this context, there are different ways to define Y with M levels. Trying to generalize the idea of Casacci and Pareto (2015), we propose: $Y = \max[\text{rank}(X_1), \ldots, \text{rank}(X_K)]$. Indeed, by means of this function, Y takes values in the following way:

$Y = 1$, corresponding to the highest level of global satisfaction, when all the elementary indicators get the highest level of satisfaction, as it should reasonably happen;

$Y = 2$, the second highest level of global satisfaction, when all the elementary indicators get at least the second highest level of satisfaction (excluding the case $Y = 1$);

…;

$Y = m$, the m-th highest level of global satisfaction, when all the elementary indicators get at least the m-th highest level of satisfaction (excluding the cases $Y = 1, \ldots, m$-1);

…;

$Y = M,$ the lowest level of global satisfaction, if at least one indicator gets the level of dissatisfaction.

Since we need to keep separate the case of overall dissatisfaction from all the others, we so modify the last level $Y = M$, and add a further one:

$Y = M$, the lowest level of global satisfaction, if at least one indicator gets the level of dissatisfaction, but not all together;

$Y = M + 1$, the level of overall dissatisfaction, if all the indicators get the level of dissatisfaction.

Formally, the Y variable can be so defined:

$$\begin{cases} Y = M + 1, & \text{if } \operatorname{rank}(X_1) = \cdots = \operatorname{rank}(X_K) = M \\ Y = \max[\operatorname{rank}(X_1), \ldots, \operatorname{rank}(X_K)], & \text{otherwise.} \end{cases}$$

Note that the Y variable is only apparently numerical; indeed it is intrinsically ordinal, its first M levels corresponding to those of the elementary indicators.

An example of this variable when $K = 2$ and $M = 4$ is illustrated in Table 3. From this example, it is also evident that the first $M - 1 = 3$ levels of Y correspond to a degree of satisfaction for all aspects, while the last two levels represent, respectively, partial satisfaction (i.e. $Y = M = 4$) and overall dissatisfaction (i.e. $Y = M + 1$=5).

Someone could disagree with this definition, considering too arbitrary the ordering of Y categories. For example, in Table 3 he could criticize that the level of global satisfaction corresponding to the association (‘quite satisfied’, ‘quite satisfied’) is higher than the one corresponding to (‘very satisfied’, ‘not very satisfied’) and to (‘not very satisfied’, ‘very satisfied’). In fact, he could assert that the discrepancy between ‘quite satisfied’ and ‘very satisfied’ for one indicator is compensated by the discrepancy between ‘quite satisfied’ and ‘not very satisfied’ for the other indicator. Hence, these associations should be considered equivalent, as it occurs for instance in Fattore et al. (2012, 2015). Then, a possible alternative definition could be $Y_{SUM} = \operatorname{rank}(X_1) + \cdots + \operatorname{rank}(X_K)$. However, such a solution should imply that Y_{SUM} is effectively a numerical variable (while we would avoid it). Secondly, this approach requires the implicit assumption that, for every elementary indicator, all distances between any two consecutive categories are equal each other. This assumption is likewise arbitrary, given the ordinal nature of data. Finally, this variable takes in general $(M - 1)K + 1$ different categories, a value that could become even large as K raises. For instance, when $M = 4$ and K varies from 2 to 4, Y_{SUM} respectively takes 7,

Table 3 Definition of the Y variable ‘degree of global satisfaction’ in the case of two four-order-levels indicators: example with five distinct levels of satisfaction

Indicator 1 Indicator 2	Very satisfied	Quite satisfied	Not very satisfied	Dissatisfied
Very satisfied	**Y = 1**			
Quite satisfied		**Y = 2**		**Y = 4**
Not very satisfied			**Y = 3**	
Dissatisfied	**Y = 4**			**Y = 5**

10 and 13 different values, whereas in the situation that will be considered in Sect. 4 (where $K = 7$) Y_{SUM} even gets 22 different levels.

Also starting from our proposal of Y, we could further detail the levels of global satisfaction by defining alternative global satisfaction variables with more categories. At this regard, two possible examples, again with $K = 2$ and $M = 4$, are shown in Tables 4 and 5. Specifically, in the first table we introduce the Y^* variable with seven levels: the first five correspond to a degree of satisfaction for all aspects, whereas the last two represent, respectively, partial satisfaction (i.e. $Y^* = 6$) and overall dissatisfaction ($Y^* = 7$). In the second table, we propose the Y^{**} variable that takes ten levels: the first six correspond to a degree of satisfaction for all aspects, while the last four represent, respectively, partial satisfaction (i.e. $Y^{**} = 7, 8, 9$) and overall dissatisfaction ($Y^{**} = 10$). Specifically, the last table corresponds to the most complex situation, where the number of levels of the global satisfaction variable coincides with the number of the K-combinations with repetitions from the set $\{1, 2,\ldots, M\}$.

It is worth noting that, by further detailing the level of global satisfaction compared to Y with $M + 1$ levels, formulas (1)–(6) could coherently modify. Consequently, the measure of the degree of global satisfaction could change. The choice of the definition of the global satisfaction variable therefore becomes crucial.

Hereafter we show how the measure of the degree of global satisfaction changes by varying the definition of this variable. Since the third index is the most complex and complete compared to the other two, we decided to base the following discussion on it only.

By applying formula (5) to Table 3 (i.e., Y with $M + 1 = 5$ levels), we have:

$$0 \le {}_{Y}IS_3(h) = \sqrt[h]{{}_{Y}F_1^h + {}_{Y}F_2^h + {}_{Y}F_3^h + {}_{Y}F_4^h} \le \sqrt[h]{4}. \tag{7}$$

Let us now consider the Y^* variable of Table 4 (with 7 levels). We have:

$$\begin{aligned} 0 \le {}_{Y^*}IS_3(h) &= \sqrt[h]{{}_{Y^*}F_1^h + {}_{Y^*}F_2^h + \cdots + {}_{Y^*}F_6^h} \\ &= \sqrt[h]{{}_{Y}F_1^h + {}_{Y}F_2^h + {}_{Y}F_3^h + {}_{Y}F_4^h + ({}_{Y}F_2 - {}_{Y^*}f_3)^h + ({}_{Y}F_3 - {}_{Y^*}f_5)^h} \le \sqrt[h]{6} \end{aligned} \tag{8}$$

being:

Table 4 Definition of the Y^* variable 'degree of global satisfaction' in the case of two four-order-levels indicators: example with seven distinct levels of satisfaction

<table>
<tr><th>Indicator 1
Indicator 2</th><th>Very satisfied</th><th>Quite satisfied</th><th>Not very satisfied</th><th>Dissatisfied</th></tr>
<tr><td>Very satisfied</td><td>$Y^* = 1$</td><td>$Y^* = 2$</td><td rowspan="2">$Y^* = 4$</td><td rowspan="3">$Y^* = 6$</td></tr>
<tr><td>Quite satisfied</td><td>$Y^* = 2$</td><td>$Y^* = 3$</td></tr>
<tr><td>Not very satisfied</td><td colspan="2">$Y^* = 4$</td><td>$Y^* = 5$</td></tr>
<tr><td>Dissatisfied</td><td colspan="3">$Y^* = 6$</td><td>$Y^* = 7$</td></tr>
</table>

Table 5 Definition of the Y^{**} variable 'degree of global satisfaction' in the case of two four-order-levels indicators: example with ten distinct levels of satisfaction

Indicator 1 Indicator 2	Very satisfied	Quite satisfied	Not very satisfied	Dissatisfied
Very satisfied	$Y^{**}=1$	$Y^{**}=2$	$Y^{**}=4$	$Y^{**}=7$
Quite satisfied	$Y^{**}=2$	$Y^{**}=3$	$Y^{**}=5$	$Y^{**}=8$
Not very satisfied	$Y^{**}=4$	$Y^{**}=5$	$Y^{**}=6$	$Y^{**}=9$
Dissatisfied	$Y^{**}=7$	$Y^{**}=8$	$Y^{**}=9$	$Y^{**}=10$

$$
\begin{aligned}
&{}_{Y^*}F_1 \equiv {}_YF_1,\ {}_{Y^*}F_2 = {}_YF_2 - {}_{Y^*}f_3,\ {}_{Y^*}F_3 \equiv {}_YF_2,\\
&{}_{Y^*}F_4 = {}_YF_3 - {}_{Y^*}f_5,\ {}_{Y^*}F_5 \equiv {}_YF_3,\ {}_{Y^*}F_6 \equiv {}_YF_4.
\end{aligned}
$$

Since the last two addends in formula (8) are non-negative, it results that: ${}_{Y^*}IS_3(h) \geq {}_YIS_3(h)$.Let us now consider the Y^{**} variable corresponding to Table 5 (with 10 levels). We have:

$$
\begin{aligned}
0 \leq {}_{Y^{**}}IS_3(h) &= \sqrt[h]{{}_{Y^{**}}F_1^h + {}_{Y^{**}}F_2^h + \cdots + {}_{Y^{**}}F_6^h}\\
&= \Big[{}_{Y^*}F_1^h + {}_{Y^*}F_2^h + {}_{Y^*}F_3^h + {}_{Y^*}F_4^h + {}_{Y^*}F_5^h + {}_{Y^*}F_6^h\\
&\quad + \left({}_{Y^*}F_4 - {}_{Y^{**}}f_5\right)^h + \left({}_{Y^*}F_6 - {}_{Y^{**}}f_8 - {}_{Y^{**}}f_9\right)^h + \left({}_{Y^*}F_6 - {}_{Y^{**}}f_9\right)^h\Big]^{\frac{1}{h}} \leq \sqrt[h]{9}.
\end{aligned} \tag{9}
$$

being:

$$
\begin{aligned}
&{}_{Y^{**}}F_1 \equiv {}_{Y^*}F_1,\ {}_{Y^{**}}F_2 = {}_{Y^*}F_2,\ {}_{Y^{**}}F_3 \equiv {}_{Y^*}F_3,\ {}_{Y^{**}}F_4 = {}_{Y^*}F_4 - {}_{Y^{**}}f_5,\\
&{}_{Y^{**}}F_5 = {}_{Y^*}F_4,\ {}_{Y^{**}}F_6 \equiv {}_{Y^*}F_5,\ {}_{Y^{**}}F_7 = {}_{Y^*}F_6 - {}_{Y^{**}}f_8 - {}_{Y^{**}}f_9,\\
&{}_{Y^{**}}F_8 = {}_{Y^*}F_6 - {}_{Y^{**}}f_9,\ {}_{Y^{**}}F_9 \equiv {}_{Y^*}F_6.
\end{aligned}
$$

It is evident that: ${}_{Y^{**}}IS_3(h) \geq {}_{Y^*}IS_3(h) \geq {}_YIS_3(h)$, for every value of h.

In other words, the absolute index of global satisfaction tends to increase as the number of global satisfaction levels increases. In particular, the difference among the values of $IS_3(h)$ corresponding to the three different definition of the variable is mostly stressed when the population is highly satisfied and is less evident when the population is partially satisfied. As a matter of fact, in the first case the percentage of individuals very or quite satisfied for all the aspects is the highest one and by definition it is included in all the addends (that are cdf's) of the summations in formulas (7)–(9). In the second case, the percentage of individuals partially satisfied is the highest one and by definition it is counted only few times in the aforementioned formulas. In normalizing $IS_3(h)$, the denominator of $is_3(h)$ increases with the number of the variable levels. However, in the case of highly satisfied population, often the increase in the denominator does not balance that in the numerator and hence the same relation existing among the values of $IS_3(h)$ could even hold for those of $is_3(h)$. Instead, in the case of partially satisfied population, the increase in the denominator often exceeds that in the numerator and hence the values of $is_3(h)$ could decrease as the number of the variable levels increases.

Until now, we have shown how formulas (5)–(6) modify by changing the variable definition, limited to the case of 2 elementary indicators. As K increases, in the most complex situation where the number of variable levels coincides with the number of the K-combinations with repetitions from the set $\{1, 2,\ldots, M\}$, the variable takes a plenty of categories compared to the case of Y with $M + 1$ levels. Consequently, even the differences between the values of $IS_3(h)$ corresponding to the two extreme situations gradually magnify as K increases. For $is_3(h)$ we can suggest remarks analogous to those aforesaid in the case $K = 2$.

Taking into account these last considerations, the most complex situation could seem preferable with respect to the other ones, since it is more sensitive to different schemes of satisfaction: indeed, it implies higher values of $is_3(h)$ in the case of highly satisfied population and lower values in the case of partially satisfied population. However, some drawbacks derive from using the variable corresponding to the most complex situation.

In particular, we can firstly observe that for $K \geq 3$ the ordering of the variable categories might become cumbersome and even more arbitrary than in the aforementioned examples of Tables 4 and 5. For example, considering the case where $K = 3$ and $M = 4$, the most complex situation implies that the global satisfaction variable assumes 20 values, whose ordering is not a trivial task.

Furthermore, except for the simplest situation (Y with $M + 1$ levels), the number of categories rises as K increases, sometimes even significantly (just starting from $K \geq 3$) and unnecessarily with an effective informative advantage (is it indeed useful to define the degree of global satisfaction with 20 or 35 or more levels?).

Based on all these considerations, we believe that a reasonable choice is to use the Y variable with $M + 1$ levels.

Applying formulas (2), (4) and (6) to this variable, we have:

$$0 \leq is_1(h) = \sqrt[h]{\frac{\sum_{l=1}^{M-1} F_l^h}{M-1}} \leq 1, \tag{10}$$

$$0 \leq is_2(h) = \sqrt[h]{\frac{\sum_{l=1}^{M-1} F_l^h + (F_{M-1} + f_M)^h}{M}} \leq 1, \tag{11}$$

$$0 \leq is_3(h) = \sqrt[h]{\frac{\sum_{l=1}^{M-1} F_l^h + \left(F_{M-1} + \sum_{i=1}^{K-1} f_{M.i} \cdot w_{M.i}\right)^h}{M}} \leq 1. \tag{12}$$

3 Simulation Studies

To evaluate the performance of the normalized indices, verifying their efficacy (that is, whether they are able to discriminate among different satisfaction schemes by assuming corresponding coherent values) and even their robustness with respect to varying number of elementary indicators, we carried out two different simulation studies.

In the first one, we verified efficacy. Here, to avoid any problems related to computational complexity, we considered a situation with three elementary indicators ($K = 3$).

Hence, we defined the following six hypothetical populations (satisfaction schemes):

1. highly satisfied population: the proportion of at least quite satisfied people for all aspects is higher than that of not very satisfied people for at least one aspect, which in turn is higher than the proportion of dissatisfied people for at least one aspect (but not for all). Furthermore, we distinguished between these two cases:
   - 1.1 the proportion of very satisfied people for all aspects is higher than that concerning quite satisfied people for at least one aspect: $\mathrm{P}(Y = 1) > \mathrm{P}(Y = 2) > \mathrm{P}(Y = 3) > \mathrm{P}(Y = 4)$;
   - 1.2 the proportion of very satisfied people for all aspects is lower than that concerning quite satisfied people for at least one aspect:$\mathrm{P}(Y = 2) > \mathrm{P}(Y = 1) > \mathrm{P}(Y = 3) > \mathrm{P}(Y = 4)$;
2. partially satisfied population: the proportion of quite or not very satisfied people for at least one aspect prevails over the other two proportions. Since partial satisfaction can mean low satisfaction or a propensity towards dissatisfaction, we distinguished between these two cases:
   - 2.1 the proportion of quite satisfied people for at least one aspect is higher than that of not very satisfied people for at least one aspect, which in turn is higher than that of very satisfied people for all aspects, which in turn is higher than that of dissatisfied people for at least one aspect (but not for all): $\mathrm{P}(Y = 2) > \mathrm{P}(Y = 3) > \mathrm{P}(Y = 1) > \mathrm{P}(Y = 4)$;
   - 2.2 the proportion of not very satisfied people for at least one aspect is higher than that of quite satisfied people for at least one aspect, which in turn is higher than that of dissatisfied people for at least one aspect (but not for all), which in turn is higher than that of very satisfied people for all aspects: $\mathrm{P}(Y = 3) > \mathrm{P}(Y = 2) > \mathrm{P}(Y = 4) > \mathrm{P}(Y = 1)$;
3. poorly satisfied population: the proportion of at most not very satisfied people for at least one aspect (but not dissatisfied for all) is higher than that of quite satisfied people for at least one aspect, which in turn is higher than the proportion of very satisfied people for all aspects. Furthermore, we distinguished between these two cases:
   - 3.1 the proportion of dissatisfied people for at least one aspect (but not for all) is higher than that concerning not very satisfied people for at least one aspect: $\mathrm{P}(Y = 4) > \mathrm{P}(Y = 3) > \mathrm{P}(Y = 2) > \mathrm{P}(Y = 1)$;
   - 3.2 the proportion of dissatisfied people for at least one aspect (but not for all) is lower than that concerning not very satisfied people for at least one aspect: $\mathrm{P}(Y = 3) > \mathrm{P}(Y = 4) > \mathrm{P}(Y = 2) > \mathrm{P}(Y = 1)$.

From a more general point of view, in generating the values for each of the three indicators coherently with the six aforementioned schemes, we considered both the situations where elementary variables are independent from each other and the one where they are not.

We generated 1000 random samples of $n = 1000$ units from each of the six hypothetical populations. For each generated sample and for several values of the h power ($h = 1, 2, \ldots, 5$), we calculated the three normalized indices (10), (11) and (12).

Figures 1, 2 and 3 show the corresponding median values of the obtained sample distributions. For each index, the median values are always coherent with the population schemes. Specifically, these values decrease going from the case 1.1 to 1.2, to 2.1, to 2.2, to 3.2, to 3.1. This result confirms the efficacy of the proposed method.

Moreover, for all the indices the median values rise as h increases.

Finally, by comparing the three indices, we can observe that:

- the is_1 index is always lower than is_2 and is_3, as we might expect, since it does not take into account people who are only partially satisfied (i.e. satisfied for at least one aspect but dissatisfied for the remaining ones);
- is_1 shows a worse performance than is_2 and is_3, being totally unable to discriminate between dissatisfied people and poorly satisfied population. In fact, in case 3.1 the median values of is_1 are identically zero (see Fig. 3), whereas we expect that a global satisfaction index be null only in the case of overall dissatisfaction;
- is_2 and is_3 present very similar patterns, especially in the cases of high and partial satisfaction;
- is_3 is always halfway between is_1 and is_2. This result confirms the better performance of is_3, since it avoids underestimating the degree of global satisfaction (as is_1 does) and overestimating it (as is_2 does in situations of partial satisfaction, since it does not take into account the number of satisfying aspects).

In short, all the three indices appear efficacious, that is able to discriminate among different satisfaction schemes; however, is_3 seems to be preferable to is_1 and is_2.

In the second simulation study, we coped with the robustness of the normalized indices with respect to the number of elementary indicators. In particular, we decided to limit the analysis to is_3, since it is the most complex and complete compared to the other two (as already noted in Sect. 2.1) and it showed itself the most efficacious too.

The idea consists in applying formula (12) to a set of K elementary indicators, from which we gradually omitted one of them, leaving at the end only two indicators. More precisely, we considered all the possible subsets containing k' indicators ($2 \leq k' \leq K$) and

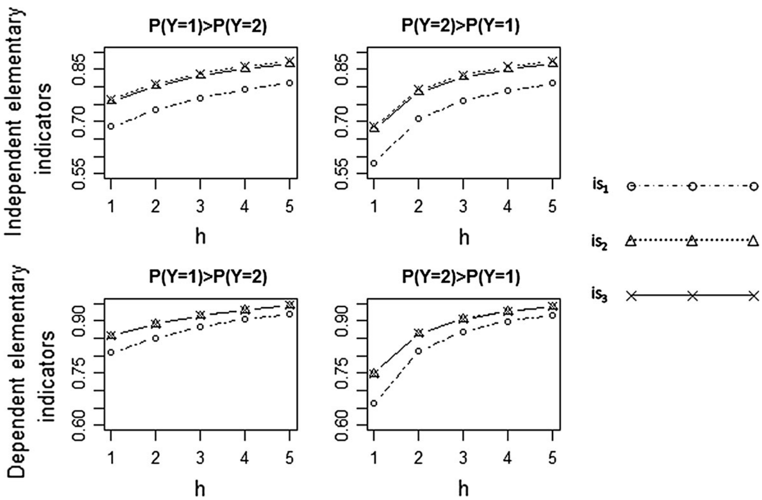

Fig. 1 Efficacy analysis: median of the sample distributions of is_1, is_2 and is_3 (independence and dependence schemes)—highly satisfied population

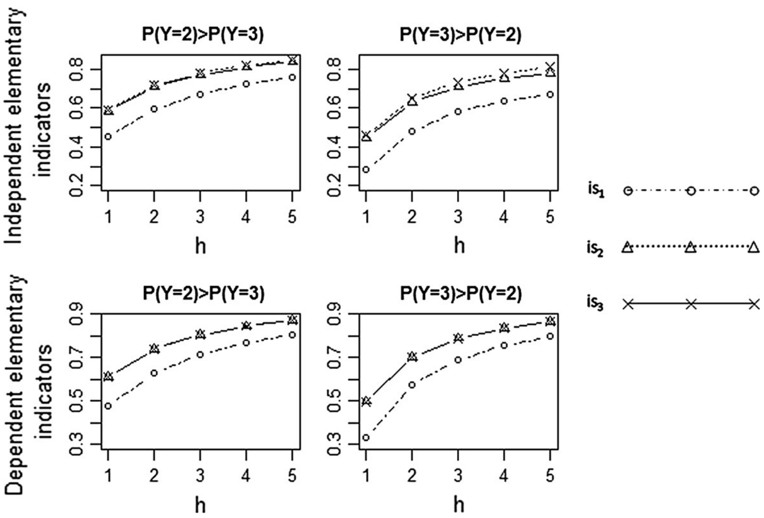

Fig. 2 Efficacy analysis: median of the sample distributions of is_1, is_2 and is_3 (independence and dependence schemes)—partially satisfied population

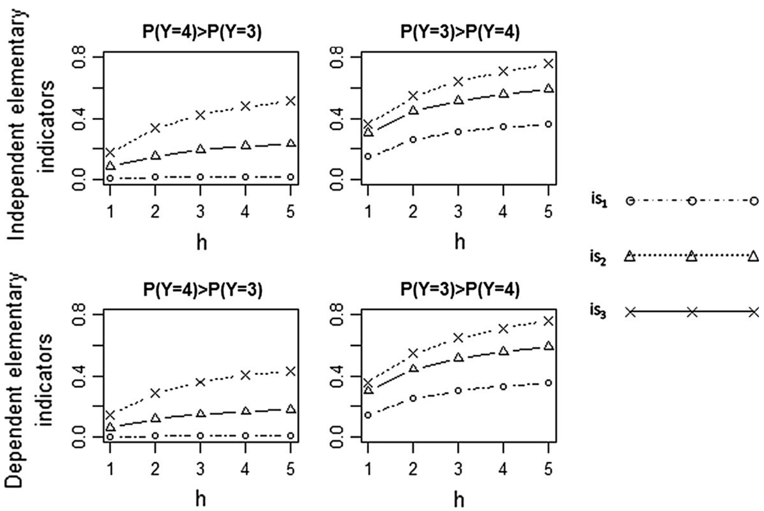

Fig. 3 Efficacy analysis: median of the sample distributions of is_1, is_2 and is_3 (independence and dependence schemes)—poorly satisfied population

for each of them we calculated the index, with $h = 1, 2, \ldots, 5$. Finally, we observed the behavior of is_3 values as k' varies.

In choosing the value of K, here we preferred to consider the same number of elementary indicators used in the empirical analysis of Sect. 4, that is $K = 7$. We generated a random sample of $n = 10{,}000$ units coherently with the aforementioned satisfaction schemes introduced in the previous efficacy analysis. However, for the sake of simplicity, here we considered only the situations 1.1 (highly satisfied population), 2.2 (partially satisfied population) and 3.1 (poorly satisfied population), and the case of independent elementary variables.

In Table 6 we show the mean values of is_3, as the number of included indicators varies. The above considerations about the behavior of is_3 as h increases still hold. On the other side, for each satisfaction scheme and for each h value, the index takes increasing values with decreasing k', as we could expect. In fact, since the number of possible associations among the categories of k' elementary indicators is $M^{k'}$ (see Sect. 2), as k' increases the probability of each association in the joint distribution becomes lower. This obviously also occurs for the associations related to the lowest levels of Y variable (very or quite satisfied people), whose corresponding cdf values are included in all the addends of is_3 formula. Notwithstanding, the changes in is_3 values are quite small (especially if $h \geq 2$) and regularly decrease going from situation 1.1 to situation 3.1. In particular, percent variations referred to two consecutive decreasing values of k' are comprised between: +1.46% and +6.44% in the scheme 1.1; +1.06% and +3.78% in the scheme 2.2; +0.02% and +2.1% in the scheme 3.1 (except for the case with $h = 1$ and $k' = 2$, where the variation is higher, due to negligible values of the index). Obviously, if percent variations are all calculated with respect to the case with $K = 7$ indicators (i.e. the first column of Table 6), their values become bigger. In particular, they are comprised between: +1.46% and +31.73% in the scheme 1.1; +1.06% and +12.19% in the scheme 2.2; +0.02% and +2.21% in the scheme 3.1 (except for the case with $h = 1$ and $k' = 2$, as for the previous variations).

These results seem to be quite comfortable and denote a substantially robustness of is_3.

4 The Analysis of Istat Data from *Multipurpose Survey on Households—Aspects of Daily Life*

To measure the degree of global satisfaction of the Italian population, we used the Istat microdata for the year 2013. To choose the aspects (variables) to be analysed, we implemented the sixth recommendation of the Final Report by the Commission on the Measurement of Economic Performance and Social Progress (Stiglitz et al. 2009): "quality of life depends on people's health and education, their everyday activities (which include the right to a decent job and housing), their participation in the political process, the social and natural environment in which they live, and the factors shaping their personal and economic security".

Consequently, we considered the following seven aspects: economic conditions, health, family relations, relationships with friends, free time, environmental conditions and work. Unfortunately, this survey does not provide data on satisfaction with other non-economic aspects.

Table 7 shows the cumulative relative frequencies for each of the seven variables. In general, Italian people appear satisfied, at least partially, for every considered aspect (on average on all variables, the satisfied account for 94.42% of the population); as a matter of fact, the percentage of dissatisfied people is relatively low, varying from 1.17% (for family relations) to 7.64% (for free time), except for economic conditions where it reaches 15.79%. The fact that economic conditions and free time are the most critical aspects could depend on the effects of the current economic crisis; moreover, for free time another reason for dissatisfaction could be the difficulty in conciliating work and family care, above all for women. The most satisfying aspect is family relations, for which the proportion of highly satisfied people (VQS index) is 93.16%. The second one is health (87.85%), in descending order followed by: relationships with friends (86.55%), environmental conditions (74.8%), work (74.38%), free time (60.69%) and economic conditions (46.56%). The last raw of Table 7

Table 6 Robustness analysis: mean values of is_3, according to the number of elementary indicators and to satisfaction schemes-percent values

Satisfaction scheme	h Power	Number of elementary indicators					
		7 (%)	6 (%)	5 (%)	4 (%)	3 (%)	2 (%)
Highly satisfied population (1.1)							
	1	65.66	68.90	72.54	76.63	81.26	86.49
	2	70.42	72.79	75.53	78.76	82.59	87.15
	3	74.04	75.84	77.98	80.57	83.76	87.76
	4	76.82	78.24	79.95	82.08	84.78	88.30
	5	78.99	80.15	81.56	83.34	85.66	88.79
Partially satisfied population (2.2)							
	1	42.92	43.71	44.52	45.39	46.40	48.16
	2	61.14	62.13	63.17	64.27	65.44	66.71
	3	69.10	70.08	71.13	72.28	73.52	74.86
	4	73.70	74.60	75.60	76.72	77.95	79.32
	5	76.78	77.59	78.51	79.57	80.77	82.14
Poorly satisfied population (3.1)							
	1	5.12	5.12	5.13	5.15	5.31	6.19
	2	10.24	10.24	10.24	10.24	10.25	10.47
	3	12.90	12.90	12.91	12.91	12.91	12.95
	4	14.48	14.49	14.49	14.49	14.49	14.50
	5	15.52	15.52	15.53	15.53	15.53	15.53

shows the DISS values obtained for each indicator, and the corresponding mean value; they are quite coherent with the findings described above in terms of the ranking of the most satisfying aspects, varying from 55.57% (economic conditions) to 80.92% (family relations).

In comparing VQS and DISS values, it is useful to remember that:

(a) the last index is more informative, since it takes into account not very satisfied people too;
(b) DISS is a quadratic average of three terms, one of which is just VQS. We can then observe that, when the percentage of not very satisfied people is almost high (about 30%), the VQS index is lower than the DISS index and underestimates the global satisfaction degree (as it occurs for economic conditions and free time). Otherwise, when the aforementioned percentage is quite low, the DISS index is lower than the VQS index, so underestimating the global satisfaction degree (as it occurs for the remaining five aspects).

Starting from the observed joint distribution of the seven variables, we determined the distribution of the Y variable (Table 8). Then we calculated the three normalized indices (10), (11) and (12), considering $h = 1, \ldots, 5$. Coherently with simulation results, all the indices show values rising with h (see Fig. 4). Furthermore, is_3 is always halfway between is_1 and is_2 and, in particular, very near to is_2, as it occurs in cases of highly and partially satisfied populations (schemes 1.1–2.2 of Sect. 3).

Referring to is_1, the global satisfaction degree is quite limited, going from 33.09% to 59.82% and only for $h \geq 3$ it does take values higher than 50%. Its values are much lower

Table 7 Cumulative relative frequencies and DISS values for seven aspects of daily life—percent values

	Economic conditions (%)	Health (%)	Family relations (%)	Relationships with friends (%)	Free time (%)	Environmental conditions (%)	Work (%)	Average (%)
Very satisfied	2.10	18.64	34.59	24.10	10.32	14.11	14.39	16.89
Quite satisfied (VQS index)	46.56	87.85	93.16	86.55	60.69	74.80	74.38	74.85
Not very satisfied	84.21	98.07	98.83	97.85	92.36	94.56	95.05	94.42
DISS index in value	55.57	76.77	80.92	76.69	64.08	70.09	70.18	70.24

Source: elaborations on Istat *multipurpose survey* data (2013)

Table 8 *Y* distribution: relative and cumulative relative frequencies—percent values

Y_i	f_i (%)	F_i (%)
1	0.27	0.27
2	24.27	24.54
3	49.93	74.47
4	25.48	99.95
5	0.05	100.00

Source: elaborations on Istat *multipurpose survey* data (2013)

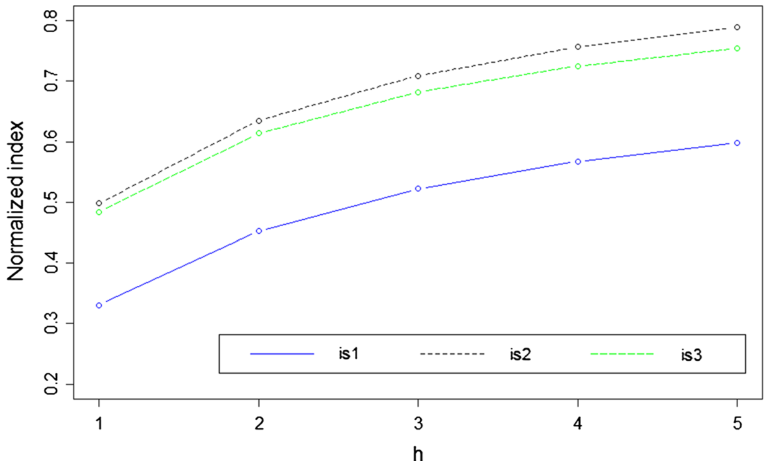

Fig. 4 Global satisfaction indices is_1, is_2 and is_3 for some aspects of daily life: the case of the Italian population

than those obtained for is_2 and is_3. The reason is that is_1 does not take into account the 25.48% of people that are only partially satisfied (i.e., dissatisfied for at least one aspect, but not for all, and satisfied for the remaining ones), as instead is_2 and is_3 do.

In any case, all the indices take quite moderate values (especially for $h \leq 2$), since almost 50% of people are not very satisfied for at least one aspect (that is $Y = 3$). In particular, is_2 varies from 49.80% to 78.95%, while is_3 from 48.42% to 75.48%. Moreover, these values are generally lower than both the average DISS index and the average VQS index (it is always true for $h \leq 3$). In fact, these two averages only take into account the marginal distributions of the seven variables, where the percentage of highly satisfied people is sufficiently high (except for economic conditions). Instead, in the joint distribution of variables, only 0.27% people are very satisfied for all the aspects and only 24.54% are at least quite satisfied for at least one aspect.

5 Conclusions

We proposed a method for measuring the degree of global satisfaction for an entire population using several elementary indicators simultaneously. The core is the introduction of a new ordinal variable based on the joint distribution of such indicators, which is able to express the individual degree of global satisfaction. We then subsequently defined three different indices (namely, is_1, is_2 and is_3), each focusing on specific subsets of individuals to be included/excluded from the analysis.

Compared to other existing methods, our procedure is interesting because:

1. it is appropriate for ordinal data, encoded on a Likert scale, since it maintains the original ordinal nature of elementary indicators, without quantifying their values nor introducing any arbitrary hypothesis about the distances between adjacent categories, as instead other methods do. Furthermore, it does not require any distributional assumptions, as instead models based on latent variable theory do;
2. it provides a condensed information, immediately useful to analysts or policy makers as composite indicators do; however, with respect to them, it does not require any choice or use of aggregation formula to synthetize elementary indicators;
3. it makes it possible to distinguish among all the different levels of global satisfaction declared by satisfied people, without aggregating them into a unique category as in the case of VQS index (percentage of very or quite satisfied people for all aspects). Moreover, in the case of is_3 index, it also makes possible to discriminate among different situations of partial satisfaction, corresponding to different numbers of satisfying aspects;
4. it allows taking into account all the aspects simultaneously. In particular, compared to other methods based on a synthesis of marginal distributions of elementary indicators, it also considers information concerning their possible associations and interactions, since it is based on their joint distribution. This makes it possible to include partially satisfied people too in the measurement of global satisfaction.

The results of the first simulation study confirm that the three indices are efficacious, that is able to discriminate among different satisfaction schemes (high, intermediate and low satisfaction). Moreover, they suggest that is_3 is preferable to is_1 and is_2, since it avoids underestimating the degree of global satisfaction (as is_1 does) and overestimating it (as is_2 does in situations of partial satisfaction, not taking into account the number of satisfying aspects). This index also appears sufficiently robust with respect to the number of elementary indicators, notwithstanding a certain variation could be expected.

Our next intent is to verify whether is_3 fulfills the main requirements that a good index should have, such as for example spatial and temporal comparability, and so on.

Some limitations of the proposed method could be the subjective choices concerning the definition of the global satisfaction variable, the value of the h exponent and the weighting structure in IS_3. Actually, all the indices take values rising with h, even if each of them seems to converge to some limiting value. Further research could then be devoted to convergence studies and to the determination of the optimal h value. Concerning the definition of the global satisfaction variable, we motivated our choice by means of some theoretical arguments, whereas for the weighting structure we suggested the most 'objective' and natural choice. Alternative definition are obviously possible and it could be interesting to evaluate their implications on is_3 values.

In this paper, according to the Likert scale used in Istat's *multipurpose survey*, we assumed that all the elementary indicators have the same number M of categories, without any neutral value (for instance, the judgement 'neither satisfied, nor dissatisfied'). Future work could concern a generalization of our methodology in the case of heterogeneous elementary indicators, with or without a neutral category.

Other interesting features to be explored regard the distributional properties of the indices for inferential analysis, and the extension to the case where variables may be affected by measurement errors.

Finally, from a practical point of view, this method can be useful in several fields of social studies, for instance in measuring customer satisfaction for a product or a service, evaluating students' or graduates' opinions about university teaching, and so on.

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