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Considering the Extremely Poor: Multidimensional Poverty Measurement for Germany

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Abstract This paper applies the Alkire and Foster (J Public Econ 95:476–487, 2011) index of multidimensional poverty to German data. This is done with respect to the politically most important dimensions of poverty mentioned in the *German Federal Government's report on poverty and wealth*. Additionally, a modification of the identification step of the Alkire–Foster index is proposed to guarantee that individuals, who are extremely poor in only few dimensions, are not omitted by the index.

Keywords Poverty index · Multidimensional poverty · Alkire–Foster index · SOEP

JEL Classification I32 · D63

1 Introduction

Starting with the Headcount measure much research on poverty measurement was done in the last century, see e.g. Atkinson (1987), Zheng (1997) and Chakravarty and Muliere (2004) for surveys on poverty measurement. One area is the development of more appropriate indices fulfilling reasonable postulates like monotonicity or transfer principle. Nevertheless, the indices have to be easily interpretable, since the recipients of poverty studies are more likely politicians than professional statisticians. This may be one of the reasons why the statistically insufficient Headcount is still the most popular poverty index. In the last decades the focus on poverty measurement has changed more and more from unidimensional to multidimensional poverty measurement, see e.g. Bourguignon and Chakravarty (2003) or Wagle (2008). This increases the gap between statistically appropriate measures and measures that are interpretable for non-professional recipients even more. Due to this change three additional questions arise: What are the (additional)

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dimensions of poverty, who is considered to be poor in the multidimensional setting and how to obtain an aggregate poverty measure?

To answer the first question we rely on the *German Federal Government's report on poverty and wealth*. Concerning the second question we adapt the approach of Alkire and Foster (2011) in which an individual is considered to be poor if he or she is deprived in at least a given number of dimensions. Since this approach may not be capable of considering individuals who are extremely poor in only a few dimensions, we propose a modification of the identification step that includes the extent of deprivation. To answer the third question we will again rely on the approach of Alkire and Foster (2011), which extends the unidimensional Foster et al. (1984) aggregation step to the multidimensional setting.[1]

This paper is organised as follows: Sect. 2 shortly reviews some multidimensional poverty measures. Section 3 describes the Alkire and Foster (2011) poverty index (henceforth AF index). In Sect. 4, we motivate and define a modified version of this poverty index. Section 5 describes the data and the choice of parameters used in Sect. 6 to apply the original and modified AF index to German data. Section 7 concludes.

2 Multidimensional Poverty Measurement

As already mentioned in the Introduction, the focus of poverty measurement has shifted towards a multidimensional perspective over the last decades. But although there is hardly any disagreement over poverty being more than just a lack of income, it is far from uncontroversial how to measure multidimensional poverty. If income is not the only dimension, which attributes have to be included in addition? How can a measure of multidimensional poverty incorporate interdependencies between those attributes? Should there be any aggregation of the different dimensions at all? All of these are normative questions which cannot be answered non-ambiguously.

Probably the most fundamental of these questions is whether one should try to aggregate the different dimensions of poverty. One could argue that these dimensions are in principle not substitutable or that one does not want to assume any valuation on implicit trade-offs between dimensions. Early advocates of this view are for example Walzer (1983) and Erikson (1993) or Ravallion (2011) for a more recent criticism of the aggregation. They argue in favour of the so called "dashboard" approach, where all dimensions are examined separately without trying to compress them into a single index.

These "dashboard" approaches do have two main drawbacks. Firstly one may argue that the benefit of incorporating possible dependencies among the dimensions outweighs the potential problems. But beside this clearly normative decision another drawback is that they are potentially much harder to interpret, being particularly problematic since one main aspect of multidimensional poverty measurement is to support policy makers. Therefore most of the empirical applications do use measures that somehow aggregate the different dimensions into a single indicator.

To obtain a single indicator of multidimensional poverty one has to aggregate not only about n individuals but also about d dimensions. The order of these two aggregations steps is exactly what distinguishes two strands within the class of single indicators. If the first step is to obtain measures for every dimensions—that means aggregating about individuals—and afterwards to aggregate these measures the resulting indicator is called a

[1] The same aggregation step is used in Bourguignon and Chakravarty (2003).

"composite" indicator.[2] Probably the best known composite indicator is the Human Poverty Index. In its original formalization the index is defined by

$$HPI_\beta = \left(\sum_{j=1}^{d} w_j p_j^\beta \right)^{\frac{1}{\beta}},$$

where p_j is the proportion of deprived individuals in dimension j, w_j are dimensional weights and β is a parameter that characterises the trade-offs between dimensions, see Sen and Anand (1997). For a discussion of the properties of composite indicators see e.g. Bossert et al. (2013) or Pattanaik et al. (2011).

Composite indicators allow for incorporating trade-offs but not interdependencies between the dimensions since they do not use information on the individual level. If the first aggregation is on individual level one can address the question if the number of deprivations an individual has to suffer has an impact on the evaluation of poverty. These approaches are what Atkinson (2003) calls "counting" approaches.[3] Thereby the counting approaches usually follow the two step method proposed by Sen (1976), who proposed that in a first step the poor individuals should be identified and in a second step their deprivations[4] should be aggregated.

For the aggregation step one has to consider the scale of measurement of the different dimensions. If at least one dimensions is only qualitative the measure can only consider the deprivation score, that is the number of dimensions in which an individual is deprived, and is therefore a "pure" counting measure. An example for such a measure is the poverty index proposed by Chakravarty and D'Ambrosio (2006), which is defined as

$$P^{CD}(\boldsymbol{X}, \boldsymbol{\pi}) = \frac{1}{n} \sum_{i=1}^{n} 1_{\{y_i > 0\}} y_i^\delta, \quad \delta \geq 1,$$

where $y_i = \sum_{j=1}^{d} 1_{\{x_{ij} < \pi_j\}}$ is the deprivation score of individual i, $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_d)$ is the vector of unidimensional poverty thresholds π_j, $j = 1, \ldots, d$, $\boldsymbol{X} = (x_{ij})_{nd}$ is a matrix with x_{ij} as the value of individual i in dimension j and δ is a parameter which determines how sensitive the measure is to higher deprivation scores.[5] Other classes of "pure" counting measures are defined e.g. by Tsui (2002) or Bossert et al. (2013).[6]

If we have qualitative data the aggregation function can incorporate this additional information and include the "degree" of deprivation. An example is the poverty measure by Chakravarty et al. (2008), which is an extension of the unidimensional poverty index by Watts (1968) and is defined as

[2] In other words a composite indicator begins with a dashboard approach and adds a aggregation function for the separate (dimensional) indicators.

[3] Composite indicators and counting approaches are different in general but may coincide in their evaluations of poverty under certain (demanding) restrictions, see e.g. Dutta et al. (2003).

[4] Throughout this paper we want to use the following notation: We use the term deprivation for a lack of an individual in a single dimension and the term poor for an individual who is considered to be (multidimensional) poor.

[5] In the original formulation the term $1_{\{y_i > 0\}}$ is not included. We want to include it here because it highlights that the measure only considers individuals who are deprived in at least one dimension.

[6] We want to focus on complete orderings due to their interpretability. See e.g. Aaberge and Peluso (2012) for a definition of partial orderings for deprivation scores.

$$P^{CDS}(\boldsymbol{X},\boldsymbol{\pi}) = \frac{1}{nd}\sum_{i=1}^{n}\sum_{j=1}^{d} 1_{\{x_{ij}<\pi_j\}} \ln\frac{\pi_j}{x_{ij}}.$$

A similar class of poverty measures is proposed by Bourguignon and Chakravarty (2003). They propose to use the aggregation function of Foster et al. (1984) and define their multidimensional poverty index to be

$$P^{BC}(\boldsymbol{X},\boldsymbol{\pi}) = \frac{1}{nd}\sum_{i=1}^{n}\sum_{j=1}^{d} 1_{\{x_{ij}<\pi_j\}}\left(1-\frac{x_{ij}}{\pi_j}\right)^{\alpha_j}, \quad \alpha_j > 1,$$

with α_j as additional parameters which allow to set higher weights to more intense deprivations.

So far we have presented different aggregation steps but not mentioned the identification step, at least not explicitly. All of the above measures imply the so called "union" approach, that is an individual is considered to be poor if he or she is deprived in at least one dimension. An alternative would be to use the "intersection" approach which assumes that an individual is poor only if he or she is deprived in all dimensions. Both approaches may seem reasonable for a small number of dimensions but tend to classify nearly all individuals or hardly any individual as poor as the number of dimensions grows. In contrast, Alkire and Foster (2011) argue that there may be a threshold for the deprivation score that an individual has to reach to be considered as poor. Consequently, they define their multidimensional poverty measure to be[7]

$$P^{AF}(\boldsymbol{X},\boldsymbol{\pi}) = \frac{1}{nd}\sum_{i=1}^{n} 1_{\{y_i \geq k\}}\sum_{j=1}^{d} 1_{\{x_{ij}<\pi_j\}}\left(1-\frac{x_{ij}}{\pi_j}\right)^{\alpha}, \quad \alpha \geq 0.$$

On the basis of this class of poverty measures Alkire and Santos (2010, 2013, 2014) have developed the "Multidimensional Poverty Index" (MPI), which has replaced the HPI in the official reports of the United Nations Development Program. Another official poverty index which uses a similar procedure is the so called "People at risk of poverty or social exlusion" (AROPE) by the European Union. This index uses a nested identification approach which is a special case of the identification step of Alkire and Foster (2011) while its aggregation step is a simple Headcount of the poor.[8]

Because of the extensive usage in official statistics and the inherent flexibility[9] we want to adopt the approach of Alkire and Foster (2011) to define an extended index of multidimensional poverty.

[7] For this overview we have excluded dimensional weights. See Sect. 3 for a definition of the AF index that incorporates dimensional weights.

[8] The AROPE index identifies an individual as poor if he or she meets at least one of the conditions income poverty, low work intensity or severe material deprivation. An individual is said to be severe materially deprived if he or she cannot afford at least four out of nine items. The AF identification step can map this procedure if $k = 4$ and the dimensional weights are $w_j = 4$ for income poverty and low work intensity and $w_j = 1$ for the nine items of severe material deprivation. See Eurostat (2012) for further information on the AROPE index.

[9] Beside the AROPE index one can easily see that for $\alpha_j = \alpha$ P^{BC} is a special case of P^{AF} with $k = 1$. But for $\delta = 1$ also counting measures like P^{CD} are nested by P^{AF}.

3 The Alkire and Foster Poverty Index

Since we propose a modification of the Alkire and Foster index, we discuss the Alkire and Foster index in more detail. Firstly we repeat some notations: Consider a population with n individuals and d dimensions like income, education or health. Let $\boldsymbol{X} = (x_{ij})_{nd}$ be a matrix with value x_{ij} for individual i in dimension j and $\boldsymbol{x}_i = (x_{i1}, \ldots, x_{id})$ vectors of individual outcomes so that $\boldsymbol{X} = (\boldsymbol{x}_1', \ldots, \boldsymbol{x}_n')'$. Let $\pi = (\pi_1, \ldots, \pi_d)$ be a vector of unidimensional poverty lines and $\boldsymbol{w} = (w_1, \ldots, w_d)$ be a vector of nonnegative dimensional weights, such that w.l.o.g. $\sum_{j=1}^{d} w_j = d$.

Alkire and Foster (2011) define a new index of multidimensional poverty using the common two step method of Sen (1976): On the first stage the (multidimensional) poor individuals are identified while on the second stage the deprivations of the (poor) individuals are aggregated. In the approach of Alkire and Foster (2011) an individual is identified as poor, if his or her values do not reach at least k out of d unidimensional poverty lines, i.e. if $y_i \geq k, k \in \{1, 2, \ldots, d\}$. Considering potentially different importance of the dimensions they obtain the multidimensional poverty identification function

$$\rho_i = \rho_{i,k}(\boldsymbol{x}_i, \boldsymbol{\pi}, \boldsymbol{w}) = \mathbf{1}_{y_i^* \geq k}, \quad k \in]0, d], \tag{1}$$

where $y_i^* = \sum_{j=1}^{d} w_j \mathbf{1}_{x_{ij} < \pi_j}$ is a weighted version of the deprivation score. For the second step Alkire and Foster (2011) use the Foster et al. (1984) aggeration procedure, i.e. they evaluate the deprivation of individual i in dimension j by the relative shortfall

$$\left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^{\alpha} := \mathbf{1}_{x_{ij} < \pi_j} \cdot \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)^{\alpha}$$

and add first across dimensions and afterwards across individuals. Altogether, the Alkire and Foster (2011) poverty index is defined by

$$P_{\alpha,k}^{AF}(\boldsymbol{X}, \boldsymbol{\pi}, \boldsymbol{w}) = \frac{1}{n} \sum_{i=1}^{n} \rho_i \cdot \left(\frac{1}{d} \sum_{j=1}^{d} w_j \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^{\alpha}\right). \tag{2}$$

Similar to the unidimensional FGT indices usually three cases for α are of special interest, namely the "Adjusted Headcount Ratio" ($\alpha = 0$), the "Adjusted Poverty Gap" ($\alpha = 1$) and a transfer sensitive index ($\alpha = 2$).[10]

At the end of this section we want to refer to some extensions and applications of the AF index, without claiming the completeness of this list. Ravallion (2011) and Alkire et al. (2011) discuss the general use of single indicators of poverty. Bennett and Mitra (2013) generalize this measure, allowing a mixture of ordinal and metric variables. Alkire and Santos (2010) adapt the measure to developing countries and obtain the so called "Multidimensional Poverty Index", which is part of the Human Development Report[11] of the United Nations. Rippin (2012) proposes the "German Correlation Sensitive Poverty Index", a modified version of the "Multidimensional Poverty Index" that includes an additional weighting component, allowing for higher importance of multiple deprivations. For other applications see for example Whelan et al. (2014) for an application to Europe,

[10] For a definition of the factorizations and a discussion of the properties of the AF index see Alkire and Foster (2011) and the discussion of the properties of the modified index at the end of Sect. 4.

[11] For information on the use of the MPI in the Human Development Report see UNDP (2014a, b).

Battiston et al. (2013) for an application to Latin American, Yu (2013) for an application to China or Mitra et al. (2013) for an analysis of the connection between disability and poverty in developing countries.

4 A Modified AF Index

To motivate our modified identification step we start with a small example: Consider $d = 5$ dimensions and set the unidimensional poverty lines to $\pi_j = 10$ for $j = 1, \ldots, 5$. Identify individual i as poor, if he or she is poor in at least $k = 3$ dimensions and set $\alpha = 1$. The following table shows the data for four individuals.

Individual	Dimension j					Poor in…	"Contribution" to the AF index:
	1	2	3	4	5	dimensions	$\rho_i \cdot \frac{1}{d}\sum_{j=1}^{d}\left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+$
$i = 1$	12	13	12	14	15	0	$0 \cdot 0.00 = 0.00$
$i = 2$	9	11	9	11	9	3	$1 \cdot 0.06 = 0.06$
$i = 3$	0	11	0	11	11	2	$0 \cdot 0.40 = 0.00$
$i = 4$	0	0	0	0	0	5	$1 \cdot 1.00 = 1.00$

Given the relevant dimensions and poverty lines, it is incontestable that individual 1 is not poor and individual 4 is poor. However, classifying individuals 2 and 3 is more complicated. Using the AF index with $k = 3$, individual 2 is poor, because he or she is deprived in three dimensions, while individual 3 is not poor, since he or she is deprived in only two dimensions. So individual 2 will contribute to the index while individual 3 will not.

We think there are dimensions where this does not make sense, e.g. individual 2 is poor, because he or she is slightly ill-nourished, slightly not healthy enough and slightly not well-educated whereas individual 3 is not poor, since he or she is well-educated but has nothing to eat and is fatally ill. Combined with the calculated lack of 0.06 for $i = 2$ and 0.40 for $i = 3$, according to our opinion, the situation of individual 2 is much better than of individual 3. So we recommend to additionally identify individual 3 as poor.

Therefore we modify the identification step of the AF index. Instead of identifying individual i as poor, if he or she is deprived in k (weighted) dimensions [see formula (1)] now individual i is also identified as poor, if his or her "contribution"[12] to the poverty index would be equal or above a threshold m,

$$\rho_i^* = \rho_{i,k,m,\beta}^*(\boldsymbol{x}_i, \boldsymbol{\pi}, \boldsymbol{w}) = \mathbf{1}_{y_i^* \geq k \,\vee\, \sum_{j=1}^{d} w_j \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^{\beta} \geq m}, \quad k, m > 0. \tag{3}$$

With this modified identification function the resulting poverty measure is

[12] In general we allow $\beta \neq \alpha$, so strictly speaking it is not necessary the contribution to the index that is used for identification. For a short discussion about the choice of the parameter β see Sect. 5.3.

$$P^{AF^*}_{\alpha,k,m,\beta}(\boldsymbol{X},\boldsymbol{\pi},\boldsymbol{w}) = \frac{1}{n}\sum_{i=1}^{n}\rho_i^* \cdot \left(\frac{1}{d}\sum_{j=1}^{d} w_j \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^{\alpha}\right). \tag{4}$$

Using this modified identification function, there are two possibilities for an individual to be considered as poor. If an individual is deprived in many dimensions, the single deprivations can be minimal, nevertheless we would consider the individual to be poor. The reason we add the second condition is to guarantee that an individual, who is extremely deprived in only a few dimensions, will be identified as poor. The new identification step is easy to interpret if $\beta = \alpha$. In this case the additional condition means that the sum of weighted relative shortfalls of individual i has to reach or exceed a threshold m. Obviously this modified identification step c.p. (weakly) increases the number of poor individuals.

Note that due to the additional character of the modified identification function the new identification step nests the old one. It simplifies to the Alkire and Foster (2011) identification step if either $m > d$ or $\beta = 0$ and $k = m$: In the first case the second condition can never be fulfilled and in the second case the two conditions are equal. On the other hand, if $k > d$, the first condition can never be fulfilled and an individual is poor only if the second condition is fulfilled.[13]

For an analysis of the properties of the modified measure we can rely on the properties of the original measure. Alkire and Foster (2011) deduce the properties of their measure by analyzing the properties of a certain combination of an identification function with an aggregation function. The benefit of this approach is that some of the properties solely rely on the aggregation function used and do not depend on the identification function. Since we have only modified the identification function, we can directly deduce that our modified measure fulfills the properties *Decomposability*, *Replication Invariance*, *Symmetry*, *Non-triviality* and *Normalization* for $\alpha \geq 0$.[14]

For the properties *Poverty focus* and *Deprivation focus* we have to ensure that the poverty measure does not change if there is an improvement in any of the dimensions of a non-poor individual or in any of the non-deprived dimensions of a poor individual. A non-poor individual does not contribute to the measure at all and the contribution of a poor individual is restricted to deprived dimensions, so these properties are fulfilled if it is impossible to become poor due to an improvement in any dimension. Since this is impossible with the identification method (3), these properties are fulfilled for our modified measure. If a poor individual lowers or removes one of his or her deprivations due to a small increment, it is clear that the poverty either remains unchanged or gets smaller, the latter is secured if $\alpha > 0$. So, together with the focus properties, the measure fulfills the properties *Weak Monotonicity*, *Dimensional Monotonicity* and *Monotonicity*, the latter for $\alpha > 0$.

The last two properties we want to mention are *Weak Rearrangement* and *Weak Transfer*, the latter for $\alpha \geq 1$. Since these are only rearrangements among the poor, the modified identification step does not affect the validity of these properties. In fact, any rearrangement among the poor is even more likely to actually lower poverty since there is an additional possibility to become non-poor due to the new identification procedure.

[13] $k > d$ is not possible in the original index of Alkire and Foster (2011) since it would lead to $P^{AF}_{\alpha,k}(\boldsymbol{X},\boldsymbol{\pi},\boldsymbol{w}) = 0$ by definition. We allow for $k > d$ because our second condition can still be fulfilled and by setting $k > d$ we can analyse the sole influence of the new condition.

[14] For a definition and a discussion of these and the following properties see Alkire and Foster (2011).

There is a recent working paper of Datt (2013) which discusses a stronger version of the transfer axiom and a "cross-dimensional convexity axiom", both of which are not fullfilled by the original AF measure. By construction our modified AF measure does not satisfy these axiomes either.

5 Data and Parameters

Using poverty measures like (2) and (4) requires a number of choices for different parameters that imply value judgements about the importance and relation of the dimensions, the extent of poverty and many more topics. Although for some parameters there is only a number of meaningful possibilities and for some other parameters there have been established specific choices in the literature, in general these choices remain somehow arbitrary.

Since measuring poverty should be the foundation of political decisions, in our opinion these choices have to be made by the society itself, at best after a broad public discussion. Task of a researcher should therefore be to point out the consequences of different choices of parameters and to provide a detailed background for the discussion.

Nevertheless, if the researcher wants to exemplify the different measures or parameters, he or she has to make some choices in the first place. This is what we want to do in this section. Therefore we choose a setting of parameters we think is meaningful in the context of poverty measurement in Germany. But again we want to emphasize, that these choices are not mandatory. All we can do is to make the process of choice transparent and thereby open for discussion.

5.1 Included Dimensions

For the choice of the dimensions, we rely on Part C of the "German federal government's 4th report on poverty and wealth" (Bundesministerium für Arbeit und Soziales 2013).[15] The report covers nine major topics, i.e. (i) distribution of material resources, (ii) employment market, (iii) child care and education, (iv) health, (v) home and rent, (vi) homelessness, (vii) prisoners and their chances on participation, (viii) social commitment and social contact and (ix) social responsibility of the rich and wealthy.

We try to build a multidimensional poverty index that includes most of these topics. Certainly (ix) is relevant for a general discussion on poverty reduction but not relevant for status quo poverty measurement. Furthermore we want to analyse poverty on an individual basis, therefore in our study (vi) is included in (v) and (vii) is assumed to be less important than the other dimensions. The rest of the topics should be included in our analysis, so our poverty measure considers up to six dimensions of poverty.

For our analysis we use data of the German Socio-economic Panel (SOEP).[16] Bringing together the dimensions and the data, we operationalize the dimensions and define unidimensional poverty lines as follows:[17]

[15] For an English summery see Federal Ministry of Labour and Social Affairs (2013). For the 1st to 3rd report see Bundesregierung (2001, 2005, 2008), respectively.

[16] Socio-economic Panel (SOEP), Data for years 1984–2011, Version 28, SOEP, 2012, doi:10.5684/soep.v28. For more information see Wagner et al. (2007, 2008).

[17] The names of the variables are those from the English version of "SOEPinfo", http://panel.gsoep.de/soepinfo2011/.

I. **Income**: *Equivalized monthly household net income*,[18] poor if the income is less than 60 % of the median income,
II. **Employment**: *Number of months registered employed within the previous year*, poor if less than 10 months employed (up to 3 months we consider unemployment to be frictional),[19]
III. **Education**: *Amount of education or training in years*, poor if less than 10.5 years of education (10.5 years correspond to a basic school degree and additional vocational training),
IV. **Health**: *Satisfaction with health*,[20] poor if less than 4,
V. **Living**: *Equivalized size of housing unit in* m^2, poor if less than $45m^2$ which is the maximum living space paid by the job center / social welfare office in Germany,[21]
VI. **Social participation**: *Number of activities per week*,[22] poor if less than one activity per week.

Clearly the determination of the poverty lines is somehow arbitrary, at least to a certain degree. If possible we tried to base our choices on existing thresholds, as for the dimensions income or employment, or on values that already reflect public value judgements, as for the living dimension. Additionally, in Sect. 6 we show the impact of alternative poverty lines on our modified measure.

5.2 Dimensional Weights

The choice of weights for the dimensions mainly reflects the relative importance of the different dimensions of poverty. There are a number of different approaches to set or derive weights, which are roughly classified as either data-driven, normative or hybrid approaches by Decancq and Lugo (2013).

In our application we want to apply three different methods to set the weights. The first is to set the weights for the different dimensions to an equal level. Certainly not an elegant approach but for a number of reasons it is the only viable way in a broad number of situations.[23]

[18] We use the modified OECD-equivalence scale, 1 to head of household, 0.3 for each child younger than approximately 14 (since only the year of birth is available), and 0.5 to all other household members.

[19] This coincides with the definition of low work intensity by the European Union which is working less than 20 % of the possible months per year, see Eurostat (2012).

[20] Self-Evaluation, measured on a scale from 0 to 10. We assume this question to be a Likert-type item, so we are able to treat it like a metric variable, see for example Traylor (1983).

[21] We construct an equivalence scale similar to Frick (1995), i.e. 1 for the first individual, $\frac{1}{3}$ for the second and third individual and $\frac{2}{9}$ for all other household members. This equivalence scale was constructed to reflect the guidelines for appropriate living space of the social welfare offices in Germany, which are approximately $45m^2$ for one individual, $60m^2$ for two individuals, $75m^2$ for three individuals and $10m^2$ for each additional household member.

[22] We construct a variable by adding up the frequencies of nine different activity variables, namely *attend cultural events*, *attend cinema, pop, jazz concerts*, *participate in sports*, *artistic activities*, *attend social gatherings*, *helping relatives, friends*, *perform volunteer work*, *participate in local politics* and *attend church or other religious events*. We set at least one time per week equal to 1, at least one time per month equal to 0.5 (since less than one time per week and at least one time per month is 1,2 or 3 times per month and the mean is approximately 2 times per month, i.e. 0.5 times per week). Similarly we set less than one time per month and more than never equal to 0.125 times per week.

[23] Often it is argued that the researcher does not want to impose a value judgement and therefore uses equal weights. But this is no proper reasoning since equal weights mean that the dimensions are of equal importance, clearly a value judgement. We would rather argue that due to data availability many of the

The second approach is to use self stated weights, in our opinion the most favourable way to set weights. Optimally the weights would come from the same survey as the data but since we do not have such weights in the SOEP data we use weights from the OECD Better Life Index[24] as a proxy. The index consists of eleven dimensions that are supposed to influence the quality of life and we use these dimensions to cover our six dimensions of poverty. The dimensions of the index are attached to the dimensions of poverty according to the following list:

(1) Civic Engagement and Government (CG), (2) Social connections (SC) ⇒ Social participation, (3) Education and Skills (ES) ⇒ Education, (4) Environment quality (EQ), (5) Health status (HS) ⇒ Health, (6) Housing (HO) ⇒ Living, (7) Income and wealth (IW) ⇒ Income, (8) Jobs and earnings (JE) ⇒ Employment, (9) Subjective well-being (SW), (10) Personal security (PS), (11) Work-Life Balance (WL).

The website allows users to set a weighting scheme according to their individual preferences. As of September 10, 2014, 6289 Germans have set their individual weights for these dimensions. We use the average weights of the dimensions to construct two weighting schemes, one for all six dimensions of our poverty measure and one for a subset that excludes the variable *social participation*.

An alternative source for self stated weights is the Eurobarometer survey,[25] an official survey to monitor the public opinion on behalf of the European commission. It includes a question about the two most important issues the respondents are facing at the moment. Although this may differ from the importance of the dimensions in general we want to consider these weights as an alternative to the self stated weights obtained from the OECD Better Life index. Unfortunately there is no answer that may serve as proxy for the dimension *social participation*, so we will only deduce weights from the Eurobarometer for the subset of five dimensions.

The last approach we want to consider is to use frequency based weights. The basic idea is that if a deprivation in a certain dimension is quite usual it does have less impact than a deprivation in a dimension where most of the population has no lack at all. Therefore the weights are set to be equal to the logarithm of the inverse of the unidimensional Headcounts.[26]

Table 1 gives an overview about the weights obtained for all dimensions and for the subset which excludes the dimension *social participation*. We do not want to analyse the different weighting schemes in detail but it is striking that the dimensions living and social participation seem to be less important than other dimensions of poverty, while health seems to be more important, independent of the actual weighting approach.

In Sect. 6 we will use the equal weighting scheme as our base scenario and illustrate the impact of the use of other weighting schemes on the AF and the modified AF index.

5.3 Other Parameters

The choices for α, k, β and m are in principal not less arbitrary than the choices for the weights, but especially for α there are some well-known choices.

Footnote 23 continued
approaches to obtain weights do not work. So we use equal weights as a arbitrary starting point which clearly reveals that setting weights is a topic that has to be analysed in more detail.

[24] See http://www.oecdbetterlifeindex.org/ for information about the OECD Better Life Index.

[25] See http://ec.europa.eu/public_opinion/index_en.htm for further information.

[26] See e.g. Cerioli and Zani (1990) or Deutsch and Silber (2005) for applications that use frequency based weights. Also note that we have used the Headcounts of 2011 since also the weights obtained from the OECD Better Life index and the Eurobarometer survey are most recent.

Table 1 Overview of different weighting schemes

Weighting scheme	Income	Employment	Education	Health	Living	Participation
Equal	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Frequency based	0.9639	1.2928	0.9118	1.0374	0.9760	0.8182
OECD better life	0.9164	0.9613	1.0971	1.1606	0.9734	0.8912
Equal	1.0000	1.0000	1.0000	1.0000	1.0000	–
Frequency based	0.9364	1.2501	0.8720	0.9976	0.9440	–
OECD better life	0.8969	0.9408	1.0737	1.1359	0.9526	–
Eurobarometer	1.2069	0.8621	1.1207	1.3793	0.4310	–

The parameter α determines how the measure reacts to changes of a poor individual in a deprived dimension. There have been established three common choices for α in the literature, which have been mentioned before and correspond to the Adjusted Headcount ($\alpha = 0$), the Adjusted Poverty Gap ($\alpha = 1$) and a transfersensitive measure ($\alpha = 2$). We will focus on these choices for α throughout the paper.

For β we will focus on the case $\beta = 1$. In general one could argue that setting $\beta = \alpha$ would be the most natural way, since in this case the identification step would mirror the contribution to the poverty measure. But for $\beta = 0$ the identification method would reduce to the original method and for $\beta = 2$ it would be difficult to interpret. In contrast, $\beta = 1$ is easy to interpret because in this case the second identification method is to compare the sum of the relative shortfalls in the different dimensions with a given threshold.

We are aware that the new threshold m is an additional normative parameter. The underlying question is which degree of individual deprivation should define an individual as poor. This problem is analogue to the choice of the poverty lines and the parameter k. All of these parameters are normative and therefore highly disputable, especially in case of relative poverty measurement for developed countries. Although the choice is highly dependent on the specific analysis at hand, we want to provide at least a rule of thumb on how to choose a sensible "starting" value for m. Note therefore that the choice of m has to be done with respect to the choice of k, since both parameters play a crucial role in determining which individuals are considered to be poor. In our analysis, setting $m = (k - 1) \cdot 0.25$ seems to be a plausible "starting" value for m. This means that an individual is considered to be poor if he or she is deprived in at least k dimensions or if the relative deprivation in $k - 1$ dimensions is on average 25 %. But keep in mind that this rule of thumb reflects both the circumstances of the specific analysis and at least to a certain degree value judgements of the authors. As for α we will provide results for different choices of these parameters and show how they influence our measure of multidimensional poverty.

6 Results of the Empirical Application

At first, we discuss the results of the well-known indices for the German data. In a second step, we illustrate the modified indices with the German data to see whether the constructed example from the beginning of Sect. 4 is relevant for real data or not.

In order to analyse multidimensional poverty over the complete time period, we will restrict the analysis in this section to five dimensions of poverty and leave out the

dimension "social participation". For descriptive statistics see Figs. 12, 13 and Tables 3, 4 in "Appendix 1". Some results for all six dimensions of poverty will be presented in Fig. 14–17 in "Appendix 2".

6.1 Results for the Well-Known Measures

First of all we take a look at the unidimensional Headcounts for the above mentioned dimensions of poverty in Fig. 1.

The Headcounts clearly show the effects of German reunification and the labour market reforms on univariate poverty. Until 1991 Fig. 1 shows Headcounts for West Germany and since 1992 for the reunited Germany. This is probably the reason for the dramatic change in education and living poverty from 1991 to 1992. It also shows an increase of employment poverty, i.e. the increasing unemployment after German reunification until chancellor Schröder's labour market reforms. Also you can see the decrease of employment poverty and the increase of income poverty in the last few years after these labour market reforms, whose different steps became law between 2003 and 2005.

Figure 2 shows the unidimensional FGT indices for the different dimensions of poverty for $\alpha = 1$ and $\alpha = 2$, which in contrast to the unidimensional Headcounts incorporate the magnitude of deprivations. This incorporation leads to a change in the order of the various dimensions. E.g. although education has the highest Headcount its unidimensional poverty is very low for $\alpha = 1$, meaning that the average deprivation of the poor is very small. For employment it is the other way around, a low Headcount comes along with a high unidimensional poverty for $\alpha = 1$. This effect intensifies with $\alpha = 2$, so in this dimension few individuals are poor but they suffer from on average very large deprivations.

Going on with the multidimensional results, Figs. 3 and 4 show the results for the AF index for different choices of k.

For $\alpha = 0$, the AF index does not report the proportion of (multidimensional) poor individuals, but the "Adjusted Headcount Ratio" described in Sect. 3. Nevertheless, the different developments of the unidimensional Headcounts are reflected in the AF index: For $k = 1$ an individual is poor, if he or she is deprived in at least one dimension. The large percentage of individuals, who are poor in living and housing, decreased dramatically

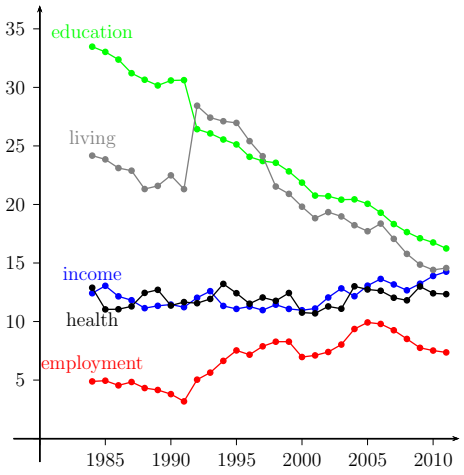


Fig. 1 Unidimensional Headcounts (in %) for income, employment, education, health and living

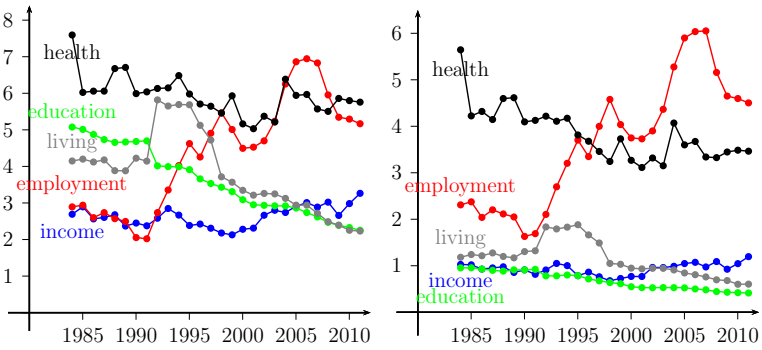


Fig. 2 Unidimensional FGT indices (in %) for $\alpha = 1$ (*left hand side*) and $\alpha = 2$ for income, employment, education, health, and living

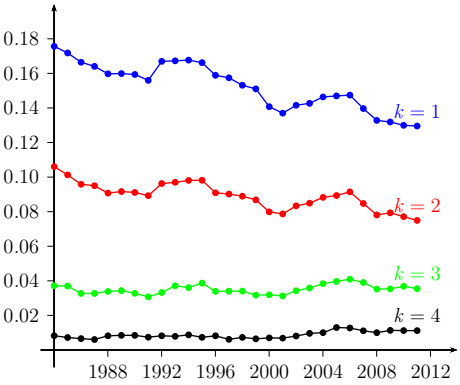


Fig. 3 Alkire–Foster indices for $\alpha = 0$ and various k

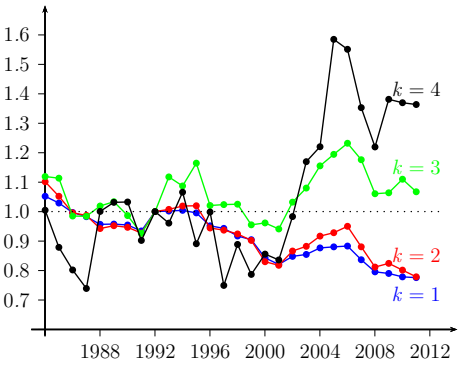


Fig. 4 Alkire–Foster indices for $\alpha = 0$ and various k, base year 1992

during the 1990s and 2000s. On the other hand poverty caused by income and unemployment increases during the early 2000s. Both developments are shown by the AF index for $k = 1$ in Fig. 3. Because of the different range of values we standardize the values of the indices using 1992 as base year in Fig. 4. After the German reunification the AF

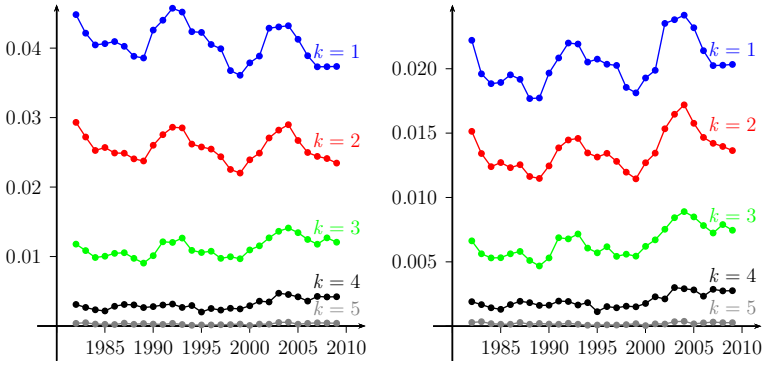


Fig. 5 Alkire–Foster indices for $\alpha = 1$ (*left*) and $\alpha = 2$ and various k

indices, where an individual is poor in at least one ($k = 1$) and at least two ($k = 2$) dimensions, decreased as described above. But if you focus only on those, who are poor in at least three ($k = 3$) and four ($k = 4$) dimension, the AF indices increased during the last decade.

Roughly speaking, if we define all individuals as poor, who are deprived in any of the dimensions ($k = 1$), poverty decreases. But the intense of poverty of those who are in a very bad situation, i.e. poor in at least three or four dimensions, rises. For similar results for $\alpha = 1$ and $\alpha = 2$ see Fig. 5.

Since the Figures of the AF indices for different α look very similar at a first glance, we now take a look on how c.p. the choice of α affects the index. Do changes of α make any difference for real data?

Of course in many situations the changes of the indices have the same sign for different choices of α, but there are also situations where you can see different behaviors. E.g. if you compare 1992 with 2005 the index decreased for $\alpha = 0$, was approximately constant for $\alpha = 1$ and increased for $\alpha = 2$, see Table 2. This means that the magnitude of suffered deprivations increased and compensated for the decrease in the number of poor individuals. This is in line with the former results, also suggesting that the situation of the extremely poor has worsened.

Now we illustrate how c.p. the choice of the weighting factors w_j affects the index. Therefore we compare the weighting scheme from the OECD Better Life index with an equal weighting scheme for $\alpha = 1$ and $k = 2$ or $k = 3$ respectively, see Fig. 6.

We can see in Fig. 6 that this special weighting scheme does not seem to affect the general development of the AF index. This is probably due to the fact that the weights we have obtained are not very different from equal weights. This will be a little bit different when we continue this discussion for the modified AF in the next subsection.

6.2 Results for the Modified Measures

Now we want to illustrate the modified AF index for several parameter specifications. In the first step, we start with various m and $k = 6$, i.e. and individual is identified as poor, if his or her illfare exceeds m and no one is identified as poor by the AF identification step, see Fig. 7.

Table 2 Alkire–Foster indices for $k = 1$ and various α

α	1992	2005	Rel. diff. (%)
0	0.1670	0.1470	−12 %
1	0.0426	0.0430	+1 %
2	0.0197	0.0238	+21 %

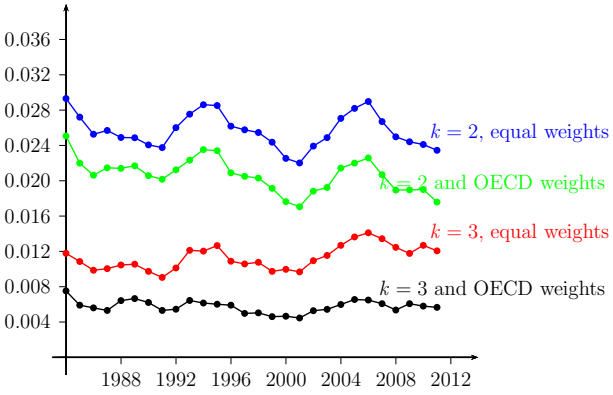


Fig. 6 Alkire–Foster indices for $\alpha = 1$ and different weighting schemes

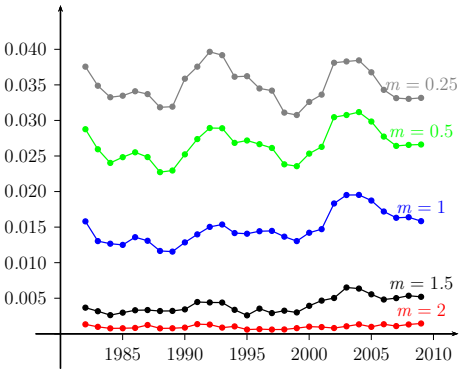


Fig. 7 Modified Alkire–Foster indices for $\alpha = 1$, $k = 6$ and various m

If an individual is identified as poor even for small illfare m, Fig. 7 shows a decrease of the index, e.g. for $m = 0.25$ the index decreases from 0.0376 in 1984 to 0.0332 in 2011. If the illfare m has to be large to count someone as poor, the index increases, for $m = 1.5$ from 0.0037 in 1984 to 0.0052 in 2011. Again we interpret this as a worsening of the situation of the extremely poor, which is in line with the results of the original AF index. Moreover, the results of the modified measure seem to be more explicit since already the identification step focusses on "extreme" poverty.

The basic idea of the modified identification step is that the original measure omits individuals, who are extremely poor in only a few dimensions. Therefore the following figure illustrates how many individuals are additionally poor, if we modify the identification step.

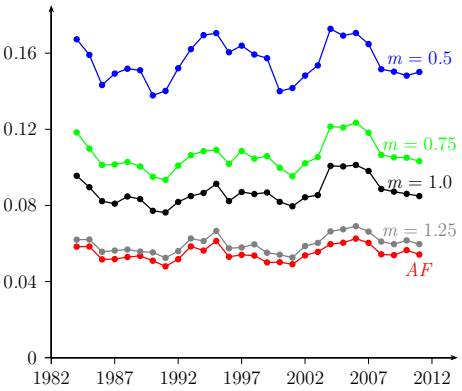


Fig. 8 Proportion of the poor for the Alkire–Foster index for $\alpha = 1$, $k = 3$ and several m for the modified index

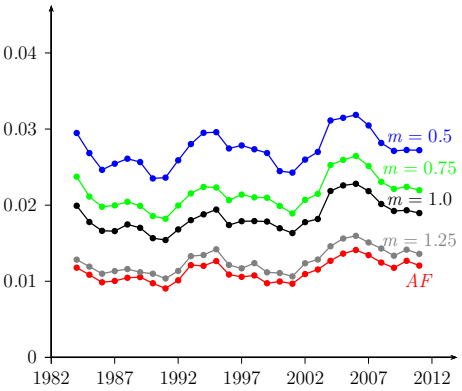


Fig. 9 Values of the Alkire–Foster index for $\alpha = 1$, $k = 3$ and various m for the modified index

Figure 8 shows an enormous difference between the number of poor defined by the AF index for $k = 3$ and the same index with the modified identification step. Of course the more restrictive the additional condition to identify the poor is, i.e. the larger m is, the less additional poor are identified. Example for $m = 1$ the new identification step means that either you are deprived in three out of five dimensions or the sum of the relative deprivations in at most two dimensions is one.[27] The results show that even for this quite restrictive choice of m the number of poor individuals nearly doubles.

Figure 9 should demonstrate how the modified identification step changes the value of the poverty index and not only the number of poor individuals. Therefore Fig. 9 shows the results for the modified AF index for $\alpha = 1$, $k = 3$ and different choices of m. Most striking is the level shift corresponding to the different numbers of poor individuals. However, the general pattern of the development of poverty does not seem to be strongly affected by the identification step.

To discuss the effect of the choice of the poverty lines we calculate the modified AF index for different poverty lines. As described in Sect. 5 our standard case is $\boldsymbol{\pi} = (0.6 \cdot x^M_{\cdot 1}, 10, 10.5, 4, 45)$, where $x^M_{\cdot 1}$ is the median income. Decreasing (increasing) a single poverty line will c.p. result in a decrease (increase) of the number of poor individuals as well as relative shortfalls and therefore of the modified poverty index. Figure 10 shows this effect for a change in the income poverty line as well as a combined change in

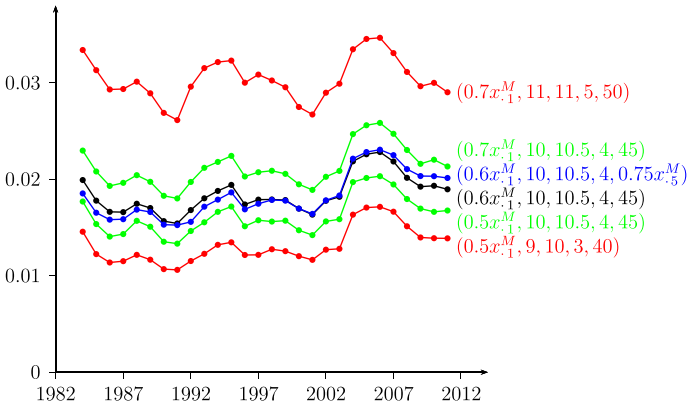


Fig. 10 Modified AF index for $\alpha = 1$, $k = 3$ and $m = 1$ for various poverty lines $(\pi_1, \ldots, \pi_5)$

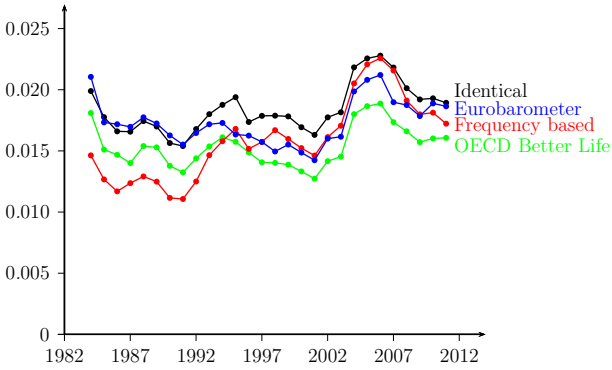


Fig. 11 Modified AF index for $\alpha = 1$, $k = 3$ and $m = 1$ for various dimensonal weights

all poverty lines. Although the extent of the shift is clearly greater if all poverty lines are affected both changes do not seem to affect the general development of the poverty index. Another change that does have an impact on the general trend is to replace the absolute threshold for the living dimension with a relative threshold.[28] Therefore we redefined the threshold from $\pi_5 = 45[m^2]$ to $\pi_5 = 0.75 \cdot x^M_{\cdot 5}$ so an individual is poor if his or her equivalized housing size is below 75 % of the median equivalized housing size. In Fig. 10 one can see that this seems to increase poverty over time relative to the absolute threshold, implicating that the housing size is increasing in general and a relative approach would lead to more deprived individuals in this dimension.

The last figure should demonstrate the effect of different dimensional weights. In the last subsection we saw that using the weights from the OECD Better Life index does decrease poverty while maintaining the general trend compared to equal weighting. Figure 11 shows a different pattern if we use weights obtained from the Eurobarometer survey or even more distinct if we use frequency based weights. To understand this pattern we

[27] This means that either the individual has nothing in one dimension or is on average 50 % below the poverty line in two dimensions.

[28] We chose living because for this dimension it might be plausible that individuals compare their situations among each other, like it is for income as well.

have to recall the results for the unidimensional FGT indices in Fig. 2. Roughly speaking we found a decreasing trend for health, education as well as living and a positive trend for income as well as employment. These trends seem to be reflected in the multidimensional indices. E.g. since the frequency based weight for employment is very high the increase in employment poverty leads to a higher growth in multidimensional poverty compared to equal weights.

7 Conclusions and Remarks

We have applied the Alkire Foster index of multidimensional poverty and defined a modified identification step. This new identification step allowed us to classify individuals with extreme deprivation in only few dimensions as poor with a very simple procedure, i.e. using the individual deprivations that are calculated for the aggregation step anyway. We have illustrated this modification with German data. This generalization came at the cost of an additional parameter m, who determines a "deprivation line", but since the choice of the parameter depends on individual deprivations it should be relatively easy to interpret.

In comparison to poverty indices used in official statistics like MPI or AROPE our index makes use of more than only binary information on the various dimensions of poverty. Although this is clearly demanding in terms of data availability and data quality – it may even be impossible if any of the considered dimensions of poverty are binary by nature – it seems to be a promising approach especially for developed countries like Germany. Additionally, if we want to enrich the analysis of poverty by taking metric data into account we may use this data not only for the aggregation but also for the identification step, being precisely the extension we have proposed for the original AF index.

Our results for the AF index show that in general multidimensional poverty in Germany seems to decrease but the situation of the extremely poor individuals seems to worsen. This is in line with the results of our modified measure, which suggest that even for restrictive choices of the new "deprivation line" the number of poor individuals significantly increases compared to the original identification procedure. But clearly the situation of extremely poor individuals has to be analysed in greater detail in future applications.

We want to finish the conclusions with some remarks about possible future research. Concerning the methodology, a dynamic extension of the modified AF measure could be an improvement, see the recent studies of Alkire et al. (2014) or Bossert et al. (2012) for the original AF measure. Also the impact of the dimensional weights could be analysed in greater detail, a problem that is not restricted to our modified measure but is present in all multidimensional poverty measures. For our first application we have used equal weights and additionally self stated dimensional weights. Other approaches to determine the dimension weights seem to be a challenging but interesting topic. Concerning the empirical application, especially a cross-country comparison seems to be promising, e.g. using EU-SILC data for a comparison of the development of extremely poor individuals across Europe. But it would also be interesting to use the modified identification approach for data of developing countries, since our motivating example at the beginning of Sect. 4 probably fits even better for poverty measurement for developing countries: In some situations being poor in two dimensions (slightly ill-nourished and slightly not well-educated) is better than being poor in one dimensions (being well educated but having nothing to eat), isn't it?!

Acknowledgments We would like to thank two anonymous referees and Karl Mosler for their very helpful contributions.

Appendix 1: Unidimensional Distributions and Descriptive Statistics

Table 3 Number of observations n and, if raising factors are used, observations $\widetilde{n}$, mean $\widetilde{\mu}$, median $\widetilde{m}$ and standard deviation $\widetilde{\sigma}$

Year	Observations		Income			Employment		Education		Health		Living	
	n	$\widetilde{n}$ (in m)	$\widetilde{\mu}$	$\widetilde{m}$	$\widetilde{\sigma}$	$\widetilde{\mu}$	$\widetilde{\sigma}$	$\widetilde{\mu}$	$\widetilde{\sigma}$	$\widetilde{\mu}$	$\widetilde{\sigma}$	$\widetilde{\mu}$	$\widetilde{\sigma}$
1984	11,518	46.9	803	716	701	11.6	1.9	10.8	2.2	6.8	2.7	59.3	21.7
1985	10,359	47.2	815	750	446	11.6	1.9	10.9	2.2	6.8	2.5	59.6	22.6
1986	10,020	47.7	863	777	704	11.6	1.8	10.9	2.2	6.7	2.5	60.0	23.0
1987	9955	48.0	894	818	626	11.6	1.8	11.0	2.3	6.7	2.4	60.3	22.0
1988	9500	48.1	919	844	560	11.6	1.8	11.0	2.3	6.5	2.4	61.1	22.6
1989	9205	48.4	965	869	592	11.7	1.8	11.0	2.3	6.5	2.5	61.9	23.3
1990	9020	49.1	1025	923	590	11.7	1.6	11.0	2.3	6.5	2.4	61.7	23.5
1991	8921	48.6	1063	972	503	11.7	1.6	11.1	2.4	6.5	2.4	62.4	24.4
1992	12,714	62.1	1026	920	527	11.6	1.8	11.2	2.4	6.6	2.4	59.4	24.5
1993	12,455	62.4	1101	1004	584	11.5	2.0	11.3	2.4	6.5	2.4	60.1	25.0
1994	12,710	62.5	1137	1023	599	11.4	2.2	11.3	2.4	6.4	2.4	60.5	25.0
1995	12,868	61.5	1194	1051	747	11.4	2.3	11.4	2.5	6.4	2.3	61.2	26.1
1996	12,534	61.3	1206	1091	659	11.4	2.2	11.4	2.4	6.4	2.3	62.0	26.0
1997	12,339	62.0	1215	1108	615	11.3	2.4	11.5	2.5	6.4	2.3	62.4	25.5
1998	13,033	58.8	1229	1119	595	11.3	2.6	11.5	2.5	6.5	2.3	64.0	25.7
1999	12,873	61.1	1270	1151	618	11.3	2.4	11.5	2.5	6.4	2.3	64.6	26.0
2000	22,115	60.5	1302	1176	679	11.4	2.3	11.6	2.5	6.7	2.3	65.5	26.3
2001	20,042	59.8	1334	1207	682	11.4	2.3	11.7	2.5	6.7	2.3	66.1	26.5
2002	21,420	59.3	1365	1200	913	11.4	2.4	11.6	2.4	6.5	2.3	65.9	26.6
2003	20,155	59.5	1399	1250	873	11.3	2.5	11.7	2.5	6.5	2.2	65.8	26.3
2004	19,604	59.9	1406	1250	967	11.2	2.7	11.7	2.5	6.4	2.3	66.7	27.2
2005	18,743	60.1	1413	1261	815	11.1	2.9	11.7	2.5	6.4	2.3	66.7	26.9
2006	19,765	59.0	1442	1267	903	11.1	2.9	11.8	2.6	6.4	2.3	67.1	27.6
2007	18,789	60.1	1466	1286	963	11.1	2.9	11.9	2.6	6.4	2.3	68.3	28.7
2008	17,557	60.0	1507	1333	984	11.2	2.7	11.9	2.6	6.4	2.2	69.2	28.5
2009	18,400	60.3	1547	1340	915	11.3	2.6	12.0	2.6	6.3	2.3	69.8	28.6
2010	16,688	59.8	1597	1400	977	11.3	2.6	12.1	2.6	6.4	2.3	70.9	29.9
2011	16,694	52.6	1619	1426	938	11.3	2.5	12.1	2.6	6.4	2.3	70.9	29.2

Appendix 2: Results for Six Dimensions

Table 4 Pairwise correlations for several years

	inc	empl	edu	health	living
1986					
inc	1	.07	.21	.04	.25
empl	.07	1	.04	.03	.08
edu	.21	.04	1	.09	.26
health	.04	.03	.09	1	.00
living	.25	.08	.26	.00	1
1991					
inc	1	.11	.35	.06	.38
empl	.11	1	.05	.06	.08
edu	.35	.05	1	.06	.25
health	.06	.06	.06	1	−.01
living	.38	.08	.25	−.01	1
1996					
inc	1	.14	.26	.06	.34
empl	.14	1	.07	.04	.09
edu	.26	.07	1	.09	.13
health	.06	.04	.09	1	−.01
living	.34	.09	.13	−.01	1
2001					
inc	1	.17	.35	.09	.37
empl	.17	1	.07	.07	.10
edu	.35	.07	1	.12	.15
health	.09	.07	.12	1	.01
living	.37	.10	.15	.01	1
2006					
inc	1	.19	.39	.11	.46
empl	.19	1	.10	.03	.15
edu	.39	.10	1	.14	.23
health	.11	.03	.14	1	.04
living	.46	.15	.23	.04	1
2011					
inc	1	.19	.41	.13	.42
empl	.19	1	.11	.05	.15
edu	.41	.11	1	.15	.20
health	.13	.05	.15	1	.04
living	.42	.15	.20	.04	1

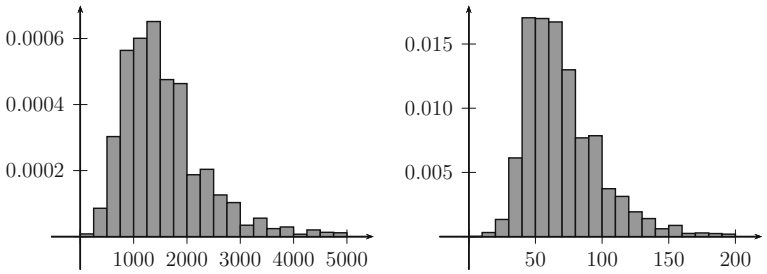


Fig. 12 Histograms for income (*left hand side*) and living, year 2011

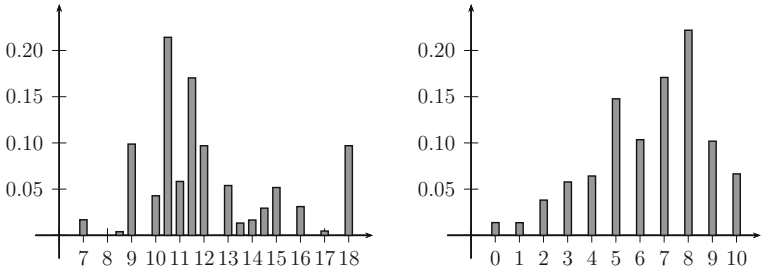


Fig. 13 Bar charts for education (*left hand side*) and health, year 2011

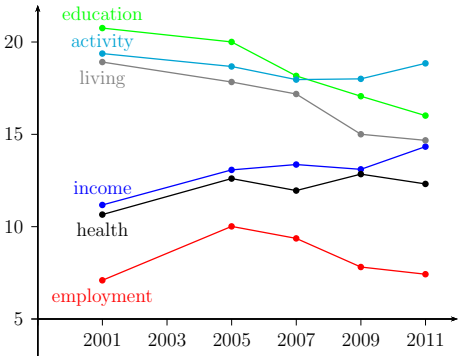


Fig. 14 Unidimensional headcounts (in %)

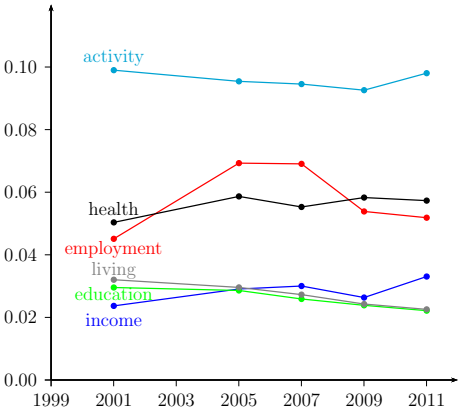


Fig. 15 Unidimensional FGT indices for $\alpha = 1$

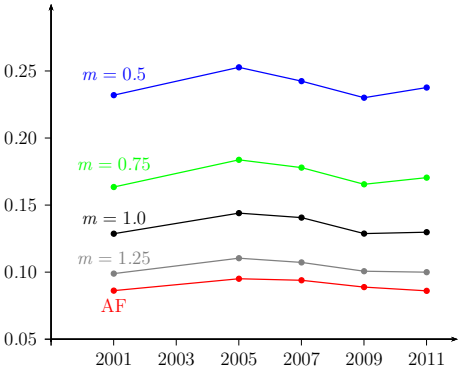
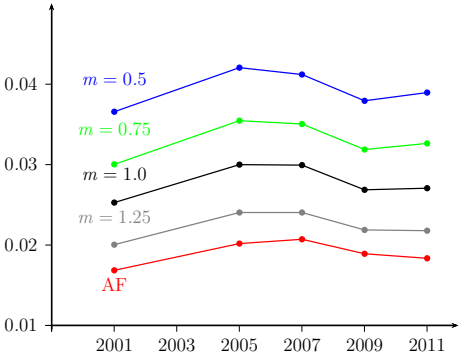


Fig. 16 Proportion of the poor for the AF and the modified index with $k = 3$, $\alpha = 1$ and for several m

Fig. 17 Values of the AF and the modified index with $k = 3$, $\alpha = 1$ and for several m



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