

# Measuring Human Development: A Multi-criteria Approach

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Accepted: 16 January 2015 / Published online: 25 January 2015  
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**Abstract** In 2010 the Human Development Index (HDI) was revised with several major changes. Many of its problems were tackled, although some drawbacks still persist. This paper proposes a multi-criteria approach to measure human development, propounding two innovations for the computation of the HDI: (1) the introduction of a double reference point scheme in the normalization; (2) an aggregation function which deals with the problem of substitutability between components. In particular, for each component of the HDI the value of each country is normalized by means of two reference values (aspiration and reservation values) by using an achievement scalarizing function that is piecewise linear. Aggregating the new normalized values, we calculate a range of indices with different degrees of substitutability: (1) a weak index that allows total substitutability; (2) a strong index that measures the state of the worst component and allows no substitutability; and (3) a mixed index that is a combination of the first two.

**Keywords** Human development · Multi-criteria approach · Double reference point methodology · Substitutability

**JEL Classifications** O15 · O57 · C02 · C44

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## 1 Introduction

The Human Development Index (HDI) is a summary measure that reflects average achievements in three basic dimensions of human development: a long and healthy life, access to knowledge, and a decent standard of living. Since 1990 the HDI has been widely used as an indicator of human well-being and progress closely related to the idea of human capabilities proposed by Sen (1985), and in broader terms than exclusively income-based progress. Despite its simplicity and limitations, the HDI has aroused great interest among researchers, practitioners and policy makers. For more than 20 years, the HDI has permitted evaluating the well-being of citizens from the perspective of human development across countries and over time, and has helped recommend policies which might lead to improve the lives of people and to enhance their choices and capabilities throughout the world, particularly in developing countries.

At the same time, since its launch, the HDI has generated in the academic field an extensive literature providing numerous critiques and proposing a number of potential improvements. The critiques have mainly focused on two major areas: (1) how to define human development and how to observe and measure its components, concentrating on topics such as the choice of indicators to reflect the human development idea accurately, and the intrinsic high correlation of all the HDI components with each other as well as with the HDI; (2) how to aggregate the different indicators to obtain a commonly acceptable composite index, centered mainly on the functional form of the HDI and aspects such as the substitutability assumptions, the normalization of indicators, the asymmetric treatment of income, and the choice of weights.<sup>1</sup>

In 2010, coinciding with the twentieth anniversary of the first global *Human Development Report* (HDR), the HDI was revised and several major changes were introduced in UNDP (2010). Though this is not the first time that the HDI was modified, it was the first time that major changes were simultaneously made to its components and to the functional form used. While life expectancy at birth was maintained as the indicator of a long and healthy life, the indicators used to measure progress in knowledge and standard of living were modified. In the knowledge dimension, mean years of schooling replaced literacy and gross enrolment was recast as expected years of schooling. To measure the standard of living, gross national income (GNI) per capita replaced gross domestic product (GDP) per capita. In addition, in each dimension the maximum values were shifted to the observed maximum, rather than having a predefined cut-off beyond which achievements were ignored. In relation to the functional form, one key modification was the replacement of the arithmetic mean of country-level attainments in the three basic dimensions for the geometric mean as the aggregation formula. In fact, the main reason given for introducing the new HDI was to avoid the past assumption of perfect substitutability between the HDI components.<sup>2</sup>

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<sup>1</sup> For a survey, see Kovacevic (2011), which was part of a comprehensive review undertaken by the Human Development Report Office (HDRO) of the United Nations Development Program (UNDP). Some significant contributions are McGillivray (1991), Desai (1991), Kelley (1991), Dossel and Gounder (1994), Gormely (1995), Ravallion (1997), Palazzi and Lauri (1998), Anand and Sen (2000), Chakravarty (2003), Chatterjee (2005), Foster et al. (2005), Chowdhury and Squire (2006), Gaertner and Xu (2006), Lind (2010), Herrero et al. (2010), De Muro et al. (2011), Nguéfack-Tsague et al. (2011), Pinar et al. (2013), Foster et al. (2012), Rende and Donduran (2013), etc.

<sup>2</sup> Let us recall that, as highlighted by authors such as Desai (1991) and Palazzi and Lauri (1998), the additive form of the HDI is problematic because it implies perfect substitution across components. It assumes that the level of priority to be given to a component is invariant to the level of attainments. In

Many of the problems pointed out by critics were tackled with the changes made to the manner in which the new HDI is calculated, although some authors consider that serious drawbacks still persist, such as some flaws in the design of the index from a purely theoretical viewpoint, the troubling tradeoffs across core dimensions of human development, and the excessively static nature of the HDI (see Ravallion 2010, 2011, 2012; Klugman et al. 2011a, b; Chakravarty 2011; Tofallis 2012; Herrero et al. 2012; Bilbao-Ubillos 2013; among others).

In general terms, many real-life problems involve tackling several criteria which are in conflict. In these problems both the criteria and the constraints that determine the feasible set of alternatives can be expressed mathematically by functions. Many methods have been developed over time. Reference point-based methods constitute an important group of techniques in this field, whose popularity and usefulness are unquestionable nowadays (see e.g. Miettinen 1999; Luque et al. 2009; and references therein). In this context, the decision maker can provide a reference value for each objective function that is considered acceptable. The important mathematical properties of these techniques—each solution obtained is efficient, and it is possible to generate every efficient solution, even the so-called non-supported solutions—have led to new proposals for more flexible models. Concretely, Wierzbicki et al. (2000) proposes the idea of the double reference point scheme, where, for each criterion, two values are introduced: an aspiration value, a level that is desirable to reach; and a reservation value, a level under which the objective function is not considered acceptable. These levels are frequently set by experts according to objective criteria, or, in their absence, in relative terms following statistical criteria. Recently, Bilbao-Terol et al. (2014) propose, among other things, using the TOPSIS methodology (Hwang and Yoon 1981) to measure the HDI. TOPSIS is also a multi-criteria technique used to rank a finite set of alternatives (countries in our case), based on the minimization of Euclidean distance from an ideal point (vector with the maximum values) and the maximization of Euclidean distance from a nadir point (vector with the minimum values in the efficient set). Ideal and nadir points provide maximum and minimum values, respectively, of the objective functions for all the efficient solutions, that is, non-dominated solutions.

In accordance with the human development paradigm, this paper introduces the multi-criteria approach in the debate about the revision of the HDI. This new methodological perspective proposes alternative normalization and aggregation formulas for the HDI, based on the double reference point methodology. In particular, for each component, the value of each country is normalized in the range  $[-1, 2]$  by means of two reference values (aspiration and reservation values) using an achievement scalarizing function which is piecewise linear. In the absence of absolute values widely accepted by the international community that can be used as desirable and reservation thresholds for each HDI component, these levels are calculated in a relative manner, taking into account for each component the situation of any country with respect to others. Aggregating the new normalized values of the components (values of the achievement scalarizing functions), we calculate three indices with different degrees of substitutability: (1) a weak index that allows full compensation between the components; (2) a strong index that measures the state of the worst component and allows no compensation; (3) and a mixed index that is a linear combination of the first two.

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Footnote 2 continued

addition, if a society were to seek policies to maximize its HDI, it might emphasize one component and disregard the others (see Klugman et al. 2011a).

This double reference point methodology goes beyond the concepts of maximum and minimum value of the indicators, and introduces a new perspective that is more in line with the human development paradigm and the capability approach. Concretely, this methodology introduces desirable and acceptable thresholds to be achieved by each component, and allows us to assess the progress in the measurement of human development depending on whether the value of the component is below the reservation level, between the reservation and aspiration levels, or above the aspiration level. This more flexible normalization allows determining  $s$  which values of the different indicators are equivalent.<sup>3</sup> In addition, the variety of indices proposed (strong, weak and mixed) offers the possibility of choosing the desired degree of substitutability. The minimization of Euclidean distance to the ideal point proposed in Bilbao-Terol et al. (2014) is in accordance with our weak index (if instead of using the Euclidean distance we use the distance  $L_1$ , both calculations would be equivalent) and the maximization of Euclidean distance to the nadir point proposed is in line with our strong index (a bad value for an indicator implies a smaller distance to the nadir point and a bad value of the strong index).

We apply the new indices proposed to 187 countries worldwide. They are calculated under the self-imposed restriction of using the data in the 2011 HDR in order to ensure the homogeneity and availability of all the values and to be able to make comparisons.

The remainder of the paper is as follows. Section 2 briefly presents some notes on the calculation of the HDI. Section 3 states our methodological approach. Section 4 displays the results for data from 2011 HDR. The final section presents our conclusions.

## 2 Calculating the HDI

The HDI is a summary composite index that measures a country's average achievements in the three core dimensions of human development<sup>4</sup>: (1) A long and healthy life, by using life expectancy at birth (LE) as an indicator (this is the only component that was not changed in 2010); (2) access to knowledge, measured as mean years of schooling (MS) and expected years of schooling (ES), the latter defined as the years of schooling that a child can expect to receive given current enrolment rates. These indicators have replaced literacy and gross enrolment rate. Both new indicators are summarized by using the geometric mean ( $S$ ); and (3) a decent standard of living, measured as the natural log of GNI per capita at purchasing-power parity (PPP) ( $Y$ ). As stated above, GNI has replaced GDP, also at PPP and logged.<sup>5</sup>

Following UNDP (2011), there are two steps to calculate the HDI. Firstly, the dimension indices are created. To this end, minimum and maximum values (goalposts) are set in order to transform the indicators into indices between 0 and 1. The maximums are the highest observed values in the time series (1980–2011) and the minimum values can be appropriately conceived as subsistence values. In particular, in the 2011 HDR the

<sup>3</sup> Note that the range normalization is a particular case of our framework.

<sup>4</sup> The HDI excludes other 'broader dimensions' of the concept of human development, such as empowerment, sustainability and equity. The 2010 HDR decided not to introduce any new dimensions in the HDI, stressing that the HDI can be characterized as an index of opportunities and freedoms, according to the two types of freedoms (opportunity freedoms and process freedoms) suggested by Sen (2002), that are valued by the human development approach (see, e.g., Klugman et al. 2011a).

<sup>5</sup> Given that the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000), the natural logarithm continues to be used to measure this HDI component.

minimum values are set at 20 years for life expectancy, at 0 years for both education variables and at \$100 for per capita GNI.<sup>6</sup>

$$I_{LE} = \frac{LE - LE^{\min}}{LE^{\max} - LE^{\min}} \quad (1)$$

$$I_S = \frac{S - S^{\min}}{S^{\max} - S^{\min}} \quad (2)$$

$$I_Y = \frac{\ln Y - \ln Y^{\min}}{\ln Y^{\max} - \ln Y^{\min}} \quad (3)$$

Having defined the minimum and maximum values and calculated the normalized dimension indices in the zero to one range, these are aggregated to produce the HDI as the geometric mean of the three dimension indices, instead of the arithmetic mean considered in the old aggregation formula. This way, in a multiplicative setting, the weights are applied by raising each variable to a power. Equal weights continue to be taken.<sup>7</sup> Thus, the HDI is calculated through the geometric mean of normalized indices measuring achievements in each core dimension.

$$\text{HDI} = I_{LE}^{1/3} \cdot I_S^{1/3} \cdot I_Y^{1/3} \quad (4)$$

As can be inferred from the foregoing, the trade-offs between components is not so troubling in this way, even though the implementation of the new functional form continues to involve implicit trade-offs. In this context, Herrero et al. (2012) highlight that although the choice of the geometric mean is certainly an important improvement, UNDP (2010) does not provide any theoretical justification of the new aggregation method. In order to justify the choice of the aggregation formula, they suggest an elementary characterization of the geometric mean following the axiomatic method, and identify the proposed indicator with a unique set of intuitive properties: neutrality, scale, and ratio consistency.

Although it is obvious that any composite index of this sort will entail potentially troubling trade-offs, as Ravallion (2010, 2012) recognizes, he highlights that the new multiplicative form appears to generate highly problematic trade-offs from the standpoint of assessing human development. In particular, he shows that the new HDI has greatly reduced its implicit weight on longevity in poor countries, and the valuations of extra schooling as a whole seem high. Ravallion (2010, 2012) and Chakravarty (2011) agree that the troubling trade-offs found in the new HDI could have been avoided to a large extent by using an alternative aggregation function from the literature, namely the generalized form of the old HDI proposed by Chakravarty (2003).

In the literature there are other significant contributions proposing alternative approaches to measuring human development. For instance, Bilbao-Ubillos (2013) propose an index called ‘Composite, Dynamic Human Development Index’, in order to palliate limitations of the HDI. This index incorporates eight different components that are significant

<sup>6</sup> UNDP (2011) reminds us that the low value for income can be justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, not reflected in the official data.

<sup>7</sup> The choice of equal weights has been widely criticized, with diverse methodologies proposed to set weights (see, e.g., Kelley 1991; Chowdhury and Squire 2006; Lind 2010; Nguiefack-Tsague et al. 2011; Tofallis 2012; Pinar et al. 2013; Foster et al. 2012), was also unchanged in 2010.

in the context of the current concept of human development (the three basic dimensions of human development already covered in the HDI, plus five other dimensions: inequality, poverty, gender situation, environmental sustainability, and personal safety) and a dynamic factor that distinguishes between countries on the basis of achievements attained. They adopt the simple additive weighting method to determine the weights subjectively and use the arithmetic mean for the aggregation of the eight components, without considering the problem of substitutability.

To the best of our knowledge, this is the first time that human development is treated by applying a multi-criteria approach based on the double reference point methodology, introducing a new perspective highly consistent with the human development paradigm. Likewise, the problem of substitutability between HDI components is tackled, offering a range of indices with different degrees of substitutability.

### 3 Methodology

#### 3.1 Multi-criteria Approach

Many real life problems involve dealing with optimization problems, in which multiple objective functions are maximized or minimized simultaneously within a feasible set of solutions or alternatives. The general form of a multi-objective optimization problem (MOP) can be represented by:

$$\begin{aligned} \max f(\mathbf{x}) &= (f_1(\mathbf{x}), \dots, f_k(\mathbf{x})) \\ \text{s.t. : } & \mathbf{x} \in X \end{aligned} \quad (5)$$

where  $\mathbf{x} = (x_1, \dots, x_n)^T$  is an  $n$ -dimensional vector of decision variables,  $X \subset \mathbb{R}^n$  is the feasible region,  $Z = f(X)$  is the feasible objective space, and  $\mathbf{z} = f(\mathbf{x})$  an objective vector where  $\mathbf{z} \in Z$  if  $\mathbf{x} \in X$  exists. The purpose is to simultaneously maximize all the  $k$  ( $k \geq 2$ ) objective functions. All the objective functions can be considered in the same sense (all maximizing or all minimizing), since minimizing an objective function is equivalent to maximizing the opposite one.

In multi-objective optimization, which generally lacks a feasible solution to simultaneously maximize all objective functions, there appears another concept of optimal where none of the components can be improved without deteriorating at least one of the others. A decision vector  $\mathbf{x}' \in X$  is called efficient or Pareto optimal of the problem MOP if there does not exist another  $\mathbf{x} \in X$  such as  $f_i(\mathbf{x}') \leq f_i(\mathbf{x})$  for all  $i = 1, \dots, k$  and  $f_j(\mathbf{x}') < f_j(\mathbf{x})$  for at least one index  $j$ . In this case,  $\mathbf{z}' = f(\mathbf{x}')$  is called nondominated objective vector. The efficient set is denoted by  $E$  and  $f(E)$  is the nondominated objective set. A decision vector  $\mathbf{x}' \in X$  is called weakly efficient or weakly Pareto optimal if there does not exist another  $\mathbf{x} \in X$  such as  $f_i(\mathbf{x}') < f_i(\mathbf{x})$  for all  $i = 1, \dots, k$ . The corresponding objective vectors are called weakly nondominated objective vectors. Note that the set of efficient solutions is a subset of weakly efficient solutions.

Since the set of non-dominated objective vectors contains more than one vector, it is useful to know the bounds for the objective vectors in the non-dominated set. Upper bounds are given by the ideal values  $\mathbf{z}^* = (z_1^*, \dots, z_k^*)$ , easily obtained by maximizing each objective function separately  $z_i^* = \max_{\mathbf{x} \in E} f_i(\mathbf{x}) = \max_{\mathbf{x} \in X} f_i(\mathbf{x})$  for all  $i = 1, \dots, k$ . However, nadir vector  $\mathbf{z}^{nad} = (z_1^{nad}, \dots, z_k^{nad})$ , which gives lower bounds for the objective vectors in

the non-dominated set  $(z_i^{nad} = \min_{\mathbf{x} \in E} f_i(\mathbf{x}) \text{ for all } i = 1, \dots, k)$ , is usually difficult to obtain (see Miettinen 1999 and references therein).

A very common way to express preferences about the efficient solutions is given by the so-called *reference point*  $\mathbf{q} = (q_1, \dots, q_k)^T$ , which consists of reference values for the objective functions. The multi-objective MOP problem and the reference point are combined in an achievement scalarizing function (ASF), which is optimized to generate (weakly) efficient solutions.

One of the most commonly used ASFs was proposed by Wierzbicki (1980):

$$s(\mathbf{q}, f(\mathbf{x}), \mu) = \min_{i=1, \dots, k} \{ \mu_i (f_i(\mathbf{x}) - q_i) \} + \rho \sum_{i=1}^k (f_i(\mathbf{x}) - q_i) \tag{6}$$

which must be maximized in the feasible region:

$$\begin{aligned} & \max s(\mathbf{q}, f(\mathbf{x}), \mu) \\ \text{s.t. : } & \mathbf{x} \in X \end{aligned} \tag{7}$$

The parameter  $\rho > 0$  is the so-called augmentation term, which must be a small value, and which assures the efficiency of the solutions generated. If the second term is not used, then only the weak efficiency of the solution is assured. The vector  $\mu = (\mu_1, \dots, \mu_k)^T$  with  $\mu_i > 0$  for all  $i = 1, \dots, k$  is formed by the weights assigned to reach the reference values, which can range from a purely normalizing coefficient to a preferential parameter (Ruiz et al. 2009; Luque et al. 2009).

Another achievement scalarizing function (Wierzbicki et al. 2000), used in both continuous and discrete programming, normalizes the objective functions (or indicators in our case) in a very appropriate way, taking into account two types of values of reference for each objective function. This type of ASF, called the double reference point (aspiration and reservation values) scheme, is based on considering an aspiration value  $z_i^a$  for each objective function  $f_i$  (it being desirable to reach that value) and a reservation value  $z_i^r$  (level under which the objective function is not considered acceptable). This type of methodology is also used in Luque et al. (2010) but for convex multiobjective problems.

In our case, let us consider the achievement scalarizing function:

$$s(f(\mathbf{x}), \mathbf{z}^a, \mathbf{z}^r) = \min_{i=1, \dots, k} \{ s_i(f_i(\mathbf{x}), z_i^a, z_i^r) \} + \rho \sum_{i=1}^k s_i(f_i(\mathbf{x}), z_i^a, z_i^r) \tag{8}$$

where  $s_i$  for all  $i = 1, \dots, k$  are the individual achievement scalarizing functions:

$$s_i(f_i(\mathbf{x}), z_i^a, z_i^r) = \begin{cases} 1 + \frac{f_i(\mathbf{x}) - z_i^a}{z_i^{max} - z_i^a} & \text{if } z_i^a \leq f_i(\mathbf{x}) \leq z_i^{max} \\ \frac{f_i(\mathbf{x}) - z_i^r}{z_i^a - z_i^r} & \text{if } z_i^r \leq f_i(\mathbf{x}) \leq z_i^a \\ \frac{f_i(\mathbf{x}) - z_i^r}{z_i^r - z_i^{min}} & \text{if } z_i^{min} \leq f_i(\mathbf{x}) \leq z_i^r \end{cases} \tag{9}$$

The values  $z_i^{max}$  and  $z_i^{min}$  are upper and lower bounds for each objective function  $f_i$  in the feasible region or even in the efficient set, if possible.  $z_i^{max} = z_i^*$  and,  $z_i^{min} = z_i^{nad}$  can be considered if available. Two parameters of the original formulation have been considered equal to 1. For more details about this ASF, see Wierzbicki et al. (2000).

This kind of ASF allows scaling all indicators in the interval  $[-1, 2]$ , so that different interpretations are given on the basis of the aspiration and reservation values. Although in the continuous case the ASF function must be maximized in the feasible region, as mentioned previously, in the discrete case (our case) it allows us to establish a ranking for the different alternatives. For our purposes, the values of the objective functions  $f_i(\mathbf{x})$  are substituted by the values of the indicators in the different alternatives (countries). It is convenient to emphasize that in our case we are not maximizing the alternatives (countries), but ranking them.

### 3.2 Application to Measure the HDI

Let us consider a total of  $N_I$  indicators and  $N_C$  the number of alternatives (countries). In our case,  $N_I = 3$  (*life expectancy at birth, combined education index, GNI per capita*) and  $N_C$  is the number of countries considered ( $N_C = 187$ ). Let us denote by  $y_{ij}$  ( $i = 1, \dots, N_C$  and  $j = 1, \dots, N_I$ ) the value of the country  $i$  and the indicator  $j$ . For each indicator it is necessary to determine whether it is of the type “more is better” (equivalent to maximizing in the continuous case) or “less is better” (equivalent to minimizing in the continuous case); in our case, the three are of type “more is better”.

For each indicator  $j$ , we have to calculate the maximum and minimum values:

$$y_j^{max} = \max_{i=1, \dots, N_C} y_{ij} \tag{10}$$

$$y_j^{min} = \min_{i=1, \dots, N_C} y_{ij} \tag{11}$$

However, these values can be modified by other values considered more appropriate.

The values of the aspiration and reservation levels, denoted by  $y_j^a$  and  $y_j^r$  respectively, are key to interpreting and analyzing the results. In the next section, we will explain which values are considered in our study.

Taking into account all the previous values calculated for each indicator  $j$ , let us consider the value given by the individual achievement scalarizing function in each alternative  $i$  (country):

$$z_{ij} = s_j(y_{ij}, y_j^a, y_j^r) = \begin{cases} 1 + \frac{y_{ij} - y_j^a}{y_j^{max} - y_j^a} & \text{if } y_j^a \leq y_{ij} \leq y_j^{max} \\ \frac{y_{ij} - y_j^r}{y_j^a - y_j^r} & \text{if } y_j^r \leq y_{ij} \leq y_j^a \\ \frac{y_{ij} - y_j^r}{y_j^r - y_j^{min}} & \text{if } y_j^{min} \leq y_{ij} \leq y_j^r \end{cases} \tag{12}$$

Given a country  $i$  and an indicator  $j$ , if  $z_{ij}$  is between  $-1$  and  $0$ , it means that the value of the indicator for this country is under the reservation value; between  $0$  and  $1$ , that it is between reservation and aspiration values; and between  $1$  and  $2$ , that it is over the aspiration value.

For each country  $i$ , let us define the *weak index* ( $W_i$ ) as the arithmetic mean of the  $N_I$  values of the indicators and the *strong index* ( $S_i$ ) as the minimum of all, that is, the worst one:



$$W_i = \frac{1}{N_I} \sum_{j=1}^{N_I} z_{ij} \tag{13}$$

$$S_i = \min_{j=1, \dots, N_I} z_{ij} \tag{14}$$

While the weak index allows compensation among different indicators (substitutability), the strong index does not allow any compensation since it represents the worst value. In case we want to assign different weights to the indicators, let  $\omega_j$  with  $j = 1, \dots, N_I$  be the weight values, which have to be strictly positive ( $\omega_j > 0 \quad \forall j = 1, \dots, N_I$ ). The weak index is calculated directly:

$$W_i = \sum_{j=1}^{N_I} \bar{\omega}_j z_{ij} \tag{15}$$

where  $\bar{\omega}_j$  is the normalized weight ( $\bar{\omega}_j = \frac{\omega_j}{\sum_{k=1}^{N_I} \omega_k} \quad \forall j = 1, \dots, N_I$ ). However, for the strong index, it is necessary to make some changes to avoid unwanted effects. Specifically, let us consider the following weights normalized by its maximum value:

$$\bar{\omega}_j = \frac{\omega_j}{\max_{k=1, \dots, N_I} \omega_k} \quad \forall j = 1, \dots, N_I \tag{16}$$

and for each country  $i$ , we define the following values:

$$\bar{z}_{ij} = z_{ij} - C_i \quad \forall j = 1, \dots, N_I \text{ with } C_i = \left[ \min_{j=1, \dots, N_I} z_{ij} \right] + 1 \tag{17}$$

where  $[ \ ]$  is the integer part of a real number. Then, the strong index is given by:

$$S_i = C_i + \min_{j=1, \dots, N_I} \bar{\omega}_j \bar{z}_{ij} \tag{18}$$

The strong index indicates that if its value is below 0, at least one indicator is under 0 (at least one indicator does not reach its corresponding reservation value). If the strong index is above 1, it means that all the indicators improve their corresponding aspiration values.

As a combination of both we propose a *mixed indicator* ( $MI_i$ ), which is a linear combination of the previous ones:

**Table 1** Goalposts for the Human Development Index in 2011 HDR

| Dimension                   | Observed maximum            | Minimum |
|-----------------------------|-----------------------------|---------|
| Life expectancy             | 83.4 (Japan, 2011)          | 20.0    |
| Mean years of schooling     | 13.1 (Czech Republic, 2005) | 0       |
| Expected years of schooling | 18.0 (capped at)            | 0       |
| Combined education index    | 0.978 (New Zealand, 2010)   | 0       |
| GNI (PPPS)                  | 107,721 (Qatar, 2011)       | 100     |

Source: UNDP (2011)

$$MI_i = \lambda W_i + (1 - \lambda)S_i \text{ with } 0 < \lambda < 1 \quad (19)$$

and reflects an intermediate state between total substitutability (weak index) and no substitutability (strong index).

### 3.3 Calculation of Aspiration and Reservation Levels

As mentioned previously, in order to apply the proposed normalization, an aspiration level and reservation level have to be defined for each component. Let us recall that a component's level of aspiration is the desirable level to be achieved by said component, whereas the level of reservation is the value below which all values are considered unacceptable (Table 1).

In our methodological proposal, these levels are key for normalization. As far as we are concerned, these values should be defined objectively in an absolute manner. However, at present there are no values widely accepted by the international community that can be used as desirable and reservation thresholds for each HDI component. This issue, which is beyond the scope of this paper, clearly constitutes a challenging and promising extension of this study. Thus, in this paper the aspiration and reservation levels have been calculated in a relative manner, considering the situation of a country with respect to the others for each component.<sup>8</sup> In particular, we use two different statistical criteria to calculate these values:

1. *Criterion I: Weighted mean of first and third group countries.* The UNDP (2011) classifies countries as *Very High Human Development*, *High Human Development*, *Medium Human Development* and *Low Human Development*. To do so, it divides the countries listed according to their HDI level into 4 equal parts. Similarly, we ranked countries according to the values of the respective components, taking as level of aspiration the corresponding mean values weighted by population of the countries with *Very High* levels for the component in question. On the other hand, as level of reservation we used the mean weighted values of the group of countries with *Medium* levels for the respective components. The figures for each component are shown in Table 2.
2. *Criterion II: First and third quartile.* The second criterion takes as level of aspiration the first quartile (value below which 75 % of the countries -127 countries- appear for each component), according to the order of the list of countries mentioned above for each component. We consider the third quartile as the reservation value; in other words, the value below which 25 % of the countries -42 countries- appear for each component. The figures are shown in Table 2.

Both criteria give us the relative position of a country with respect to the others. A priori no criterion is preferred to any other, and we work with both in parallel.

## 4 Results

### 4.1 Calculation of Normalized Components

In addition to the aspiration and reservation values, for the normalization of the components, we need a maximum and a minimum value for each indicator. For our calculations,

<sup>8</sup> In part, this is an empirical criterion similar to the one used by UNDP (2011) to set the minimum and maximum values (goalposts), according to the situation of countries worldwide in recent years.

**Table 2** Maximum and minimum and reference values

|                       | Life expectancy | Ln GNI per capita | Combined Education Index |
|-----------------------|-----------------|-------------------|--------------------------|
| Max                   | 83.394          | 11.587            | 15.709                   |
| Min                   | 20              | 4.605             | 0                        |
| Aspiration values I   | 80.000          | 10.416            | 13.381                   |
| Reservation values I  | 66.825          | 8.173             | 9.111                    |
| Aspiration values II  | 76.128          | 9.729             | 11.852                   |
| Reservation values II | 64.228          | 7.728             | 7.131                    |

we consider the maximum and minimum goalposts for each indicator used to calculate the official HDI. The respective maximum and minimum values for each component are shown in Table 2.

After calculating the necessary parameters, we obtained the normalized components by applying Eq. (12). Since we are working with two criteria to calculate aspiration and reservation values, we obviously obtained two different results. In Table 3 we show the components normalized for a selection of countries<sup>9</sup> according to reference values calculated by means of criterion I and criterion II.

Table 3 shows, for example, that Japan's normalized life expectancy value is 2. This means that its value for this component coincides with the maximum considered. Indeed, let us recall that in this case the maximum considered for this component was taken from the value registered in Japan in 2011 (see Table 1). In respect to the other results, normalized values above 1 mean that countries are above the aspiration level for that component, whereas values below 0 indicate that they are below the reservation value. If we look at Table 3—Criterion I, we can see, for example, that the United States is above the level of aspiration in attainments in education and income, although in respect to life expectancy it is below said level, indicating a very high, though slightly imbalanced, level of human development. Nigeria, on the other hand, shows values below the level of reservation for all the components in both tables, thus indicating low but relatively balanced levels of human development in the three core dimensions.

#### 4.2 Calculation of Strong, Weak and Mixed Indices

Let us calculate the weak and strong indices using the Eqs. (13–18). Although our methodology allows establishing different weights for each component, in this paper we have considered the same weights<sup>10</sup> for all components, in line with the position followed by the UNDP in calculating the official HDI. To calculate the mixed index with the Eq. (19), we use a value of  $\lambda = 0.5$ .

<sup>9</sup> For the presentation of the results, we have selected the 10 most populated countries, which represent about 60 % of world population. These countries, listed on the basis of their HDI, are distributed amongst the 4 groups of countries as defined in the 2011 HDR: *Very High Human Development* (United States, Japan), *High Human Development* (Russian Federation, Brazil), *Medium Human Development* (China, Indonesia, India, Pakistan) and *Low Human Development* (Bangladesh, Nigeria).

<sup>10</sup> We have carried out a sensitivity analysis of the weights of the components, showing that for a slight modification of one of them (for instance, 10 %) the ranking is almost equal (90 % of countries maintain their position or vary one or two positions), whereas major changes in the weights involve greater variations in the rankings. These results are in line with the role of the weights in the achievement scalarizing functions (see Luque et al. (2009) or Ruiz et al. (2009) for more details).

**Table 3** Normalized components

|                     | Life expectancy | Life expectancy (normalized) | Ln GNI per capita | Ln GNI per capita (normalized) | Combined Education Index | Combined Education Index (normalized) |
|---------------------|-----------------|------------------------------|-------------------|--------------------------------|--------------------------|---------------------------------------|
| <i>Criterion I</i>  |                 |                              |                   |                                |                          |                                       |
| Min                 | 20.000          |                              | 4.605             |                                | 0.000                    |                                       |
| Reservation         | 66.825          |                              | 8.173             |                                | 9.111                    |                                       |
| Aspiration          | 80.000          |                              | 10.416            |                                | 13.381                   |                                       |
| Max                 | 83.394          |                              | 11.587            |                                | 15.709                   |                                       |
| United States       | 78.531          | 0.889                        | 10.669            | 1.216                          | 14.095                   | 1.307                                 |
| Japan               | 83.394          | 2.000                        | 10.383            | 0.985                          | 13.248                   | 0.969                                 |
| Russian Federation  | 68.823          | 0.152                        | 9.586             | 0.630                          | 11.769                   | 0.623                                 |
| Brazil              | 73.488          | 0.506                        | 9.226             | 0.469                          | 9.944                    | 0.195                                 |
| China               | 73.456          | 0.503                        | 8.920             | 0.333                          | 9.344                    | 0.054                                 |
| Indonesia           | 69.366          | 0.193                        | 8.220             | 0.021                          | 8.760                    | -0.039                                |
| India               | 65.438          | -0.030                       | 8.151             | -0.006                         | 6.748                    | -0.259                                |
| Pakistan            | 65.437          | -0.030                       | 7.844             | -0.092                         | 5.789                    | -0.365                                |
| Bangladesh          | 68.944          | 0.161                        | 7.333             | -0.236                         | 6.232                    | -0.316                                |
| Nigeria             | 51.879          | -0.319                       | 7.635             | -0.151                         | 6.631                    | -0.272                                |
| <i>Criterion II</i> |                 |                              |                   |                                |                          |                                       |
| Min                 | 20.000          |                              | 4.605             |                                | 0.000                    |                                       |
| Reservation         | 64.228          |                              | 7.728             |                                | 7.131                    |                                       |
| Aspiration          | 76.128          |                              | 9.729             |                                | 11.852                   |                                       |
| Max                 | 83.394          |                              | 11.587            |                                | 15.709                   |                                       |
| United States       | 78.531          | 1.331                        | 10.669            | 1.506                          | 14.095                   | 1.581                                 |
| Japan               | 83.394          | 2.000                        | 10.383            | 1.352                          | 13.248                   | 1.362                                 |
| Russian Federation  | 68.823          | 0.386                        | 9.586             | 0.929                          | 11.769                   | 0.982                                 |
| Brazil              | 73.488          | 0.778                        | 9.226             | 0.749                          | 9.944                    | 0.596                                 |
| China               | 73.456          | 0.775                        | 8.920             | 0.595                          | 9.344                    | 0.469                                 |
| Indonesia           | 69.366          | 0.432                        | 8.220             | 0.246                          | 8.760                    | 0.345                                 |
| India               | 65.438          | 0.102                        | 8.151             | 0.212                          | 6.748                    | -0.054                                |
| Pakistan            | 65.437          | 0.102                        | 7.844             | 0.058                          | 5.789                    | -0.188                                |
| Bangladesh          | 68.944          | 0.396                        | 7.333             | -0.127                         | 6.232                    | -0.126                                |
| Nigeria             | 51.879          | -0.279                       | 7.635             | -0.030                         | 6.631                    | -0.070                                |

We show the values of these indices for the group of countries selected using values of reference I and II (Table 4), denoted as DRP-WI (Double Reference Point—Weak Index), DRP-SI (Double Reference Point—Strong Index) and DRP-MI (Double Reference Point—Mixed Index).

Table 4 shows that the DRP-SI -which measures the state of the worst component and allows no compensation between components- is much stricter than the DRP-WI -which

allows compensation among components-, and is lower in all cases. The DRP-MI, in turn, always presents values between the other two indices.

#### 4.3 Rankings and Differences in Respect to HDI Rank

If we take into account country rankings for each new index calculated (DRP-WI, DRP-SI, DRP-MI), we should analyze the consistency of those rankings with HDI country rankings, and the differences in the number of positions between said rankings for each country. The results for the group of countries selected obtained using criteria I and II appear in Table 5 (the position in the corresponding ranking appears in parenthesis). To be more precise, the HDI values (ranks in parentheses) are displayed in the second column; the weak, strong and mixed indices in the third, fourth and fifth columns, respectively, whilst the differences of rank between the official HDI and our indices are shown in the last three columns. The new indices are calculated using both criteria I and II.

In general, the correlation between HDI's ranking of countries and the rankings according to DRP-WI, DRP-SI and DRP-MI is obviously quite high, in particular if we bear in mind that we are only working with three components and are analyzing a relatively large sample ( $N = 187$ ). Specifically, using criterion I for the reference values, Spearman's correlation coefficients  $\rho$  are 0.997, 0.976 and 0.994, respectively, and using criterion II, they are 0.997, 0.974 and 0.992. On the other hand, Kendall's correlation coefficients  $\tau$  are 0.958, 0.874 and 0.939 (criterion I) and 0.958, 0.867 and 0.93 (criterion II).

**Table 4** DRP-WI, DRP-SI and DRP-MI

| Country             | DRP-WI | DRP-SI | DRP-MI |
|---------------------|--------|--------|--------|
| <i>Criterion I</i>  |        |        |        |
| United States       | 1.137  | 0.889  | 1.013  |
| Japan               | 1.318  | 0.969  | 1.143  |
| Russian Federation  | 0.468  | 0.152  | 0.310  |
| Brazil              | 0.390  | 0.195  | 0.293  |
| China               | 0.297  | 0.054  | 0.176  |
| Indonesia           | 0.058  | -0.039 | 0.010  |
| India               | -0.098 | -0.259 | -0.179 |
| Pakistan            | -0.162 | -0.365 | -0.263 |
| Bangladesh          | -0.130 | -0.316 | -0.223 |
| Nigeria             | -0.247 | -0.319 | -0.283 |
| <i>Criterion II</i> |        |        |        |
| United States       | 1.473  | 1.331  | 1.402  |
| Japan               | 1.571  | 1.352  | 1.461  |
| Russian Federation  | 0.766  | 0.386  | 0.576  |
| Brazil              | 0.708  | 0.596  | 0.652  |
| China               | 0.613  | 0.469  | 0.541  |
| Indonesia           | 0.341  | 0.246  | 0.293  |
| India               | 0.086  | -0.054 | 0.016  |
| Pakistan            | -0.010 | -0.188 | -0.099 |
| Bangladesh          | 0.048  | -0.127 | -0.039 |
| Nigeria             | -0.126 | -0.279 | -0.203 |

Table 5 Rankings and differences

| Country             | HDI   | DRP-WI | DRP-SI | DRP-MI | HDI rank—<br>DRP-WI rank | HDI rank—<br>DRP-SI rank | HDI rank—<br>DRP-MI rank |
|---------------------|-------|--------|--------|--------|--------------------------|--------------------------|--------------------------|
| <i>Criterion I</i>  |       |        |        |        |                          |                          |                          |
| United States       | 0.910 | 1.137  | 0.889  | 1.013  | (16)                     | (16)                     | -10                      |
| Japan               | 0.901 | 1.318  | 0.969  | 1.143  | (3)                      | (3)                      | 9                        |
| Russian Federation  | 0.755 | 0.468  | 0.152  | 0.310  | (86)                     | (80)                     | -5                       |
| Brazil              | 0.718 | 0.390  | 0.195  | 0.293  | (86)                     | (83)                     | -2                       |
| China               | 0.687 | 0.297  | 0.054  | 0.176  | (100)                    | (100)                    | 1                        |
| Indonesia           | 0.617 | 0.058  | -0.039 | 0.010  | (131)                    | (121)                    | -7                       |
| India               | 0.547 | -0.098 | -0.259 | -0.179 | (136)                    | (138)                    | 0                        |
| Pakistan            | 0.504 | -0.162 | -0.365 | -0.263 | (144)                    | (147)                    | 1                        |
| Bangladesh          | 0.500 | -0.130 | -0.316 | -0.223 | (141)                    | (143)                    | 5                        |
| Nigeria             | 0.459 | -0.247 | -0.319 | -0.283 | (157)                    | (152)                    | -1                       |
| <i>Criterion II</i> |       |        |        |        |                          |                          |                          |
| United States       | 0.910 | 1.473  | 1.331  | 1.402  | (16)                     | (11)                     | -12                      |
| Japan               | 0.901 | 1.571  | 1.352  | 1.461  | (3)                      | (6)                      | 9                        |
| Russian Federation  | 0.755 | 0.766  | 0.386  | 0.576  | (73)                     | (89)                     | -7                       |
| Brazil              | 0.718 | 0.708  | 0.596  | 0.652  | (85)                     | (78)                     | -1                       |
| China               | 0.687 | 0.613  | 0.469  | 0.541  | (98)                     | (91)                     | 3                        |
| Indonesia           | 0.617 | 0.341  | 0.246  | 0.293  | (126)                    | (115)                    | -2                       |
| India               | 0.547 | 0.086  | -0.054 | 0.016  | (137)                    | (134)                    | -3                       |
| Pakistan            | 0.504 | -0.010 | -0.188 | -0.099 | (146)                    | (145)                    | -1                       |
| Bangladesh          | 0.500 | 0.048  | -0.127 | -0.039 | (141)                    | (139)                    | 5                        |
| Nigeria             | 0.459 | -0.126 | -0.279 | -0.203 | (157)                    | (158)                    | -1                       |

For example, Table 5—Criterion I shows how the United States descends 13 positions in the DRP-SI country ranking as compared to the HDI rank, influenced by its worse relative situation in terms of life expectancy; whereas Nigeria, with less inequality between components than other countries with similar levels of human development, rises 12 positions. Likewise, the cases of the Russian Federation and Indonesia are interesting. With respect to both criteria, the former loses 20 and 29 positions respectively in the DRP-SI ranking as compared to the HDI ranking, also affected by its relatively low attainments in health, whereas the latter rises 16 and 17 positions.<sup>11</sup>

In order to facilitate the analysis of the results, the differences between the HDI and DRP-WI and DRP-SI country ranks have been represented graphically (Fig. 1). As can be seen, the distribution in Fig. 1a, b, on one hand, and Fig. 1c, d, on the other, is similar, with no significant differences in respect to country rankings between the two criteria used to determine the values of reference.<sup>12</sup> In this sense, in order to avoid reiterations and since that the correlation between the results of both criteria is very high, hereafter we focus our analysis on the results obtained with the values of reference calculated using criterion I (the results for the other criterion may be obtained from the authors upon request).

In respect to the differences in country positions between the HDI and DRP-WI rankings, it should be noted that the maximum variation of positions is (+13), whereas it reaches (−60) if we compare HDI and DRP-SI rankings. Furthermore, in the first case 80 % of countries change positions in the range of [−5, 5], whereas in the second case only 40 % of countries are in that range. This clearly shows that the difference of positions of the HDI ranking in respect to DRP-SI's are greater than in respect to DRP-WI's, which seems to confirm that the valuations of the level of human development stemming from the official HDI are more similar to the values of our weak index than to those of our strong index.

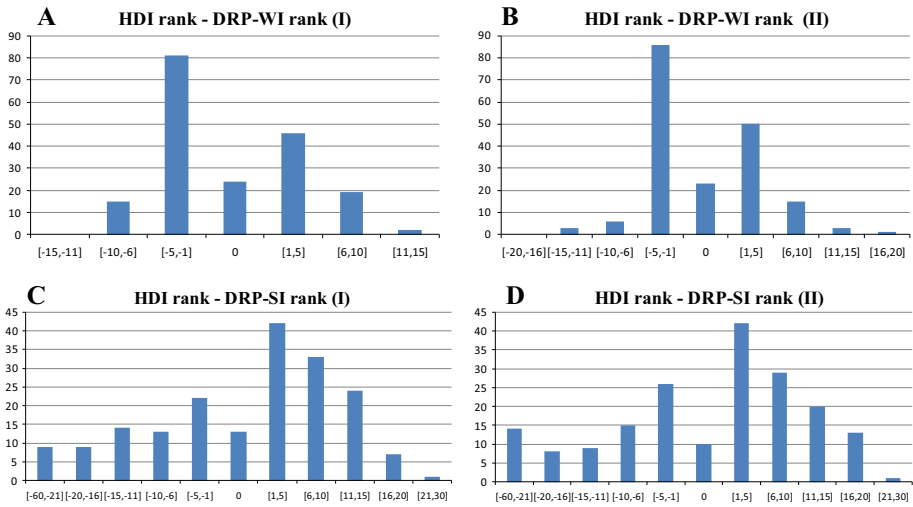
#### 4.4 Analyzing Differences Between DRP-WI Rank and DRP-SI Rank

In order to compare a composite index with total substitutability and one that does not allow any substitutability between components, the country ranks resulting from DRP-WI and DRP-SI should also be compared. Figure 2 shows the differences in country rank positions between the alternative indicators using reference values I.

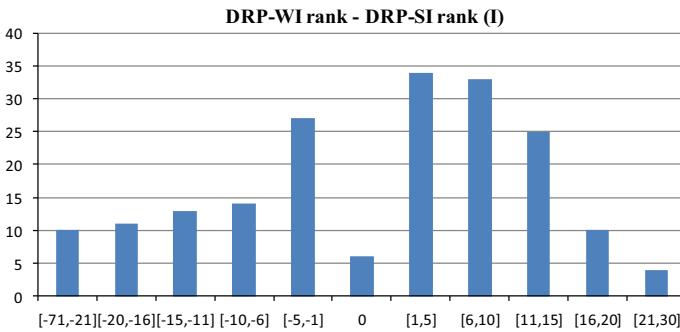
The graph shows, first of all, that 6 countries maintain their position in both rankings: Eritrea, Iran, Lesotho, Mali, Mauritania and Norway (with first place in the HDI, it maintains that position in all the indices calculated). Likewise, note should be taken of the countries which lose more than 20 positions with DRP-SI as compared to DRP-WI, and in some cases up to 71 positions (left side of Fig. 2). These countries are Qatar, Kuwait, Cuba, Oman, Bhutan, Botswana, South Africa, Andorra, Kazakhstan and Georgia, all having very imbalanced levels in respect to health, education and income achievements. On the other hand, the maximum number of positions a country has gained with the DRP-SI is 28, the following four having improved more than 20 positions: Saint Vincent and the Grenadines, Indonesia, Azerbaijan and Kiribati (right side of Fig. 2). For these countries, the three components show highly similar levels, meaning that their human development is very balanced.

<sup>11</sup> The results of Table 5 for all countries are available from the authors upon request.

<sup>12</sup> Specifically, Spearman's correlation coefficient  $\rho$  amongst the ranks of countries listed according to the DRP-WI using criteria I and II is 0.999, and 0.992 amongst the DRP-SI ranks using both criteria.



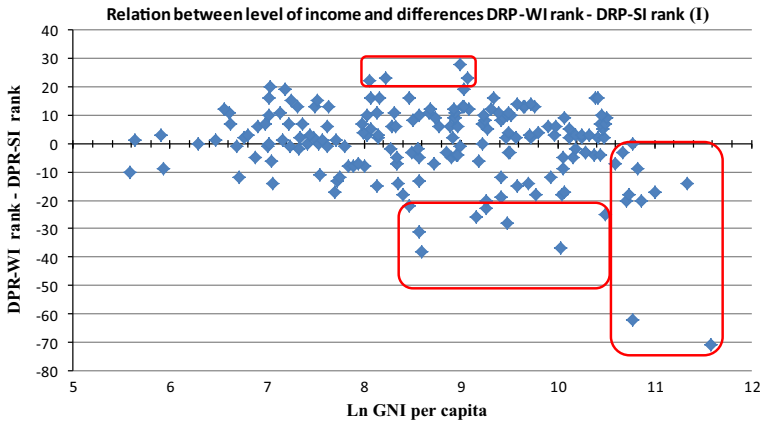
**Fig. 1** a Differences between HDI rank and DRP-WI rank (I). b Differences between HDI rank and DRP-WI rank (II). c Differences between HDI rank and DRP-SI rank (I). d Differences between HDI rank and DRP-SI rank (II)



**Fig. 2** Differences between DRP-WI rank and DRP-SI rank (I)

In order to assess the degree of imbalance amongst components of countries on the basis of their level of income, Fig. 3 relates the difference of positions in the DRP-WI and DRP-SI rankings to the level of the countries' GNI per capita. In general, differences of rank increase as country income level rises, both in positive and negative terms, this difference tending to be solely negative in countries with a high level of income. In this sense, 3 areas of the graph are noteworthy: the upper area, where we find positive differences of more than 20 positions which begin at a level that corresponds to a GNI per capita of \$3,140 (constant 2005 PPP\$), and which includes Kiribati, Indonesia, Azerbaijan and Saint Vincent and the Grenadines; a lower middle area, where we find the first negative differences of more than 20 positions, which occur starting at a GNI per capita of \$4,780 (constant 2005 PPP\$), with Georgia, Bhutan, Cuba, South Africa, Kazakhstan, Botswana, Oman and Andorra; and lastly, a striking lower area to the right, where starting at a level of GNI per capita of \$39,924 (constant 2005 PPP\$) all the countries (except Norway, with a





**Fig. 3** Relation between level of income and DRP-WI rank—DRP-SI rank differences (I)

**Table 6** Differences DRP-WI rank—DRP-SI rank and normalized components

| HDI rank | Country      | Income Group        | DRP-WI rank | DRP-SI rank | DRP-WI rank—DRP-SI rank | Life expectancy (normalized) | Ln GNI per capita (normalized) | Combined Education Index (normalized) |
|----------|--------------|---------------------|-------------|-------------|-------------------------|------------------------------|--------------------------------|---------------------------------------|
| 37       | Qatar        | High income         | 26          | 97          | -71                     | 0.876                        | 2.000                          | <b>0.056</b>                          |
| 63       | Kuwait       | High income         | 50          | 112         | -62                     | 0.588                        | 1.308                          | <b>-0.049</b>                         |
| 51       | Cuba         | Upper middle income | 42          | 80          | -38                     | 0.935                        | <b>0.189</b>                   | 0.943                                 |
| 89       | Oman         | High income         | 85          | 122         | -37                     | 0.467                        | 0.831                          | <b>-0.112</b>                         |
| 141      | Bhutan       | Upper middle income | 135         | 166         | -31                     | 0.027                        | 0.179                          | <b>-0.447</b>                         |
| 118      | Botswana     | High income         | 111         | 139         | -28                     | <b>-0.291</b>                | 0.581                          | 0.303                                 |
| 123      | South Africa | Upper middle income | 116         | 142         | -26                     | <b>-0.300</b>                | 0.438                          | 0.343                                 |
| 32       | Andorra      | High income         | 30          | 55          | -25                     | 1.276                        | 1.066                          | <b>0.421</b>                          |
| 68       | Kazakhstan   | Upper middle income | 78          | 101         | -23                     | <b>0.015</b>                 | 0.488                          | 0.797                                 |
| 75       | Georgia      | Upper middle income | 67          | 89          | -22                     | 0.525                        | <b>0.133</b>                   | 0.816                                 |

difference of 0) show negative rank differences, namely Switzerland, United States, Hong Kong, China (SAR), Brunei Darussalam, Norway, Kuwait, Luxembourg, Singapore, United Arab Emirates, Liechtenstein and Qatar.

To underscore the imbalances existing between the different core dimensions in countries with negative rank differences of more than 20 positions between DRP-WI and DRP-SI, Table 6 shows the normalized components, highlighting in bold for each case the one which is in a worse situation. Countries are ordered according to their rank differences, also specifying the income group to which they belong pursuant to the World Bank classification (2013), which distinguishes 4 categories: low-income, lower-middle-income, upper-middle-income, and high-income.

If we observe the situation of the different components in the countries with the greatest imbalance amongst core dimensions, note should be taken, first of all, of countries such as Qatar and Kuwait, which are well ranked by DRP-WI basically due to their high value of GNI per capita, descending 71 and 62 positions, respectively, in the DRP-SI ranking. In both cases, the other components do not reach the aspiration value, with Qatar very close to the reservation value in education and Kuwait even below said value. Other countries with significant imbalances where education is the component at the lowest level are Oman, Bhutan and Andorra. On the other hand, countries whose health component is in relatively more unfavorable situation include Botswana, South Africa and Kazakhstan. Note should also be taken of countries such as Cuba and Georgia, in which the results for the income component are clearly the worst.

In general terms, it is worth pointing out that out of 32 countries with negative differences of more than 10 positions, 29 are high-income (19) or upper middle-income (10) countries and 23 have a very high (11) or high (12) HDI. Meanwhile, out of 32 countries with positive differences of more than 10 positions, 27 are upper middle-income (16) and lower middle-income (11) countries, although they present diverse HDI levels (7 low, 8 middle, 13 high, 4 very high). Both groups of countries are located across different continents, without any particular pattern being detected in respect thereto.

## 5 Conclusions

The problem of trade-offs between components is present to a greater or lesser extent in the majority of composite indices. One of major critiques of the original HDI was the perfect substitutability between different dimensions of well-being. The new HDI has replaced the arithmetic mean with the geometric mean as the aggregation formula of country-level attainments in health, education and income, reaching a compromise between the extremes of perfect substitutability and no substitutability. However, some leading authors, such as Ravallion (2010, 2012), point out that there are still considerable troubling trade-offs that might involve inappropriate implications in terms of development policy.

This paper introduces a multi-criteria approach in the debate about the measurement of human development, providing two major innovations for the computation of the HDI: a new normalization more in line with the human development paradigm, and alternative aggregation formulas that provide a variety of composite indices with different degrees of substitutability. Concretely, we implement a multi-criteria approach based on the double reference point methodology (aspiration and reservation). In order to be able to compare, we use the same indicators as the new HDI and data from the 2011 HDR. Nevertheless, we take advantage of a different normalization: for each indicator the value of each country is normalized by means of two reference values (aspiration and reservation values) by using

an achievement scalarizing function which is piecewise linear. In the absence of commonly agreed objective reference values, we use two relative reference values for each indicator, taking into account that the reservation value can be interpreted as a minimal level of acceptable achievement for an indicator across countries, and the aspiration value as a minimal level of desirable achievement.

As in the official HDI, each core dimension has been equally weighted, although our methodology would enable adopting any alternative weighting scheme. Aggregating the normalized values of the achievement scalarizing functions, we calculate three composite indicators: (1) a weak index that allows total substitutability; (2) a strong index that measures the state of the worst component and allows no substitutability; (3) and a mixed index that is a linear combination of the first two and would allow different degrees of substitutability. In contrast with the conventional range  $[0, 1]$  between maximum and minimum values of the HDI, these indices range between  $-1$  and  $2$ , so that if their values are between  $-1$  and  $0$ , it means that the value of the indicator for this country is under the reservation value; between  $0$  and  $1$ , that they are between reservation and aspiration values; and between  $1$  and  $2$ , that they are over the aspiration value. This normalization could be moved to the range  $[0, 1]$  by means of a linear transformation and the results would be identical (the same ranks for the different countries). However, the interpretation of said results, taking into account whether the corresponding value is in  $[-1, 0]$ ,  $[0, 1]$  or  $[1, 2]$ , would not be as intuitive.

As an application of this new approach, we have calculated these indices for 2011 and analyzed and compared the resulting country ranks with the official HDI. We observe that the results of the HDI are closer to the results of the weak index than to those of the strong index. On comparing the country ranks of the strong index with those of HDI and the weak index, we note that when a country moves down in the strong index ranking with respect to the HDI ranking or weak index ranking, it is due to imbalances between HDI dimensions; that is, at least one component is significantly worse than the others. The countries that have the biggest imbalances between components include Qatar, Kuwait, Cuba, Oman, Bhutan, Botswana, South Africa, Andorra, Kazakhstan and Georgia. They are all placed in the upper-middle- and high-income groups. On the contrary, when a country moves up in the strong index ranking, it is because its normalized values are very similar; that is, its human development is very balanced. This category includes, amongst others, Saint Vincent and the Grenadines, Indonesia, Azerbaijan and Kiribati, in addition to Norway, which maintains the first position with all the indices analyzed.

We should not forget that in 1990 the first global HDR already addressed the importance of seeking a balance in priorities across dimensions of human development (UNDP 1990). If every core dimension has the same significance in terms of human development, it should be desirable to achieve balanced development across dimensions. From the point of view of development policy, it seems rational that the worse the deprivation is in a particular core dimension, the more urgent should be the efforts to improve achievements in that dimension. Our proposal would strengthen Mahbub ul Haq's point that the HDI 'emphasizes sufficiency rather than satiety' (ul Haq 1995, p. 49). Therefore, in addition to the progress of human development as a whole, it is worth taking into consideration the state and evolution of the weakest dimension for each country.

This new methodological perspective is more in line with the human development paradigm and the capability approach, opening the door to whole new possibilities for future research. In this context, beyond the concepts of maximum and minimum value of the indicators, objective aspiration and reservation values would enrich the debate on the HDI. The international community should tend to set objective aspiration and reservation

values worldwide, not only according to an empirical justification, but also according to ethical and political justifications.

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