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Human Development Index Based on ELECTRE TRI-C Multicriteria Method: An Application in the City of Recife

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Abstract The Human Development Index (HDI) is an indicator that measures individuals' welfare through three dimensions: health, education and income. Since its conception, the HDI has been a focus of attention for various segments of society such as politicians, professionals in the media, policy makers, academics and ordinary citizens. The index, however, has received several criticisms over the years, the compensatory effect between the dimensions being the main one. In this context, this paper puts forward an alternative approach for calculating the Municipal Human Development Index, thereby mitigating some criticisms of the index and supporting public decision making. For this purpose, the ELECTRE TRI-C multicriteria method was used, in order to attenuate the compensatory effect, to reduce calculation problems and to allow comparison year by year. An application was conducted in the city of Recife, Pernambuco, in order to demonstrate the benefits of the proposed approach. As a result, a more adequate classification of the regions in four levels of human development was obtained.

Keywords Human Development Index · Multicriteria decision aid · ELECTRE TRI-C · Decision making

1 Introduction

The Human Development Index (HDI) is an index published annually by the United Nations Development Programme (UNDP) with the purpose of measuring individuals well-being. Since its inception, in 1990, the HDI has suffered several criticisms, as set out

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in Srinivasan (2012), Ravallion (2011), Chakravarty (2011), Sagar and Najam (1998), Ivanova et al. (1999) and Kelley (1991). Such criticisms include the compensatory effect between dimensions, the incorrect measurement of the income dimension, the use of inappropriate indicators, redundancy and poor data.

Despite these criticisms, the HDI has persisted as an important instrument for measuring human development (HD). The index prompts discussions about human development and is used as a basis for various decisions, in addition to ensuring that the significance of human welfare is not limited to economic factors.

Lopez-Calva and Ortiz-Juarez (2012) point out that the HDI has a strong political content because it allows the evaluation of public policies and the budget allocation. According to Rende and Donduran (2013), however, little attention has been given to the political implications that the index can raise. According to these authors, much is said about the countries that rise or fall in the rankings, but little is discussed about how countries can change their policies or how they can learn from more developed countries.

Normally, the HDI is used to assess the human development of countries around the world. However, as stated by Anand and Sen (1994), there is also a need to examine the development situation in more detailed levels. When the index is calculated for countries, it is only useful for international comparisons. Therefore, its disaggregation is important to reveal the disparities within nations. It was for this context that the Municipal Human Development Index (MHDI) was created.

Pagliani (2010) asserts that the indices measured in smaller dimensions have played a valuable role, because they contribute to a better understanding of local human development. The assessment of the HDI at more detailed levels is extremely important: the detailed HDI can be used, for example, in state or municipal decision making, thus helping to define the areas that should receive public resources.

The present study, therefore, is intended to suggest an alternative methodology for calculating the MHDI, taking into account the objections found in the literature. The proposed approach aims to support public decision making at municipal or state levels by using a more detailed calculation of human development. Hence, MCDA was used, in particular the ELECTRE TRI-C method. The methodology was applied in Recife, in order to evaluate the well-being of citizens of this city.

This paper is structured as follows: Sect. 2 presents a theoretical reference about HDI; Sect. 3 comments on why MCDA is used and describes the ELECTRE TRI-C method; Sect. 4 puts forward the new approach and its application in Recife, together with the results; Sect. 5 discusses the application performed; and Sect. 6 draws some conclusions.

2 Human Development Index

The HDI is calculated by grouping three dimensions considered essential to human development—longevity (or health), knowledge and standard of living (or income). In agreement with the 1990 Human Development Report (HDR), if these three dimensions are not satisfied in an individuaĺs life, other aspects may be inaccessible (UNDP 1990). Each of those dimensions is evaluated by well-being indicators, which form sub-indices and are used to calculate the HDI—as shown in Fig. 1.

Longevity is represented by the life expectancy at birth indicator. The 1990 HDR affirms that high life expectancy allows people to develop their skills and use their talents for a longer period. Moreover, longevity is linked to adequate nutrition and good access to hospital assistance.

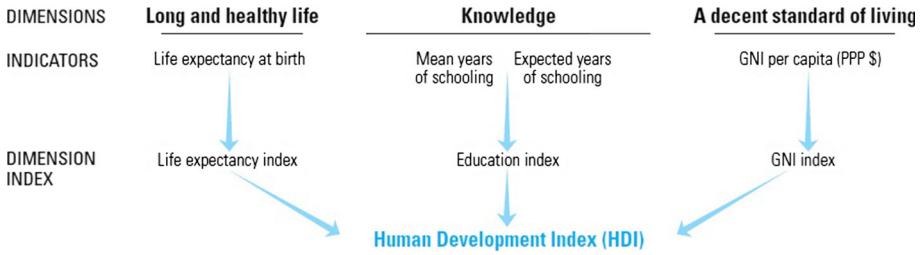

Fig. 1 A graphical representation of HDI (UNDP 2011)

Education is also considered an important factor for human beings because of the relationship between literacy and human productive capacity. Thus the higher the knowledge level an individual has, the greater the probability of a higher quality of life. This dimension is measured by two indicators: mean years of schooling and expected years of schooling.

The economic dimension is evaluated by income that, as per Anand and Sen (2000), is an instrument to achieve other purposes because other dimensions of life are dependent on economic factors. The role of the income dimension in the index, therefore, is to represent other important aspects of human development, which are not covered by longevity and education. Currently, the indicator used is Gross National Income (GNI) *per capita* expressed in Purchasing Power Parity—a conversion factor that allows price comparisons between countries.

The task of choosing the dimensions that comprise the index is complex, such as selecting the indicators which will represent them. According to Klugman et al. (2011), the UNDP team admits that other relevant aspects could be included in the HDI. However, the unavailability of data and divergent concepts impedes the consideration of other factors.

2.1 Calculation of HDI

The calculation of HDI, as described in the 2011 HDR, can be divided into two stages. The first phase involves the creation of dimension indices and the second includes the aggregation of such sub-indices.

In the first stage, indices must be calculated for each dimension. Initially, the values of the indicators are normalized because they are expressed in different scales. For this reason, minimum and maximum limits are determined for each criterion. The maximum value is the highest one observed in the historical data. The minimum limit is predetermined: a value considered insufficient for human development.

The sub-indices must be estimated according to Eq. 1.

$$\text{Dimension index} = \frac{\text{real value} - \text{minimum value of the dimension}}{\text{maximum value of the dimension - minimum value of the dimension}} \quad (1)$$

Note that, since the education dimension has two indicators, Eq. 1 is used for each indicator separately and thereafter they are aggregated by a geometric mean. Another peculiarity occurs with the income dimension, because all values suffer logarithmic transformation.

During the second phase, the sub-indices are aggregated using a geometric mean, as shown in Eq. 2.

$$\mathrm{HDI} = \left(\mathrm{I}_{\mathrm{life}}^{1/3} \times \mathrm{I}_{\mathrm{education}}^{1/3} \times \mathrm{I}_{\mathrm{income}}^{1/3} \right) \tag{2}$$

At the end of these two stages, the HDI is obtained, which must be in a range between zero and one. An area is considered more developed, the nearer to one its index reaches. After having obtained the indices of all areas, they are arranged in decreasing order and classified as showing very high, high, medium or low human development. The classes are cut into quartiles.

2.2 Criticism of HDI

Since its first publication, the HDI has been the subject of many studies, which raise several criticisms, considering various aspects. Stanton (2007) claims that the criticisms of the index can be classified into five categories, namely: poor data, inadequate indicators, problems with general formula, incorrect measurement of income dimension and redundancy.

A brief synthesis of the criticisms is shown in Table 1. This table summarizes the questions, found in the literature, showing the reasons for these questions and the authors responsible for each one.

The existence of such criticism, nevertheless, does not affect the importance of the HDI. Even though some adjustments are needed, the index is widely used for many different purposes. The criticisms are well received by the UNDP team because they contribute to improving the HDI, as reported by Klugman et al. (2011).

2.3 Municipal Human Development Index

Nowadays, the HDI is the most used indicator for measuring human development, as stated in Harttgen and Klasen (2012), Hoyland et al. (2012), Wolff et al. (2011), Stapleton and Garrod (2007) and Ivanova et al. (1999). This index has a clear focus on public policies, according to Hagerty et al. (2001), because it evaluates trends in development and compares areas. These authors assess the validity and the usefulness of the index for public policy, under 14 criteria.

The index is calculated for countries annually by UNDP, and serves as a basis for international comparisons. The HDI, however, is used to help decision making or as an instrument for public policy development. For this reason, it is interesting that the index be measured at more detailed levels. In this context, therefore, the MHDI was drawn up.

The MHDI offers a more comprehensive result of human development, and has an impact on policy dialogues and decisions. For UNDP (2000), the MHDI can be considered as an important reference tool for planning in cities. The importance of the MHDI can be exemplified when decision makers decide to use it in order to target public actions on different areas, according to Sant'Anna et al. (2011).

Agostini and Richardson (1997) also affirm that the HDI applied in cities is a useful tool for politicians concerned about making their municipalities better places to live in. In consonance with these authors, the municipal index can support decisions about how to ensure employment opportunities, health and education for citizens.

Table 1 Criticisms of Human Development Index

Criticisms	Justification	Authors
Inadequate dimensions and indicators	The variables used to compile the index do not correctly represent human development	Anand and Sen (2000), Ranis et al. (2006), Sagar and Najam (1998), Kelley (1991)
Poor data	Several problems are found in these data, such as: different measurements between countries, outdated data, incomplete coverage and biases	Murray (1991), Tokuyama and Pillarisetti (2009), Srinivasan (2012), Wolff et al. (2011)
Redundant index	The index presents a high correlation with its indicators. Therefore, it does not offer significant additional information	McGillivray (1991), Cahill (2005), Kelley (1991), Ivanova et al. (1999)
Different measurement of income dimension	Decreasing returns, by logarithmic transformation, should be considered for all or none of the dimensions	Kelley (1991), Sagar and Najam (1998), Noorbakhsh (1998)
Weights assigned to dimensions	The establishment of equal weight is not appropriate and is determined arbitrarily	Kelley (1991), Lind (2010), Wu et al. (2014), Cherchye et al. (2011)
Disproportional scale of indicators	The index is sensitive to the scales of the indicators, which are disproportional for each dimension	Kelley (1991), Ravallion (2012), Ivanova et al. (1999), Noorbakhsh (1998)
Disregard of disparity within countries	A single value is considered for a whole country, without taking into account differences that exist within it	Sagar and Najam (1998), Grimm et al. (2008), Harttgen and Klasen (2012), Kelley (1991), Bilbao-Ubillos (2013)
Aggregation procedure	The adoption of a geometric mean is not enough to solve compensation problems	Ravallion (2011), Ravallion (2012), Chakravarty (2011), Wu et al. (2014), Cherchye et al. (2011)

Some studies have been devoted to how the HDI is calculated in sub-international units. Examples are articles by Das (2008) and Indrayan et al. (1999) which estimate human development for areas in India; Sant'Anna et al. (2011) make an assessment of Brazil municipalities; and Silva and Ferreira-Lopes (2013) measure the HDI for 30 regions of Portugal.

3 Multicriteria Decision Aid

In the decision making process it is rare for there to be situations in which only one viewpoint is sufficient to assess a problem. Thus, when dealing with social decision problems (Munda 2012), there is a need to incorporate various aspects into the decision framework, thus opening space for multicriteria decision aid (MCDA).

Belton and Stewart (2002) describe MCDA as a collection of formal approaches that take into account multiple criteria in helping individual or group decisions. Vincke (1992) emphasizes that the objective of multicriteria support is to provide the decision maker with some tools that allows the resolution of a decision problem in which there are many points of view that must be considered, and which are often mutually conflicting.

MCDA, however, does not attempt to indicate a solution which should be considered as absolute truth, as Belton and Stewart (2002) and Roy (1996) assert. Instead, the intention of using it is to offer recommendations to the decision maker and to permit learning about the problem that is being dealt with.

In the literature, many multicriteria methods can be found. According to Roy (1996), the multicriteria methods may be classified into three major families: the single-criterion synthesis approach, the outranking synthesis approach and interactive local judgment.

3.1 ELECTRE TRI-C Method

The ELECTRE TRI-C method, proposed by Almeida-Dias et al. (2010), belongs to the outranking synthesis approach. This method is recommended for situations in which the alternatives can be allocated into predefined classes by evaluating multiple criteria. In this method, the classes are ordered and the outranking relation between the alternatives and the reference actions is explored. The outranking relation is a binary relation between two alternatives (a and a'), denoted by aSa', where a is at least as good as a'.

This approach was chosen for the HDI measurement because it can provide many advantages, mainly when considering an evaluation of municipalities—this will be discussed in detail in Sect. 5. To apply the ELECTRE TRI-C method a set of alternatives, denoted by A, must be defined. These alternatives will be distributed in ordered classes (C_q) according to a set of criteria.

In the context of this study, the alternatives will be the districts of a Brazilian city and the classes will be the human development levels, comprising four ordered levels established by UNDP. Moreover, the criteria will be the human development indicators, such as income, education, and health. The criteria will be useful to classify the districts into the four human development levels. Further, a set of weights should be defined for the criteria, representing the voting power of each criterion.

As previously stated, the ELECTRE TRI-C method explores the outranking relation between alternatives and reference actions. In this way, the alternatives will be allocated to the classes by a comparison with the reference actions. The set of references actions,

denoted by B, characterizes the norms of each class. In the measurement of HDI, the reference actions should be interpreted as the expected performances for each human development level. Therefore, for an area to be designated in a class, it must have enough conditions to be allocated according to the reference actions of the class.

When alternatives, criteria, weights, and reference actions have already been set, the process of classification can be started. Firstly, the concordance and discordance indexes must be calculated, with the purpose to assess the outranking relation. The concordance index, $C(a, a')$, represents the sum of all the criteria in which a is at least as good as a'. In this sense, the concordance index sum the votes in favor of preferring alternative a in relation to reference action a'. Therefore, when the alternative is always as good as the reference action, the concordance index is equal to 1.

On the other hand, the discordance index $d_j(a, a')$ evaluates the arguments that are against the claim that a is at least as good as a', for each criterion. Consequently, the higher the discordance index, the opposition that a outranks a' is more significant for a given criterion. When the strength of this opposition is greater than a critical level, referred to as a veto threshold, then it cannot be said that alternative a outranks reference action a'. The discordance index is measured by the difference between the performances of alternatives and the reference actions. When this difference is greater than the veto threshold, the discordance index is 1, not allowing the assertion that a outranks a'.

In brief, the concordance index sum the weights in which alternative a is at least better than reference action a'. Nevertheless, the discordance index measures the opposition about the affirmation that a outranks a'. To assure the outranking relation, therefore, two conditions must be observed: the concordance index must be greater than the required level (credibility level) and the discordance index must be lower than the veto threshold.

In order to validate or not the outranking relation a credibility level, denoted by λ, should be chosen. This number ranges between 0.5 and 1. The credibility level is the minimum acceptable value for the force of the statement that a outranks a', taking into consideration all criteria of the problem. For example, when a credibility level of 0.7 is chosen, this means that at least 70 % of the votes are needed to legitimize the outranking relation between the alternative and the reference action.

In the next steps, the credibility index, denoted by $\sigma(a,a')$, must be obtained. The credibility index represents the strength of the assertion that alternative a outranks reference action a', taking into account all the criteria. This force is related to the weight of criteria and can range between 0 and 1. This index is measured based on the concordance and the discordance indexes measured before. When the concordance index is greater than the discordance index, the credibility index is equal to the concordance index. However, when the concordance index is lower than the discordance index, the credibility index suffers a penalty in accordance with this difference. Finally, the credibility index will be useful to classify the alternatives.

The ELECTRE TRI-C calculations enable the impreciseness of the data to be dealt with, by considering the preference, indifference and veto thresholds. These thresholds are important for building binary relations between alternatives and actions reference and for measuring concordance and discordance indexes. The thresholds represent the imperfect nature of the assessments, as per Figueira et al. (2005). They affirm that these parameters are used when the evaluation of two alternatives justifies the preference in favor of one of the two (a preference threshold) or when this shows the indifference between two alternatives (an indifference threshold). Moreover, the veto threshold expresses the power assigned to a criterion to be against the claim that "a outranks a'". These thresholds,

therefore, should be established to consider the doubts and the imperfection in the decision maker's evaluation.

The recommendation of the multicriteria method is a classification, which is formed from two joint rules: descending and ascending rules. For the two rules, initially must be found the first class in which the credibility index satisfies the credibility level. Next, two joint procedures are applied, where one rule analyzes alternatives from the highest to the lowest classes, while the other does the opposite.

In the descending rule, alternatives must be allocated to a class which ranges from the highest to the lowest. Analyzing the credibility index and the credibility level, if an alternative can be allocated in the highest or the lowest classes, it should be directly allocated to this class. However, when the class is intermediate, the alternative can be allocated to the class in question or to the one above, depending on the analysis of adjacent classes.

In the ascending rule, the allocations are made by analyzing the lowest to the highest classes. Analyzing the credibility index and the credibility level, if an alternative can be allocated in the highest or the lowest classes, it should be directly allocated to this class. However, when the class is intermediate, the alternative can be allocated to the class in question or to the lower one, depending on the analysis of adjacent classes.

After applying that procedure, therefore, two classifications are obtained. These two rules are taken in order to verify the highest and lowest possible classes to allocate an alternative. In some situations, the class can be the same. As the notion of classes boundaries are not defined in this method, the final allocation may consider two consecutive classes.

More information about the ELECTRE TRI-C method can be found in "Appendix 2", in which its formulas are also given.

Then, in the measurement of HDI, the districts (alternatives) are compared with the reference actions of each human development class. In this work, the UNDP classification was maintained with four human development levels: very high, high, medium and low. The criteria for analysis are the indicators of human development, defined by UNDP.

The concordance index can be obtained by summing all criteria weights in which the districts have performances at least as good as the reference actions. In other hand, to calculate the discordance index must be considered the situations in which the reference actions have better perform than the districts. In addition, for the discordance index, a veto threshold should be assessed. When the difference between a reference action and an alternative is greater than the veto threshold, this alternative cannot outrank this reference action.

Next, the credibility index must be measured according to the indices of concordance and discordance. The credibility index expresses the force of the statement that an alternative outranks a reference action. Therefore, in the HDI measurement, one district cannot be allocated in one class if the district performance is not as good as the reference action of this class.

Finally, the rules will be obtained through the construction of the ascending and descending rules. For both classifications, the process is initiated with the comparison between the credibility index and the credibility level. In the descending rule, the classes can be analyzed from the best to the worst. In other words, the classes can be observed in the following order: very high, high, medium and low. Considering this order, the first class in which the credibility index satisfies the credibility level must be found. If that is the highest or the lowest class, the district can be classified as very high or low human

development, respectively. However, when an intermediate class is found, the analysis for the classification are deeper.

In the ascending rule, the classes can be analyzed from the worst to the best. Considering this order, the first class in which the credibility index satisfies the credibility level must be found. If that is the highest or the lowest class, the district can be classified as very high or low human development, respectively. However, when an intermediate class is found, the analysis for the classification are deeper.

In the next section, more details about the application are given and the results are shown.

4 Evaluation of MHDI in Recife Based on ELECTRE TRI-C

This study is intended to suggest an alternative approach for calculating the MHDI, in which the criticisms in the literature are attenuated. For this purpose, the ELECTRE TRI-C method was used. The methodology can be applied to any region, whether a country, a state or a city, provided that the data necessary are available.

Given the importance of the MHDI, as this paper has made clear, an example is given below of applying this different methodology to a city. Besides illustrating the methodology, the application aims to evaluate the advantages and limitations of the proposal. Recife, the state capital of Pernambuco in Brazil was chosen. The analysis was performed in subdivisions of the city, called Human Development Units (HDU). The HDUs are 62 areas defined by Recife City Hall and correspond to places that aggregate and/or subdivide neighborhoods in accordance with the socioeconomic characteristics of their residents.

Lopez-Calva and Ortiz-Juarez (2012) emphasize that the measurement of the development at household level is more appropriate than those made at municipal, regional or state levels. The reason is that the household analysis minimizes losses due to the inequalities in the areas. In the case of Recife, however, it is not possible to do this kind of analysis, because the data are not available. These authors cite the difficulty of using household data, given the difficulty of finding indicators at this level.

In this context, the best units of analysis for Recife are the HDUs, because the most detailed existing data are on this level. Such areas have a greater degree of homogeneity than the neighborhoods, nevertheless there is certain heterogeneity in the HDUs. Figure 2 shows the division of Recife into its HDUs.

The 2005 Atlas of Human Development in Recife was used as the data source for this study. This publication provides the most diverse demographic, economic and social characteristics of Recife, and uses the 2000 census as its reference base. This data source is the most currently available one, because the data from the 2010 census have not been published at this level of detail.

Some of the indicators used in the suggested methodology has changed due to the unavailability of data. Life expectancy at birth is the only indicator that has not changed. To assess the income dimension, the GNI *per capita* was replaced by Gross Domestic Product (GDP) *per capita*. The mean years of schooling indicator was changed to mean years of schooling of those aged 25 or older. Finally, the values for the expected years of schooling had to be measured, since there was not a parameter to replace it.

The unavailability of the expected years of schooling indicator, however, was not a problem. The Atlas of Recife does not provide this value, but has all the numbers necessary to derive it. According to the United Nations Educational, Scientific and Cultural Organization (UNESCO) (2011), there are two ways to calculate the expected years of

schooling. One is by multiplying the rate of each level of education by the duration of school levels.

The Atlas data used were the percentage of 4–6 year-old children in school, the percentage of 7–14 year-old children who are attending elementary school, the percentage of 15–17 year-olds who are attending high school and the percentage of 18–24 year-olds who

Fig. 2 Recife Human Development Units (Atlas of Human Development in Recife 2005)

Table 2 Reference actions

Classes	Reference actions	Criteria			
		Life expectancy at birth	Mean years of schooling	Expected years of schooling	GDP *per capita* (US$)
	b0	0	0	0	0
C1	b1	64	6	10	112.57
C2	b2	67	8	12	180.11
C3	b3	71	9	14	247.65
C4	b4	75	10	16	315.19
	b5	100	20	25	2,251.34

are attending college. These rates were multiplied by the duration of each level of education viz., 3, 9, 3 and 4 years, respectively.

The reference actions adopted in this application are given in Table 2. They contain representative characteristics of each class. In all cases, it has been determined what values would be expected for areas belonging to the classes.

The reference actions for the education indicators were established according to the periods of compulsory education determined by Brazilian laws (Brazil 1996). The values for life expectancy have taken into account the national average for this indicator. In addition, the reference to the GDP *per capita* was defined taking as a basis the indigence and poverty levels, established by a Brazilian government institute, the Institute of Applied Economic Research (2006).

The necessary parameters needed to apply the methodology were determined based on the performance range of each criterion. These parameters have also been varied in order to evaluate the use of the methodology and observe its impact on the results. The weights of criteria were defined by the revised Simos' procedure, which is proposed in Figueira and Roy (2002). In this application, it is possible to consider different weights for the dimensions, unlike what happens in the methodology of UNDP. Table 3 shows the weights established for the criteria, for this case, as well as the indifference (q_j), preference (p_j) and veto thresholds (v_j).

The indifference threshold (q_j), according to Mousseau and Slowinski (1998), specifies the largest difference between the performance of two alternatives that preserves the indifference between them, on a given criterion. Thus, for example, it was considered appropriate to define this threshold as $q_j = 2$ for life expectancy at birth. This means, for instance, that the comparison of two areas where this is 70 and 72 years old could be seen as indifferent, in this indicator.

Table 3 Weights and indifference, preference and veto thresholds

Criteria	Parameters			
	w_j	q_j	p_j	v_j
Life expectancy at birth	0.292	2	3	3
Mean years of schooling	0.208	1	2	2
Expected years of schooling	0.083	1.5	2	2
GDP *per capita* (US$)	0.417	45	90	90

The preference threshold (p_j) is understood by Mousseau and Slowinski (1998) as the smallest difference between two alternatives that is compatible with the preference in favor of one of these alternatives, according to a given criterion. In this problem, for life expectancy, the parameter was determined as $p_j = 3$, representing that if the life expectancy in two areas is 70 and 73 years old respectively, the second area will be strictly preferred to the first.

Finally, there is the veto threshold (v_j). This threshold expresses the power assigned to a criterion to be against the claim that one alternative outranks another, as per Figueira et al. (2005). The veto occurs when the difference between the alternative and the reference action is greater than the threshold. In this case, the veto threshold established for life expectancy at birth is $v_j = 4$. This means that if the life expectancy in two places is 70 and 74 years old, respectively, the first cannot outranks the second, even if it has a good performance in other criteria.

The thresholds for the three other criteria may be interpreted similarly.

The initial credibility level adopted in this application was 0.7, but it was simulated between 0.6 and 0.8 to verify the possible modifications in the results. Note that this parameter can only assume values between 0.5 and 1. The variation did not result in significant alterations in the results, because just one HDU changed its class in the different situations.

The results will be presented in Sub–Sect. 4.1

4.1 Results

The result of the method was to distribute the HDUs into four human development levels, as shown in Table 4 which also shows the number of units allocated to each class and the code of the areas allocated to each class.

Using the alternative methodology, the result was that 31 HDUs (50 %) had their human development class altered. In all of them, they were re-allocated to a lower class. A high rate of changes was expected, since the two methods are very different. In the ELETRE TRI-C the classification is made in relation to appropriate performances (reference actions), while in the classical approach there is a defined number of units for each class. Therefore, in the new methodology, the two highest HD levels have the smallest number of areas, because these classes require much higher performances on the indicators, which is not the reality in many regions of Recife. Thus, the low and the medium classes contain most of the HDUs.

According to UNDP methodology, low human development class corresponds to the first quartile. In the suggested methodology, nevertheless, be classified as low level has a

Table 4 HDUs classification by ELECTRE TRI-C

Classes	Number of units	HDU code
Very high human development	10	3; 7; 15; 16; 17; 28; 29; 47; 48; 49
High human development	10	2; 9; 26; 27; 33; 36; 40; 43; 50; 51
Medium human development	13	6; 18; 20; 25; 30; 34; 35; 38; 39; 42; 52; 53; 56
Low human development	29	1; 4; 5; 8; 10; 11; 12; 13; 14; 19; 21; 22; 23; 24; 31; 32; 37; 41; 44; 45; 46; 54; 55; 57; 58; 59; 60; 61; 62

different meaning. When an alternative belongs to a class it means that the alternative attends the expectation of this class, in accordance with the reference actions defined by decision maker. In this simulation, to belong to a low human development class means that the expected performances are, approximately: life expectancy at birth of 64 years, mean of 6 years of schooling, expected years schooling of 10 years and US$112.57 of GDP *per capita*. The same relation can be made for each human development class, taking into account the values of Table 2. Moreover, if the alternative is on the low class it is because the alternative didn't have the level performance of the classes above that class. That is, the alternatives do not have the minimum required performance to be allocated in the medium human development class.

Table 5 makes a comparison between the numbers of HDUs allocated to each class for the two approaches. As can be seen, in the ELECTRE TRI-C method, the number of HDUs in each class is variable, unlike what occurs in the UNDP methodology. This happens because, in the multicriteria method, the areas are not divided into quartiles, but by assessing their adequacy for the reference actions. Thus, in the suggested methodology, it is not possible to predict how many HDUs will be assigned to each class.

The focus of the article is not to compare the two approaches, but the intention is to show that some problems in measuring HDI can have an impact on the classification and, for certain types of HDI measurements, such complications may be attenuated by using the ELECTRE TRI-C method. The difference in the results, therefore, it is expected because the concepts of the two approaches are very distinct. The emphasis in this case should be on the new classification of the areas, which shows a much harsher reality than the approach of UNDP.

The proposed methodology is more interesting when evaluating municipalities, because the results can be used in public decision making. In municipalities, the homogeneity of the scenarios is higher, and thus the parameters of the method can be defined in a more consistent manner. Moreover, the given information is more detailed and therefore more useful for municipal analysis—in this case, improvement actions can be assigned according to the performance of each district.

Is important to note that ELECTRE TRI-C is a new approach in which uncertain or imprecise situations in the decision making process can be considered. Furthermore, using this approach is possible to avoid a purely mathematical analysis, in which the decision maker can set the performance indicators for each level of human development and define the parameters of the evaluation.

On looking at the results of the proposed methodology as set out in Table 5, it can be seen that there are fewer areas allocated to the higher classes. Under the methodology proposed, the high and very high human development classes contain only 16.13 % of the HDUs, each. On the other hand, the low human development class comprises almost half of the units namely 29 out of 62 i.e. 46.77 % of them. The medium development class

Table 5 Allocation of HDUs to human development classes

Classes	UNDP methodology	ELECTRE TRI-C
Very high human development	16	10
High human development	16	10
Medium human development	15	13
Low human development	15	29

contains 20.97 % of the districts. The units were classified in this way because their performance indicators did not justify allocating them to higher classes.

Is important to emphasize that must be not think that the method has a pessimistic view of human development. Any results will depend on the choices of the decision maker about the reference actions and parameters. In this application, patterns were used to choose the reference action, such as indulgence and poverty levels and education levels established by Brazilians laws. So, in accordance with this considered patterns, the classification tends to allocate the areas of Recife in lower classes.

In a real context, the decision maker will be more or less stringent in accordance with the reality of his area For example, if the decision maker has high expectations when defining reference actions, many alternatives will be classified in the lower classes because these alternatives will not have enough performance to be included in the higher classes. Therefore, the choices about reference actions and parameters will directly influence the classification of alternatives.

Figure 3 shows the map of Recife divided into its HDUs, classified according to their human development levels, using the ELECTRE TRI-C method. These areas are shaded in various tones of gray scale: the darker the color of a region, the higher the degree of human development.

On the other hand, Fig. 4 shows the same map, but classifying the human development of the districts was conducted as per the methodology of UNDP. On looking at both maps, the results before and after applying the proposed methodology can be compared. The maps were compiled with the assistance of ArcGIS 10.1—this software allows a spatial analysis of the results.

On looking at the two figures is easy to see that Fig. 3 is lighter than Fig. 4, indicating that the classification is more accurate and for this reason fewer areas are classified as high development. In both Figures, it can be seen that there are two areas where the HDI of the population is significant, but in Fig. 3 note that these areas are smaller and more sharply delineated.

Sensitivity analysis was conducted to verify the stability of the model and to observe how it behaves when some parameters are varied. Hence, modifications were made to criteria weights in order to evaluate the behavior of the method. This is an important question, because one of the criticisms raised against the index is related to the weights of the criteria.

Therefore, four different situations were considered. First, all dimensions were deemed to have the same weight (1/3); secondly, the same weight was set for all indicators (1/4); thirdly, the income dimension weight was reduced by 20 %; and finally the life expectancy weight was reduced by 20 %. Even with these modifications, the method proved to be stable, since only one HDU changed its class in three of these situations. In the last case, nevertheless, there was no change of class to any area. Given these evaluations, it can be concluded that the method used to classify the HDUs of Recife is stable, since the changes in the parameter of the model did not interfere significantly in its results.

5 Discussion

The Atlas of Human Development in Recife does not only disseminate economic, social and demographic data. This document also publishes the HDI of the city. The propagation of such information is relevant because various policy decisions are based on the HDI, and comparisons between the regions can be made.

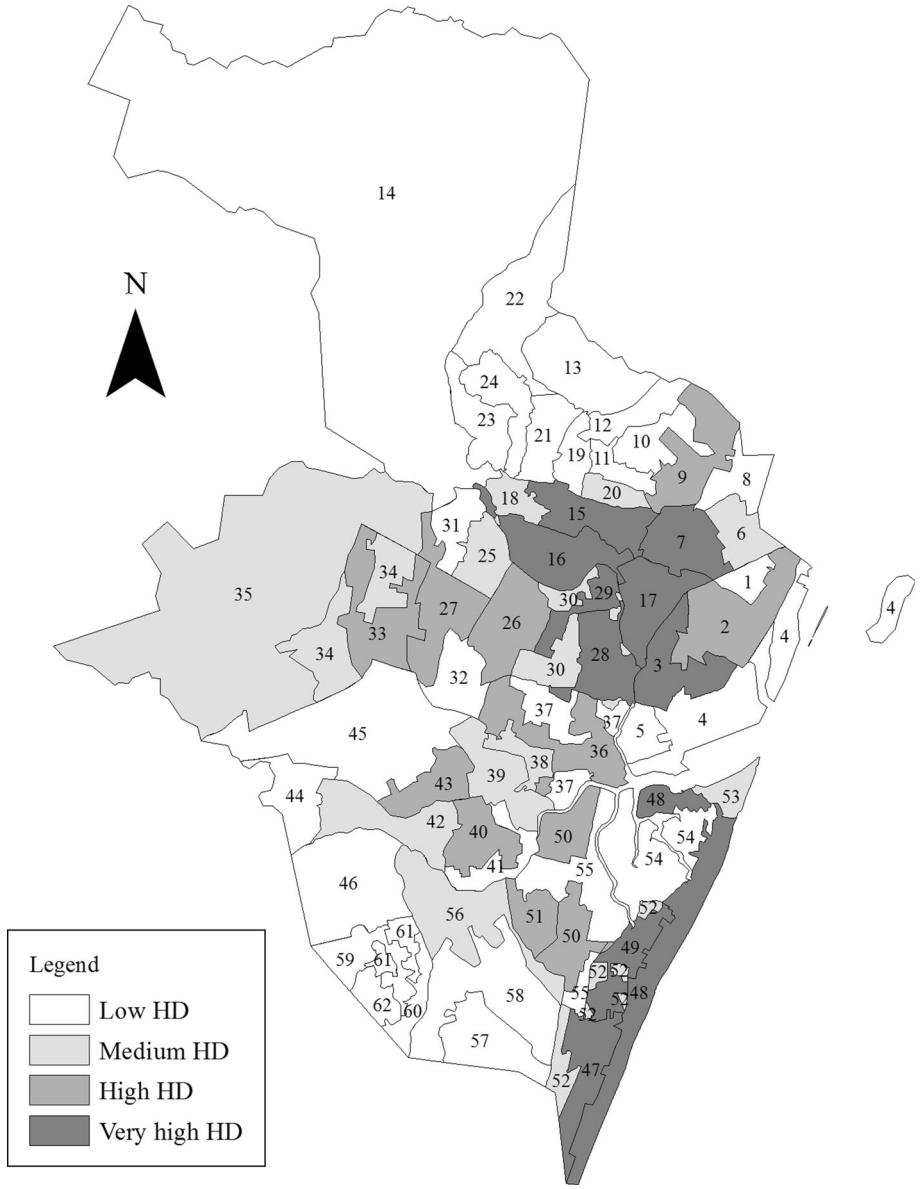

Fig. 3 Division of HDUs according to human development levels (ELECTRE TRI-C method)

The measurement of this index, however, has several problems, as discussed in Sub-Sect. 2.2. Perhaps, in the case of Recife, the main complication of the HDI may be the division into quartiles to determine the classes. This separation suggests that half of the HDUs are labeled as having high or very high human development. This, however, is not the reality of Recife.

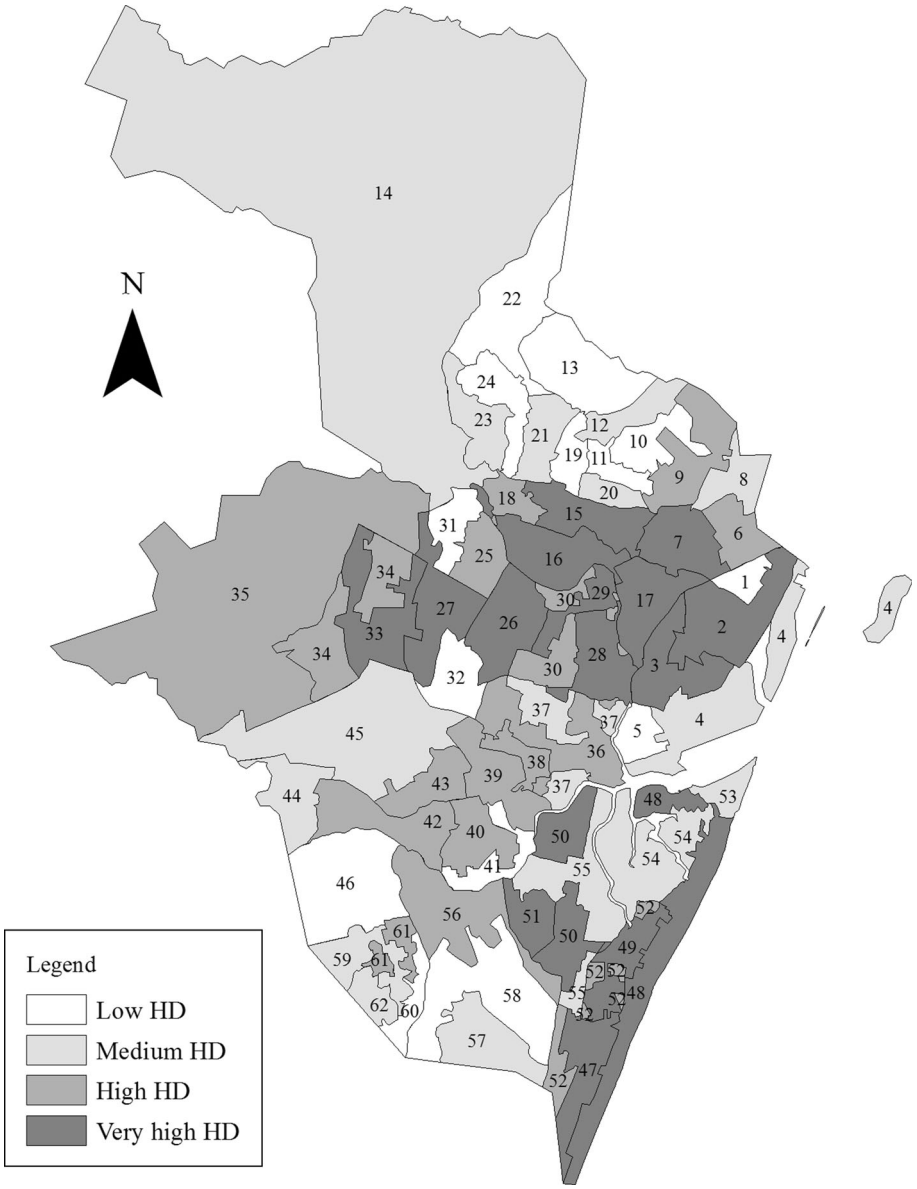

Fig. 4 Division of HDUs according to human development levels (UNDP methodology)

The UNDP classification leads to a false impression of great human development in Recife. Some regions have unsatisfactory performance indicators, but are still said to have high human welfare. It is not hard to find examples to ratify this assertion.

HDUs 27 and 51, for example, which include Engenho do Meio, Iputinga, Cordeiro and Ipsep neighborhoods, are identified as having very high human development as per the UNDP classification. These regions, nevertheless, have a monthly *per capita* income of

US$ 216.10 and US$ 190.16, respectively. However, it can be seen that the classification of both units is not consistent with the reality, since the income of both are close to the poverty line (US$ 140.03). Thus, HDUs 27 and 51 HDUs should not be assigned to the very high human development class.

Similarly, HDUs 38 and 61, which cover the neighborhoods of Afogados and COHAB, can also be cited. They have a monthly *per capita* income of US$ 75.06 and US$ 80.77, respectively, but they are considered as being of high human development. The situation of the high human development class is worrying. The average of the class in the income *per capita* is within the poverty line range. While the poverty line varies between US$ 70.02 and US$ 140.03, the average of this class is US$ 120.40. Thus, it can be said that the division of human development areas according to quartiles does not classify them correctly.

If the HDUs in which the *per capita* income is below the poverty line could not be allocated to one of the two highest classes, a large change would happen in those classes. In the high human development class, 10 of the 16 units would suffer change: that is, 63 % of them would be placed in a lower class.

Another contradiction can be seen regarding the mean years of schooling. The high human development class, for example, has just one HDU in which this average is greater than 9 years—the duration of elementary school. This means that almost all units have a lower education level than the period indicated as compulsory by Brazilian law no. 9.394 of December 20, 1996.

By applying ELECTRE TRI-C to distribute HDUs into human development classes, the distortions described above could be attenuated. Altogether, 31 HDUs changed their class, all of them being downgraded.

The problem mentioned earlier about UNDP's classification was reduced by applying ELECTRE TRI-C. With respect to *per capita* income, there is no inconsistency in the results from the proposed method. The allocation of the HDUs into development classes is consistent with the poverty and indigence levels.

In the high class, there is no unit where the monthly income is below poverty line—US$ 140.03. After the reclassification, the mean income *per capita* of this class is US$ 192.35. Moreover, the medium human development class does not present any HDUs with an income below the indigence level, as happened in the other approach. On using the multicriteria method, the average income for this group was found to be US$ 105.64.

Changes can also be found in relation to education. As expressed before, in almost all regions identified as being of high human development, the level of schooling did not exceed 9 years. After using ELECTRE TRI-C, however, the average schooling for this class was 9.157 years. In addition, in the high and very high classes, there is no HDU in which the expected years of schooling is less than 12 years—a period that includes basic education (elementary and high school).

Moreover, the division into quartiles can cause problems in the comparison between subnational measurements. Depending on the area in which the human development is being measured, the development classes may present distinct averages to income, education and health. For instance, if the HDI is measured according to UNDP methodology, in Brazil and in Norway (which is much richer than Brazil), the averages of *per capita* income will certainly be different in each class. The income averages from Norway will be much higher than Brazilian averages, in each class. Therefore, this example leads to an important question: what does each human development class really mean according to the UNDP approach? What levels of income, education and health are related to each class? Through the UNDP methodology there is no way to respond to this question. The division

in quartiles is not stable and consequently the average of indicators to each class will vary depending on the area that is being assessed.

The problems with the HDI methodology, however, are not only a result of the division into quartiles. There is a disagreement about the logarithmic transformation too, which is only applied to the income dimension. With this transformation, the utility of additional increments of income is lower for higher values. There is, therefore, discussion about the necessity of such transformation to income—or considering this transformation for all dimensions.

The reason for the use of this transformation is that increases in income have greater utility when income is lower. Thus, a growth of US$ 100 for anyone who has an income of US$ 200 is more significant than an increase of the same amount for those who earn US$ 500. This is an understandable thought, but it would also be consistent for the other dimensions. In the case of education, for example, the utility of 2 years of study is not the same for an illiterate person and a college graduate—the knowledge acquired by the illiterate in those 2 years brings a greater utility than for the graduate.

On the other hand, the logarithmic transformation provides a false impression of equity and, therefore, should be avoided. Taking as an example two areas in which the incomes are US$ 100 and US$ 1,000, there is the conviction that the scenario between these two is distinct, since the difference between the incomes is US$ 900. With the logarithmic transformation, however, US$ 100 is transformed into 4.605 and US$ 1,000 is modified for 6.908—and, with these values, there is the impression that the income of these two places is more similar.

By using ELECTRE TRI-C, however, the discussion of the logarithmic transformation loses meaning because such adaptation is unnecessary in the method. This happens because the indicators are measured in absolute values, rather than by increments in relation to a minimum level—which occurs with the geometric mean. This, then, is one more advantage of using the multicriteria method to measure HDI.

Another criticism of the HDI is the compensatory effect among the indicators. An attempt to solve this problem has been made, when the aggregation procedure was changed from an arithmetical to a geometric mean. Some authors, however, even with this change, insist that the compensatory effect remains present in the calculations.

The problem of the compensatory effect is that some areas can be classified into higher levels because their performance is satisfactory in one dimension even if it is not as good in the other dimensions. In the case of Recife for example, the compensatory effect can be evidenced through HDU 61. According to the UNDP methodology, this unit is classified as having a high level of human development. This area, however, does not have good levels of income or schooling expectancy. The *per capita* income for this area is US$ 80.77 and the expected years of schooling are only 13.86. Nevertheless, this HDU has a life expectancy that is above of the average and the mean years of schooling very close to the average—such performances led to the unit being classified as highly developed, due to the compensatory effect.

In accordance with the classification made by ELECTRE TRI-C, on which the areas are classified in accordance with the comparison between their performance and the reference actions, HDU 61 was indicated as having low human development. A major characteristic of this method is its non-compensatory effect, which represents an advantage over the UNDP measurement. ELECTRE TRI-C is a non-compensatory approach, given that incremental performance of one criterion (the GDP indicator, for instance) it is not directly considered in the overall view of the criteria, because the criteria are analyzed in an isolated manner and there is no aggregation of the indicators.

Another advantage of the ELECTRE TRI-C method is the possibility of comparison between calendar years, whether or not the alternatives are included or excluded. In the UNDP methodology this evaluation is not recommended because the ranking is relative and depends on the values of other alternatives. In this case, therefore, whether by considering fewer or more locations, it is not possible to know if an upgrade/downgrade occurred due to a better/worse performance or due to the inclusion of new alternatives (leading to a change in an existing alternative's relative position). On the proposed methodology, however, this sensitivity does not exist because the result is not a ranking (it is a classification) and because the classification is done by comparing the alternatives with the reference actions.

The disproportional scale of indicators is also a HDI problem that can be mitigated by using the multicriteria method. This disproportionality makes the index more sensitive to the indicators that have a smaller range. Thus, changes in indicators with a shorter range cause a larger impact on the index value, while alterations in indicators with a higher range do not have an impact of the same intensity.

Due to the disproportional scales, for example, the increase of one unit in the mean years of schooling (a range of almost 9 years) would result in a higher change on the index than the increase of one unit in the *per capita* income (a range of approximately US$ 800). This issue, however, is not observed in the ELECTRE TRI-C method because the classification of areas is obtained by comparing the alternatives and the reference actions, thereby forming a binary relation between them.

A further benefit with the alternative approach is the possibility of assigning different weights to the indicators, which is not possible with the UNDP methodology. By using ELECTRE TRI-C the decision maker can give different weights to the dimensions or indicators, which can be interpreted as a voting power. This is a significant feature because the decision maker may assume that one dimension or indicator is more relevant than others in the measurement of human development.

Finally, it can be pointed out that the ELECTRE TRI-C method enables one area to be sorted individually, which is not possible with the approach of UNDP. By the conventional method, many areas must be analyzed to obtain the sorting, because initially a ranking is made of all alternatives. In the multicriteria method, however, the districts are compared only with the reference actions, permitting, therefore, only one alternative to be evaluated at a time.

Because of this peculiarity, a public manager can obtain the level of human development of his municipality, by providing just the data for this area. Thus, there is no need to provide information for other places to obtain the classification, since the locations are compared with the reference actions, and not with other areas. The classification, therefore, may be used for monitoring the performance of a municipality, considering the reference actions as goals to be implemented and thus higher levels of development can be achieved.

On looking at the Fig. 3, it can be seen that there are two dark patches, which means that there are two very high human development regions—one in the south and another in the north. The south area mainly covers the Boa Viagem neighborhood, while the north area includes several districts. This region comprises: Boa Vista, Ilha do Leite, Paissandu, Encruzilhada, Hipódromo, Rosarinho, Torreão, Casa Amarela, Tamarineira, Casa Forte, Parnamirim Jaqueira, Monteiro, Graças, Aflitos, Derby, Espinheiro, Madalena, Ilha do Retiro, Prado, Torre and Zumbi.

Those two high development regions indicated by the suggested method are ones in which a better quality of life for the citizens is perceived. It can thus be verified that the new classification is more faithful to reality than the one proposed by UNDP. The

traditional methodology placed very humble neighborhoods of the city in higher classes, such as Ipsep, COHAB and Ibura. This cannot be seen in the ELECTRE TRI-C results.

Accordingly, it is clear that methods other than that adopted by UNDP may be more appropriate to define the human development levels of HDUs in Recife. Many criticisms of HDI, for instance, may be attenuated by using the ELECTRE TRI-C multicriteria method. The traditional procedure sets the class cut-off points by quartiles, and does not take the characteristics of a region into account. Thus, the development scenario is false, one that is without harmony between the reality of the population and the classification.

The use of the ELECTRE TRI-C method, in contrast, may reduce this problem. The consideration of reference actions and the veto threshold enable a more efficient allocation of the HDU, thus leading to a more realistic classification.

Furthermore, as previously mentioned, this methodology can attenuate many difficulties in the measurement of the HDI. The first contribution that can be highlighted is the end of the compensatory effect between the dimensions, since ELECTRE TRI-C is a non-compensatory method. What can also be emphasized is the minimization of calculation problems, such as the differential measurement of income dimension, the determination of maximum and minimum limits and the disproportional scales of the indicators. Moreover, the method still brings other benefits, such as the consideration of different weights for the dimensions and the ability to rank only one area.

The method also enables the index to be compared between years, since the classification problematic is not sensitive to the entry of new alternatives. This fact is not possible when the UNDP methodology is adopted. Finally, there is no more a purely mathematical analysis: the method considers human development profiles for each class (through the reference actions), which are defined according to the decision maker's experience.

The proposed methodology, however, has some limitations. The first one is the determination of several parameters as input of the method, such as thresholds and reference actions. Another problem is the loss of information because the UNDP methodology provides a ranking of the regions, while the proposed offers a classification.

Moreover, another disadvantage is the loss of the simplicity in index measurements. By adopting the ELECTRE TRI-C method, the calculations for obtaining the HDI have become more complex, but this is not necessarily a problem. The biggest problem is the incorrect classification of the areas, because decisions are made based on such classification. To resolve the complexity of the calculations, some computational resources can be used to classify the districts by using a multicriteria method.

If they have the correct classification of areas, public decision makers can use the MHDI to support their decision processes. Gill and Hall (1997) argue that the allocation of public resources is a complex process that causes many debates. Therefore, indicators can be used as a basis for planning, the intention being that the governments will be more transparent in their decisions. Thus, the benefits provided can be justified and allocating resources becomes more objective rather than being based merely on political criteria. In this context, therefore, the MHDI can be used to support public decision making.

Therefore, the new approach can be used by public managers, in order to evaluate the districts or even to allocate resources. The index utility is maximized when it is measured in smaller dimensions, since the homogeneity of the areas increases. Thus, the new approach to MHDI can be used mainly for measurements within and between the municipalities—which will provide valuable information for the managers.

Finally, it can be said that a correct classification of the development areas is extremely important. Policymakers could allocate public resources more efficiently and prioritize less favored areas if they had the correct human development classification for the districts.

Society can also make use of such information, by demanding higher political support for the localities classified as having low human development. In addition, companies can use this classification to determine the targets of philanthropic actions. In applications of MHDI, the decision maker can be a mayor, a governor, other public stakeholders, or group of them.

6 Conclusions

The HDI is an indicator of human well-being involving educational, health and economical factors in its measurement. The UNDP has published this index for all countries of the world, annually for more than 20 years.

The assessment of the HDI at more detailed levels, however, is highly relevant. Such information is a powerful tool that can be used by politicians, executives and ordinary citizens. The MHDI can be used, for example, in state or municipal decision making, thus helping to define the areas to which public resources should be targeted.

With this in mind, the ELECTRE TRI-C method was applied to measure the MHDI in Recife. The method changed the classification of 31 HDUs, which corresponds to 50 % of the total. It should be observed that none of changes led to an HDU being placed in a higher class.

The method allowed a more realistic classification of the districts, without the distortions found in the UNDP methodology. Two large areas were highlighted by the proposed method, namely those with high or very high human development. These areas have the greatest purchasing power and the best infrastructure in Recife.

Appendix 1: Recife Human Development Units

Table 6 gives information about the Recife Human Development Units, showing their codes and the neighborhoods which are part of them.

Table 6 Recife Human Development Units

Code	HDUs	Code	HDUs
1	Santo Amaro: Santo Amaro; João de Barros	32	Torrões: Torrões; Vietnã
2	Santo Amaro; Soledade—Naval; Operária	33	Várzea; Cidade Universitária
3	Boa Vista; Ilha do Leite; Paissandu	34	Várzea: Brasilit, Campo Banco; V. Arraes
4	Recife Centro—Coelhos; Comunidade Pilar	35	Várzea; Caxangá—Rosa Selvagem
5	Ilha Joana Bezerra; Coque	36	Afogados; Mustardinha; San Martin
6	Campo Grande: Estrada de Belém; Ilha Joaneiro	37	Bongi; San Martin: Afogados; Mustardinha
7	Encruzilhada; Hipódromo; Rosarinho; Torreão	38	Afogados: Mangueira; Vila do Siri
8	Campo Grande; Campina Barreto	39	Estância, San Martin; Jiquiá
9	Água Fria; Arruda; Porto da Madeira; Cajueiro	40	Areias—Ines Andreazza; Vila Cardeal; Silva
10	Água Fria; Fundão: C. Amarela; Fundão Fora	41	Areias: Caçote; Beirinha; J.Uchôa; Rio/Iraque
11	Santa Terezinha; B. do Hemetério—C. Amarela	42	Barro; Sancho; Tejipió
12	Beberibe; Linha do Tiro—Casa Amarela	43	Jardim São Paulo: Areias; Barro; JSPaulo
13	Dois Unidos: Dois Unidos; Linha do Tiro	44	Curado, Coqueiral and Totó—Cavaleiro
14	Apipucos; Dois Irmãos; Sítio dos Pintos; Guabiraba	45	Curado; J. São Paulo—Planeta dos Macacos
15	Casa Amarela; Tamarineira	46	Barro—Tejipió; Pacheco; Vila dos Milagres
16	Casa Forte; Parnamirim; Jaqueira; Monteiro	47	Boa Viagem—Setúbal
17	Graças; Aflitos; Derby; Espinheiro	48	Boa Viagem; Pina—Orla; Av. Herc. Bandeira
18	Casa Amarela: Alto Mandu; Sta. Isabel	49	Boa Viagem—Shopping
19	Alto J. Bonifácio; M. da Conceição—C. Amarela	50	Imbiribeira—Lagoa Araçá; Masc. de Morais
20	Alto José do Pinho; Mangabeira—C. Amarela	51	Ipsep—Av. Jean Emile Favre; Av. Recife
21	Vasco da Gama—Casa Amarela	52	Boa Viagem: Entra-Apulso; Ilha do Destino
22	Brejo da Guabiraba; B. de Beberibe—C. Amarela	53	Brasília Teimosa; Pina—Brasília Teimosa
23	Córrego do Jenipapo; Macaxeira—C. Amarela	54	Pina: Pina; Encanta Moça; Ilha de Deus
24	Nova Descoberta—Casa Amarela	55	Imbiribeira: Sítio Grande; Aritana
25	Iputinga: Alto do Céu; Monsenhor Fabrício	56	Ibura—Av. Dom Hélder Câmara; Vila do Sesi
26	Cordeiro—Av. do Forte; Exposição dos Animais	57	Ibura; Jordão—Alto da Jaqueira

Table 6 continued

Code	HDUs	Code	HDUs
27	Engenho do Meio; Cordeiro; Iputinga—Bom Pastor	58	Ibura; Jordão—Ibura; Jordão
28	Madalena; Ilha do Retiro; Prado	59	COHAB—Lagoa Encantada; Monte Verde
29	Torre; Zumbi	60	COHAB—UŔs 10; 4; 5; R Franc. Vitoriano
30	Madalena; Torre: Sítios Cardoso; Berardo	61	COHAB—UŔs 1; 2; 3
31	Iputinga—Vila União; AP Detran	62	COHAB—UR 5; Três Carneiros

Appendix 2: ELECTRE TRI-C Method

This appendix offers additional information about ELECTRE TRI-C multicriteria method, such as its formulas.

The ELECTRE TRI-C is a sorting method that involves ordered classes and each one of these classes are represented for only one reference action. Basically, outranking relations between alternatives and reference actions are explored, with the intend to find suitable classes to each alternative. The method results in two classifications and these classifications must be analyzed together in the recommendation of the decision process.

The set of alternatives is denoted by A and must be known a priori. So, a set of alternatives $A = \{a_1, a_2, \ldots, a_i\}$ is given and should be distributed in completely ordered classes $\{C_1, \ldots, C_h, \ldots, C_q\}$, where C_1 is the worst class and C_q is the best. The objective of the method is assign the alternatives to the set of classes, according to a set of criteria $F = \{g_1, \ldots, g_j, \ldots, g_n\}$. The set of criteria has a vector of weights that can be interpreted as a voting power, denoted as w_j, such that $w_j > 0, j = 1, \ldots, n$ and assuming $\sum_{j=1}^{n} w_j = 1$.

The set of reference actions $B = \{a'_0, a'_1, \ldots, a'_h, \ldots, a'_q, a'_{q+1}\}$ should be determined a priori too, and they represent the classes in which the alternatives will be distributed. Note that the number of reference actions is equal to $q + 2$ and actions a'_0 and a'_{q+1} represent the worst and the best performances in each criterion, respectively.

Firstly, the comprehensive concordance index $C(a, a')$ should be calculated, as shown in Eqs. 3 and 4, respectively. This index considers all criteria in which the relation a outranks a' is valid. Equation 3 involves the preference threshold (p_j) and the indifference threshold (q_j).

$$\mathbf{C_j}\left(\mathbf{a}, \mathbf{a}'\right) = \begin{cases} 0 & \text{if} \quad \mathbf{g_j}\left(\mathbf{a}'\right) - \mathbf{g_j}(\mathbf{a}) \geq \mathbf{p_j}\left(\mathbf{a}'\right), \\ 1 & \text{if} \quad \mathbf{g_j}\left(\mathbf{a}'\right) - \mathbf{g_j}(\mathbf{a}) \leq \mathbf{q_j}\left(\mathbf{a}'\right), \\ \dfrac{\mathbf{p_j}\left(\mathbf{a}'\right) + \mathbf{g_j}(\mathbf{a}) - \mathbf{g_j}\left(\mathbf{a}'\right)}{\mathbf{p_j}\left(\mathbf{a}'\right) - \mathbf{q_j}\left(\mathbf{a}'\right)} & \text{otherwise.} \end{cases} \tag{3}$$

$$\mathbf{C}\left(\mathbf{a}, \mathbf{a}'\right) = \frac{\sum_{\mathbf{j} \in \mathbf{F}} \mathbf{w_j} \mathbf{C_j}\left(\mathbf{a}, \mathbf{a}'\right)}{\sum_{\mathbf{j} \in \mathbf{F}} \mathbf{w_j}} \tag{4}$$

Subsequently, the partial discordance index $d_j(a, a')$ must be calculated, as shown in Eq. 5. This index considers all criteria against the affirmation a outranks a', and take in account the veto threshold (v_j).

$$\mathbf{d_j}\left(\mathbf{a}, \mathbf{a}'\right) = \begin{cases} 1 & \textbf{if} \quad \mathbf{g_j}(\mathbf{a}) - \mathbf{g_j}\left(\mathbf{a}'\right) < -\mathbf{v_j}, \\ \dfrac{\mathbf{g_j}(\mathbf{a}) - \mathbf{g_j}\left(\mathbf{a}'\right) + \mathbf{p_j}}{\mathbf{p_j} - \mathbf{v_j}} & \textbf{if} \quad -\mathbf{v_j} \leq \mathbf{g_j}(\mathbf{a}) - \mathbf{g_j}\left(\mathbf{a}'\right) < -\mathbf{p_j}, \\ 0 & \textbf{if} \quad \mathbf{g_j}(\mathbf{a}) - \mathbf{g_j}\left(\mathbf{a}'\right) \geq -\mathbf{p_j}. \end{cases} \tag{5}$$

After, the credibility index $\sigma(a,a')$ can be measured as per Eq. 6. This index is intended to measure the strength of the assertion that alternative a outranks reference action a',

$$\begin{aligned} &\sigma\left(\mathbf{a}, \mathbf{a}'\right) = \mathbf{c}\left(\mathbf{a}, \mathbf{a}'\right) \prod_{\mathbf{j}=1}^{\mathbf{n}} \mathbf{T_j}\left(\mathbf{a}, \mathbf{a}'\right), \\ &\text{where } T_j(a, a') = \begin{cases} \dfrac{1 - d_j(a, a')}{1 - c_j(a, a')} & \text{if} \quad d_j(a, a') > c_j(a, a') \\ 1 & \text{otherwise} \end{cases} \end{aligned} \tag{6}$$

Given the credibility index, a credibility level, denoted by λ, should be defined, representing the minimum value of $\sigma(a,a')$ in order to validate or not the outranking relation.

The recommendation of the ELECTRE TRI-C method is a classification, which is formed from two joint rules: descending and ascending. These classifications are made in accordance with the two procedures described below:

Descending rule: evaluating the worst to the best class, the first class a'_t that satisfies $\sigma(a, a'_t) \geq \lambda$ must be found. Given t:

- For $t = q$, select C_q as a possible class to which to assign alternative a.
- For $0 < t < q$, if $\rho(a, a'_t) > \rho(a, a'_{t+1})$, select C_t as a possible class to which to assign alternative a. Otherwise, select C_{t+1}. Consider $\rho(a, a'_k) = \min\{\sigma(a, a'_k), \sigma(a'_k, a)\}$.
- For $t = 0$, select C_1 as a possible class to which to assign alternative a.

Ascending rule: evaluating the best to the worst class, the first class b_k that satisfies $\sigma(a'_k, a) \geq \lambda$ must be found. Given k:

- For $k = 1$, select C_1 as a possible class to which to assign alternative a.
- For $1 < k < (q + 1)$, if $\rho(a, a'_k) > \rho(a, a'_{k-1})$, select C_k as a possible class to which to assign alternative a. Otherwise, select C_{k-1}. Consider $\rho(a, a'_k) = \min\{\sigma(a, a'_k), \sigma(a'_k, a)\}$.
- For $k = (q + 1)$, select C_q as a possible class to which to assign alternative a.

References

Agostini, S. J., & Richarson, S. J. (1997). A Human Development Index for U.S. Cities: Methodological issues and preliminary findings. *real state. Economics, 25*(1), 13–41.

Almeida-Dias, J., Figueira, J. R., & Roy, B. (2010). Electre Tri-C: A multiple criteria sorting method based on characteristic reference actions. *European Journal of Operational Research, 204*(3), 565–580.

Anand, S., & Sen, A. (1994). Human Development Index: Methodology and measurement. Technical Report HDOCPA-1994-02. United Nations Development Programme. http://hdr.undp.org/en/reports/global/hdr1994/papers/oc12.pdf. Accessed 02 October 2013.

Anand, S., & Sen, A. (2000). The income component of the Human Development Index. *Journal of Human Development, 1*(1), 83–106.

Belton, V., & Stewart, T. J. (2002). *Multiple criteria decision analysis*. Boston: Kluwer Academic Publishers.

Bilbao-Ubillos, J. (2013). Another approach to measuring human development: The composite dynamic Human Development Index. *Social Indicators Research, 111*(2), 473–484.

Brazil. (1996). Lei no. 9.394 de 20 de Dezembro de 1996: Estabelece as Diretrizes e Bases da Educação Nacional. Presidência da República, Casa Civil, Subchefia para Assuntos Jurídicos. www.planalto.gov.br/ccivil_03/leis/l9394.htm. Accessed 13 October 2013.

Cahill, M. B. (2005). Is the Human Development Index redundant? *Eastern Economic Journal, 31*(1), 1–5.

Chakravarty, S. R. (2011). A reconsideration of the tradeoffs in the new Human Development Index. *The Journal of Economic Inequality, 9*(3), 471–474.

Cherchye, L., Ooghe, E., & Puyenbroeck, T. V. (2011). Robust human development rankings. *The Journal of Economic Inequality, 6*(4), 287–321.

Das, S. (2008). Human well being: A decile group analysis on indian household data. *Social Indicators Research, 87*(3), 461–472.

Figueira, J., Mousseau, V., & Roy, B. (2005). ELECTRE methods. In J. Figueira, S. Greco, & M. Ehrgott (Eds.), *Multiple criteria decision analysis: State of the art surveys* (pp. 133–156). Boston: Springer.

Figueira, J., & Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *European Journal of Operational Research, 139*(2), 317–326.

Gill, P., & Hall, P. (1997). Playing numbers Or politics? Approaches to the prioritization of development needs in South Africa. *Social Indicators Research, 41*(1–3), 251–278.

Grimm, M., Harttgen, K., Klasen, S., & Misselhorn, M. (2008). A Human Development Index by income groups. *World Development, 36*(12), 2527–2546.
Hagerty, M. R., Cummins, R. A., Ferriss, A. L., Land, K., Michalos, A. C., Peterson, M., et al. (2001). Quality of Life Indexes for national policy: Review and agenda for research. *Social Indicators Research, 55*(1), 1–96.
Harttgen, K., & Klasen, S. (2012). A household-based Human Development Index. *World Development, 40*(5), 878–899.
Hoyland, B., Moene, K., & Willumsen, F. (2012). The tyranny of International Index Rankings. *Journal of Development Economics, 97*(1), 1–14.
Indrayan, A., Wysocki, M. J., Chawla, A., Kumar, R., & Singh, N. (1999). 3-Decade trend in Human Development Index in India and Its major states. *Social Indicators Research, 46*(1), 91–120.
Institute of Applied Economic Research. (2006). Radar Social 2006: Condições de Vida no Brasil. Report. Directory of Social Studies. http://www.ipea.gov.br/portal/index.php?option=com_content&view=article&id=5511. Accessed 07 October 2013.
Ivanova, I., Arcelus, F. J., & Srinivasan, G. (1999). An assessment of the measurement properties of the Human Development Index. *Social Indicators Research, 46*(2), 157–179.
Kelley, A. C. (1991). The Human Development Index: "Handle with Care". *Population and Development Review, 17*(2), 315–324.
Klugman, J., Rodríguez, F., & Choi, H. J. (2011). The HDI 2010: New controversies. *Old Critiques. Journal of Economics Inequality, 9*(2), 249–288.
Lind, N. (2010). A calibrated Index of Human development. *Social Indicators Research, 98*(2), 301–319.
Lopez-Calva, L. F., & Ortiz-Juarez, E. (2012). A household-based distribution-sensitive Human Development Index: An empirical application to Mexico Nicaragua and Peru. *Social Indicators Research, 109*(3), 395–411.
McGillivray, M. (1991). The Human Development Index: Yet another redundant composite development indicator? *World Development, 19*(10), 1461–1468.
Mousseau, V., & Slowinski, R. (1998). Inferring an ELECTRE TRI model from assignment examples. *Journal of Global Optimization, 12*(2), 157–174.
Munda, G. (2012). Choosing aggregation rules for composite indicators. *Social Indicators Research, 109*(3), 337–354.
Murray, C. J. L. (1991). Development data constraints and the Human Development Index. Discussion Paper No. 25. United Nations Research Institute for Social Development. http://www.unrisd.org/80256B3C005BCCF9/%28httpPublications%29/C1CA9D172A23FB6880256B67005B6239?OpenDocument. Accessed 04 October 2013.
Noorbakhsh, F. (1998). The Human Development Index: Some technical issues and alternative indices. *Journal of International Development, 10*(5), 589–605.
Pagliani, P. (2010). Influence of Regional, National and Sub-national HDRs. Human Development Research Paper 2010/19. United Nations Development Programme. http://hdr.undp.org/en/reports/global/hdr2010/papers/HDRP_2010_19.pdf. Accessed 01 October 2013.
Prefeitura do Recife (2005). Atlas of Human Development in Recife. Atlas. Recife City Hall and UNDP. http://www.recife.pe.gov.br/pr/secplanejamento/pnud2006/downloads.html. Accessed 28 September 2013.
Ranis, G., Stewart, F., & Samman, E. (2006). Human development: Beyond the Human Development Index. *Journal of Human Development, 7*(3), 323–358.
Ravallion, M. (2011). The Human Development Index: A response to Klugman, Rodriguez and Choi. *Journal of Economics Inequality, 9*(3), 475–478.
Ravallion, M. (2012). Troubling tradeoffs in the Human Development Index. *Journal of Development Economics, 99*(2), 201–209.
Rende, S., & Donduran, M. (2013). Neighborhoods in development: Human Development Index and Self-organizing Maps. *Social Indicators Research, 110*(2), 721–734.
Roy, B. (1996). *Multicriteria methodology for decision aiding*. London: Kluwer Academic Publishers.
Sagar, A. D., & Najam, A. (1998). The Human Development Index: A critical review. *Ecological Economics, 25*(3), 249–264.
Sant'Anna, A. P., Ribeiro, R. O. A., & Dutt-Ross, S. (2011). Employing the components of the Human Development Index to drive resources to educational policies. *Social Indicators Research, 104*(3), 523–532.
Silva, R., & Ferreira-Lopes, A. (2013). A Regional Development Index for Portugal. *Social Indicators Research,*. doi:10.1007/s11205-013-0455-z.
Srinivasan, T. N. (2012). Human development: A new paradigm or reinvention of the wheel? *American Economic Review, 84*(2), 238–243.

Stanton, E. A. (2007). The Human Development Index: A history. University of Massachusetts Amherst, Political Economy Research Institute. http://www.peri.umass.edu/fileadmin/pdf/working_papers/working_papers_101-50/WP127.pdf. Accessed 04 October 2013.

Stapleton, L. M., & Garrod, G. D. (2007). Keeping things simple: Why the Human Development Index should not diverge from its equal weights assumption. *Social Indicators Research, 84*(2), 179–188.

Tokuyama, C., & Pillarisetti, J. R. (2009). Measuring and monitoring human welfare: How credible are the data in the UNDP's human development reports? *Journal of Economic and Social Measurement, 34*(1), 35–50.

United Nations Development Programme (UNDP). (1990). Concept and measurement of human development. Human Development Report. UNDP. http://hdr.undp.org/en/reports/global/hdr1990/chapters/. Accessed 30 September 2013.

United Nations Development Programme (UNDP). (2000). National Human Development Report Bulgaria 2000; The Municipal Mosaic. Human Development Report. UNDP. http://hdr.undp.org/en/reports/national/europethecis/bulgaria/bulgaria_2000_en.pdf. Accessed 02 October 2013.

United Nations Development Programme (UNDP). (2011). Relatório do Desenvolvimento Humano; Sustentabilidade e Equidade: Um Futuro Melhor para Todos. Human Development Report. UNDP. http://hdr.undp.org/en/media/HDR_2011_PT_Complete.pdf. Accessed 30 September 2013.

United Nations Educational, Scientific and Cultural Organization (UNESCO). (2011). EFA Global Monitoring Report; The Hidden Crisis: Armed Conflict and Education. Report. UNESCO. http://unesdoc.unesco.org/images/0019/001907/190743e.pdf. Accessed 10 October 2013.

Vincke, P. (1992). *Multicriteria decision-aid*. New York: Wiley.

Wolff, H., Chong, H., & Auffhammer, M. (2011). Classification, detection and consequences of data error: Evidence from the Human Development Index. *The Economic Journal, 121*(553), 843–870.

Wu, P. C., Fan, C. W., & Pan, S. C. (2014). Does Human Development Index provide rational development rankings? Evidence from efficiency rankings in super efficiency model. *Social Indicators Research, 116*(2), 647–658.