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SOIL MECHANICS

SETTLEMENT AND LONG-TERM BEARING CAPACITY OF A CIRCULAR FOUNDATION ON AN ELASTIC-VISCOPLASTIC BASE

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The paper presents a numerical analytic solution for the calculation of settlement and long-term bearing capacity of a circular foundation on an elastic–viscoplastic base. The work is a development of the previously obtained solution [1] in the elastic–plastic formulation. The Kelvin–Voigt model was used to calculate the volume component of deformation. For the deviatoric component the A. Z. Ter-Martirosyan model was used.

Introduction

The current Russian standards do not provide clear guidelines, based on appropriate formula for taking into account the rheological properties that are inherent in most types of dispersed soil. In design practice, rheological properties often remain outside the main research program. Nevertheless, consideration of rheological behavior is important, otherwise it could be necessary to carry out additional special tests, calculations, and modifications in the subsequent stages of the foundation design procedure.

The development and improvement of the existing methods of foundations calculation taking into account the rheological behavior of soils, as well as new model solutions for soil deformation under a long-term load, are in the mainstream of modern soil mechanics.

This paper deals with the numerical analytic solution for calculation of settlement and long-term bearing capacity of a circular foundation on an elastic–viscoplastic base. The work is a development of the solution [1] obtained previously in the range of elastic–plastic approaches.

Methods

When considering the stress–strain behavior of a circular foundation on a semi-infinite elastic medium, the relationships obtained by integration of Boussinesq expressions [2] were used:

$$\sigma_z = p_0\left[1 - z^3\left(r^2 + z^2\right)^{-\frac{3}{2}}\right]; \tag{1}$$

$$\sigma_r = \sigma_t = \frac{p_0}{2}\left[1 + 2\nu - 2\left(1+\nu\right)z\left(r^2 + z^2\right)^{-\frac{1}{2}} + z^3\left(r^2 + z^2\right)^{-\frac{3}{2}}\right]; \tag{2}$$

$$\sigma_m = \frac{2p_0\left(1+\nu\right)}{3}\left[1 - \frac{z}{\sqrt{z^2 + r^2}}\right], \tag{3}$$

where σ_z, σ_r, σ_t and σ_m are vertical, radial, tangential, and medium stresses respectively; p_0 is the distributed load; z is the vertical coordinate along the foundation axis (depth); r is the foundation radius; and ν is the coefficient of lateral soil expansion.

We could use Eqs. (1)–(3), as the stress states of elastic and viscoelastic soil foundations are identical [3–5]. The identity of the stress state of media in accordance with the theory of elasticity and of the

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Boltzmann–Volterra hereditary theory, when Poisson's coefficient is constant, was proved by Arutyunyan [5, 6].

Note that such an identity may not be provided for the case of nonlinearity. Studying the possible influence of this aspect is a promising task for future investigations.

To describe the viscoelastic behavior of soils, the system of Hencky's physical equations was used [7, 8]:

$$\dot{\varepsilon}_x = \dot{\chi}\left(\sigma_x - \sigma_m\right) + \dot{\chi}^* \sigma_m;\ \dot{\gamma}_{xy} = 2\dot{\chi}\tau_{xy};$$
$$\dot{\varepsilon}_y = \dot{\chi}\left(\sigma_y - \sigma_m\right) + \dot{\chi}^* \sigma_m; \dot{\gamma}_{yz} = 2\dot{\chi}\tau_{yz}; \tag{4}$$
$$\dot{\varepsilon}_z = \dot{\chi}\left(\sigma_z - \sigma_m\right) + \dot{\chi}^* \sigma_m;\ \dot{\gamma}_{zy} = 2\dot{\chi}\tau_{zy},$$

where $\dot{\varepsilon}_x$, $\dot{\varepsilon}_y$, $\dot{\varepsilon}_z$, σ_x, σ_y, σ_z are the linear components of strain rate and normal stresses for *x*, *y*, and *z* axes respectively; $\dot{\gamma}_{xy}$, $\dot{\gamma}_{yz}$, $\dot{\gamma}_{zy}, \tau_{xy}, \tau_{yz}, \tau_{zy}$ are the shear strain rates and shear stresses for directions *xy*, *yz*, and *zy* respectively; $\dot{\chi}, \dot{\chi}^*$ are the deviatoric and volumetric strain components respectively, defined as:

$$\dot{\chi} = \frac{\dot{\gamma}_i}{2\tau_i} = \frac{f\left(\tau_i, \sigma_m, \mu_\sigma, t\right)}{2\tau_i};\ \dot{\chi}^* = \frac{\dot{\varepsilon}_m}{\sigma_m} = \frac{f\left(\tau_i, \sigma_m, \mu_\sigma, t\right)}{\sigma_m}, \tag{5}$$

where $\dot{\varepsilon}_m$ is the medium linear strain rate, $\dot{\gamma}_i$ is the shear strain intensity rate, μ_σ is the Nadai–Lode factor, *t* is time, and τ_i is the shear stress intensity:

$$\tau_i = \frac{\sigma_1 - \sigma_3}{\sqrt{3}} = \frac{\sigma_z - \sigma_r}{\sqrt{3}} = \frac{\sigma_z - (3\sigma_m - \sigma_z)/2}{\sqrt{3}} = \frac{3(\sigma_z - \sigma_m)}{2\sqrt{3}}. \tag{6}$$

The Kelvin–Voigt model was chosen for the volumetric strains [8]:

$$\sigma_m = \sigma_m^{elast} + \sigma_m^{visc} = K\varepsilon_m + \eta_v \dot{\varepsilon}_m, \tag{7}$$

where *K* and η_v are the volumetric deformation modulus and soil viscosity respectively.

The expression for volumetric strain and its rate is written in the form [8]:

$$\varepsilon_m = \frac{\sigma_m}{K}\left(1 - e^{(-K/\eta_v)t}\right); \tag{8}$$

$$\dot{\varepsilon}_m = \frac{\sigma_m}{K}\left(\frac{-K}{\eta_v} e^{(-K/\eta_v)t}\right). \tag{9}$$

From Eq. (5) we obtain:

$$\dot{\chi}^* = \frac{1}{K}\left(\frac{-K}{\eta_v} e^{(-K/\eta_v)t}\right). \tag{10}$$

The model for the shear component of deformation should describe the creep behavior under long-term shear stresses, as observed from the experimental results obtained by S. S. Vyalov, G. I. Ter-Stepanyan, S. R. Meschyan, N. M. Maslov, and others (Fig. 1) . The model should describe all stages of the deformation: steady creep, steady flow, and progressive failure.

It should be noted that the progressive creep model overestimates the deformations under the tangential stresses less than $-0.5\tau_s$, where τ_s is the limit value of shear stresses under short-term loading (Fig. 1).

The model of A. Z. Ter-Martirosyan [3] meets the conditions mentioned above:

$$\dot{\gamma}(\sigma_m) = \frac{\tau - \tau^*}{\eta_\gamma(\sigma_m)}\left(\frac{e^{-\alpha\varepsilon_z}}{a} + \frac{e^{\beta\varepsilon_z}}{b}\right), \tag{11}$$

where $\dot{\gamma}(\sigma_m)$ is the shear strain rate; $\eta_\gamma(\sigma_m)$ is the shear soil viscosity; τ is shear stress; τ^* is the limit value of shear stresses; $\alpha, \beta, a,$ and *b* are the hardening (softening) parameters of clayey soil determined by the results of the kinematic shear test.

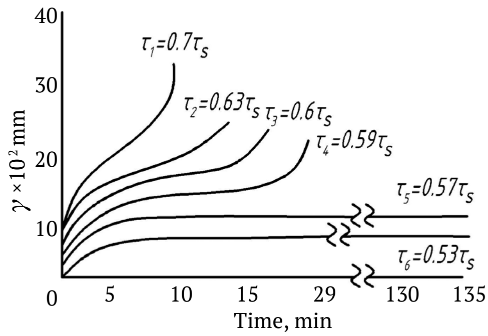

Fig. 1. Creep curves for the plastic clays under different shear stresses (chosen as a proportion of their limit value under short-term loading) [8].

The shear stress should be greater than τ^*, otherwise the shear strain rate is zero. Determination of τ^* can be carried out according to the method [9]; it is also possible following [10] as the first approximation:

$$\tau^* = \sigma_m \tan\varphi + c_c, \tag{12}$$

where φ is the internal friction angle and c_c is the structural cohesion.

According to N. N. Maslov's classification [10], the structural cohesion is inherent in "rigid" and "hidden plastic" clayey soils. For soils of liquid consistency $c_c = 0$ in most cases. It should be noted that according to recent studies [11], the influence of the viscous cohesion (part of the cohesion occurring at kinematic shear) should be additionally considered.

To use Eq. (11) in the system of Hencky's equations, the equivalent of the shear stresses' limit value τ^* in the principal stresses should be subtracted from $\sigma_1 - \sigma_3$:

$$\dot{\varepsilon}_z = \frac{\sigma_z - \sigma_m - \frac{2}{3}\left(\sigma_m \tan\varphi + c_c\right)}{\eta_\gamma(\sigma_m)} \left(\frac{e^{-\alpha\varepsilon_z}}{a} + \frac{e^{\beta\varepsilon_z}}{b} \right). \tag{13}$$

Then the expression for the rate parameter of the deviatoric component of deformation could be expressed as:

$$\dot{\chi}^* = \frac{1}{\eta_\gamma} \left(\frac{e^{-\alpha\gamma}}{a} + \frac{e^{\beta\gamma}}{b} \right). \tag{14}$$

The velocity of the vertical component of deformation could be written as follows:

$$\dot{\varepsilon}_z = \frac{\sigma_z - \sigma_m - \frac{2}{3}\left(\sigma_m \tan\varphi + c_c\right)}{\eta_\gamma\left(\sigma_m\right)} \left(\frac{e^{-\alpha\varepsilon_z}}{a} + \frac{e^{\beta\varepsilon_z}}{b} \right) + \frac{\sigma_m}{K} \left(\frac{-K}{\eta_v} e^{(-K/\eta_v)t} \right). \tag{15}$$

The results obtained by integrating Eq. (15), however, do not fully reflect the behavior of the foundation, as the limit state is completely determined by the magnitude of shear strain at the point with the maximum $\sigma_z - \sigma_m - \frac{2}{3}\left(\sigma_m \tan\varphi + c_c\right)$. In reality, the shear strains at this point are limited by the passive earth pressure (Fig. 2).

To determine the value of + in the first approximation, this paper proposes to introduce a special function into (15):

$$p_h(z, \varepsilon_z) = \begin{cases} p_{h,elast}(z, \varepsilon_z) \text{ when } p_{h,elast}(z, \varepsilon_z) < \lambda\gamma_s\left(h + z\right) \\ \lambda\gamma_s\left(h + z\right) \quad \text{when } p_{h,elast} \geq \lambda\gamma_s\left(h + z\right) \end{cases}, \tag{16}$$

where λ is the lateral pressure coefficient (in the first approximation it can be taken as equal to the passive pressure coefficient); γ is the unit weight of the soil.

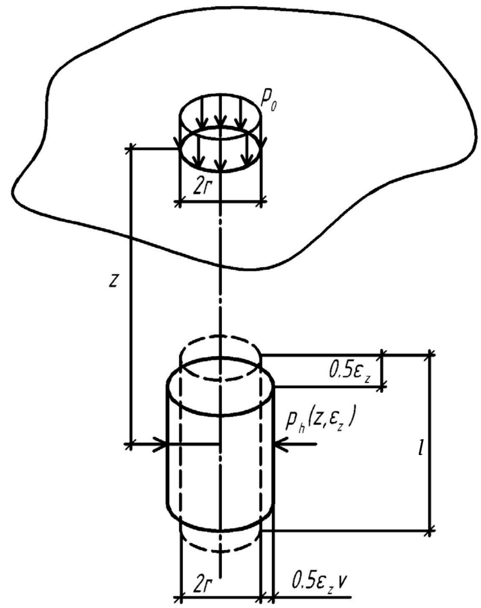

Fig. 2. Calculation diagram demonstrating the effect of the surrounding soil rebound on the strains.

The following function can be obtained by solving the Lame problem according to the known method [12]:

$$p_{h,elast}(z,\varepsilon_z)=\left(v\varepsilon_z+2p_b k^2\frac{r}{\left(k^2-1\right)E}\right)\Big/\left(\left(1-v+k^2\left(v+1\right)\right)\frac{r}{\left(k^2-1\right)E}\right), \tag{17}$$

where v is Poisson's ratio; E is the deformation modulus; $p_b=\lambda_0\gamma_s(h+z)$ is the horizontal pressure outside the foundation influence zone; λ_0 is the lateral soil pressure coefficient at rest; k is the coefficient, which can be assumed to be 6; and r is the foundation radius.

Thus, the expression for the vertical component of the strain rate is as follows:

$$\dot{\varepsilon}_z=\frac{\sigma_z-\sigma_m-\frac{2}{3}\left(\sigma_m\mathrm{tg}\varphi+c_c\right)-p_h\left(z,\varepsilon_z\right)}{\eta_\gamma(\sigma_m)}\left(\frac{e^{-\alpha\varepsilon_z}}{a}+\frac{e^{\beta\varepsilon_z}}{b}\right)+\frac{\sigma_m}{K}\left(\frac{-K}{\eta_v}e^{\frac{-K}{\eta_v}t}\right). \tag{18}$$

The solution for $\varepsilon_z(t)$ can be obtained by numerical integration in Mathcad. The foundation settlement can be determined by layer-by-layer summation within the depth of the compressible layer h_s.

In the present work, a series of calculations was performed using the parameters: η_γ = 1.157×10^5 kPa·days, K = 3500 kPa, γ = 18 kN/m^3, φ = 10°, c_c = 10 kPa, η_v = 2.187×10^6 kPa·days, a = 15, b = 60, α = 280, and β = 45.

The calculation results for a 10-m diameter foundation at a depth of 1 m are shown in Figs. 3a and b.

For a constant value of the distributed load when the foundation diameter increases, a sharper increase in settlement at the initial section with time and a longer period before the onset of the failure stage owing to the increase in medium stresses are observed (Fig. 3c).

The obtained shape of the plots (excluding the plots describing the limit state) is consistent with the observed settlement curves for the buildings in St. Petersburg [13].

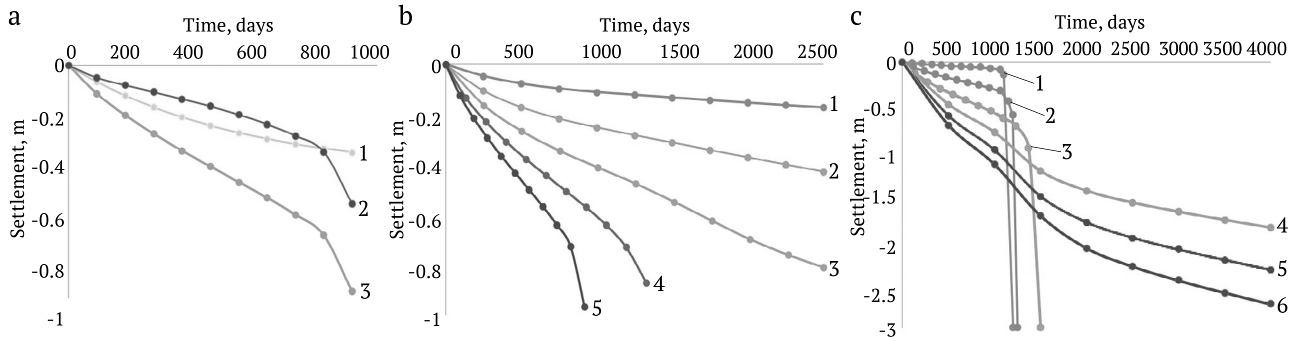

Fig. 3. Plots of settlement as a function of time: a) diameter d = 10 m, pressure P = 400 kPa (1 is the volumetric settlement component, 2 is the shear settlement component, and 3 is the total settlement); b) d = 10 m, P = 100 kPa (1), P = 200 kPa (2), P = 300 kPa (3), and P = 400 kPa (4); c) P = 400 kPa, d = 1 m (1), d = 5 m (2), d = 10 m (3), d = 15 m (4), and d = 25 m (5).

Conclusions

The obtained solution allows the behavior of a circular foundation under vertical load to be described, taking into account the rheological properties of the foundation soil. Depending on the magnitude of the applied load, the solution can describe decaying, steady, and progressive settlement.

The decaying settlement can be described by taking into account the growth of radial stresses as the axial shear strain increases.

When the foundation diameter increases during a constant distributed load, the solution shows a sharp increase in settlement in the initial part of the settlement-time graph, as well as an increase in the long-term bearing capacity due to an increase in the medium stresses.

The accuracy of the solution directly depends on the adopted mechanical parameters of rheological models. It is necessary to develop a methodology for determining these parameters based on the results of laboratory tests of soils.

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