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# CALCULATION OF BUILDING SETTLEMENT INDUCED BY TUNNELING BASED ON AN EQUIVALENT BEAM MODEL

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*Tunneling-induced ground movement may excessively deform the overlying structure. It is necessary to analyze the soil–structure interaction mechanism and calculate the settlement in advance to avoid potential damage to an adjacent building. The present study adopts the Gaussian distribution law of a greenfield and the equivalent beam method in considering a building's bending and shearing deformation, the ground's residual deformation, and the potential gap between the building and ground and analyses the relationship between the building settlement, the building load, and the number of floors. It is found that the settlement of masonry buildings decreases with an increase in the number of floors, gaps generally appear for low-rise buildings, and the settlement of framed buildings can generally be estimated as greenfield settlement. The effectiveness of the proposed method is verified in a case study.*

## Introduction

Tunneling in urban areas possibly damages the overlying building structures. Burland and Boone [1, 2] first focused on tunneling-induced deformation; their approaches suppose the building is completely flexible. However, studies [3, 4] have found that the subsidence of a building is much less than that of a greenfield, and the effects of the building load, especially stiffness, should not be neglected. Methods of numerical simulation [5, 6] require the determination of many parameter values for the specific project. On the basis of Burland's theory, Potts and Franzius [7, 8] proposed a relative stiffness method, which divides a building into convex and concave portions. Zhou [9] pointed out that a method that neglects the integrity of a building overestimates rigid-body rotation. Giardina [10] compared several relative-stiffness methods that could not directly reproduce building deformation. Studies [1, 7, 11] have simplified buildings into an equivalent elastic beam structure, which can easily be used to consider the interaction between buildings and soil. Maleki [12] confirmed that the calculation results of the equivalent beam and real geometry of the building were similar. For many buildings, there is no strong connection between the foundations and soil, and the differential settlement of the building foundation and soil creates a gap at the interface [12]. Additionally, the irrecoverable residual deformation of the soil at the bottom of the building which is usually ignored could affect the calculation of the settlement of the building.

In this paper, considering the bending and shearing effects of a building, the mechanism of interaction between the building and ground, the residual deformation of the ground and the potential gap, a curve of the settlement of the building induced by the construction of a tunnel is obtained.

## Greenfield Settlement

The Gaussian distribution curve for surface settlement caused by tunnel excavation [13] is:

$$f(x)=\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{x^2}{2i^2}),\qquad i=K_g z_0, \tag{1}$$

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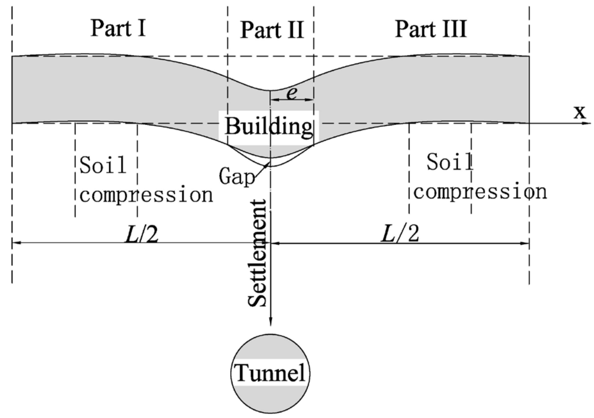


**Fig. 1.** Calculation sketch.

where $x$ is the horizontal distance from the tunnel axis, $f(x)$ is the ground surface settlement at position $x$, $A$ is the excavation area, $V_1$ is the ground volume loss rate, $i$ is the width coefficient of the ground settlement trough, $K_g$ is the width parameter of the surface settlement trough of the greenfield, and $z_0$ is the burial depth of the tunnel axis.

## Building Stiffness

The building is simplified as a Timoshenko beam structure with a certain thickness on the ground soil for consideration of shearing and bending stiffness. According to Potts [7], the vertical section of a masonry building deforms according to the assumption of plane cross-section. The stiffness of a masonry building is calculated as

$$C = (m+1)G_{\text{slab}}A_{\text{slab}}, \tag{2}$$

$$D = E_{\text{slab}}\sum_{1}^{m+1}\left(I_{\text{slab}} + A_{\text{slab}}H_m{}^2\right), \tag{3}$$

where $C$ is the shearing stiffness of the building, $D$ is the bending stiffness of the building, $m$ is the number of stories, $A_{\text{slab}}$ is the cross-sectional area of the slab, $G_{\text{slab}}$ is the shearing modulus of the slab, $E_{\text{slab}}$ is Young's modulus of the slab, $I_{\text{slab}}$ is the vertical moment of the slab section, and $H_m$ is the vertical distance from the neutral axis of the section of the building to the neutral axis of the slab section.

If there are cracks in the walls of a masonry building or the construction is a framed structure, the walls less constrain the bending slabs and the bending stiffness of the building cannot be calculated completely by assuming a flat section. If the walls do not constrain the bending slabs, the bending stiffness is

$$D = \left(m+1\right)E_{\text{slab}}I_{\text{slab}}. \tag{4}$$

## Building Settlement

As shown in Fig. 1, part of the ground is in a further compressed state while part is in a rebound state, and there may be a gap at the bottom of the building. Due to the effect of unrecoverable residual deformation, it is necessary to adopt the double ground reaction modulus. The ratio of the load and compression deformation is denoted as $k$, and the ratio of the load and rebound deformation is denoted as $k_r$; and according to the standard of the American Association of State Highway and Transportation Officials, $k_r = 1.77k$. The building is simplified as a Timoshenko beam, where the part in contact with the ground is a foundation beam while the part suspended is an ordinary beam. According to Timoshenko's theory of two generalized displacement beams, the displacement satisfies Eq. (5). Including the mechanism of interaction between the building and ground and Eq. (1), the displacement of the ordinary beam satisfies Eq. (6) while the displacement of the foundation beam satisfies Eq. (7):

$$-\frac{d}{dx}(D\frac{d\psi}{dx})-C(\frac{dw}{dx}-\psi)=0, \tag{5}$$

$$-\frac{d}{dx}\left[C(\frac{dw}{dx}-\psi)\right]=q, \tag{6}$$

$$w=\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{x^2}{2i^2})+\frac{1}{k}\frac{d}{dx}\left[C(\frac{dw}{dx}-\psi)\right], \tag{7}$$

where $w$ and $\psi$ are respectively the settlement and deflection of the beam, $C$ and $D$ are respectively the shear stiffness and bending stiffness of a beam of unit width, $q$ is the weight of the building per unit area, $k$ is the ground reaction modulus, and $k = k_r$ as the ground soil compresses.

According to Eqs. (5) and (7), the differential equation of the foundation beam section is

$$\frac{d^4w}{dx^4}-\frac{k}{C}\frac{d^2w}{dx^2}+\frac{k}{D}w=\left(\frac{k}{D}+\frac{k}{Ci^2}-\frac{kx^2}{Ci^4}\right)\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{x^2}{2i^2}). \tag{8}$$

If the beam and ground are separated, and then according to Eqs. (7) and (8) and symmetry, the ordinary beam (i.e., the equation for part II in the figure) is expressed as

$$w(x)=\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{e^2}{2i^2})-\frac{q}{k}+\frac{5e^4q}{24D}+\frac{e^2q}{2C}-\left(\frac{e^2q}{4D}+\frac{q}{2C}\right)x^2+\frac{q}{24D}x^4+\left(\frac{e^2}{2D}-\frac{1}{2D}x^2\right)M_e, \tag{9}$$

$$M_e=-Dw''(e)-\frac{qD}{C}, \tag{10}$$

where $M_e$ is the moment at the boundary point, i.e., the moment when $x = e$.

Relative to the ground soil before tunnel excavation, the two sides of soil at $x = e_2$ and $x = e_3$ are respectively in compression and rebound states. The shearing force and bending moment at the edge of the beam are zero. Adopting the initial parameter method and continuity of beam deformation gives

$$w(e)=\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{e^2}{2i^2})-\frac{q}{k}, \tag{11}$$

$$w''(e)-\frac{1}{e}w'(e)=\frac{e^2q}{3D}, \tag{12}$$

$$w'''(e)-\frac{k}{C}w'\left(e\right)=\frac{ek}{Ci^2}\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{e^2}{2i^2})+\frac{eq}{D}, \tag{13}$$

$$w''(L/2)-\frac{k}{C}w(L/2)+\frac{k}{C}\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{L^2}{8i^2})=0, \tag{14}$$

$$w'''(L/2)-\frac{k}{C}w'(L/2)-\frac{Lk}{2Ci^2}\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{L^2}{8i^2})=0, \tag{15}$$

$$w(e_2)=\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{e_2^{\,2}}{2i^2}), \tag{16}$$

$$w(e_3)=\frac{AV_1}{i\sqrt{2\pi}}\exp(-\frac{e_3^{\,2}}{2i^2}). \tag{17}$$

According to the above conditions, the settlement curve of the building is obtained by fitting using numerical calculation software. In the calculation, when there is no gap at the bottom of the building, accord-

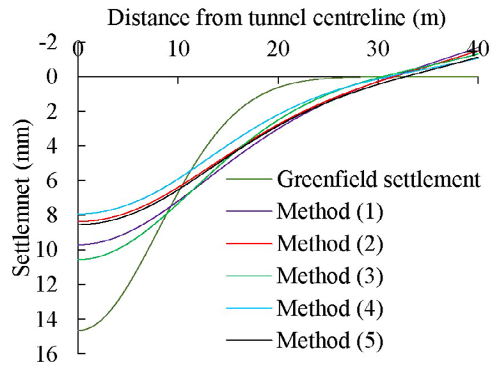


**Fig. 2.** Building settlement curve.

ing to Eq. (8), the curve is obtained from the boundary conditions (14) and (15) and the symmetry of settlement; when there is a gap at the bottom of the building, according to Eqs. (8) and (9), the curve is obtained from boundary conditions (11)-(17) and the symmetry of settlement.

## Analysis of the Proposed Method

A three-storey masonry building is taken as an example. The storey height is 2.8 m and the slab thickness is 0.12 m. The settlement of the building is stable before excavation. The load of the building on the ground soil $q$ is 40 kPa. According to Eqs. (2) and (3), $C$ = 4.4e9 N/m and $D$ = 1.08e11 N·m$^2$/m. The ground reaction modulus in compression $k$ is 1.5e7 N/m$^3$ while the reaction modulus in rebound $k_r$ is 4.5e7 N/m$^3$. The tunnel diameter is 5 m, the tunnel axis depth is 20 m, the settlement tank width parameter $K_g$ is 0.5, and the ground volume loss rate $V_1$ is 1%. The settlement curve of the building is calculated using five methods: (1) the building and ground are considered inseparable, and the ground reaction modulus is $k$; (2) the building and ground are considered separable, and the ground reaction modulus is $k$; (3) the building and ground are considered inseparable, and the ground reaction modulus is $k_r$; (4) the building and ground are considered separable, and the ground reaction modulus is $k_r$; and (5) the building and ground are considered separable, and the ground reaction moduli in soil compression and rebound are $k$ and $k_r$ respectively. The settlement curves of the building obtained using the five methods are shown in Fig. 2. The greenfield settlement curve is also included in the figure.

Among the five methods, Method (5) considers all influencing factors, shown in Fig. 1, and the maximum settlement is 8.5 mm, and by contrast, the deviation resulting from assuming the building completely flexible reaches 42%, and in Methods (1)-(4), the deviation resulting from using different reaction modulus reaches 6% and 8%, the deviation resulting from the gap reaches 15% and 34%. Obviously, the potential gap and reaction modulus strongly affect the estimation of settlement, which cannot be ignored.

## Parametric Studies

The load provided by a three-storey building is generally 40-200 kPa. A parameter sensitivity analysis is presented for the above three-storey building example. The critical load at which exactly no gaps appear at the bottom of the building is 103 kPa, and when the load is lower than the critical value, with an increase in the load, the building settlement increases, the length of the gap decreases, and when the load is above the critical value, the building settlement curve no longer changes with a varying load (Figs. 3a and 4). It is seen that within a certain range, the settlement of the building increases with the building weight. The bottom of the relatively heavy building is less prone to have a gap. If there is no gap, the change in building weight does not affect the settlement.

Buildings with different numbers of storeys have different building stiffness and loads acting on the ground. The load for each storey is set at 10 kPa. The settlement of masonry and framed structure build-

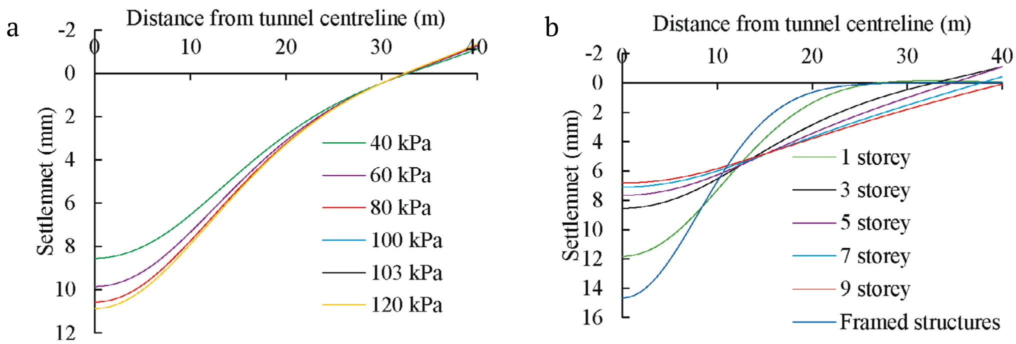


**Fig. 3.** Settlement curves for: a) different building loads, b) different numbers of storeys.

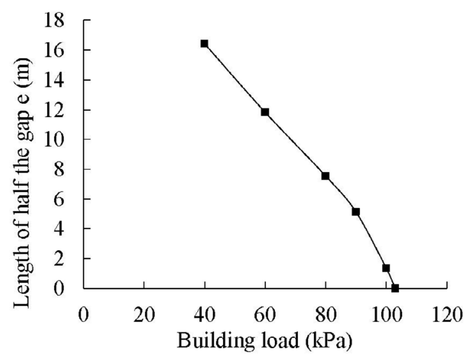


**Fig. 4.** Variation in the gap length with an increase in building weight.

ings with different storeys are calculated as shown in Fig. 3b. For the masonry buildings, as the number of building storeys increases from one to nine, the building stiffness increases, the building curvature decreases, and the maximum settlement decreases from 11.8 to 6.8 mm, which is a change of 74%, and in a certain range, the gap length decreases with an increase in the building load. Moreover, the gap length obviously increases with the building stiffness, and the low stiffness has a stronger effect on the one-storey building, and as for buildings with three to nine storeys or more, the load is the main factor responsible for the gap length decreasing with an increase in the storey number, and gaps are not prone to occur at the bottom of high-rise buildings. It is seen that the ground soil of the higher-rise masonry building generally rebounds in the middle and compresses at the ends compared with the ground soil in the initial state. For the frame structures, owing to their small bending stiffness, the buildings are basically flexible, and the settlement law of the buildings is similar to the greenfield settlement law.

## Case Study

Frischmann [3] reported the actual observation settlement curve of Mansion House caused by tunnel excavation. The diameter of the tunnel was 3.05 m, the distance from the bottom of the building to the axis of the tunnel was 12.4 m, and the ground volume loss rate of the tunnel excavation was 2.6%. The ground soil was London clay, for which the ground reaction modulus $k$ = 2.5e7 N/m$^3$ and $k_r$ = 4.4e7 N/m$^3$. The building stiffnesses was calculated as previously described as $C$ = 6.6e9 N/m and $D$ = 4.35e11 N·m$^2$/m, and the building load $q$ was 60 kPa. As shown by the calculation results in Fig. 5, the maximum settlement is 4.76 mm and there is a 16 m gap at the bottom of the building. The comparison between the settlement curve calculated using the method proposed in this paper and the measurement shows that the two results are similar but have certain differences. There are two main reasons for this error. First, the building has existed for a long

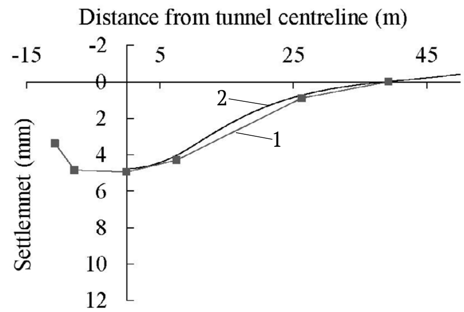


**Fig. 5.** Measured (1) and calculated (2) settlement of Mansion House, London.

time and the building stiffness has decreased after several deformations and repairs. Second, the tunnel is not directly below the middle of the building, and the inclination of the building varies.

## Conclusions

1. Following tunnel excavation, there may be a gap at the bottom of a building and the ground soil may compress or rebound relative to the initial state. It is erroneous to assume that the building and ground soil will remain in contact or that only using the Winkler modulus is sufficient in calculating the settlement of the building. It is necessary to consider the effect of a potential gap and use a double ground reaction modulus.

2. Within a certain range, the settlement of the building increases with the building weight, and a relatively heavy building is less prone to have a gap at the bottom. If there is no gap, a change in building weight has no effect on the settlement.

3. For masonry buildings, the settlement decreases with an increase in the number of storeys. A gap generally appears for a low-rise building. In the case of a higher-rising building, there is less likely to be a gap between the building and ground and the ground soil generally rebounds in the middle and compresses at the two ends. For a framed building, the bending stiffness is usually small, and the bottom of the building is not prone to having a gap, and the settlement can generally be estimated as greenfield settlement.

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