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BEARING CAPACITY ASSESSMENT OF SOIL FOUNDATION

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Bearing capacity calculations for soil foundation beds are analyzed using modern numerical methods based on limit equilibrium theory. The authors examine the results of elastoplastic solutions employing the finite element method (FEM) with the use of PLAXIS and MIDAS software packages. It is shown that the stress fields determined via the FEM do not always satisfy the equilibrium requirements. The stability calculation methods implemented in the Optum G2 are characterized. Finally, the paper proposes additions to the existing regulatory procedure for calculating the bearing capacity of foundation beds.

Introduction

Solutions to bearing capacity problems of soil foundation beds are primarily based on the limit equilibrium theory (LET) [1], whose solutions are included in Russian design regulations [2], as well as successfully applied in practice. New LET solutions have recently emerged to further develop the concept [2]. In addition, while the use of the finite element method (FEM) to describe the limit state of soils remains understudied, it is already commonly used in geotechnical calculations. The Optum G2 software increasingly used in conjunction with the FEM in design practice and scientific research also implements a fundamentally different approach to solving limit equilibrium problems by applying linear and nonlinear programming methods.

Main Solutions to the Bearing Capacity Problems of Foundation Beds

Let us consider only static bearing capacity problems for plane strain conditions. The most common mechanisms here include those identified by Hill (Fig. 1a) and Prandtl (Fig. 1b).

While Hill's mechanism assumes no friction at the loading plate-soil interface, Prandtl's mechanism, which is more commonly used in practice, implies shear stresses, which are obtained from the solution.

The first solutions to the bearing capacity problems of foundation beds were obtained in the 1920s. Ludwig Prandtl [3] and Hans Reissner [4] provided formulas for the ultimate pressure on a granular (cohesive) weightless soil bed (unit weight of soil – $\gamma = 0$; internal friction angle – $\varphi \neq 0$; specific cohesion – $c \neq 0$) and on a perfectly cohesive weightless soil bed ($\gamma \neq 0$, $\varphi = 0$, $c \neq 0$). In both cases, Hill and Prandtl's mechanisms yield the same ultimate load values. In 1938, these solutions were generalized by V. Novotortsev [5] for the inclined load case.

The general packing theory for solving static problems in a granular medium pertaining to the limit equilibrium of soils was developed by V. Sokolovskii between 1939 and 1942 [1]. In addition, a numerical solution was obtained for a loading plate on a granular bed ($\gamma \neq 0$, $\varphi \neq 0$, $c \neq 0$) according to Hill's mechanism. Subsequently, solutions were proposed according to Prandtl's mechanism: for a perfectly granular loaded soil bed by M. Malyshev in 1959 [6] and for the general case of a granular loaded soil bed by Yu. Solov'ev in 1979 [7]. In 1983, A. Stroganov published a graph-analytical solution to the same problem taking into account the inclination of the ultimate pressure resultant [8], whose results were included in [2].

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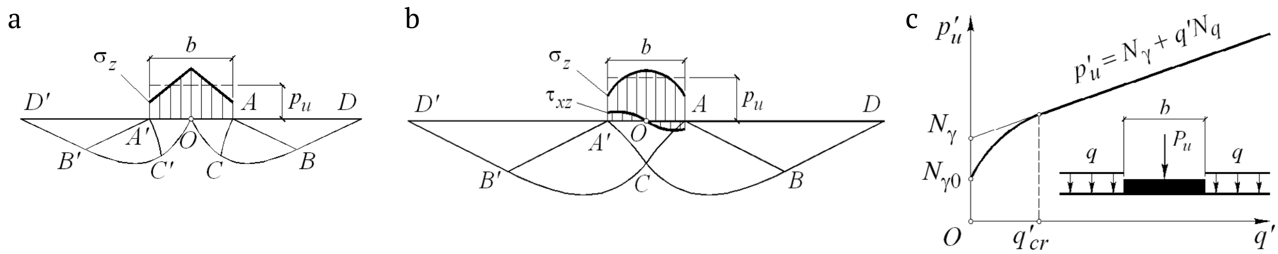

Fig. 1. Solution to the bearing capacity problem of foundation beds: a) Hill's mechanism, b) Prandtl's mechanism, c) graph.

In 2010–2012, a closed-form solution to the bearing capacity problem of a granular loaded soil bed was obtained [9, 10]. This solution provided a theoretical justification for Terzaghi's three-term expression for the ultimate pressure:

$$p_u = \gamma b N_\gamma + q N_q + c N_c, \tag{1}$$

where b is a footing width, $q = \gamma' d$ is a lateral vertical surcharge (γ' is unit weight of soil above the foundation footing, d is a foundation depth; N_γ, N_q, and N_c are bearing capacity factors determined as follows:

$$\begin{aligned} N_\gamma &= \frac{1+\sin\varphi\cos 2\alpha}{4\cos^2\varphi}[2f_\sigma \cos(\alpha+\mu)\cos\varphi - \cos(2\alpha+\varphi)], \\ N_q &= \frac{1+\sin\varphi\cos 2\alpha}{1-\sin\varphi} e^{(\pi-2\alpha)\tan\varphi}, \\ N_c &= (N_q - 1)\cot\varphi, \end{aligned} \tag{2}$$

where $f_\sigma = \frac{e^{3(\pi/2-\alpha)\tan\varphi}}{2\sin\mu} + \frac{1}{1+8\sin^2\varphi}\left[\cos(\alpha+3\mu) + 2\cos(\alpha-\mu) + e^{3(\pi/2-\alpha)\tan\varphi}(\sin 3\mu - 2\sin\mu)\right]$,

$\mu = \pi/4 - \varphi/2$ is an angle between the direction of σ_1 and slip lines, α is an angle between the Oz-axis and the direction of σ_1, which is related to the inclination angle of the reduced ultimate pressure resultant δ to the vertical at the loading plate-soil interface by the following expression: $\sin(2\alpha - \delta) = \sin\delta/\sin\varphi$.

Reduced ultimate pressure commonly used in LET solutions is determined by the reduced normal and shear τ_u components, while in this case

$$\tau_u = p'_u \tan\delta \equiv (p_u + c\,\mathrm{ctg}\,\varphi)\tan\delta, \tag{3}$$

where p_u is an actual vertical component of the ultimate pressure.

In (2), the factors N_q and N_c are in agreement with Novotortsev's solution [5].

The limited scope of application of Eq. (1) can be observed from the graph showing the dependence of the ultimate pressure on the lateral surcharge q' in relative reduced variables (Fig. 1c):

$$p'_u = (p_u + c\cot\varphi)/(\gamma b),\ q' = (q + c\cot\varphi)/(\gamma b).$$

It can be seen in Fig. 1c that the graph $p'_u(q')$ becomes curvilinear under a lateral surcharge load not exceeding a certain value q'_{cr}. Thus, the solution for a non-surcharge perfectly granular soil bed ($\gamma \neq 0$, $\varphi \neq 0$, $q' = 0$) yielding $N_{\gamma 0}$ is of particular importance in the LET. This solution is usually obtained using a special system of equations for a granular wedge [1]. The solution to this problem is considered in the works by V. Sokolovskii, H. Lundgren and K. Mortensen, M. Malyshev, Yu. Solov'ev and A.Karaulov, V. Fedorovskii, and elsewhere [6, 11, 12]. Of note is the article by Victor Fedorovskii [11] providing the most general solution to this problem for inclined load taking eccentricity into account, which is given for both Hill and Prandtl's mechanisms. In order to calculate $N_{\gamma 0}$, it is possible to use an approximation dependence obtained in [11, 12]:

$$N_{\gamma 0} = N_\gamma / k_{\gamma 0},\ k_{\gamma 0} = 1.865 - 0.708\varphi.$$

Bearing Capacity Calculation as per [2]

According to [2], bearing capacity should be calculated using LET methods. In this case, two principal cases are distinguished: stabilized and unstabilized states.

Stabilized state. For this state, Coulomb's law provides the basic physical equation expressing a relationship between the ultimate shear τ_n and normal σ_n stresses across the shear plane:

$$\tau_n = \sigma_n \tan\varphi + \mathrm{c}.$$

The bearing capacity of a foundation bed is determined as the vertical component of the ultimate pressure resultant of a rectangular foundation acting on bearing soil:

$$N_u = b'l'(\gamma b'\xi_\gamma N_\gamma + q\xi_q N_q + c\xi_c N_c), \tag{4}$$

where b' and l' are reduced width and length of the footing; ξ_γ, ξ_q, and ξ_c are shape (spatiality) factors, depending on $\eta = l'/b'$, N_γ, N_q, and N_c are bearing capacity factors determined depending on the angles φ and δ.

Let us consider the feasibility of developing this procedure.

1. The reduced parameters b' and l' are used to factor in the eccentricity of load application. It is assumed that the ultimate load N_u centrally applied over the width b' can be considered as eccentric for the width b. Thus, with $M = N_u(b - b')/2$, the actual width b and reduced width b' will be related by $b' = b - 2e_b$ at an eccentricity of $e_b = M/N$ [13]. The numerical solution in its strict static formulation is considered in [11] to determine the ultimate pressure taking eccentricity into account.

2. The shape factors given in [2] are as follows:

$$\xi_\gamma = 1 - 0.25/\eta,\ \xi_q = 1 + 1.5/\eta,\ \xi_c = 1 + 0.3/\eta. \tag{5}$$

However, the 1975 regulations state that:

$$\xi_\gamma = 1 + 0.25/\eta. \tag{6}$$

A fundamental question arises as to whether the ultimate pressure exerted on a non-surcharge granular soil bed decreases or increases when switching from a square plate load to a strip load. A review of works on this problem is provided, for example, in [6]. It seems that the most reasonable approach is proposed in [14], which consists in determining the ultimate load applied to a rectangular plate using a combination of plane and axisymmetric LET solutions. In [15], a solution was obtained, whose results can be conveniently represented in the following form:

$$\xi_\gamma = 1 + a_\gamma / \eta,\ \xi_q = 1 + a_q / \eta,\ \xi_c = 1 + a_c / \eta, \tag{7}$$

where $a_\gamma = 1.26\tan\varphi - 0.519$, $a_q = 0.689\tan\varphi + 0.056$, and $a_c = 0.559\tan\varphi + 0.163$.

The "+" sign for ξ_γ in Eq.(7) coincides with that used in Eq.(6). The experimental justification of this result given in [15] is rather convincing, even though it runs counter to the opinion of several researchers.

3. N_γ, N_q, and N_c given in [2] are calculated depending on the inclination angle of the reduced ultimate pressure δ to the vertical, which is determined from (3).

The ultimate shear stress at the interface with the soil can be expressed as the inclination angle of actual δ_{act} rather than reduced pressure to the vertical:

$$\tau_u = p_u \tan\delta_{\text{act}}. \tag{8}$$

By comparing Eq.(8) with Eq.(3), we obtain:

$$\tan\delta = \frac{p_u}{p_u + c\cot\varphi}\tan\delta_{\text{act}}.$$

4. Given the analytical solution of Eq.(2), we can determine N_γ and N_q for the limiting case at $\delta = \varphi$, which was not the case in [2]:

$$N_\gamma = \frac{1}{4}\sin 2\varphi,\ N_q = 2\cos^2 \mu e^{2\mu\tan\varphi}.$$

Thus, when estimating the bearing capacity according to the normal component of the ultimate pressure, it is recommended to assume $\delta \leqslant \delta_{cr}$ instead of $\tan\delta \leqslant \sin\varphi$, where

$$\delta_{cr} = \arctan\left[\frac{\gamma b\sin 2\varphi + 8(q + c\operatorname{ctg}\varphi)e^{2\mu\tan\varphi}}{\gamma b\sin 2\varphi + 8(q + c\cot\varphi)e^{2\mu\tan\varphi} - 4c\cot\varphi}\tan\varphi\right].$$

At $\delta > \delta_{cr}$, the bearing soil strength is verified for in-plane shear at the interface with the soil (according to Coulomb's law).

5. It is shown in [10] that q'_{cr} (see Fig. 1c) can be assumed to be equal to unity for practical purposes. At , a correction factor k_q can be introduced for the first summand in Eq. (1):

$$k_q = \frac{N_{\gamma 0}}{N_\gamma} + \frac{q'}{q'_{cr}}\left(1 - \frac{N_{\gamma 0}}{N_\gamma}\right)\left(2 - \frac{q'}{q'_{cr}}\right).$$

Another solution was proposed by V. Fedorovskii in [16].

Unstabilized state. For this case, the strength condition of slowly compacted saturated clay soils is adopted in [2], which is written as combined (total) stresses:

$$\tau_n = (\sigma_n - u)\tan\varphi + c,$$

where u is excess pore pressure, φ and c are strength characteristics of saturated soil determined in consolidated tests.

In [2], the bearing capacity of such foundation beds is defined by the Prandtl-Reissner solution for a perfectly cohesive loaded medium. Essentially, this means accepting the hypothesis that, when loaded to the limit state, saturated soil has no internal friction, while its strength is consistently characterized by a single parameter, i.e., specific cohesion determined in unconsolidated-undrained tests.

It seems reasonable to adopt the concepts of initial and final bearing capacity that are more adequate to the actual soil performance [10].

Analysis of FEM Application

The FEM used to determine the ultimate pressure revealed that the settlement-load dependence $s(p)$ extends far beyond the limits corresponding to the exact LET solution and that equilibrium conditions are significantly violated in several areas of the foundation bed [17].

In 2010–2014, a debate questioning this point of view was initiated [18]. The main objection consisted in the revised graph $s(p)$, according to which the ultimate load value corresponded to the soil LET.

In 2019, V. Fedorovskii [19] conclusively demonstrated that, for a plane problem under an associated plastic flow rule, PLAXIS yields estimates of the ultimate load acting on the plate that are very similar to those of the LET. However, some aspects require further discussion.

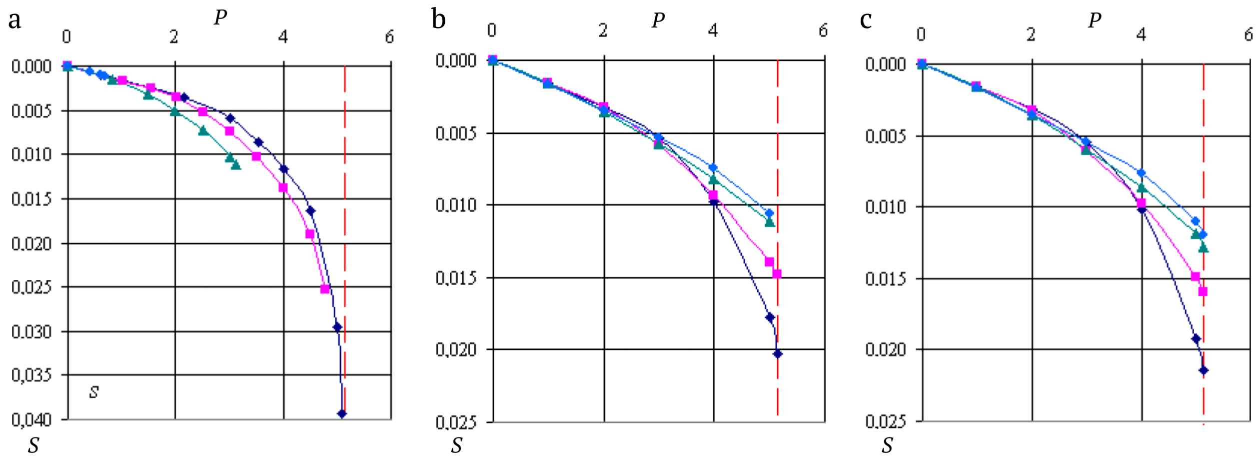

Fig. 2. Settlement-load graphs for the loading plate problem: a) using PLAXIS ($\psi = 0$), b) using PLAXIS ($\psi = \varphi$), and c) using MIDAS ($\psi = \varphi$), ♦) $\varphi = 10°$, ■) $\varphi = 20°$, ▲) $\varphi = 30°$, ●) $\varphi = 40°$.

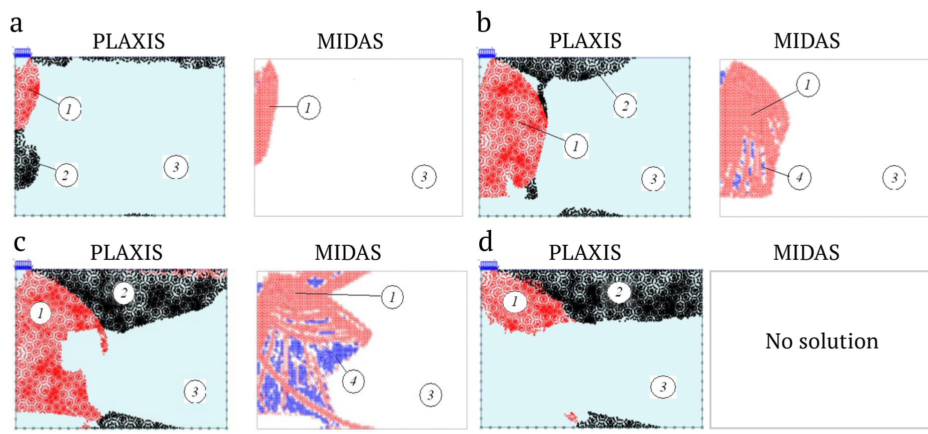

Fig. 3. Development of plastic zones beneath the plate ($\varphi = 30°$) under increasing load: a) $p = 2$, b) $p = 4$, and c) $p = 5.14$; 1) plastic strains, 2) tension, 3) elastic zone, 4) unloading area.

Let us consider the problem of a loading plate on a perfectly elastoplastic weightless soil bed according to the Mohr-Coulomb failure criterion. We will study this problem with the help of PLAXIS and MIDAS software packages. The calculations are performed in relative variables: unit of length b and unit of stress γ_b. Soil characteristics are as follows: $E = 1000$, $v = 0.3$; $\varphi = 0°$, $c = 1$; $\varphi = 10°$, c = 0.616; $\varphi = 20°$, c = 0.347; $\varphi = 30°$, c = 0.171; and $\varphi = 40°$, c = 0.0683. For the sake of a clear comparison, such a value of c is adopted that the ultimate load amounts to 5.14 for each φ at $q = 0$. The dilatancy angle ψ is assumed to be equal to zero or φ.

As follows from the $s(p)$ graphs obtained via the FEM using PLAXIS at $\psi = 0$ (Fig. 2a), PLAXIS at $\psi = \varphi$ (Fig. 2b), and MIDAS at $\psi = \varphi$ (Fig. 2c), the curves exhibit significant differences.

At small φ, the graph has a section that is fairly consistent with the experimental pattern indicating a settlement increase at small load increments in the final stage of the plate load test. However, the larger φ is, the less distinct this section becomes; at $\varphi = 40°$, it disappears altogether.

Development of plastic zones. Figure 3 shows plastic zones obtained using the PLAXIS and MIDAS software packages at relative loads p of 1–5.14. All calculations were performed for the associated plastic flow rule.

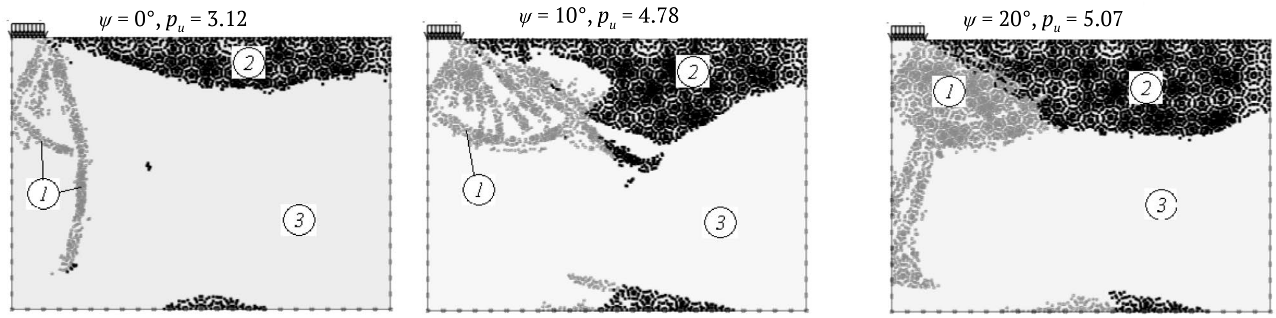

Fig. 4. Plastic and tensile zones at $p = p_u$ and $\varphi = 30°$ for different dilatancy angles ψ using PLAXIS.

Two points are of particular note here:

1) after their initial appearance, failure areas immediately move downward to a considerable depth, which requires experimental verification,

2) when strength loss occurs, the failure areas "spontaneously" acquire a form that corresponds approximately both to the actual pattern of foundation bed failure and to the outlines of limit equilibrium areas from LET solutions.

The PLAXIS and MIDAS results raise several issues. If the plastic zones "approximately" correspond to the experimental data and the LET solutions, the question arises as to why the ultimate load is so accurate. Moreover, it is not clear how to interpret the "spontaneous" partial unloading of the foundation bed at a depth where plastic strains are observed prior to reaching the ultimate load.

This unloading could be attributed to the fact that lateral heave becomes possible when the ultimate pressure is reached.

The downward development of plastic zones can be explained relying on the results obtained for the non-associated rule. Figure 4 presents these zones calculated in PLAXIS at $\varphi = 30°$, as well as dilatancy angles of $\psi = 0°$, $\psi = 10°$, and $\psi = 20°$.

Although the LET provides a solution extending the construction of the ultimate stress field deeper into the foundation bed, it does not affect the ultimate pressure value; thus, due to the obvious lack of practical relevance, it has not become widespread. The elastoplastic FEM solution may be seen as "attempting to reproduce" exactly this result.

The problem of equilibrium. When using the simplest finite elements in the plate problem, the graph $s(p)$ is known to exhibit values far exceeding the theoretical value of p_u. Attempts to explain this result led to the idea of a direct equilibrium analysis of the stress field obtained in elastoplastic FEM solutions. However, the graph $s(p)$ has recently acquired its target form in finite element solutions, including in PLAXIS and MIDAS [18, 19]. This achievement is attributed to the transition to higher-order elements [18]. However, the above-mentioned issues prompted the authors to revisit the problem of equilibrium.

Figure 5 shows two tested zones beneath the loading plate and residual graphs (in %) when analyzing the equilibrium of force projections on the horizontal and vertical axes, as well as moments relative to the center of the selected rectangles.

The obtained results raise two relevant issues.

1. It should be considered how significant a 5% residual value can be if the equilibrium of nodal forces is reduced to fractions of a percent in the program settings.

2. If a sufficiently large zone is detected (No. 15 in Fig. 5b) in which the equilibrium of force projections on the vertical axis is violated by 16% in MIDAS and by 18% in PLAXIS, the question arises as to how well such a solution meets the problem statement, i.e., whether such a solution is correct.

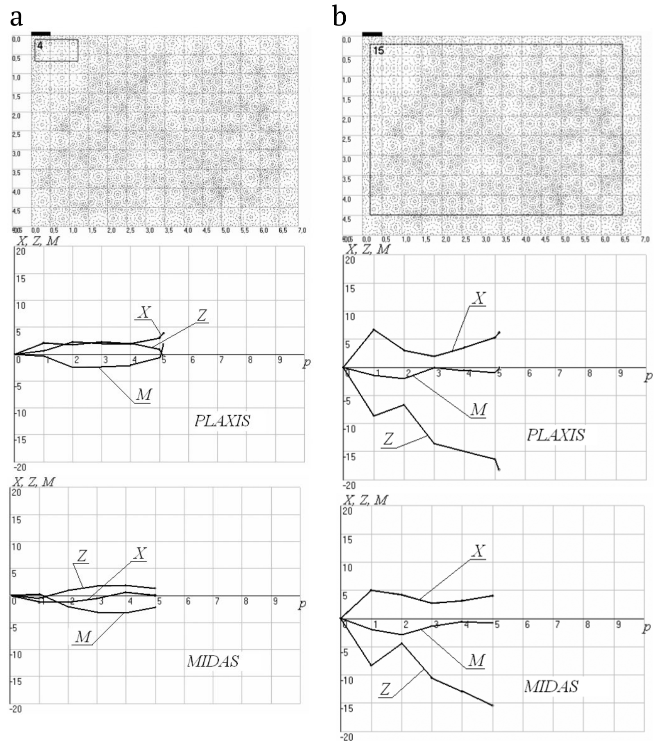

Fig. 5. Equilibrium analysis of zones beneath the loading plate drawing on PLAXIS and MIDAS calculations.

Thus, when using the FEM in practical stability calculations, it is necessary:

– to compare, whenever possible, FEM results with those of classical LET solutions or verified grapho-analytical solutions,

– to analyze character and dynamics in the development of plastic zones,

– to analyze the equilibrium of individual zones and the entire computational domain as per [2].

Application of Linear and Nonlinear Programming Methods in Limit Analysis

The violation of equilibrium in FEM solutions can be attributed to the FEM equations guaranteeing the equilibrium of nodal forces, while the stress field is determined through recalculation depending on the element stiffness, i.e., in a sense on an arbitrary value.

In limit analysis, this problem is solved by means of linear and nonlinear programming methods [20, 21]. By limit analysis, the authors of the method understand the determination of the lower (static solution) and upper (kinematic solution) bounds of the ultimate load drawing on the basic theorems of plasticity theory.

In [20, 21], the authors explicitly formulate requirements for determining the lower bound of ultimate pressure: intra-element equilibrium; equilibrium at the boundaries between elements (not only at nodes as in the FEM); equilibrium at the boundaries of the computational domain (boundary conditions); values of the yield function that are below or equal to zero.

This method is implemented in the Optum G2 software demonstrating a higher reliability in the determination of ultimate loads as compared to the FEM. However, this method also yields controversial results for a number of problems (e.g., the rock pressure problem). When calculating the bearing capacity of a single loading plate, the considered method produces consistently good results.

Conclusions

1. Regulatory documents [2] are generally consistent with the current level of LET development (in terms of bearing capacity calculations for foundation beds). However, it would be reasonable to extend the regulatory procedure with solutions for the stabilized and non-stabilized states of foundation beds.
2. The FEM used to calculate the bearing capacity of foundation beds requires increased attention, especially with regard to the character and development of plastic zones, as well as the fulfillment of equilibrium conditions.
3. Application of linear and nonlinear programming methods to the limit equilibrium problems of soil masses constitutes the most promising area of development in the theory of soil stability. However, the application of these methods to stability problems remains to be verified, while the classical LET solutions are still regarded as standard.

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