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PREDICTING THE SETTLEMENT AND LONG-TERM BEARING CAPACITY OF A BASE OF FOUNDATION OF FINITE WIDTH

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The article shows that if the model proposed by A. Z. Ter-Martirosyan is used to describe shear deformations in a soil medium, and the Kelvin-Voigt model is used to describe volume deformations, the calculated settlement-time dependence is nonlinear and has a double curvature. The article presents an analytical solution to the problems of predicting settlement and long-term bearing capacity of a base of foundation of finite width using the above models.

Introduction

In [1], the authors considered the problem of the stress-strain state (SSS) of the base of foundations of finite width, taking into account the elastoplastic properties of soils during shear and nonlinearity during volume deformation. As a result, a solution that allows constructing a family of settlement–load ($S-p$) curves with double curvature at different acting loads was obtained.

In this study, in order to take into account the propagation in time of deformations of soils with the creep property, the rheological model proposed by A. Z. Ter-Martirosyan [2] is used as a calculation model to describe shear, and the elastic-viscous Kelvin-Voigt model is used to describe volume deformations [3].

Determining the Creep of Base Soils During Shear

In the simplest case (τ = const), the rheological model proposed by A. Z. Ter-Martirosyan [2] is described by an equation of the form

$$\dot{\gamma} = \frac{\tau - \tau^*}{\eta_\gamma(\sigma_m)}\left(\frac{e^{-\alpha\gamma}}{a} + \frac{e^{\beta\gamma}}{b}\right), \qquad (1)$$

where τ and τ^* are the effective and limiting shear stresses; $\dot{\gamma}(\gamma)$ is the angular deformation rate, depending on the value of the accumulated deformation γ, $\eta_\gamma(\sigma_m)$ is the initial shear viscosity of the soil, which in the general case depends on the mean stress σ_m; α, β, a, and b are the hardening (softening) parameters of clay soil, which are determined by the results of the kinematic shift ($\dot{\gamma}$ = const). The expression in brackets is the hardening (softening) function, where γ is essentially a measure of hardening according to the terminology of creep by Yu. N. Rabotnov [4, 5].

According to Eq. (1), the shear deformation rate depends nonlinearly on the accumulated shear deformation. Eq. (1) allows describing the $\gamma - t$ dependence at different τ as a curve with double cur-

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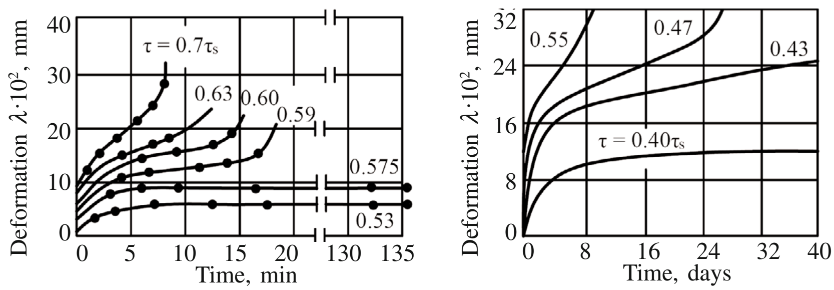

Fig. 1. Creep curves of plastic clays during shear according to S. S. Vyalov [6].

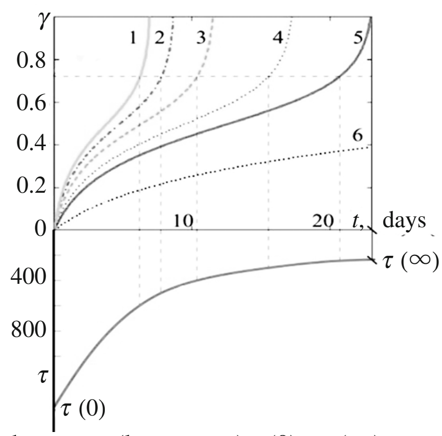

Fig. 2. Long-term strength curves (lower part) $\tau(0) \to \tau(\infty)$ at $t \to \infty$, constructed according to Eq. (1), and $\gamma - t$ creep curves: 1) τ_1, 2) τ_2, 3) τ_3, 4) τ_4, 5) τ_5, 6) τ_6.

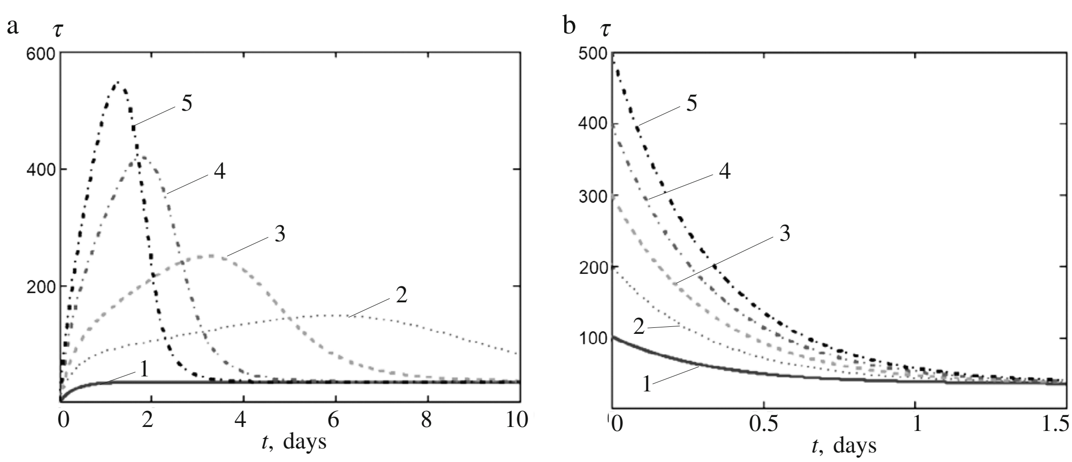

Fig. 3. Curves: a) $\tau - t$ according to test results in kinematic mode at different $\dot{\gamma}$ = const, b) relaxation of the shear stress $\tau(t)$ at different initial shear stresses $\tau(0)$ and at $\gamma(t)$ = const, 1) $\dot{\gamma}_1$, 2) $\dot{\gamma}_2$, 3) $\dot{\gamma}_3$, 4) $\dot{\gamma}_4$, 5) $\dot{\gamma}_5$.

vature, which is typical for many types of soils with rheological properties (Fig. 1). This dependence includes three stages: the initial, nonlinear intermediate with a steady shear rate and the final, developing with an increasing rate and transferring to the stage of progressive destruction. Moreover, based on Eq. (1), it is possible to construct $\tau - t$ curves during kinematic shear ($\dot{\gamma}$ = const), as well as relaxation curves $\tau(0) \to \tau(t)$ at $\gamma(t)$ = const (Figs. 2 and 3). Curves of a similar form were obtained based on the laboratory tests results of various authors [6, 2].

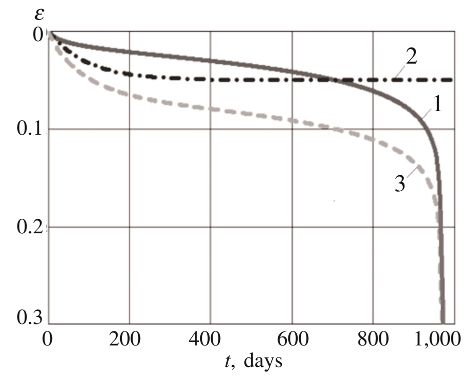

Fig. 4. Deformations of the sample: 1) shear (according to the model by A. Z. Ter-Martirosyan), 2) volume (according to the Kelvin-Voigt model), 3) $\varepsilon_z - \mathrm{t}$ total deformations (according to Eq. (5)).

It is important that for the first time all these curves are described based on a unified formula of Eq. (1) with the same parameters α, β, a, and b (for example, A. R. Rzhanitsyn [7] proposed to describe separately the initial, intermediate and continuous stages of creep curves with double curvature).

Determining the Volume Deformation of Soils

As a calculation model for describing the volume deformation of the base soils, the Kelvin-Voigt rheological equation [3] is taken in the form

$$\sigma_m = \sigma_m^e + \sigma_m^v = K(\sigma_m)\varepsilon_m + \eta_v \dot{\varepsilon}_m, \tag{2}$$

where η_v is the volume viscosity, σ_m^e and σ_m^v are the elastic and viscoelastic stresses, respectively.

At $\varepsilon_m(t \approx 0) = 0$, the following equation is obtained:

$$\varepsilon_m(t) = \frac{\sigma_m}{K_{(\sigma_m)}}(1 - e^{-Kt/\eta v}). \tag{3}$$

In this case, the volume deformation is attenuated and its rate decreases:

$$\dot{\varepsilon}_m(t) = \frac{\sigma_m}{K_{(\sigma_m)}}\left(\frac{K}{\eta_v} e^{-Kt/\eta v}\right). \tag{4}$$

As a calculation model for describing the relationship between stresses and deformations and their rates, it is advisable to use the Hencky's system of equations [8], which represents any linear deformation as a sum of shear and volume components.

Determining Total Deformations

In the case under consideration, the deformation rate $\dot{\varepsilon}_z$ in the soil layer under the impact of σ_z and σ_m and with the possibility of the deformations development $\varepsilon_x \neq 0$ and $\varepsilon_y \neq 0$ during elastic-viscous volume deformation can be represented as

$$\dot{\varepsilon}_z = \frac{\sigma_z - \sigma_m}{\eta_Y(\sigma_m)}\left(\frac{e^{-\alpha\varepsilon_z}}{a} + \frac{e^{\beta\varepsilon_z}}{b}\right) + \frac{\sigma_m}{K_{(\sigma_m)}}\left(-\frac{K}{\eta_v} e^{-K/\eta v}\right). \tag{5}$$

The first term of this equation is transcendental, and for its integration let us use the MathCad software package. The integral of the second term is determined by Eq. (3). The results of constructing the graphs of the dependence of shear and volume deformation components on time according to Eqs. (1) and (3), as well as the total graph for sample of Eq. (5), are shown in Fig. 4.

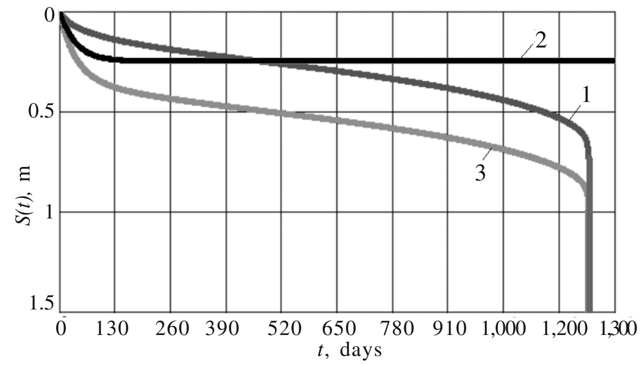

Fig. 5. Settlements of the base: 1) shear component, 2) volume component, 3) total settlements $S'(t)$, determined by Eq. (5).

The graphs were constructed using the MathCad software package with the following parameters: η_γ = 1.157·10^5 kPa·day, σ_z = 580 kPa, σ_m = 400 kPa, a = 1.2, b = 60, α = 171, β = 40, K = 8,000 kPa, and K/η_v = 0.0106 day^{-1}.

Predicting the Settlement and Long-Term Bearing Capacity of the Base

It is known that the stress state of elastic and elastic-viscous bases coincide [2, 9]. To describe the stress state of a linearly deformable soil base under the action of a load p = const distributed over a band b = 2**a** (plane problem), the Flamant formulas are used [10]:

$$\sigma_x = \frac{p}{\pi}\left[\arctan\frac{a-x}{z} + \arctan\frac{a+x}{z}\right] + \frac{2apz(x^2-z^2-a^2)}{\pi\left[(x^2+z^2-a^2)^2+4a^2z^2\right]}, \tag{6}$$

$$\sigma_z = \frac{p}{\pi}\left[\arctan\frac{a-x}{z} + \arctan\frac{a+x}{z}\right] - \frac{2apz(x^2-z^2-a^2)}{\pi\left[(x^2+z^2-a^2)^2+4a^2z^2\right]}, \tag{7}$$

$$\sigma_m = \frac{2p(1+v)}{3\pi}\left[\arctan\frac{a-x}{z} + \arctan\frac{a+x}{z}\right], \tag{8}$$

$$\sigma_z - \sigma_m = \frac{2p}{\pi}\left(\frac{az}{a^2+z^2} + \frac{1-2v}{3}\arctan\frac{z}{a}\right). \tag{9}$$

Based on these formulas, using Hencky's equations and Eq. (5), it is possible (similarly to [8]) to determine the total settlement of a layer of finite thickness as the sum of the volume and shear components:

$$\sum S'(t) = \sum S_\gamma(t) + \sum S_v(t). \tag{10}$$

The results of the calculations performed to determine the total settlement (layer with thickness h = 10 m and width b = 40 m, distributed load p = 400 kPa, soil parameters are the same as those given above for the sample) are shown in Fig. 5.

As expected, the use of the Kelvin-Voigt model [3] to describe the volume deformations of the base soils and the use of the model proposed by A. Z. Ter-Martirosyan [2] to describe the shear deformations, leads to a "total settlement–time" curve, which has a double curvature and at a certain point in time transfers to the stage of progressive settlement.

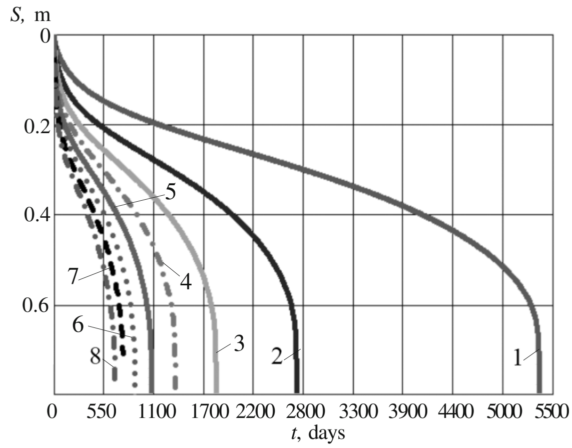

Fig. 6. Total settlement of the base $S-t$ at various loads from the foundation: 1) p_1 = 100 kPa, 2) p_2 = 200 kPa, 3) p_3 = 300 kPa, 4) p_4 = 400 kPa, 5) p_5 = 500 kPa, 6) p_6 = 600 kPa, 7) p_7 = 700 kPa, 8) p_8 = 800 kPa.

Analysing the graphs of settlement $S(t)$ at different values of the distributed load from the foundation p (Fig. 6), it is possible to construct the graph of the long-term strength of the base under consideration.

Conclusions

1. Under the impact of a distributed load on a foundation of finite width, a complex and inhomogeneous SSS arises in the elastic-viscous base. In this case, the stress distribution can be taken according to Flamant [9].

2. For soils with rheological properties in case of changes in volume and form (shear), when determining the components of deformations and their rate, it is convenient to use the system of Hencky's physical equations, which allows determining the linear deformation in any direction as a sum of the volume and shear components of this deformation.

3. The use in the system of Hencky's physical equations of the calculation formulas formulated based on the Kelvin-Voigt rheological model for describing the volume deformation and on the model by A. Z. Ter-Martirosyan for shear deformations leads to the "total deformations - time" curve with double curvature.

4. Summing the settlements of the base layers determined based on these deformations and the stresses distribution according to Flamant leads to the base settlement - time curve with a double curvature as well.

5. Analysing the graphs of settlement at various loads, it is possible to construct a graph of the long-term bearing capacity of the base.

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