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BEARING CAPACITY OF SOIL BASE AND SETTLEMENT OF FOUNDATIONS OF FINITE WIDTH

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The analytical solution of the problem of the bearing capacity of the soil base and the foundation settlement of a limited width is presented, considering the soil's own weight and residual stresses (in the presence of over-compacted soils). The Flamant's solutions (plane problem) are used for the stress state calculation. The Hencky's equations are used to determine the stress – strain relationships and the influence of the average stress on the shear and the volume strain modulus.

Introduction

Considering the approaches described in [1-3], if the shear deformations in the soil medium are described by S. P. Timoshenko's elastic-plastic model [4], and the volumetric deformations are described by S. S. Grigoryan's nonlinear model, then the finally calculated settlement – load ($S - p$) relationship could be plotted as a non-linear curve having a double curvature.

Within the framework of the current regulatory documents [5], the stress-strain curves ($\varepsilon - \sigma$), based on the results of soil tests, allow us to determine the soil's deformation modules E_{0_i} depending on stresses σ_{zi}, changing with depth along the foundation vertical axis z. These modules, in accordance with the Hooke equation, determine the deformation of the soil layers $\varepsilon_{zi} = 0{,}8(\sigma_{zi}/E_{0i})$ (i is the number of the current layer from 1 to N), and the foundation settlement under loads less than the normative soil resistance R is defined as the sum of the layer's settlements along the central vertical axis (under the compression conditions $\varepsilon_x = 0$, $\varepsilon_y = 0$). Then $S = \sum_{i=1}^{n} \varepsilon_{zi} \Delta h_i$.

Thus, assuming that the stress state of the base is determined by Flaman [6], it is possible to indirectly take into account the nonlinearity of deformation of the foundation soils.

In this paper, the system of Hencky's equations [7] is used, which describes ε_z as the sum of shear and volumetric deformations ($\varepsilon_z = \varepsilon_{z\gamma} + \varepsilon_{zv}$):

$$\varepsilon_z = \frac{\sigma_z - \sigma_m}{G(\sigma_m, \tau_i / \tau_i^*)} + \frac{\sigma_m}{K(\sigma_m)}, \tag{1}$$

where $G(\sigma_m, \tau_i/\tau_i^*)$ and $K(\sigma_m)$ are the shear and volumetric strain modulus, depending on the mean stress σ_m and the ratio of the intensity of tangential stresses τ_i to its limit value τ_i^*, where $\tau_i^* = \sigma \mathrm{tg}\varphi + c$ (φ is the angle of internal friction and c is cohesion).

In the special case, when $G = \text{const}$ and $K = \text{const}$, Eq. (1) transforms into the system of Hooke equations. The parameters $G(\sigma_m, \tau_i/\tau_i^*)$ and $K(\sigma_m)$ are determined by the results of standard triaxial tests (Fig. 1).

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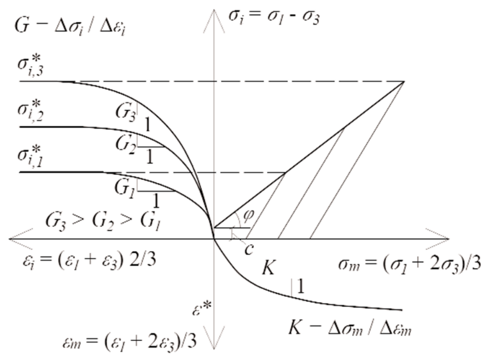

Fig. 1. Standard triaxial tests results of the soils under the kinematic loading mode ($\dot{\varepsilon}_1$ = const or $\dot{\sigma}_1$ = const).

Equation (1) allows one to consider the naturally different forms of the shear and volumetric deformation curves (not only the always attenuating $\varepsilon_m - \sigma_m$ but also the non-attenuating progressive $\varepsilon_i - \sigma_i$); see Fig. 1, upper left and lower right segments.

Moreover, the Hencky's equations make it possible to predict the settlement for both $p < R$ and $p > R$.

Foundation Settlement and the Soil Base Bearing Capacity Prediction Based on the Hencky's Equation

Let us consider a computational model of the soil base as a linearly deformable half-space under the action of a distributed load (p = const) over a strip of the limited width $b = 2a$ (flat problem). According to the Flamant's formulas [6]:

$$\sigma_x = \frac{p}{\pi}\left[\arctan\frac{a-x}{z} + \arctan\frac{a+x}{z}\right] + \frac{2apz(x^2 - z^2 - a^2)}{\pi\left[(x^2+z^2-a^2)^2 + 4a^2z^2\right]};$$
$$\sigma_z = \frac{p}{\pi}\left[\arctan\frac{a-x}{z} + \arctan\frac{a+x}{z}\right] - \frac{2apz(x^2 - z^2 - a^2)}{\pi\left[(x^2+z^2-a^2)^2 + 4a^2z^2\right]}; \tag{2}$$

$$\sigma_m = \frac{2p(1+\nu)}{3\pi}\left[\arctan\frac{a-x}{z} + \arctan\frac{a+x}{z}\right]; \tag{3}$$

$$\sigma_z - \sigma_m = \frac{2p}{\pi}\left(\frac{az}{a^2+z^2} + \frac{1-2\nu}{3}\arctan\frac{z}{a}\right). \tag{4}$$

The isolines of σ_z, σ_m, and $\sigma_z - \sigma_m$ for the strip load (a = 20 m and p = 400 kPa) according to (1)-(4) are shown in Figs. 2a, 2b, and 2c.

V. A. Florin [8] compiled extensive tables to determine $\sigma_{x/p}$, $\sigma_{z/p}$, and $\sigma_{m(z/p)}$, for the Hencky's equations:

$$\varepsilon_x = \chi(\sigma_x - \sigma_m) + \chi^*\sigma_m;\quad \gamma_{xy} = 2\chi\tau_{xy},$$
$$\varepsilon_y = \chi(\sigma_y - \sigma_m) + \chi^*\sigma_m;\quad \gamma_{yz} = 2\chi\tau_{yz}, \tag{5}$$
$$\varepsilon_z = \chi(\sigma_z - \sigma_m) + \chi^*\sigma_m;\quad \gamma_{zx} = 2\chi\tau_{zx},$$

where
$$\chi = \frac{\gamma_i}{2\tau_i} = \frac{f(\tau_i, \sigma_m, \mu_\sigma)}{2\tau_i}, \chi^* = \frac{\varepsilon_m}{\sigma_m} = \frac{f^*(\tau_i, \sigma_m, \mu_\sigma)}{\sigma_m}, \tag{6}$$

τ_i is the intensity of tangential stresses, and μ_σ is the Nadai-Lode parameter [6].

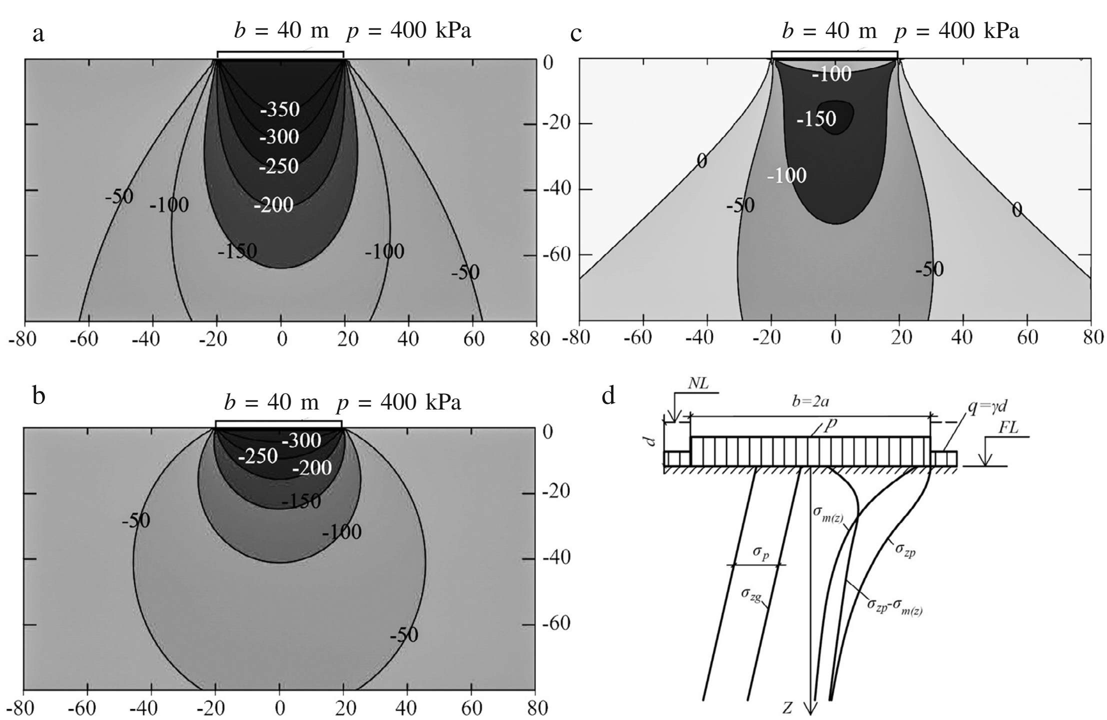

Fig. 2. The isolines of σ_z (a), σ_m (b), and $\sigma_z - \sigma_m$ (c) and the scheme for calculation of the shear and volumetric deformations (Hencky's model) (d)

The Hencky's equations for χ = 1/2G and χ^* = 1/K,, where $G = E/2(1 + v)$ and $K = E/2(1 + v)$ are transformed into the system of Hooke equations.

We take the dependence proposed by S. S. Grigoryan as a computational model for nonlinear volume deformations:

$$\varepsilon_m(\sigma_m) = \varepsilon^*(1 - e^{-\alpha\sigma_m}). \tag{7}$$

The secant modulus of the volumetric strain is determined as

$$K = \frac{\sigma_m}{\varepsilon_m} = \frac{\sigma_m}{\varepsilon^*(1 - e^{-\alpha\sigma_m})} \tag{8}$$

when $\sigma_m \to \infty$ and $\varepsilon_m \to \varepsilon^*$.

To describe the elastic-plastic deformation of clay soil during shear, the formula of S. P. Timoshenko [4], adopted to the soil media, is used [2]:

$$\gamma_i = \frac{\tau_i}{G^e} \frac{\tau_i^*}{\tau_i^* - \tau_i}, \tag{9}$$

$$\tau_i^* = (\sigma_m + \sigma_g)\mathrm{tg}\varphi_i + c_i, \tag{10}$$

where φ_i and c_i are the limit strength parameters corresponding to the limit straight line at the plane $\tau_i - \sigma_m$ (Fig. 2); σ_g is the natural gravity stress,

for the normally compacted soils

$$\sigma_g = \gamma h; \tag{11}$$

for the overcompacted soils

$$\sigma'_g = \gamma h + \sigma_p, \tag{12}$$

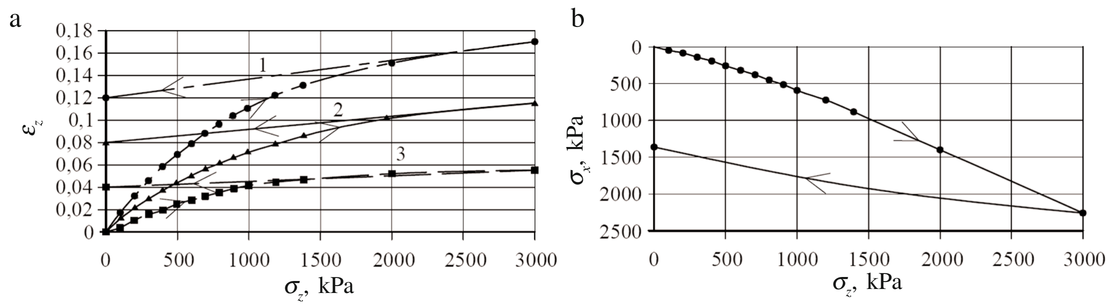

Fig. 3. The vertical pressure σ_z, versus: a) total ε_z (1); volumetric $\varepsilon_{z,v}$ (2), and shear $\varepsilon_{z,\gamma}$ (3) components of the total strain, b) lateral pressure σ_x.

where γ is the specific gravity of the soil; h is the depth, where τ_i^* is measured; σ_p is the residual stress in over-compacted soils, determined by the compression tests (Cazagrande method [2, 9]).

For the secant shear modulus $G = \tau_i/\gamma_i$ from Eq. (9) we get

$$G = G^e\left(1 - \frac{\tau_i}{\tau_i^*}\right), \tag{13}$$

where $G = G^e$ when $\tau_i = 0$ and $G \to 0$ when $\tau_i \to \tau_i^*$.

The calculation scheme for shear and volumetric deformations using. (1), (8), and (13) is shown in Fig. 2d, where *NL* and *FL* are the marks of the natural terrain and the bottom of the foundation.

Forecast of a Linearly Deformable Base Settlement

In the simplest case, when there is a linear relationship between stresses and deformations with G and K parameters, the settlement can be determined by an analytical solution for the z axis ($x = 0$):

$$S = \int_0^{h_a} \frac{\sigma_m}{K} dz + \int_0^{h_a} \frac{\sigma_z - \sigma_m}{2G} dz, \tag{14}$$

where h_a is the thickness of the compressible layer; σ_z and σ_m are determined by Eqs. (2) and (3), respectively.

The stresses σ_m and $\sigma_z - \sigma_m$ change with depth along the z axis ($x = 0$) in accordance with Eqs. (3) and (4).

Substituting Eq. (3) in the first integral of Eq. (14), we obtain the part of the foundation settlement corresponding to the volumetric component of the linear deformation $\varepsilon_{z,v}$

$$S_v = \frac{4p(1+v)}{3\pi K}\left[h_a \arctan\frac{h_a}{a} + \frac{a}{2}\ln\frac{a^2 + h_a^2}{a^2}\right]. \tag{15}$$

Substituting Eq. (4) into the second integral of Eq. (14), we obtain the foundation settlement caused by the shear component of the linear deformations $\varepsilon_{z,\gamma}$:

$$S_\gamma = \frac{p}{3\pi G}\left[(1-2v)h_a \arctan\frac{h_a}{a} + (2-v)a\ln\frac{a^2 + h_a^2}{a^2}\right]. \tag{16}$$

It can be seen from Eqs. (15) and (16) that S_γ and S_v depend nonlinearly on the geometric parameters (a and h_a).

The results for the example with a = 40 m, h_a = 80 m, v = 0,33, K = 40000 kPa, p = 400 kPa, G^e = 5113 kPa showed that S_v = 38.8 cm and S_γ = 96.7 cm, i.e., the settlement caused by the shear component S_γ is 2.49 times greater than S_v!

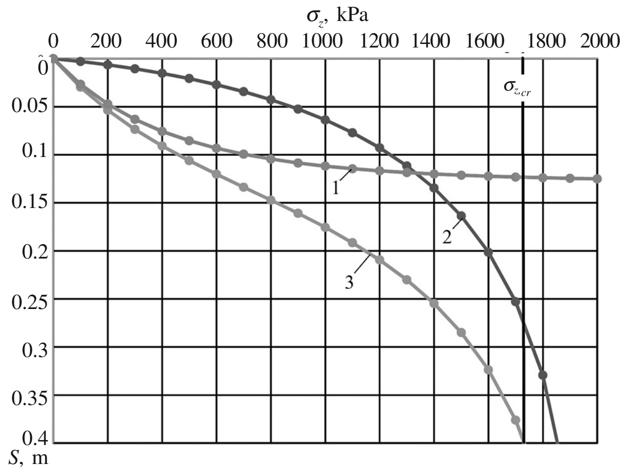

Fig. 4. The soil layer (h = 8 m) settlement calculated by Eqs. (17) and (18): 1) $\varepsilon_{z,v}$; 2) $\varepsilon_{z,\gamma}$; 3) $\varepsilon_z = \varepsilon_{z,v} + \varepsilon_{z,\gamma}$.

Nonlinear Deformation of a Soil Layer of Limited Thickness

Deformations and settlements of a soil layer under the compression conditions ($\varepsilon_x = \varepsilon_y = 0$) are considered in [10] (Fig. 3). The total deformations in Fig. 3a (ε_z) are calculated by (14). The residual ε_z and σ_x are clearly obtained during the elastic unloading. In addition, the $\varepsilon_z - \sigma$ curve has a decaying character with increasing σ.

It can be shown that if horizontal displacements of the layer have no special limitations ($\varepsilon_x \neq 0$), the $\varepsilon_z - \sigma$ curve could not only be decaying but also have a non-attenuating, progressive character, depending on the value of σ_{zp} and the ratio τ_i/τ_i^* (Fig. 4).

Settlement of a Nonlinearly Deformable Soil Layer Allowing Free Horizontal Deformations

Substituting $G(\sigma_m, \tau_i)$ and $K(\sigma_m)$ in Eq. (1), we get

$$\varepsilon_{z,v} = \varepsilon^*(1 - e^{-\alpha\sigma_m}), \tag{17}$$

$$\varepsilon_{z,\gamma} = \frac{\sigma_z - \sigma_m}{2G^e(1 - \tau_i/\tau_i^*)}, \tag{18}$$

where $\tau_i = (\sigma_1 - \sigma_3)/\sqrt{3} = (\sigma_z - \sigma_x)/\sqrt{3}$ $(x = 0)$; $\tau_i = (\sigma_m + \sigma_g)\mathrm{tg}\varphi\ + c$; σ_{xp} and σ_{zp} are determined from Eq. (2).

Similarly, we can determine the components of $\varepsilon_{x,v}$ and $\varepsilon_{x,\gamma}$.

From the analysis of Eqs. (17) and (18), it follows that the volumetric component of the strains $\varepsilon_{z,v}$ with the growth of σ_z will have a decaying character at $\sigma_m \to \infty$; $\varepsilon_{zv} \to \varepsilon^*$. At the same time, with the growth of σ_z, the value of $\varepsilon_{z,\gamma}$ will initially grow linearly and then pass into the stage of progressive growth, since $\varepsilon_{z\gamma} \to \infty$ at $\tau_i \to \tau_i^*$. Consequently, the graph of the total strain will have a double curvature, i.e., $\varepsilon_z(\sigma_z)$ has a decaying character in the initial section when $\tau_i < \tau_i^*$, and then at $\tau_i \to \tau_i^*$ it passes into the stage of progressive deformation (see Fig. 4).

The solution with $\varepsilon^* = 0.016$, $\alpha = 0.005$, $G^e = 50{,}000$ kPa, $v = 0.3$, $\varphi = 25^{\circ}$, $c = 10$ kPa, and $h = 8$ m has confirmed this analysis.

Based on Eqs. (17) and (18), it is possible to draw up the isolines of ε_{zv} = const and $\varepsilon_{z\gamma}$ = const, as well as $\varepsilon_z = \varepsilon_{zv}$ and $\varepsilon_{z\gamma}$ = const for the given parameters p, a, G, K, φ, and c, and the isolines of the stress components. As expected, a nonuniform stress state occurs in the soil base (i.e., induced anisotropy) [6].

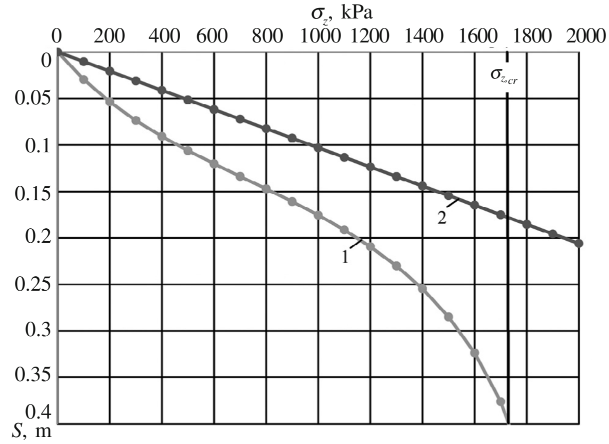

Fig. 5. The settlement of the foundation of limited width (nonlinearly deformable soil layer), calculated according to Eqs. (17) and (18), including (1) and neglecting (2) horizontal displacements of the layers.

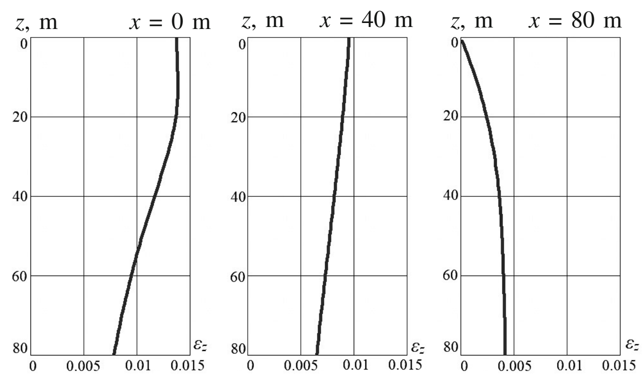

Fig. 6. The axial deformations ε_z at various verticals $x > 0$.

A comparison of the total settlement of a layer of limited thickness obtained by Eqs. (17) and (18), when the horizontal displacements are limited (or not limited), demonstrates the significant difference in the results (Fig. 5).

Deformations at the Vertical Surfaces and Calculation of the Settlement

The total deformations of the base at different verticals $x \geq 0$ are as follows:

$$S = \int_0^{h_a} \varepsilon_{zvi} dz + \int_0^{h_a} \varepsilon_{z\gamma i} dz = S_v + S_\gamma, \tag{19}$$

where h_a is the distance between $z = 0$ and the lower boundary of the compressible thickness.

The total deformations $\varepsilon_z = \varepsilon_{z,v} + \varepsilon_{z\gamma}$ when $h_a = 80$ m at different verticals $x \geq 0$, with the parameters $\varepsilon^* = 0.016$, $\alpha = 0.005$, $v = 0.3$, $G^e = 50{,}000$ kPa, $\varphi = 25^\circ$, and $c = 10$ kPa are shown in Fig. 6.

Additionally, to Eq. (18), $\tau_i = f(\sigma_{xp}, \sigma_{zp}$, and $\tau_{xzp})$ should be defined [2] as

$$\tau_i = \frac{\sigma_1 - \sigma_3}{\sqrt{3}} = \frac{2}{\sqrt{3}} \left[\frac{(\sigma_{zp} - \sigma_{xp})^2}{2} + \tau_{xzp} \right]^{1/2}. \tag{20}$$

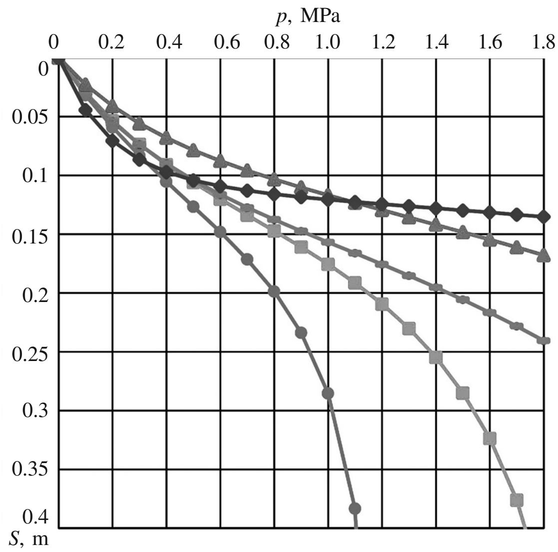

Fig. 7. $S-p$ graphs along the $x = 0$ axis, with different parameters of deformability and strength.

The results of the calculation of the settlement for a layer with the thickness $h = 8$ m and different parameters of deformability (G^e, ν, ε^*, and α) and strength (φ and c) are shown in Fig. 7.

The settlement – load ($S-p$) curves calculated according to the Coulomb Mohr elastic-plastic model with different deformability and strength parameters [2] turn out to be like the curves calculated on the basis of the Hencky's model (see Fig. 7).

Conclusions

1. In this paper the line deformation of soil $\varepsilon_z(\sigma, \tau)$ is represented in the framework of the Hencky's model as the sum of the volumetric component, described by a nonlinear dependence on the mean stress, and the shear component according to the elastic-plastic model of Tymoshenko. Consequently, in dependence on the combination of the input parameters, the calculation model represents well the real strain–stress and the settlement-load curves, including not only decaying but also non-attenuating, progressive stages (double curvature trajectories).

2. If the horizontal movements of the soil are restricted ($\varepsilon_x = 0$) the calculation model inevitably leads to the attenuation of ε_z deformations with an increase in σ_z.

REFERENCES

1. S. S. Vyalov, *Rheological Bases of Soil Mechanics*, Vyscshaya Schkola, Moscow (1978).
2. Z. G. Ter Martirosyan and A. Z. Ter Martirosyan, *Mechanics of Soils*, ASV, Moscow (2020).
3. S. B. Ukhov et al., *Mechanics of Soils, Bases, and Foundations*, Vyscshaya Schkola, Moscow (2007).
4. S. P. Timoshenko and J. Gudier, *Theory of Eelasticity*, Nauka, Moscow (1975).
5. SP 22.13330.2016. Foundations of Buildings and Structures. Updated version of Construction Rules and Regulations 2.02.01 83* (2016).
6. Flamant, *Comptes Rendus*, Paris (1892).
7. H. Hencky, "Zur Theorie plastischer Deformationen und der hierdurch im Material hervorgerufenen Nachspannungen," *ZaMM*, **4**, No.4, 323-334 (1924).
8. V. A. Florin, *Fundamentals of Soil Mechanics*, Gosstroizdat, Moscow (1961).
9. A. Casagrande, "The determination of the pre consolidation load and its practical significance," *Proc. ICOSMFE*, Cambridge, No. 3, 60-64 (1936).
10. I. N. Luzin, "The stress-strain state of the foundations of deep foundations on the homogeneous and inhomogeneous over-compacted ground half-space," Doctoral thesis, Moscow, 2016.