# *SOIL MECHANICS*

# **ANALYSIS OF THE RESPONSE OF PILE GROUPS CONSIDERING PILE**−**CAP**−**SOIL INTERACTION IN LAYERED SOIL**

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*The shear displacement method was used to simulate the response of pile groups considering pile-cap-soil interaction in layered soil. The pile cap was modeled using the finite element method with 16-node degenerate elements and the pile-pile interaction was assumed to be in a linear elastic state. The effect of the reinforcement effect between plies on the response of pile groups was considered. The obtained calculation results were compared with values estimated in finite element and field tests, and the feasibility of the proposed model for the analysis of a pile group and piled raft foundation was verified.*

# **Introduction**

The conventional method of designing a piled raft foundation is that the entire load is shared by the piles and the contribution of the raft is neglected. With respect to the conventional approach, a more rational and economical solution can be obtained by accounting for the contribution of the raft. Analyses on pile-soil interaction had increasingly received attention recently. Three-dimensional finite element analysis on pile−raft−soil interaction was proposed by Ottaviani [1]. Trochanis et al. [2] and De Sanctis and Mandolini [3] analyzed the pile group using the three-dimensional finite element method and accounted for the nonlinear behavior of the soil. Barvashov and Boldyrev [4] proposed a method of analyzing a rigid spatial pile foundation with "high-" and "low-profile" rafts of an arbitrary platform. However, the above-mentioned methods are difficult to apply because of their complex modeling and computational complexity.

Zhang and Zhang [5] proposed a simplified analysis method for the analysis of the response of a single pile. Another method has been presented for investigating the response of drilled piles depending on the effects of upward or downward loading [6]. However, that method was not used to analyze the response of pile groups or a piled raft foundation. Zhang et al. [7] studied a piled raft foundation with different lengths and diameters of piles, while Zhang and Zhang [8] studied a piled raft foundation with different lengths of piles. Babanov and Shashkin [9] analyzed the performance of the pile foundation with low and high rafts, considering an elasto-plastic bed with independent strain hardening. They employed an approximate approach based on an interaction factor to study the nonlinear behavior of pile groups with a rigid cap and a piled raft foundation.

Under the assumption that the pile has no effect on the free displacement field of soil around piles, Poulos [10] studied the pile-pile interaction factor using the boundary element method. Chow [11] showed that the pile side and the pile-pile interaction was elastic. Ter-Martirosyan et al. [12] investigated the interaction between a large-scale model of a combined pile-shell foundation and a clayey soil



**Fig. 1.** Element division of piles, the pile cap, and soil.

bed. Zhang et al. [13, 14, 15] pointed out that a reinforcement effect between plies existed and the freeform deformation of soil was restricted in actual engineering.

The present paper divides the piled raft foundation into a pile-soil system and raft (pile cap) system in the process of practical analysis. Pile spacing is about 5 to 6 times the pile diameter and pilepile interaction is found to be elastic generally. The raft is modeled employing the finite element method with 16-node degenerate elements, and the pile-pile interaction is assumed to be in a linear elastic state. The shear displacement method is used to analyze the pile-raft (pile cap)-soil interaction in layered soil, and the effect of the reinforcement between plies on settlement is considered.

### **Analysis on pile-cap-soil interaction in layered soils**

The positions of piles are divided into pile elements, and the total number of pile elements is m. To consider pile-soil interaction, the positions of soil between pile elements are divided into soil elements, and the total number of soil elements is *n*. The total number of raft and pile cap elements is  $m + n$ . Pile shaft elements are divided according to the soil layers, the element thickness is  $h_i$ , and the total number of layers is  $n_1$ , as shown in Fig. 1.

The contact force between a raft or pile cap and ground is simulated using a continuous function, and 16-node degenerate isoperimetric elements [16] are used to analyze the raft with arbitrary shape and variable thickness. According to the principle of virtual work, the stiffness matrix of the 16 node degenerate isoperimetric element is expressed as

$$
\left[K_{\rm r}\right] = \iiint\limits_{\rm V} \left[B\right]^{\rm T} \left[D\right] \left[B\right] \left|J\right| d\xi d\eta d\zeta,\tag{1}
$$

where [*B*] is the element strain-displacement related matrix,  $|J|$  is the Jacobi determinant and [*D*] is the elastic coefficient matrix. For isotropic material, [*D*] is expressed as

$$
[D] = \begin{bmatrix} d_{11} & d_{12} & d_{13} & & & \\ d_{12} & d_{11} & d_{13} & & & \\ d_{13} & d_{13} & d_{33} & & & \\ & & & d_{44} & & \\ & & & & d_{44} & & \\ & & & & & d_{44} & \\ & & & & & & d_{44} \end{bmatrix} .
$$
 (2)

Using three-dimensional elasticity theory, the values of  $d_{11}$ ,  $d_{12}$ ,  $d_{13}$ ,  $d_{33}$  and  $d_{44}$  can be computed as

$$
d_{11} = d_{33} = \frac{E(1-v)}{(1+v)(1-2v)}, d_{12} = d_{13} = \frac{Ev}{(1+v)(1-2v)}, d_{44} = \frac{E}{2(1+v)},
$$
\n(3)

where *E* is Young's modulus, *v* is Poisson's ratio, and  $\lambda$  is generally larger than 1.

The assumption of a plate is introduced into the stress-strain relationship, and the elastic coefficient matrix [*D*] of plate is obtained as

$$
d_{11} = \frac{E}{1 - v^2}, \ d_{12} = \frac{Ev}{1 - v^2}, \ d_{13} = 0, \ d_{33} = \frac{\lambda E}{1 - v^2}, \ d_{44} = \frac{E}{2(1 + v^2)}.
$$
\n<sup>(4)</sup>

Under the assumption that compression in the thickness direction of the raft can be neglected, the raft element becomes a degenerate isoperimetric element, and the actual number of degrees of freedom of the element reduces from 48 to 24.

Under the assumptions that there is no relative slip and the interaction between a pile and soil is elastic, the control equation of pile shaft settlement  $w(z)$  is

$$
\frac{\partial^2 w(z)}{\partial z^2} - \frac{k_z w(z)}{E_p A_p} = 0.
$$
\n(5)

The general solution of pile shaft settlement  $w(z)$  can be expressed as

$$
w(z) = c_1 e^{\mu z} + c_2 e^{-\mu z}.
$$
\n(6)

The axial force of the pile shaft  $P(z)$  can be expressed as

$$
P(z) = -E_p A_p \mu (c_1 e^{\mu z} - c_2 e^{-\mu z}),
$$
\n(7)

where  $c_1$  and  $c_2$  are constants;  $\mu = \sqrt{k_z/E_p A_p}$ ;  $E_p$  and  $A_p$  are the elasticity modulus and cross-sectional area of the pile, respectively. The spring stiffness of soil around piles  $k_z$  [17] is calculated as  $k_z = 2\pi G_s / \ln(r_m/r_0)$ , where  $G_s$  is the shear modulus of the soil;  $r_0$  is the radius of the pile;  $r_m$  is the negligible influence radius of shear deformation and is defined by  $r_m = 2.5l(1 - v)$ , where *l* is the pile length.

Pile elements are divided according to the soil layers, the element thickness is  $h_i$  and the total number of layers is  $n_1$ . Boundary conditions of Eq. (5) are

$$
w_{ii}(z)|_{z=h_i} = w_{iib}
$$
  
\n
$$
P_{ii}(z)|_{z=h_i} = -P_{iib}
$$
 (8)

The values of  $c_1$  and  $c_2$  can be obtained using Eqs. (6), (7) and (8). The relationship of the settlement and axial force between the top and bottom of pile element *i* can be calculated according to

$$
\begin{Bmatrix} w_{ii}(z) \\ P_{ii}(z) \end{Bmatrix}_{t} = \left[ T_e^1 \right]_i \begin{Bmatrix} w_{ii}(z) \\ P_{ii}(z) \end{Bmatrix}_{b}, \qquad (9)
$$

where  $[T_e^1]$  is the transfer matrix, which can be expressed in the form

$$
\left[T_e^1\right]_i = \begin{bmatrix} \cosh(\mu_i h_i) & \sinh(\mu_i h_i) / E_p A_p \mu_i \\ E_p A_p \mu_i \sinh(\mu_i h_i) & \cosh(\mu_i h_i) \end{bmatrix},
$$
\n(10)

where  $\mu_i = \sqrt{k_{zi} / E_p A_p}$  and  $k_{zi} = 2\pi G_{si} / \ln(r_{mi}/r_0)$ .

The settlement of the pile end can be calculated as

$$
w(l) = \eta \frac{P(l)(1 - v_{\rm b})}{4r_0 G_{\rm sb}} = \frac{P(l)}{k_{\rm b}},\tag{11}
$$

where  $G_{sh}$  and  $v_h$  are the shear modulus and Poisson's ratio of soil around the pile end, respectively;  $\eta$ is the influence coefficient of the embedded depth of the pile, whose value ranges approximately from 0.85 to 1.0; and  $k<sub>b</sub>$  is the spring stiffness of soil around the pile end.

According to the equilibrium condition and continuity condition of each element, the relationship between the pile top settlement and pile top load and the relationship between the pile end settlement and pile end load can be expressed as

$$
\begin{Bmatrix} w(0) \\ P(0) \end{Bmatrix} = \begin{bmatrix} T_c^{-1} \end{bmatrix} \begin{Bmatrix} w(l) \\ P(l) \end{Bmatrix},\tag{12}
$$

where  $T_{\rm e}^1$  =  $\prod_{\alpha=1}^{n_1} (\Gamma_{\rm e}^1)$  $\left[T_{\rm e}^{\,1}\right] = \prod_{i=1}^{n_{\rm 1}} \left(\left[T_{\rm e}^{\,1}\right]\right)_i.$ 

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The stiffness of a single pile can be expressed by

$$
K = \frac{P(0)}{w(0)} = \frac{\left[T_e^{1}\right](2,1) + \left[T_e^{1}\right](2,2)k_b}{\left[T_e^{1}\right](1,1) + \left[T_e^{1}\right](1,2)k_b}.
$$
\n(13)

The settlement and axial force of any section *j* of the pile under vertical load  $P_0$  on the pile top are expressed as

$$
\begin{Bmatrix} w_{jj}(z) \\ P_{jj}(z) \end{Bmatrix} = \prod_{i=1}^{j} \left( \begin{bmatrix} T_e^1 \end{bmatrix} \right)_i \begin{Bmatrix} \frac{P_0}{K} \\ \frac{P_0}{R} \end{Bmatrix} . \tag{14}
$$

A plastic zone forms to a certain depth  $l_1$  with an increase in load. At a depth  $l_1$ , the axial force and settlement are denoted  $P_c(l_1)$  and  $w_c(l_1)$ , respectively, while the load and settlement of the pile top are denoted  $P_t(0)$  and  $w_t(0)$ , respectively, and can be expressed as

$$
\begin{cases}\nP_{\rm t}(0) = \prod_{i=1}^{j_1} (\left[T_{\rm e}^{-1}\right]_i) P_0 + \sum_{h_i}^{l_1} \int_0^{h_i} 2\pi r_0 \tau_{\rm f} dz \\
w_{\rm t}(0) = \prod_{i=1}^{j_1} (\left[T_{\rm e}^{-1}\right]_i) \frac{P_0}{K} + \left[\prod_{i=1}^{j_1} (\left[T_{\rm e}^{-1}\right]_i) P_0 + \sum_{h_i}^{l_1} \int_0^{h_i} 2\pi r_0 \tau_{\rm f} dz\right] l_1 / E_{\rm p} A_{\rm p}\n\end{cases} (15)
$$

where  $j_1$  is the total layer presented as elastic while  $\tau<sub>i</sub>$  is the ultimate shear strength of soil layer *i*.

The nonlinear flexibility of a single pile is then

$$
\delta_{\rm nl} = \frac{w_{\rm t}(0)}{P_{\rm t}(0)} = \frac{\prod_{i=1}^{j_{\rm t}} \left(T_{\rm e}^1\right]_{i} P_{\rm f}}{\prod_{i=1}^{j_{\rm t}} \left(T_{\rm e}^1\right]_{i} P_{\rm 0} + \sum_{h_{i}}^{l_{\rm t}} \int_{0}^{h_{i}} 2\pi r_{\rm 0} \tau_{\rm f} dz} + \frac{l_{\rm l}}{E_{\rm p} A_{\rm p}}.
$$
\n(16)

When the entire soil around piles is in the plastic stage, the nonlinear flexibility of a single pile can be expressed as

$$
\delta_{\rm nl} = \frac{\prod_{i=1}^{n_{\rm l}} \left( T_{\rm e}^{-1} \right) \frac{P_0}{K}}{\prod_{i=1}^{n_{\rm l}} \left( T_{\rm e}^{-1} \right) \cdot P_0 + \sum_{h_i}^{l} \int_0^{h_i} 2\pi r_0 \tau_{\rm fi} dz} + \frac{l}{E_{\rm p} A_{\rm p}}.
$$
\n(17)

The nonlinear stiffness of a single pile can be expressed by  $K_{n1} = \delta_{n1}^{-1}$ .

Pile-pile, pile-soil, soil-pile and soil-soil interactions are shown in Fig. 2, where  $\delta_{p_\text{p},ee}$  (Fig. 2a) and  $\delta_{\eta_{\text{R}}ee}$  (Fig. 2b) are the settlement of piles *e* and *f* due to unit force  $P_e = 1$  acting on the top of pile *e*, respectively;  $\delta_{ps,fe}$  (Fig. 2c),  $\delta_{ss,ee}$ , and  $\delta_{ss,fe}$  (Fig. 2d) are the settlement of pile *f*, soil elements *e* and *f* due to uniform load  $P_e = 1/A_e$  acted on area  $A_e$  of soil element *e*.

Chow [11] showed that the pile-pile interaction is elastic. The assumption of elastic pile-pile interaction is thus accepted in this paper. The settlement of passive pile *f* caused by active pile *e* with a pile spacing of *r* is denoted  $w_{fe}(z)$ , while the settlement of soil around passive pile *f* is denoted  $w_{s}(r, z) = \psi(r)w_{ee}(z)$ , where  $\psi(r)$  is an attenuation coefficient of the displacement field of soil around piles as r increases [18].

The control equation of passive pile f shaft settlement is expressed as

$$
\frac{\partial^2 w_{fe}(z)}{\partial z^2} - \frac{k_z w_{fe}(z)}{E_p A_p} = -\psi(r) \frac{k_z w_{ee}(z)}{E_p A_p}.
$$
\n(18)

The general solution of Eq. (18) can be expressed in the form

$$
w_{fe}(z) = c_3 e^{\mu z} + c_4 e^{-\mu z} - \frac{z \mu \psi(r) (c_1 e^{\mu z} + c_2 e^{-\mu z})}{2} = c_3 e^{\mu z} + c_4 e^{-\mu z} + \frac{z \psi(r) P_{ee}(z)}{2E_p A_p}.
$$
\n(19)



**Fig. 2.** Schematic diagrams of interactions: a) pile−pile ; b) pile−soil; c) soil−pile interaction; d) soil−soil.

The axial force of passive pile f is evaluated according to

$$
P_{fe}(z) = -E_p A_p \frac{\partial w_{fe}(z)}{\partial z} = -E_p A_p \mu (c_3 e^{\mu z} - c_4 e^{-\mu z}) + \frac{\psi(r)}{2} [zk_z w_{ee}(z) - P_{ee}(z)].
$$
\n(20)

The pile elements are divided according to the soil layers, the element thickness is  $h_i$ , and the total number of layers is  $n_1$ . One obtains

$$
\begin{Bmatrix} w_{fe}(z) \\ P_{fe}(z) \end{Bmatrix}_{i,b} = \begin{bmatrix} T^1 \end{bmatrix}_{i} \begin{Bmatrix} w_{fe}(z) \\ P_{fe}(z) \end{Bmatrix}_{i,t} + \begin{bmatrix} T^2 \end{bmatrix}_{i} \begin{Bmatrix} w_{ee}(z) \\ P_{ee}(z) \end{Bmatrix}_{i,t},
$$
\n(21)

where 
$$
\left[T^1\right]_i = \begin{bmatrix} \cosh(\mu_i h_i) & -\sinh(\mu_i h_i)/E_p A_p \mu_i \\ -E_p A_p \mu_i \sinh(\mu_i h_i) & \cosh(\mu_i h_i) \end{bmatrix}
$$
, (22)

$$
\left[T^{2}\right]_{i} = \frac{\psi(s)}{2} \left[\mu_{i}E_{p}A_{p}\left[\mu_{i}h_{i}\cosh(\mu_{i}h_{i}) + \sinh(\mu_{i}h_{i})\right] \frac{1}{E_{p}A_{p}}\left[h_{i}\cosh(\mu_{i}h_{i}) - \frac{1}{\mu_{i}}\sinh(\mu_{i}h_{i})\right]\right].
$$
 (23)

According to the equilibrium condition and continuity condition of each element, the relationship between settlement and load can be expressed in the form

$$
\begin{Bmatrix} w_{fe}(l) \\ P_{fe}(l) \end{Bmatrix} = \begin{bmatrix} T^1 \end{bmatrix} \begin{Bmatrix} w_{fe}(0) \\ P_{fe}(0) \end{Bmatrix} + \begin{bmatrix} T^2 \end{bmatrix} \begin{Bmatrix} w_{ee}(0) \\ P_{ee}(0) \end{Bmatrix},
$$
\n(24)

where  $[T^1] = \prod_{r=1}^{n_1} (\lceil T^1 \rceil)$  and  $\lceil T^2 \rceil = \sum_{r=1}^{n_1}$ 

The coefficient of interaction between active pile *e* and passive pile *f* is defined by  $\alpha_{\epsilon}$ . One obtains

$$
\alpha_{fe} = \frac{\text{The settlement of passive pile } f \text{ caused by unit force acted on active pile } e}{\text{The settlement of active pile } e \text{ caused by unit force acted on active pile } e}.\tag{25}
$$

If the settlement of the top of active pile *e* is  $w_{ee}(0) = 1$ , then the load on the top of pile *e* is  $P_{ee}(0) = K$ . If the load on the top of passive pile *f* is  $P_{fe}(0) = 0$ , then the settlement of the top of pile *f* is  $w_{fe}(0) = \alpha_{fe}$ .

Combining these special conditions and Eq. (24) yields

1

 $1 - \frac{1}{2} \int \frac{1}{2} \ln \left( \frac{1}{2} \right) \cdot \ln \left( \frac{1}{2} \right) \cdot \left($  $\prod_{i=1}^{n_1} ([T^1]_i)$  and  $[T^2] = \sum_{i=1}^{n_1} [T^2]_{n_1} \dots [T^2]_i [T^2]_{i-1} \dots [T^2]_1$ .

 $\left[T^1\right] = \prod_{i=1}^{n} \left(\left[T^1\right]_i\right)$  and  $\left\lfloor T^2\right] = \sum_{i=1}^{n} \left[\left[T^2\right]_{n_1} \dots \left[T^2\right]_i\right]$   $\left\lfloor T^2\right]_{i-1} \dots \left[T^2\right]$ 

$$
w_{fe}(l) = \left[T^{1}\right](1,1)w_{fe}(0) + \left[T^{1}\right](1,2)P_{fe}(0) + \left[T^{2}\right](1,1)w_{ee}(0) + \left[T^{2}\right](1,2)P_{ee}(0)
$$
  
\n
$$
P_{fe}(l) = \left[T^{1}\right](2,1)w_{fe}(0) + \left[T^{1}\right](2,2)P_{fe}(0) + \left[T^{2}\right](2,1)w_{ee}(0) + \left[T^{2}\right](2,2)P_{ee}(0)
$$
\n(26)

i.e., 
$$
k_{\mathbf{b}} = \frac{\alpha_{\mathbf{f}e} [T^1](1,1) + [T^2](1,1) + K [T^2](1,2)}{\alpha_{\mathbf{f}e} [T^1](2,1) + [T^2](2,1) + K [T^2](2,2)}.
$$
 (27)

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The coefficient of interaction of active pile *e* and passive pile *f* can be evaluated from Eq. (27) as

$$
\alpha_{fe} = \frac{[T^2](2,1) - [T^2](1,1)k_b + K\{[T^2](2,2) - [T^2](1,2)k_b\}}{k_b[T^1](1,1) - [T^1](2,1)}.
$$
\n(28)

The stiffness of pile-pile interaction is expressed by

$$
k_{pp,fe} = \begin{cases} K/\alpha_{fe} , & (r_0 < r < r_m) \\ 0 , & (r > r_m) \end{cases} \tag{29}
$$

We assume  $\alpha_e$  is equal to 1 when  $e = f$ . The settlement and axial force of any section *j* of a passive pile under arbitrary vertical load  $P_{\rho}$  on active pile  $e$  is calculated as

$$
\begin{Bmatrix} w_{fe}(z) \\ P_{fe}(z) \end{Bmatrix}_{j} = \prod_{i=1}^{j} ([T^{1}]_{i}) \begin{Bmatrix} \alpha_{fe} P_{e} / K \\ 0 \end{Bmatrix} + \sum_{i=1}^{j} [T^{2}]_{j} ... [T^{2}]_{i} [T^{2}]_{i-1} ... [T^{2}]_{1} \begin{Bmatrix} P_{e} / K \\ P_{e} \end{Bmatrix}.
$$
 (30)

Under the assumption that all soil elements are in an elastic stage and the nonlinear influence can be ignored,  $k_{sp,fe}$  is equal to  $k_{ps,fe}$ . One obtains

$$
k_{sp,fe} = \begin{cases} \psi(r) \, K \, , & (r_0 < r < r_m) \\ 0 \, , & (r > r_m) \end{cases} \tag{31}
$$

The displacement solution under a vertical concentrated load *P* acting on the surface of a homogeneous isotropic elastic half-space can be expressed by

$$
w = \frac{(1 - v^2)P}{\pi E_0 \sqrt{x^2 + y^2}}.
$$
\n(32)

When vertical uniform load  $P(x, y)$  is applied to the surface of an area  $\Omega$  (2*a*  $\times$  2*b*), the settlement of an arbitrary point on the surface is calculated using the area  $\Omega$  integral:

$$
w = \frac{(1 - v^2)}{\pi E_0} \int_{-a}^{a} \int_{-b}^{b} \frac{P(\xi, \eta)}{\sqrt{(x - \xi)^2 + (y - \eta)^2}} d\xi d\eta.
$$
 (33)

An approximate expression of Eq. (33) is obtained by isometric transformation. The value of  $\delta_{s,ee}$  is evaluated using a uniform load  $P_e = 1/A_e$  acting on area  $A_e$  in the actual computation. All elements of the matrix flexibility  $[\delta_{\alpha}]$  of soil-soil interaction can therefore be calculated directly:

$$
\delta_{ss,ee}(z) = \frac{1 - v^2}{\pi a E_s (1 - 2v)} \bigg[ m \ln(\sqrt{1 + n^2} + n) + \ln(\sqrt{1 + m^2} + m) \bigg],\tag{34}
$$

$$
\delta_{ss,fe}(z) = \begin{cases}\n\frac{1 - v^2}{\pi E_s (1 - 2v)\sqrt{(x_e - x_f)^2 + (y_e - y_f)^2}}, (r_0 < r < r_m) \\
0, & (r > r_m)\n\end{cases}
$$
\n(35)

where  $m = a/b$  and  $n = b/a$ , with *a* and *b* being the long and short lengths of the soil element respectively.

## **Analysis of the response of pile groups considering pile-raft (pile cap)-soil interaction**

The matrix stiffness of a pile-soil sustaining system is denoted  $[K_{\scriptscriptstyle{sp}}]$ . One obtains

$$
\left[K_{sp}\right] = \begin{bmatrix} k_{pp} & k_{ps} \\ k_{sp} & k_{ss} \end{bmatrix} . \tag{36}
$$



**Fig. 3.** Plane layout of piles.

The balance equation of the pile-soil sustaining system is expressed as

$$
\begin{bmatrix}\nk_{pp,11} & \dots & k_{pp,1m} & k_{ps,1m+1} & \dots & k_{ps,1m+n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
k_{pp,m\,1} & \dots & k_{pp,m\,m} & k_{ps,m\,m+1} & \dots & k_{ps,m\,m+n} \\
k_{sp,m+11} & \dots & k_{sp,m+1m} & k_{ss,m+1m+1} & \dots & k_{ss,m+1m+n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
k_{sp,m+n\,1} & \dots & k_{sp,m+n\,m} & k_{ss,m+n\,m+1} & \dots & k_{ss,m+n\,m+n}\n\end{bmatrix}\n\begin{bmatrix}\nw_1 \\ \vdots \\ w_m \\ w_m\n\end{bmatrix}\n=\n\begin{bmatrix}\nR_1 \\ \vdots \\ R_m \\ R_m\n\end{bmatrix},
$$
\n(37)

where  $k_{\text{p},\text{e},\text{e}}$  is the matrix stiffness of a pile.

 $L_{pp,ee}$  is the matrix strifferent vector and reaction vector of a pile-soil sustaining system are denoted  $\bar{U}$  and  $\bar{U}$ *R*, respectively. The relationship between the displacement vector and reaction vector can be expressed by

$$
\left[K_{sp}\right]\left\{\bar{U}\right\} = \left\{R\right\}.\tag{38}
$$

The matrix stiffness of the raft or pile cap and the total load are respectively denoted  $[K_{r}]$  and *Q*, and the balance equation of raft can thus be expressed by

$$
[K_r][U] = \{Q\} - \{R\}.
$$
\n(39)

According to continuity condition  $\overline{U} = U$ , the equation of the pile-raft (pile cap)-soil interaction is

$$
\left[K_r + K_{sp}\right] \left\{ U \right\} = \left\{ Q \right\}.
$$
\n
$$
(40)
$$

The incremental form of Eq. (41) is expressed in the form

$$
\left[K_r + K_{sp}\right]_t \left\{\Delta U\right\}_t = \left\{\Delta Q\right\}_t. \tag{41}
$$

The settlement of an arbitrary pile in the pile group is calculated as

$$
w_{pe} = \sum_{e=1}^{m} \alpha_{fe} p_e / K_e = \begin{cases} K_e = K, & (e \neq f) \\ K_e = K_{nl} , & (e = f) \end{cases}, \quad (42)
$$

# **Case study**

A model experiment was carried out by He and Jin [19]. The length and width of the test flume were 18 and 6 m, respectively. The pile cap was 3.4 m in length, 1.0 m in width, and 40 cm in thickness. Model piles were seamless pipes having a diameter of 114 mm and length of 2 m. The aspect ratio was 17.5, and the pile spacing was 400 mm (about 3.5 times the pile diameter). The distance between the peripheral of the pile cap and the center of a pile was 100 mm, and 27 piles were located under the pile cap. The physical and mechanical parameters of each soil in the model test are given in Table 1.





**Fig. 4.** Comparison of predicted and measured behaviors of pile groups: 1) present calculated results; 2) calculated results [20]; 3) measured results [19].

Computed and measured load settlement curves of the pile groups are compared in Fig. 4. It is seen that the load-settlement relationship at the pile head obtained using the present method is generally consistent with the measurements obtained by He and Jin [19] and results computed by Shang et al. [20] at all loading levels.

# **Conclusions**

A shear displacement method was used to simulate the response of pile groups considering pilecap-soil interaction in layered soils. The pile cap was modeled using the finite element method with 16 node degenerate elements, and the pile-pile interaction was assumed to be in a linear elastic state. The effect of the reinforcement effect between plies on the response of pile groups was considered. The calculation results obtained in the present paper were compared with values estimated in the finite element and field tests, and the feasibility of the proposed model for the analysis of a pile group and piled raft foundation was verified.

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