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RESPONSE OF A SINGLE PILE SUBJECTED TO TENSION LOAD BY USING SOFTENING MODELS

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This paper proposes a single exponential model and a double exponential model to simulate the relationship between pile displacement and unit skin friction developed at the pile-soil interface. Based on the proposed two models, two simple analytical approaches were presented for nonlinear analysis of the response of a single tension pile. A highly effective iterative computer program was developed by using the load transfer method. Comparisons of the present computed results, the calculated results derived from other analytical methods, and measured results were made to demonstrate the reliability of the proposed analytical methods.

Introduction

Tension piles installed as the foundation of high-rise buildings have been widely used to resist uplift forces. The applied uplift load was resisted by skin friction developed along the pile-soil interface. Analyses on the response of tension pile has been carried out during the last five decades. Poulos and Davis [1], O'Neill and Reese [2], and Ramasamy et al. [3] reported that the unit skin friction of tension pile was smaller than that of compression pile. Prediction of the bearing capacity of an uplift pile has been paid more attention in pile foundation designs. The analytical methods [4-9], the model tests [10, 11], and the numerical methods [12, 13] have been used to capture the behavior of a single pile subjected to tension load. However, few studies have been concerned with the details of the softening characteristic of skin friction. To get a better understanding of the shaft resistance degradation, the softening characteristic of skin friction should be analyzed in detail. Therefore, there is a need to develop a more suitable calculation method for the analysis of the tension pile response by using softening models of skin friction.

This paper proposes two analytical approaches to analyze the response of a single pile subjected to tension load by using the load transfer method. Comparisons of the present computed results, the calculated results derived from other analytical methods, and measured results were made to demonstrate the reliability of the proposed analytical methods.

Softening model of skin friction

The results of load tests presented by Zhang et al. [14] were chosen to check the reliability of the softening behavior of skin friction. The observed and the theoretical relationship between τ_s / τ_{su} and S_s / S_{su} of tension piles are shown in Fig. 1, where τ_s is the measured unit skin friction, τ_{su} is the limiting unit shaft resistance, S_s is the measured pile deformation, and S_{su} is the measured pile displacement corresponding to the ultimate skin friction.

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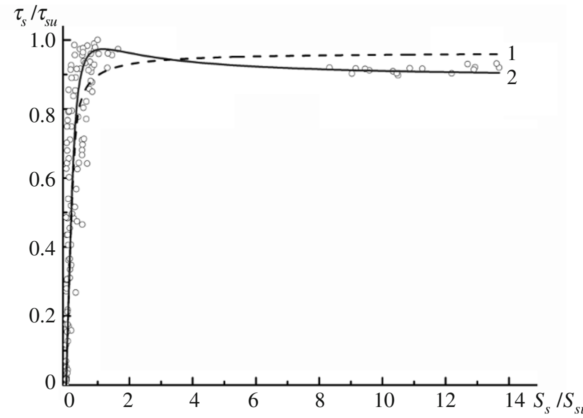

Fig. 1. Observed and theoretical relationship between τ_s/τ_{su} and S_s/S_{su}:
1) $\tau_s/\tau_u = \dfrac{S_s/S_{su}}{0.0761+1.038(S_s/S_{su})}$, $R^2=0.7674$, 2) $\tau_s/\tau_u = \dfrac{(S_s/S_{su})(0.1334+0.1386(S_s/S_{su})}{(0.1334+0.3952(S_s/S_{su})}$, $R^2=0.9032$.

Figure 1 shows that a hyperbolic model is not suitable for simulating the post-peak response of skin friction and has a lower accuracy (R^2 = 0.7674, where R^2 is the goodness of fit and is used to describe how well the hyperbolic model fits a set of observations.). The relationship between skin friction and corresponding shear displacement follows a softening model when the skin friction is fully mobilized, and is in a good agreement with the results of the field tests. Therefore, there is a need to analyze the relationship between unit skin friction and pile-soil displacement by using a softening model, when skin friction is fully mobilized.

Analysis of response of a single pile subjected to tension load

For the load transfer method, the skin resistance of the tension pile was modeled as a series of elastic elements supported by discrete nonlinear vertical springs (see Fig. 2). The relationship between pile displacement and unit skin friction was simulated by using the present softening models. Taking the self weight of the pile into account, the axial force $P(z)$ at a given z can be calculated as

$$P(z)=P_t-U\int_0^L \tau_s(z)dz-\gamma_p A_p z, \tag{1}$$

where U is the pile circumference, L is the pile length, $\tau_s(z)$ is the shaft shear stress at a given depth z, P_t is the pile head load, γ_p is the unit weight of the pile, whose value can be adopted as 25kN/m^3 in practice, and A_p is the cross-sectional area of the pile.

The pile displacement $S_s(z)$ at a given depth z can be computed by

$$S_s(z)=S_t-\frac{1}{E_pA_p}\int_0^L P(z)dz, \tag{2}$$

where E_p is the pile elastic modulus, and S_t is the pile head displacement.

The relationship between axial force $P(z)$ and pile displacement $S_s(z)$ can be obtained by differentiating Eq. (2)

$$dS_s(z)=-\frac{P(z)}{E_pA_p}dz. \tag{3}$$

Based on the equilibrium condition of a pile element (see Fig. 2), the relationship between axial force $P(z)$ and unit skin friction $\tau_s(z)$ at a given z can be written as

$$dP(z) + U\tau_s(z)dz + \gamma_p A_p dz = 0. \tag{4}$$

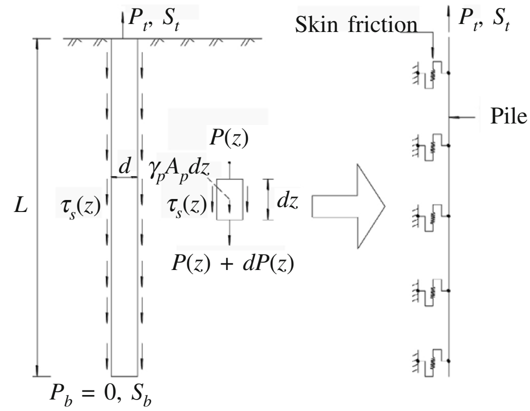

Fig. 2. Calculation model of a single pile subjected to tension load by using the load transfer method.

Softening model I of skin friction

As suggested by Zou et al. [15], the softening behavior between skin friction and pile displacement can be described using the single exponential model and can be expressed in the following form

$$\tau_s(z) = \tau_{su}\left(\frac{S_s(z)}{S_{su}}\right)^c \exp\left[1-\left(\frac{S_s(z)}{S_{su}}\right)^c\right], \tag{5}$$

where c is the softening coefficient, whose value is assumed to be in the range 0.40 to 0.45 (silty clay), 0.45 to 0.48 (clay), 0.48 to 0.52 (mucky soil), and 0.52 to 0.55 (mud). It is well known that the construction methods, pile types, soil types, stratigraphy, and loading procedure have an influence on the interface behavior of an actual pile at a site and may influence the magnitude of the parameter S_{su}. For practical purposes, the value of S_{su} is commonly determined experimentally or by back-analysis of field load test results. Two full-scale loading tests on tension piles showed that the values of S_{su} were in the range of 5.33 mm to 10.9 mm (about 0.67% to 1.34% of the pile diameter) in different soils [14]. However, as to the piles subjected to compressive load, a series of field tests showed that the values of S_{su} were in the range of 5 to 25 mm for bored piles in different soils (e.g., mud, clay, sandy silt, and silty clay) [9, 16, 17]. In the analytical approach, the effective stress method can be employed to predict the value of τ_{su} in the drained condition and can be computed by

$$\tau_{su} = K_0\left(\frac{K}{K_0}\right)\tan\left[\varphi\left(\frac{\delta}{\varphi}\right)\right]\sigma'_v, \tag{6}$$

where δ is the friction angle of pile-soil interface, φ is the effective angle of internal friction of the surrounding soil, σ'_v is the effective overburden pressure at the depth under consideration, K is the lateral earth pressure coefficient, and K_0 is the in situ earth pressure coefficient approximately estimated by $K_0 = 1 - \sin\varphi$ for normally consolidated soils. Suggested values of K and δ were summarized in [16].

Note that equation (6) is used to calculate the limiting skin friction of compression pile, however, previous work showed that the ultimate skin friction of tension pile was smaller than that of compression pile, and the ratio of the limiting skin friction of the tension pile to the ultimate shaft resistance of the compression pile was in the range 0.49 to 0.84 [1, 3, 17, 18].

The relationship between pile displacement and unit skin friction can be simulated by using the softening model presented in Eq. (5). Substituting Eqs. (3) and (5) into Eq. (4), we can write the governing equation for pile-soil interaction as

$$P(z)dP(z)=E_pA_p\left\{U\tau_{su}(\frac{S_s(z)}{S_{su}})^c\exp[1-(\frac{S_s(z)}{S_{su}})^c]+\gamma_pA_p\right\}dS_s(z). \tag{7}$$

The axial force $P(z)$ at a given depth z, can be computed by

$$P(z)=\sqrt{-2UE_pA_p\tau_{su}S_{su}\left\{\frac{S_s(z)}{S_{su}}+[\frac{S_s(z)}{S_{su}}]^{1-c}\right\}\exp\left\{1-[\frac{S_s(z)}{S_{su}}]^c\right\}+2E_p\gamma_pA_p^2S_s(z)+C_1}, \tag{8}$$

where the boundary conditions $P(L) = 0$ and $S_S(L) = S_b$, C_1 is an integral constant

$$C_1=2UE_pA_p\tau_{su}S_{su}\left\{\frac{S_b}{S_{su}}+(\frac{S_b}{S_{su}})^{1-c}\right\}\exp\left\{1-(\frac{S_b}{S_{su}})^c\right\}-2E_p\gamma_pA_p^2S_b. \tag{9}$$

Substituting Eq. (9) into Eq. (8), we can express the axial force $P(z)$.

Softening model II of skin friction

The double exponential model suggested by Van Der Veen [19] can be used to analyze the response of a single pile subjected to tension load. The double exponential model can be expressed as

$$P(z)=P_{u\max}\{1-\exp[-\beta S_s(z)]\}, \tag{10}$$

where $P_{u\max}$ is the ultimate uplift resistance, and β is the attenuation coefficient.

The relationship between $P(z)$ and $S_s(z)$ can be obtained by differentiating Eq. (10)

$$dP(z)=\beta P_{u\max}\exp[-\beta S_s(z)]dS_s(z). \tag{11}$$

Substituting Eqs. (11) and (3) into Eq. (4), we can express the unit skin friction at a given depth z

$$\tau_s(z)=\frac{\beta P_{u\max}^2}{E_pA_pU}[\exp(-\beta S_s(z))-\exp(-2\beta S_s(z))]-\frac{\gamma_pA_p}{U}. \tag{12}$$

The limiting unit skin friction

$$\tau_{su}=\frac{\beta P_{u\max}^2}{4E_pA_pU}-\frac{\gamma_pA_p}{U}. \tag{13}$$

The initial stiffness of the double exponential model k_{ini} can be computed as

$$k_{ini}=\frac{d\tau_s(z)}{dS_s(z)}\Big|_{z=0}=\frac{\beta^2P_{u\max}^2}{E_pA_pU}-\frac{\gamma_pA_p}{U}. \tag{14}$$

Following the suggestion of Zhu [20]

$$k_{ini} = K_s/U, \tag{15}$$

where $K_s = K_1K_2/(K_1 + K_2)$ is the effective shear stiffness at the pile-soil interface, K_2 is the shear stiffness of pile-soil interface, which can be obtained from Table 1 [21], and K_1 is the shear stiffness of pile, whose value can be calculated using the following Eq. [22]:

$$K_1 = 2\pi G_s/\ln2, \tag{17}$$

where G_s is the shear modulus of the pile.

We can compute the $P_{u\max}$ and β by

$$P_{u\max}=4U(\tau_{su}+\frac{\gamma_pA_p}{U})\sqrt{\frac{E_pA_p}{K_s+\gamma_pA_p}}; \tag{18}$$

.

TABLE 1

Lithology	K_2, GPa
Hard rock	5.00-10.00
Soft rock	1.50-3.00
Decomposed rock	1.00-2.00
Mudstone	1.20-2.50
Diluvium sand	0.40-0.70
Gravel sand	0.40-0.70
Diluvium clay	0.40-1.00
Alluvium sand	0.05-0.20

$$\beta = \frac{K_s + \gamma_p A_p}{4U(\tau_{su} + \frac{\gamma_p A_p}{U})}. \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (13), we can compute the unit skin friction

$$\beta = \frac{K_s + \gamma_p A_p}{4U(\tau_{su} + \frac{\gamma_p A_p}{U})}. \tag{20}$$

The governing equation for pile-soil interaction can be obtained by substituting Eqs. (3) and (20) into Eq. (4)

$$P(z)dP(z) = E_p A_p [U\tau_s(z) + \gamma_p A_p] dS_s(z); \tag{21}$$

$$P(z) = \sqrt{\frac{E_p A_p U}{\beta}(4\tau_{su} + \frac{\gamma_p A_p}{U})\exp[-\beta S_s(z)]\left\{\exp[-\beta S_s(z)] - 2)\right\} + C_1}, \tag{22}$$

where the boundary conditions $P(L) = 0$ and $S_S(L) = S_b$, C_1 can be calculated by

$$C_1 = \frac{E_p A_p U}{\beta}(4\tau_{su} + \frac{\gamma_p A_p}{U})\exp(-\beta S_b)[2 - \exp(-\beta S_b)]. \tag{23}$$

Note that the mobilized pile base load may not be equal to zero due to the suction at the pile tip. However, the suction is small and difficult to determine in practice. Therefore, for practical purposes, it is commonly assumed that the force at the tip of a tension pile is equal to zero. Substituting Eq. (23) into (22), we can write $P(z)$ at a given depth z.

Algorithm for analysis of response of a tension pile embedded in layered soils

The response of a single pile subjected to tension load can be analyzed by using the computational flow chart (Fig.3). Assuming a series of different values of pile end settlement S_{bn}, a load-displacement curve can be obtained by using the computational flow chart. In Fig.3, L_n is the length of the pile segment n, d is the pile diameter, and S_{tn} is the displacement at the top of the pile segment n.

Actually, a single pile subjected to tension load is very likely to be pulled out together with surrounding soils. The failure of different types of tension pile may occur in the soil or along the pile-soil interface. However, it is difficult to determine the possible failure mode occurring in the soil. Therefore, this paper was only focused on the possible failure mode occurring along the pile-soil interface. This may underestimate the tension pile capacity, which was acceptable for practical purposes.

Case study

To check the reliability of the proposed method, the present approach was applied to analyze the response of a single pile subjected to tension load reported in the literature. The case history analyzed,

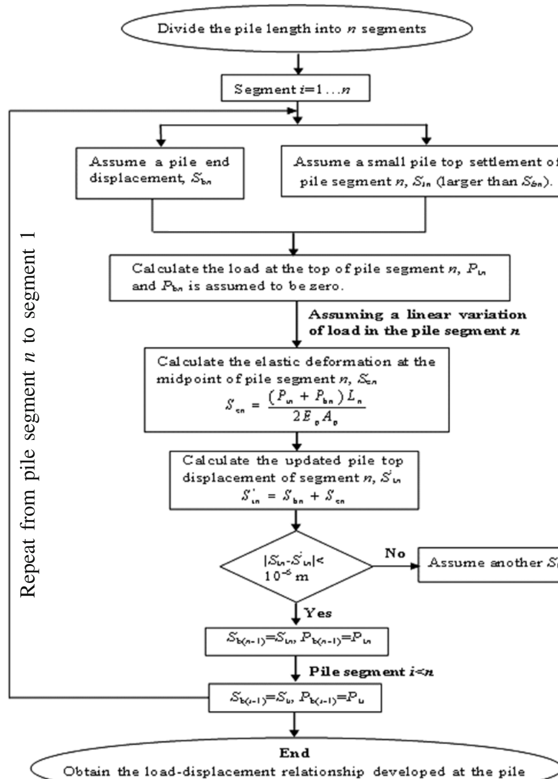

Fig. 3. Computational flow chart for the analysis of the response of a single pile.

regarding the full scale test, was reported by Sowa [23] on a reinforced concrete pile embedded in fine sand deposit overlain by a few feet of clayey silt to silty clay. The pile diameter was 0.53 m, and the pile length was 12 m. The elastic modulus for the reinforced concrete pile was taken as 30 GPa, and the unit weight of the reinforced concrete pile was 25 kN/m^3. The groundwater table elevation in this site was 1.2 m below the ground surface, and the saturated unit weight was adopted as 18.4 kN/m^3. The angle of shearing resistance of the soil φ was adopted as 30°. The shear modulus of the soil was taken as 2.0 MPa as suggested by Sowa [23].

In the analytical method I, the values of the softening coefficients c can be taken as 0.40. Based on the suggested values of K and δ (see Table 2 [19]), the value of δ/φ is assumed to be 0.75, and the value of K/K_0 is adopted as 2.0. The values of S_{su} are assumed to be 3.71 mm (0.7%d), 5.30 mm (1.0%d), and 6.36 mm (1.2%d) for the softening model I. In the analytical approach II, the shear modulus of the reinforced concrete pile can be taken as 10 GPa. Based on Table 1, K_2 can be adopted as 0.5 and 1.0, and K_1 can be calculated by using Eq. (17). As stated previously, the limiting unit skin friction of the tension pile can be adopted as 0.7 times the value estimated from Eq. (6). In practice, the ultimate unit skin friction of each pile segment can be adopted as an average value of the limiting shaft resistance of a recommended soil depth.

Comparisons of the computed results of Goel and Patra [7], the pile response derived from the present methods, and the measured single pile response given by Sowa [23] are shown in Fig. 4. Figure 4 shows that when the pile head load is smaller than 200 kN, there is generally good agreement among the response at the pile head calculated by using the present approach I, the measured results [23], and the computed results [7, 24]. When the pile head load is larger than 200 kN, the displacement of the tension pile derived from the presented approach I is smaller than the measured value [23] and the cal-

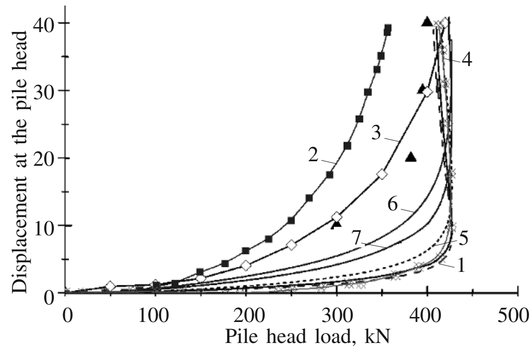

Fig. 4. Load-displacement curves of a single uplift pile. Calculated values from present model I: 1) S_{su} = 3.71 mm; 2) R_{sf} = 0.90 [26]; 3) S_{su} = 5.30 mm; 4) β_s = 0.90, S_{su} = 5.30 mm; 5) S_{su} = 6.36 mm; and 6, 7) from model II K_2 = 0.50, 1.00; ▲) measured values [25].

culated value [7]. A softening model and a hyperbolic model were adopted to simulate the relationship between pile-soil relative displacement and unit skin friction, as suggested in [24]. However, the softening model presented in this paper agrees better with the well-documented field experimental data than the methods described by Zhang et al.

It can also be observed that the load-displacement curves at the pile head derived from the approach II are generally consistent with the measured results [23] and the computed results [7] at all loading levels. Note that the pile displacement estimated from the presented approaches I and II increases with increasing value of S_{su} and K_2 under the same loading level. The pile head load decreases slightly due to the skin friction degradation when the skin friction along the pile depth is fully mobilized. This analysis demonstrates that the proposed approach is efficient and suitable for the analysis on the response of a single pile subjected to tension load.

Conclusion

This paper proposed a single exponential model and a double exponential model to simulate the relationship between pile displacement and unit skin friction developed at the pile-soil interface. Based on the proposed two models, two simple analytical approaches were presented for nonlinear analysis of the response of a single tension pile. A highly effective iterative computer program was developed by using the load transfer method. To demonstrate the reliability of the proposed analytical methods, comparisons of the present computed results, the calculated results derived from other analytical methods, and measured results were made.

This analysis shows that the pile displacement estimated from the presented methods increases with increasing pile displacement corresponding to the limiting unit skin friction S_{su} and decreases with increase in the shear stiffness of the pile-soil interface K_2. The pile head load decreases slightly due to the skin friction degradation when the total skin friction is fully mobilized. However, in general, the parameters K_2 and S_{su} have a small influence on the response of the tension pile in the presented approaches I and II.

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