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DESIGN

PROPAGATION CONDITIONS OF RAYLEIGH WAVES IN NONHOMOGENEOUS SATURATED POROUS MEDIA

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Based on Biot's theory of saturated porous media, the propagation of Rayleigh waves in nonhomogeneous saturated porous media is studied. The frequency equation of Rayleigh waves in inhomogeneous saturated porous media is derived in which the variation of shear modulus is taken into account, and the existence conditions are also given. It is pointed out that the shear modulus of the material parameters is a function of depth via a theoretical derivation, and the final expression of the solid skeleton and fluid displacement in the medium is obtained.

1. Introduction

The study of wave propagation and dynamic response is of particular importance in a number of engineering problems. In general, two main groups of waves, body waves and surface waves, can be observed in an elastic medium. The former propagate inside an unbounded domain, such as dilatational waves and shear waves, whereas the later develop when the medium is stratified or possesses a free surface, such as Love waves and Rayleigh waves. Due to the smaller attenuation of the energy during propagation along the surface than that of the body waves in the interior of the medium, Rayleigh waves hold more potential to destroy vibration to the structures [1]. Because most geological materials can be treated as some kind of nonhomogeneous fluid-saturated porous media, investigation of the characteristics of Rayleigh waves in nonhomogeneous fluid-saturated porous media has significant practical meaning in many fields such as earthquake engineering, petroleum industry, and geotechnical engineering.

Due to the different material properties and the different motions of the solid and liquid constituents in a fluid-saturated porous medium, it is inadequate to study the mechanical behavior of fluid saturated porous materials by using classical continuum mechanics, which portrays a fluid-saturated porous medium as a single-phase solid material [2]. To overcome these deficiencies, Biot [3-4] used Lagrange's equations to derive a set of coupled differential equations governing the motions of solid and fluid phases, known as the theory of poroelasticity, in which the inertial, viscous, mechanical coupling, and compressibility of the solid particles and fluid were taken into account. Geetsman and Smit [5] not only developed the Biot theory and made the stress-strain relationship of two-phase porous media perfect and practical, but also laid the foundation for the study of elastic wave propagation in two-phase media. Considering only one type of compression waves in the characteristic equations, Jones [6] studied Rayleigh waves in macroscopic isotropic homogeneous saturated soils. Therefore, the potential function is not the actual solution to the problem. Considering two kinds of compression waves in the transversely isotropic saturated soil, Tajuddin [7] established the characteristic equation of Rayleigh waves.

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He considered the propagation of Rayleigh waves on the stress-free surface of a viscoelastic porous solid saturated with viscous fluid. The dispersion equation was obtained in a complex irrational form. The effects of wave frequency, frame permeability, and pore tortuosity on particle motion were discussed by Sharma [8]. All these studies on Rayleigh waves in fluid-saturated porous media are focused mainly on the homogeneously isotropic waves. However, nonhomogeneity is an important characteristic for sediments in some circumstances. For example, different sedimentation conditions, including geography, environments, and types of climate, lead to intensive nonhomogeneity along the depth of the soil layer and rock stratum on the Earth's crust. In general, the physical and mechanical properties of the foundation, such as shear modulus, porosity, and permeability, are functions of the depth [9]. For some kinds of soil, the material properties also change dramatically with depth when the soil layer is thick [10]. Based on the theory of elasticity for single-phase continuous media, many investigations on the propagation characteristics of Rayleigh waves in inhomogeneous media have been presented [11-14]. Obviously, the effects of nonhomogeneity of the solid skeleton on the propagation of Rayleigh waves is an important problem that should be clarified.

In this paper, the propagation of Rayleigh waves in nonhomogeneous saturated porous media is studied based on Biot's theory of saturated porous media. Firstly, the frequency equation of Rayleigh waves in the inhomogeneous saturated porous media is derived in which the variation of shear modulus is taken into account, and the existence conditions are also given. Secondly, it is pointed out that the shear modulus of the material parameters is a function of depth via a theoretical derivation, and the expression of shear modulus is calculated. Finally, the final expression of the solid skeleton and fluid displacement in the medium are given.

2. Notation and formulas

Within the framework of Biot's theory, the basic equations for a saturated soil are as follows:

Constitutive law:

$$\sigma_{ij} = \frac{2G(x_3)\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij} + 2G(x_3)\varepsilon_{ij} - \alpha p \delta_{ij}; \tag{1}$$

$$p = M\zeta - \alpha M \varepsilon_{kk}. \tag{2}$$

Strain-displacement relationship:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

Equations of motion:

$$\sigma_{ij,j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i, \tag{4}$$

$$-p_i = \rho_f \ddot{u}_i + m\ddot{w}_i + b\dot{w}_i. \tag{5}$$

In the above equations, σ_{ij} (i, j = 1, 3) and p are the stress components in the porous aggregate and fluid pressure, respectively, u_i represents the components of displacements of the solid skeleton, w_i represents the components of displacement of the fluid relative to the solid skeleton, $e = u_{i,i}$, $\zeta = -w_{i,i}$, summation is over repeated indices. ε_{ij} denotes the strain components of the solid skeleton, and b is a parameter accounting for internal friction due to the relative motion between the solid and the pore fluid ($b = 1/k_f$, where k_f is the dynamic permeability); α and M are the Biot parameters considering compressibility of the two-phase material; $\alpha = 1 - K/K_s$ and $1/M = (\alpha - n)/K_s + n/K_f$, where K, K_s, and K_f are the bulk modulus of the solid skeleton, solid particles, and pore fluid; n is the soil porosity; ρ is the density of the porous aggregate and is given by $\rho = (1 - n)\rho_s + n\rho_f$, where ρ_s and ρ_f are densities of solid phase and liquid phase, respectively; $m = \rho_f/n$ is a parametric representation related to the mass density of pore fluid and pore geometry.

3. Rayleigh waves in a nonhomogeneous saturated porous medium

We consider the propagation of plane waves in a nonhomogeneous saturated porous medium with a plane boundary. The solution of Eqs. (4) and (5) must be obtained with suitable boundary conditions, namely that the bounding surface is stress free, so that we can obtain the disturbance caused by such a deformation. Taking the bounding surface to coincide with the $x_1 - x_2$ plane, with positive direction of the x_3 axis towards the interior of the solid, we can assume the displacement vector to be

$$\begin{aligned} &u_1 = A\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)}, u_2 = 0, u_3 = B\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)};\\ &w_1 = C\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)}, w_2 = 0, w_3 = D\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)}. \end{aligned} \tag{6}$$

The strain components (3) become

$$\begin{aligned} &\varepsilon_{11} = iKA\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)},\ \varepsilon_{33} = -K_1B\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)},\\ &\varepsilon_{13} = \frac{1}{2}(-K_1A + iKB)\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)},\ \varepsilon_{22} = \varepsilon_{12} = \varepsilon_{23} = 0. \end{aligned} \tag{7}$$

Substituting these values in the constitutive law relation (1) and (2), we write the nonvanishing components of the stress as

$$\begin{aligned} &\sigma_{11} = \left[iKA\left(\frac{2G\upsilon}{1-2\upsilon} + 2G + \alpha^2 M\right) - K_1B\left(\frac{2G\upsilon}{1-2\upsilon} + \alpha^2 M\right) + i\alpha MKC - \alpha MK_1D\right]\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)};\\ &\sigma_{33} = \left[iKA\left(\frac{2G\upsilon}{1-2\upsilon} + \alpha^2 M\right) - K_1B\left(\frac{2G\upsilon}{1-2\upsilon} + 2G + \alpha^2 M\right) + i\alpha MKC - \alpha MK_1D\right]\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)};\\ &\sigma_{13} = G\left[-K_1A + iKB\right]\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)};\\ &p = \left[-iMKC + K_1MD - i\alpha MKA + \alpha MK_1B\right]\mathrm{e}^{-K_1x_3}\mathrm{e}^{i(Kx_1-\omega t)}. \end{aligned} \tag{8}$$

Substituting (8) in the equations of motion (4) and (5), we see that the equation corresponding to $i = 2$ is identically satisfied, while the equations corresponding to $i = 1, 3$, after eliminating the exponential function in which the variation of shear modulus is taken into account, reduce to

$$\begin{aligned} &iK\left[iKA\left(\frac{2G\upsilon}{1-2\upsilon} + 2G + \alpha^2 M\right) - K_1B\left(\frac{2G\upsilon}{1-2\upsilon} + \alpha^2 M\right) + i\alpha MKC - \alpha MK_1D\right] +\\ &+ \left[-K_1A + iKB\right]G' - K_1G\left[-K_1A + iKB\right] + \rho\omega^2 A + \rho_f\omega^2 C = 0;\\ &iKG\left[-K_1A + iKB\right] - 2G'K_1B + \rho\omega^2 B + \rho_f\omega^2 D -\\ &-K_1\left[iKA\left(\frac{2G\upsilon}{1-2\upsilon} + \alpha^2 M\right) - K_1B\left(\frac{2G\upsilon}{1-2\upsilon} + 2G + \alpha^2 M\right) + i\alpha MKC - \alpha MK_1D\right] = 0;\\ &iK\left[i\alpha MKA - \alpha MK_1B + iMKC - K_1MD\right] + \rho_f\omega^2 A + m\omega^2 C + ib\omega C = 0;\\ &-K_1\left[i\alpha MKA - \alpha MK_1B + iMKC - K_1MD\right] + \rho_f\omega^2 B + m\omega^2 D + ib\omega D = 0. \end{aligned} \tag{9}$$

Solving Eqs. (9) we obtain

$$\left(K_1A + iKB\right)\left[\left(-K^2G + K_1^2G - K_1G' + \rho\omega^2\right)\left(m\omega^2 + ib\omega\right) - \rho_f^{\,2}\omega^4\right] = 0; \tag{10a}$$

$$\left(K_1C + iKD\right)\left[\rho_f^{\,2}\omega^4 - \left(-K^2G + K_1^2G - K_1G' + \rho\omega^2\right)\left(m\omega^2 + ib\omega\right)\right] = 0, \tag{10b}$$

which are similar, so we take Eq. (10a) as an example. The first expression on the left-hand side of (10a) equated to zero does not give any solution. However, it shows that the arbitrary constants are not distinct but are related to each other. The second expression is

$$\left(-K^2G+K_1^2G-K_1G'+\rho\omega^2\right)\left(m\omega^2+ib\omega\right)-\rho_f^2\omega^4=0, \tag{11}$$

and can be written as follows

$$K_1^2G-K_1G'-K^2G+Q=0, \tag{12}$$

where $Q=\dfrac{\left(m\omega^2+ib\omega\right)\rho\omega^2-\rho_f^2\omega^4}{m\omega^2+ib\omega}$.

Since it is quadratic in K_1, there are two roots, K_2 and K_3, which are given by

$$K_2=\frac{G'+\sqrt{G'^2-4G\left(Q-K^2G\right)}}{2G},\ K_3=\frac{G'-\sqrt{G'^2-4G\left(Q-K^2G\right)}}{2G}, \tag{13}$$

subject to the condition

$$G'^2-4G\left(Q-K^2G\right)\geq 0, \tag{14}$$

where the elastic constant G is a function of x_3.

In view of (13), the displacement vector (6) becomes

$$\begin{aligned} &u_1=\left(A_1\mathrm{e}^{-K_2x_3}+A_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)}, u_3=\left(\frac{iK_2}{K}A_1\mathrm{e}^{-K_2x_3}+\frac{iK_3}{K}A_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)};\\ &w_1=\left(C_1\mathrm{e}^{-K_2x_3}+C_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)}, w_3=\left(\frac{iK_2}{K}C_1\mathrm{e}^{-K_2x_3}+\frac{iK_3}{K}C_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)}. \end{aligned} \tag{15}$$

The nonvanishing components of strain and stress are now given by

$$\begin{aligned} &\varepsilon_{11}=iK\left(A_1\mathrm{e}^{-K_2x_3}+A_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)};\\ &\varepsilon_{33}=-\frac{i}{K}\left(A_1K_2^2\mathrm{e}^{-K_2x_3}+A_2K_3^2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)};\\ &\varepsilon_{13}=-\left(K_2A_1\mathrm{e}^{-K_2x_3}+K_3A_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)}, \end{aligned} \tag{16}$$

$$\begin{aligned} &\sigma_{11}=\frac{i}{K}\left[\begin{array}{l}\left(\dfrac{2G\upsilon}{1-2\upsilon}\beta_1+2GK^2+\alpha^2M\beta_1\right)A_1\mathrm{e}^{-K_2x_3}+\alpha M\beta_1C_1\mathrm{e}^{-K_2x_3}\\ +\left(\dfrac{2G\upsilon}{1-2\upsilon}\beta_2+2GK^2+\alpha^2M\beta_2\right)A_2\mathrm{e}^{-K_3x_3}+\alpha M\beta_2C_2\mathrm{e}^{-K_3x_3}\end{array}\right]\mathrm{e}^{i(Kx_1-\omega t)};\\ &\sigma_{33}=\frac{i}{K}\left[\begin{array}{l}\left(\dfrac{2G\upsilon}{1-2\upsilon}\beta_1-2GK_2^2+\alpha^2M\beta_1\right)A_1\mathrm{e}^{-K_2x_3}+\alpha M\beta_1C_1\mathrm{e}^{-K_2x_3}\\ +\left(\dfrac{2G\upsilon}{1-2\upsilon}\beta_2-2GK_3^2+\alpha^2M\beta_2\right)A_2\mathrm{e}^{-K_3x_3}+\alpha M\beta_2C_2\mathrm{e}^{-K_3x_3}\end{array}\right]\mathrm{e}^{i(Kx_1-\omega t)};\\ &\sigma_{13}=-2G\left(K_2A_1\mathrm{e}^{-K_2x_3}+K_3A_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)};\\ &p=-\frac{i}{K}\left(\alpha M\beta_1A_1\mathrm{e}^{-K_2x_3}+\alpha M\beta_2A_2\mathrm{e}^{-K_3x_3}+M\beta_1C_1\mathrm{e}^{-K_2x_3}+M\beta_2C_2\mathrm{e}^{-K_3x_3}\right)\mathrm{e}^{i(Kx_1-\omega t)}, \end{aligned} \tag{17}$$

where $\beta_1 = K^2 - K_2^2$ and $\beta_2 = K^2 - K_3^2$.

For a Rayleigh wave in saturated soil, the boundary conditions of permeability to be satisfied by the stress on the stress-free boundary $x_3 = 0$ are

$$\sigma_{13} = 0,\ \sigma_{13} = 0,\ p = 0. \tag{18}$$

This would mean that

$$K_2 A_1 + K_3 A_2 = 0;$$
$$\left(\frac{2G\upsilon}{1-2\upsilon}\beta_1 - 2GK_2{}^2 + \alpha^2 M\beta_1\right)A_1 + \left(\frac{2G\upsilon}{1-2\upsilon}\beta_2 - 2GK_3{}^2 + \alpha^2 M\beta_2\right)A_2 + \alpha M\beta_1 C_1 + \alpha M\beta_2 C_2 = 0; \quad (19)$$
$$\alpha\beta_1 A_1 + \alpha\beta_2 A_2 + \beta_1 C_1 + \beta_2 C_2 = 0.$$

From Eqs. (19) we get the frequency equation of Rayleigh waves when the boundary conditions are permeable

$$2G\left(\frac{\upsilon}{1-2\upsilon}K^2 + \frac{1-\upsilon}{1-2\upsilon}K_2K_3\right)(K_2 - K_3) = 0. \quad (20)$$

For a Rayleigh wave in saturated soil, the boundary conditions of impermeabity to be satisfied by the stress on the stress-free boundary $x_3 = 0$ are

$$\sigma_{13} = 0,\ \sigma_{33} = 0,\ w_3 = 0. \quad (21)$$

This would mean that

$$K_2 A_1 + K_3 A_2 = 0;$$
$$\left(\frac{2G\upsilon}{1-2\upsilon}\beta_1 - 2GK_2{}^2 + \alpha^2 M\beta_1\right)A_1 + \left(\frac{2G\upsilon}{1-2\upsilon}\beta_2 - 2GK_3{}^2 + \alpha^2 M\beta_2\right)A_2 + \alpha M\beta_1 C_1 + \alpha M\beta_2 C_2 = 0; \quad (22)$$
$$K_2 C_1 + K_3 C_2 = 0.$$

As above, from Eqs. (22) we get the frequency equation of Rayleigh waves when the boundary conditions are impermeable:

$$\left[\left[\frac{2G\upsilon}{1-2\upsilon}\left(K^2 + K_2K_3\right) + 2GK_2K_3 + \alpha^2 M\left(K^2 + K_2K_3\right)\right]A_1 + \left(K^2 + K_2K_3\right)\alpha MC_1\right](K_3 - K_2) = 0. \quad (23)$$

To sum up, the frequency equation of Rayleigh waves, regardless of whether the surfaces are permeable or completely impermeable, is satisfied if and only if

$$K_2 = K_3 . \quad (24)$$

From Eqs. (24), (13), and (14) we get

$$K_2 = K_3 = \frac{G'}{2G}, \quad (25)$$

subject to the condition

$$G'^2 + 4G^2K^2 - 4GQ = 0. \quad (26)$$

If ρ is assumed constant, then (26) gets the solution

$$G = C_1 e^{2\sqrt{c-K^2}x_3}, \quad (27)$$

where $c = Q/G$.

The expressions for displacement, after taking into consideration (24), are given by

$$u_1 = Ae^{-K_2x_3}e^{i(Kx_1-\omega t)};\ \ u_3 = \frac{iK_2}{K}Ae^{-K_2x_3}e^{i(Kx_1-\omega t)}; \quad (28)$$
$$w_1 = Ce^{-K_2x_3}e^{i(Kx_1-\omega t)};\ w_3 = \frac{iK_2}{K}Ce^{-K_2x_3}e^{i(Kx_1-\omega t)}.$$

Expanding Eq. (28) in the complex exponential using Euler's formula and taking the real part, we obtain the final expression of displacement:

,

$$u_1 = A\mathrm{e}^{-K_2 x_3}\cos\left(Kx_1-\omega t\right), u_3 = \frac{K_2}{K}A\mathrm{e}^{-K_2 x_3}\sin\left(Kx_1-\omega t\right);$$
$$w_1 = C\mathrm{e}^{-K_2 x_3}\cos\left(Kx_1-\omega t\right), w_3 = \frac{K_2}{K}C\mathrm{e}^{-K_2 x_3}\sin\left(Kx_1-\omega t\right). \quad (29)$$

4. Conclusions

In this paper, based on Biot's theory of saturated porous media, the propagation of Rayleigh waves in nonhomogeneous saturated porous media is studied in which the variation of shear modulus is taken into account. We obtain the following conclusions:

1. Regardness of whether the surfaces are permeable or completely impermeable, the existence conditions of frequency equation Rayleigh waves in inhomogeneous saturated porous media is satisfied if and only if $K_2 = K_3$.

2. If ρ is assumed constant, the material elastic parameters shear modulus in a nonhomogeneous saturated porous medium is a function of depth. The expression of the shear modulus is calculated.

3. We obtained the final expression of the solid skeleton and fluid displacement in the nonhomogeneous saturated porous medium. From the final displacement expression, it is obvious that the path of any particle in the nonhomogeneous saturated porous medium is an ellipse, which agrees with the classical result. In addition, from the final displacement expression we see that the displacement attenuation factor in accordance with the distribution law of depth is closely related to the material elastic parameters and the frequency. With increase in depth the x_3 direction, the amplitude with the exponential $\mathrm{e}^{-K_2 x_3}$ is attenuated, and the energy of Rayleigh waves decreases rapidly.

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