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RHEOLOGICAL PROPERTIES OF SOIL SUBJECT TO SHEAR

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A rheologic equation is proposed for description of the shear deformations of an incompletely saturated clayey soil; the equation was derived on the basis of modification of Maxwell's rheologic model. It is demonstrated that the proposed equation describes creep, relaxation, and kinetic shear for the same parameters, including transient, steady-state, and progressive creep as a function of tangential-stress intensity.

Investigations of clayey soils subject to shear [1-16] indicate that in generalized form, the rheologic curves can be represented by creep curves under a static load (τ = const) (Fig. 1, a), by $\tau(t) = f(\dot{\gamma}, \sigma)$ curves under a kinematic loading regime ($\dot{\gamma} = \text{const}, \ \dot{\varepsilon}_1 = \text{const}$) (Fig. 1, b), and by $\tau(t) = f(y_0, \sigma)$ stress-relaxation curves for an assigned fixed deformation ($y(0)$ = const, $\tau(t) \neq$ const) (Fig. 1, c), where τ and σ are the tangential and compressive stresses, γ and $\dot{\gamma}$ are the deformation and its rate, v is the rate of angular deformation, and t is time.

In [1-16], these curves are described for each case by empirical relationships based on rheologic models.

In this study, an equation is proposed for description of the shear deformations of a clayey soil exhibiting clearly expressed rheologic properties; it is derived on the basis of modification of Maxwell's rheologic model in which the threshold of creep τ^*, and the hardening and loosening during shear are defined more precisely as a function of the accumulated shear strain (Fig. 2).

The idea of simultaneous hardening and loosening of a clayey soil during deformation has been repeatedly utilized by S. S. Vyalov, M. N. Gol'dshtein, and G. I Ter-Stepanyan, and has acquired experimental confirmation in [1-5].

In codifying these investigations, Vyalov [1] noted that creep of soils is accompanied by mutually opposing hardening and stratification phenomena of the soil. If hardening predominates, deformations will attenuate, and if stratification predominates, non-attenuating creep resulting in failure of the soil will develop in the latter. Studying the kinetics of the deformations and structural changes, Vyalov developed a kinematic theory of the strength and creep of soil on the basis of Ya. I. Frekel's molecular theory of flow.

The equation that we are proposing applies to the theory of flow, when the strain rate is summed from the elastic $\dot{\gamma}^e$ and viscoplastic $\dot{\gamma}^{vp}$ strain rates $\dot{\gamma} = \dot{\gamma}^e + \dot{\gamma}^{vp}$

$$\dot{\gamma} = \frac{\tau - \tau^*}{\eta(t)} \left(\frac{e^{\alpha\gamma}}{a} + \frac{e^{-\beta\gamma}}{b} \right) + \frac{\dot{\tau}}{G}, \tag{1}$$

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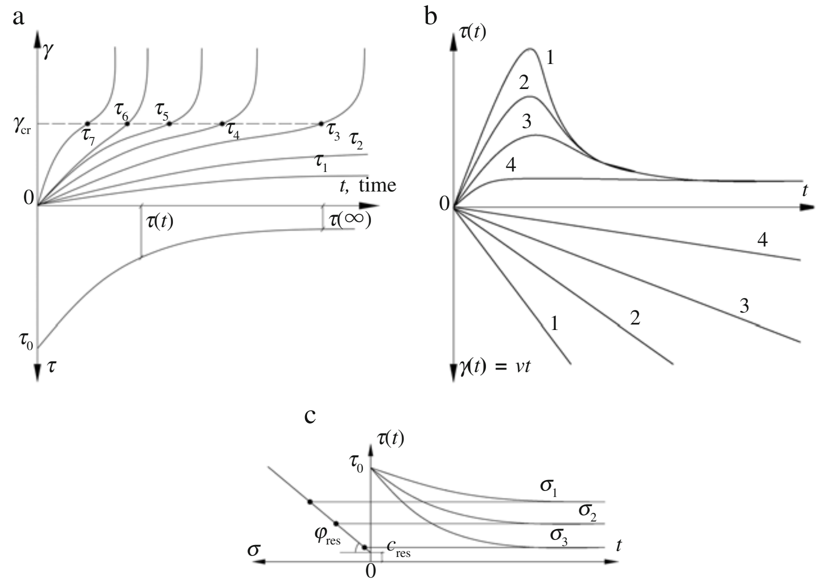

Fig. 1. General appearance of rheologic curves: a) creep and long-term strength ($\tau_1 < \tau_2 \ldots < \tau_7$ are critical τ values when γ_{cr} = const); b) tangential stresses $\tau(t)$ for different constant shear-strain rates ($\dot{\gamma}$ = const); here, $\dot{\gamma}_1 > \dot{\gamma}_2 > \dot{\gamma}_3 > \dot{\gamma}_4$; c) relaxation of shear stresses for different σ values (on right), and limiting line of residual (long-term) shear strength (on left).

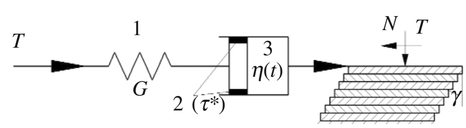

Fig. 2. Maxwell's rheologic model with consideration of hardening, loosening, and structural shear strength: 1) elastic element; 2) structural strength; 3) viscous element.

where a, b, α, and β are hardening and loosening parameters, G is the shear modulus, and τ^* is the creep threshold

$$\tau^* = \sigma' \tan \varphi + c(t), \tag{2}$$

where σ' is the effective stress, and $c(t)$ is the time-dependent cohesion.

Equation (1) is written similarly for triaxial compression, if the subscript i is added to all parameters, denoting conversion to strain rates γ_i, and shear stresses τ_i and τ_i^*.

Let us examine the rheologic processes on the basis of (1).

Creep and long-term strength

For a constant coefficient of cohesion ($c(t)$ = const) and dilatational strain, analysis of Eq. (1) indicated that the critical value of the angular strain γ_{cr}, which is obtained from the condition of constancy of acceleration $\ddot{\gamma}$ = 0, are constant at the deflection points on the creep curves (see Fig,. 1, a), upper portion) and are defined by the equation

$$\gamma_{cr} = \frac{1}{\alpha + \beta} \ln \frac{a\beta}{\alpha b} = \text{const}, \tag{3}$$

and the corresponding critical stresses τ_{cr} and times t_{cr} will depend on τ and γ_{cr}, i.e., $\tau_{cr} = f(\tau, \gamma_{cr})$ and $t_{cr} = f(\tau, \gamma_{cr})$.

The time t_{cr} required for development of a deflection point on the creep curve can be determined from the intersection of the lines $\gamma(t)$ and γ_{cr} = const (see Fig. 1). Consequently, each τ will have its corresponding τ_{cr} and t_{cr}.

Using (1) and (3), the parameters τ_0 and τ_∞ on the long-term-strength curve can therefore be determined on the basis of the parameters obtained from the creep curve (see Fig. 1, a, lower portion).

Equation (1) can be used to analyze results of soil tests conducted under laboratory conditions. By analogy with (1), the equation

$$\dot{\gamma} = \frac{\tau - \tau^*}{\eta} \left(\frac{e^{\alpha_1 t}}{a_1} + \frac{e^{-\beta_1 t}}{b_1} \right) + \frac{\dot{\tau}}{G} \tag{4}$$

can be used to describe creep processes in a soil mass.

When τ = const,

$$\dot{\gamma} = \frac{\tau - \tau^*}{\eta} \left(\frac{e^{\alpha_1 t}}{a_1} + \frac{e^{-\beta_1 t}}{b_1} \right). \tag{5}$$

Solution of (5) can be represented as

$$\gamma(t) = \frac{\tau - \tau^*}{\eta} \left(\frac{e^{\alpha_1 t}}{\alpha_1 a_1} - \frac{e^{-\beta_1 t}}{\beta_1 b_1} \right), \tag{6}$$

where a_1, α_1, b_1, γ_1, and β_1 are time-dependent hardening and softening parameters, and η is the viscosity.

Calculations performed on the basis of (5) indicate that the expression $\gamma(t)$ has a double curvature as in the case of (1), i.e., describes attenuating, non-attenuating, and progressive creep as a function of the level of stress τ, and the parameters a_1, b_1, α_1, and β_1 (Fig. 3).This result is dictated by the difference in the exponential functions within the parentheses in (4), the first of which describes hardening, and the second loosening.

Consequently, Eqs. (1) and (5) are analogous, since their solution leads to the same results.

For use of (5) in the solutions of boundary problems, it is necessary to determine the parameters a_1, b_1, α_1, and β_1 from experiments, which may differ from the parameters in (1).

Kinematic shear

Deviator loading of a specimen after hydrostatic compression at a constant axial-strain rate $\dot{\varepsilon}_1$ = const is one of the most widely utilized triaxial tests of soils. For simple shear (distortion) and a kinematic loading regime ($\dot{\gamma}$ = const), Eq. (1) assumes the form

$$\dot{\gamma} = \frac{\tau - \tau^*}{\eta} \left(\frac{e^{\alpha v t}}{a} + \frac{e^{-\beta v t}}{b} \right) + \frac{\dot{\tau}}{G}, \tag{7}$$

where v is the angular-strain rate $\dot{\gamma} = v$ = const.

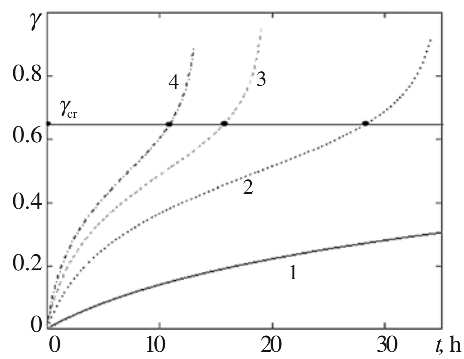

Fig. 3. $\gamma - t$ curves for clayey soil subject to various tangential stresses τ (1-4) under simple shear from (1) for assigned parameters α, β, a, b, and η, and when $\tau > \tau^*$.

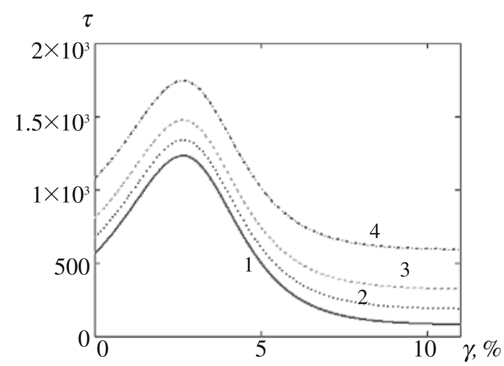

Fig. 4. τ (kPa) $-\gamma$ (%) curves in kinematic loading regime ($\dot{\gamma}$ = const) for different hardening loads σ (1-4).

After transforming (7), we have:

$$\dot{\tau} + \frac{\tau G}{\eta}\left(\frac{e^{\alpha v t}}{a} + \frac{e^{-\beta v t}}{b}\right) = v\eta G + \frac{\tau^* G}{\eta}\left(\frac{e^{\alpha v t}}{a} + \frac{e^{-\beta v t}}{b}\right). \tag{8}$$

Solution (8), which is obtained with use of the MathCad software package for different shear-strain rates $\dot{\gamma}_1, \dot{\gamma}_2 \ldots \dot{\gamma}_n$, makes it possible to construct a family of $\tau(t)-\gamma$ curves (Fig. 4). Analyses indicated that they have extrema corresponding to the maximum τ_{max}, and minimum τ_{min} in a characteristic time t_{cr} = const, and a common asymptote. It is obvious that $\tau_{max}(\sigma)$ and $\tau_{min}(\sigma)$ curves can be constructed from these curves when the assigned $\dot{\gamma}$ = const.

Stress relaxation

Equation (1) describes the stress relaxation when $\dot{\gamma} = 0$, i.e., when $\gamma(t) = \gamma(0)$ = const, and the initial stress $\tau_0 > \tau^*$. In that case, solution (1) takes on the form

$$\tau(t) = \tau_{res}(1 - e^{-At}) + \tau_0 e^{-At}, \tag{9}$$

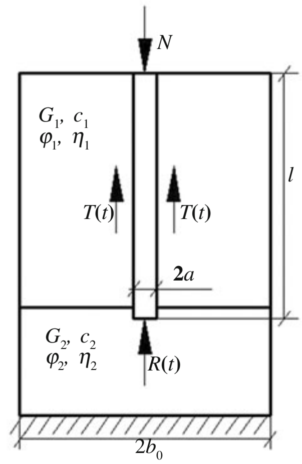

Fig. 5. Computational diagram of interaction between pile and two-layer soil mass: G, φ, c, and η are, respectively, deformation, strength, and viscosity parameters of soils.

where $A = \dfrac{G}{\eta}\left(\dfrac{e^{\alpha\gamma_0}}{a} + \dfrac{e^{-\beta\gamma_0}}{b}\right)$, and $\tau_{\text{res}} = f(\sigma)$ is the residual strength. (10)

The limiting line of residual strength can be determined from the stress-relaxation curve for different compressive stresses σ (see Fig. 1, c, left side).

Solution of some problems of applied soil mechanics

When a pile interacts with the surrounding soil exhibiting rheologic properties, the problem reduces to determination of the distribution law governing the constant force N on the pile between the resistance against the lateral surface $T(t)$ and under the lower end of the pile $R(t)$ (Fig. 5), whereupon

$$N = T(t) + R(t), \tag{11}$$

where $N = \pi a_0^2 p_1$; $T = 2\pi a_0^2 l$; $R = \pi a_0^2 p_2$; a_0 and b_0 are the radii of the pile and its zone of influence, l is the length of the pile, and p_1 and p_2 are the stresses at the level of the head and beneath its heel.

To solve this problem, the settlements of the pile due to the effect of forces $T(t)$ and $R(t)$ should be determined, and equated, assuming that the compression modulus E_p of the pile is much greater than the compression modulus of the surrounding soil E_s, i.e., $E_p > E_s$. Let us examine different cases for a two-layer mass, the upper layer of which exhibits type-(4) elastoviscous properties, and the lower layer elastoplastic and viscous properties.

The settlement rate of the pile under the action of friction force $T(t)$ can be determined from the solution that we have obtained on the assumption of a shear mechanism for displacement of the soils around the pile, and with disregard of dilatational strains [11]. When $\tau^* = 0$, we have

$$\dot{S}_T = \frac{a\tau_a}{\eta_1(t)}\ln(b_0/a_0) + \frac{a_0\dot{\tau}_a}{G_1(t)}\ln(b_0/a_0), \tag{12}$$

where $\dot{S}_T$ is the settlement rate of the pile, $\tau_0 = T/2\pi al$, $\dot{\tau}_a$ is the rate of change in τ_a, and,

$$\eta_1(t) = \eta_1 \Bigg/ \left(\frac{e^{\alpha_1 t}}{a_1} + \frac{e^{-\beta_1 t}}{b_1} \right). \tag{13}$$

The settlement rate of the pile due to the force $R(t)$ can be determined from the solution concerning the impression of a circular rigid plate in an elastic medium

$$\dot{S}_T = \dot{p}_2 \frac{\pi a_0 (1-\nu_2) K_1}{4G_2}, \tag{14}$$

where $K_1 \le 1$ is a coefficient that takes into account the depth of load application on the plate, and p_1 and $\dot{p}_2$ are the load on the plate and its rate of change, respectively.

Comparing (12) and (14) with consideration of (11), we obtain

$$\frac{a_0^2 (p_1 - p_2)}{2l\eta_1(t)} \ln(b_0/a_0) - \dot{p}_2 \frac{a_0^2 \ln(b_0/a_0)}{2lG_1} = \dot{p}_2 \frac{\pi a (1-\nu^2) K_1}{4G_2}. \tag{15}$$

After certain transformations, we have

$$\dot{p}_2 + p_2 P(t) = p_1 Q(t), \tag{16}$$

where

$$\begin{aligned} &P(t) = \frac{B(t)}{A};\ Q(t) = \frac{D(t)}{A}; \\ &A = \frac{\pi(1-\nu^2)K_1}{4G_2} + \frac{a_0}{2l} \frac{\ln(b_0/a_0)}{G_1}; \\ &B(t) = \frac{a_0}{2l} \frac{\ln(b_0/a_0)}{\eta_1(t)};\ D(t) = \frac{a_0}{2l} \frac{p_1 \ln(b_0/a_0)}{\eta_1(t)}. \end{aligned} \tag{17}$$

Solution (16) with the initial condition $p_2(0) = 0$ (obtained with use of the MathCad software package) indicated that p_2 varies over time at a different rate, and tends to constant values (Fig. 6). The settlement of the pile can also be determined from (14), substituting $p_2(t)$ for $\dot{p}_2(t)$. It is obvious that $S(t) \to S_\infty = \text{const}$.

The settlement rate of the bed soils beneath the heel of the pile can be determined, assuming an elastoplastic bed in first approximation

$$\dot{S}_r = \dot{p}_2 \frac{\pi a (1-\nu_2)}{4G_2} \frac{p_2^*}{p_2^* - p_2}, \tag{18}$$

where p_2^* is the limiting load on the bed soil, as determined from [11].

It follows from (1) that when $p_2 \to p_2^*$, $\dot{S} \to \infty$.

Comparing (4) with (18), and considering (11), we obtain the nonlinear differential equation in terms of p_2:

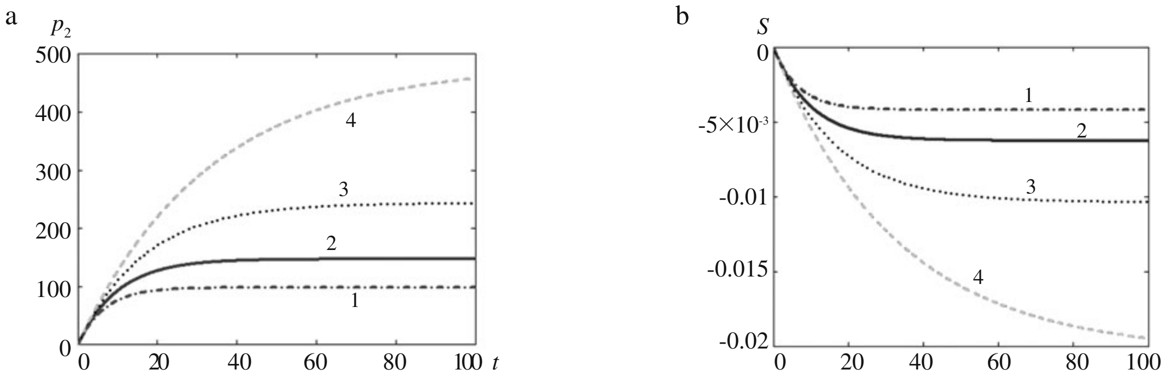

Fig. 6. Plots showing dependencies of p_2 on t (a), and S on t (b) based on (16) and (14), respectively, for assigned parameters 1, 2, 3, and 4, which are introduced to (15).

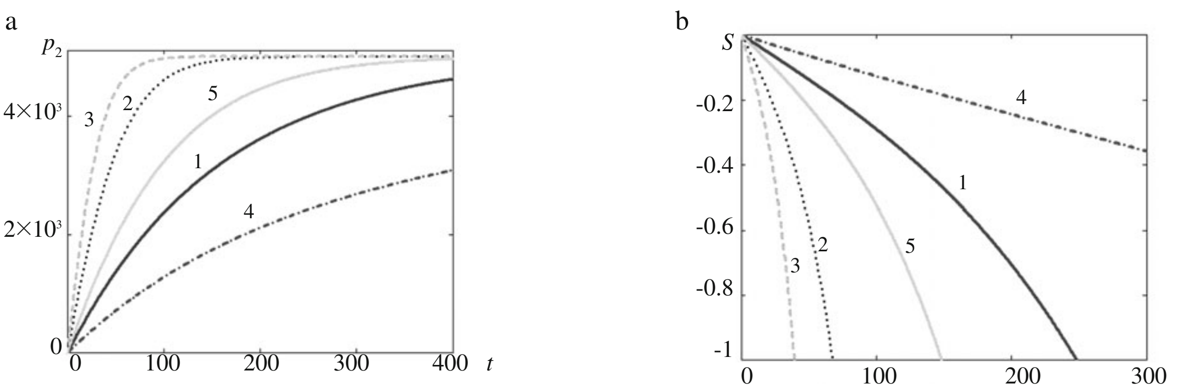

Fig. 7. Plots showing dependence of p_2 on t (a) based on (19), and S on t (b) based on (18) for different viscosity and elasticity parameters of soil around pile and elastoplastic properties under heel of pile (1-4).

$$\frac{a_0^2(p_1-p_2)}{2l\eta_1(t)}\ln\left(b_0/a_0\right)-\dot{p}_2\frac{a_0^2\ln(b_0/a_0)}{2lG_1}=\dot{p}_2\frac{\pi a(1-\nu^2)}{4G_2}\frac{p_2^*}{p_2^*-p_2}. \tag{19}$$

In solving (19), it was established that p_2 evolves over time at a different attenuating rate, and tends to constant values (Fig. 7), while in contrast to Fig. 6, b, the settlement develops at an attenuating and non-attenuating rate, depending on the intensity of the applied load $p_1 = N/\pi a^2$ (Fig. 7, b).

Basic conclusions

1. A rheologic equation in which the hardening and softening of a clayey soil are defined more precisely is proposed for description of shear deformations on the basis of a modified Maxwell model.

2. Analysis of the equation indicated that for a constant load, it describes attenuating, non-attenuating, and progressive creep of soils, and also processes of stress relaxation and shear deformation in a kinematic loading regime.

3. When the equation is used in the problem of interaction between a pile and the surrounding soil, distribution of the forces on the pile between the lateral surface and under its lower end will occur over time, and may lead to attenuation, or non-attenuation of the pile's settlement, depending on the load applied and the parameters of the soil around the pile and under its heel.

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