Soil Mechanics and Foundation Engineering, Vol. 46, No. 5, 2009

DISCUSSION OF CONSTRUCTION RULES AND REGULATIONS

EXPERIMENTAL AND THEORETICAL RESEARCH ON ANALYTICAL MODELS OF PILED-RAFT FOUNDATIONS

UDC 624.131:624.159

V. A. Barvashov and G. G. Boldyrev
Scientific-Research Institute of Foundations and Underground Structures, Moscow, Russia; and, Penza State University of Architecture and Construction, Penza, Russia.

Several methods for the design of pile foundations, including those recommended in Building Code 50-102-2003 [1], are analyzed,. Tests of a model of a pile foundation in a pan with freeze-frame photographs of the displacements of sand grains, and their digital processing by the method of particle imaging velocimetry (PIV) are described. A method of analyzing a rigid spatial pile foundation with "high-" and "low-profile" rafts of arbitrary planform, which makes it possible to obtain a uniform load distribution on the piles without use of finite-element software is proposed on the basis of experimental results. The method is implemented in the form of design software in the MathCad system. Examples of the analysis are presented.

Inaccuracy and Suitability of Analytical Models (AM)

"*All models are inaccurate, but some of them are useful," or "... all models are inaccurate; the practical question is how inaccurate should they be in order to be useful*" – these are oft-quoted statements of the familiar British mathematician George E. P. Box [2], V. Pareto's's principle "*there are few significant factors, but trivial factors abound" ("principle 20/80"*) has pointed the way to more precise definition of AM: significant factors (20%) should be evaluated possibly more accurately, and the inaccurate ones (80%) with a much lower accuracy. To evaluate the importance of some factor, it must be determined just how *sensitive* the modeling system (in geotechnics, this is the "bed-foundation-structure") is to variations in this factor [3].

These statements are critical to geotechnics, where building/structure settlements calculated in accordance with recommendations set forth in standard documents may be two-three times lower than actual values [4], and a deviation of 1.5 times is considered successful, and where conservative design solutions predominate; foundation failures are therefore extremely rare. Conservatism and wastefulness, however, are not always identified with enhanced reliability, and are not a replacement for scientific-based search for optimization of design solutions.

In this connection, let us examine three AM of pile foundations, which are recommended in [1], from the standpoint of *inaccuracy, usefulness, and importance* of assumptions adopted, and the *sensitivity* of the "bed-foundation-structure" system.

Methods for Analysis of Pile Foundations [1 and 5]

The AM of a "conventional" foundation is *inaccurate, but is widely used* in design. The *inaccuracy* consists in the fact that there is no finite compressible stratum H under the tips of the piles, but

Translated from Osnovaniya, Fundamenty i Mekhanika Gruntov, No. 5, pp. 32-39, September-October, 2009.
0038-0741/09/4605-0207

this AM *is calibrated* (extremely approximately) for analysis of settlements based on data derived from numerous observations of the settlement of structures, and is widely used for design analyses. As for a foundation on the natural bed, H is determined from *actual* contact stresses along the lower surface of the foundation (for a pile foundation, the lower surface implies the ends of the piles) in accordance with recommendations given in [5]. This means that in determining H, the weight of the *interpile soil* should be considered in the weight of the conventional foundation; this results in the following *paradox*: H, the weight of the conventional foundation, and the settlements increase with increasing pile length. Ultimately, this paradox can be avoided, if the weight of the interpile soil is disregarded in the determination of H; this, however, contradicts recommendations [5], and, moreover, may even yield $H \leqslant 0$. The condition $H \geqslant 4 + 0.1b$, which is close to the recommendations outlined in Construction Rule and Regulation 2.02-01-83 [6] with respect to selection of the thickness of the compressible layer for sandy soils, is therefore artificially introduced in [5]. Let us point out that in practice, it is precisely the recommendations in [6], and not [5], that are more frequently used for determination of H, especially for lengthy pile foundations.

Methods for calculating the settlement of a pile foundation with consideration of the mutual effect of piles in a group and piled-raft foundations [1] are nearly indistinguishable, and use the same Table 7.19, which contains 120 values of the coefficient $R_s = R(n, l/d, \lambda, a/d)$ in 10 rows, where n is the number of piles, l/d is the ratio of the length of the pile to its diameter, λ is the ratio of the elastic modulus of the concrete to the compression modulus of the soil, and a/d is the ratio of the distance between the axes of the piles to their diameter. As a matter of fact, this table unifies three different tables for three pairs of values (λ, l/d). The table on the left ($\lambda = 100$) is apparently compiled for *soil-cement* piles, and that on the right ($\lambda = 10{,}000$) for *steel* piles (not tubular!).

It is easy to convince oneself that the interpolation formula $R_s(n) = 0.5R_s(100)\log n$ in this table is correct for any n; only *one of the ten rows* for $n = 100$, which contains 12 figures, is therefore sufficient.

The middle table ($\lambda = 1{,}000$) corresponds to reinforced-concrete piles, i.e., a *total of four useful* values in the table for the four a/d values between which it is possible to interpolate. Only one pair of parameter values is given: $\lambda = 1{,}000$ and $l/d = 25$, i.e., the interpolation of R_s for other pairs of λ and l/d values is impossible.

A lack of recommendations for determination of the average compression modulus E_s of the soil "at a depth to B," equal to the width of the slab, renders this method of analysis *unsuitable*, since E_s can be obtained by subjective averaging; this is as much as is desired. In Section 7.4.13, therefore, it is recommended that a conventional foundation be analyzed for verification. *But which of the two results obtained is correct?*

In Section 7.4.14 of [1], loads on the edge and corner piles are designated conditional: $P_k = 2P_{\text{avg}}$ and $P_y = 3P_{\text{avg}}$, respectively, where P_{avg} is the average load on a pile. Firstly, however, P_k and P_y should not exceed the limiting load on a pile, and, secondly, it is very critical that the equilibrium conditions of the slab be fulfilled. A paradoxical situation may then also arise as, for example, for a 7×7 group of piles, where the total load on the foundation is $49P_{\text{avg}}$, but according to the above-indicated recommendations, approaches $60P_{\text{avg}}$ on the edge and corner piles, i.e., each internal pile should take up a *pull-out* (!) load of $0.3P_{\text{avg}}$ for fulfillment of equilibrium conditions.

Mathematical Modeling of Piled-Raft Foundations (PRF). The dependence of the stiffness K_{pr} of a PRF on the stiffness of the piles k_p and raft k_r, which are defined as the ratio of the load P on an element to its settlement s (kN/m) is examined in a number of publications. P is that load applied to the pile, and $P = pA$ to the raft, where p is the average load on a pile, which is distributed over the area A that covers this pile.

Clancy and Randolph [7], and Randolph [8] derived a number of empirical formulas linking K_{pr}, k_p, and k_r, and indicated that for large groups of piles, irrespective of their spacing and stiffness, the empirical formula

$$K_{pr} = \frac{1 - 0.6(k_r / k_p)}{1 - 0.64(k_r / k_p)} k_p,$$

from which it follows that $K_{pr} \sim k_p$ is acceptable, i.e., the stiffness of the PRF approaches the stiffness of the piles, and the role of the stiffness of the raft is *insignificant.*

These authors have also derived a simple formula for the coefficient α_{rp} of mutual pile interaction, and a formula for the ratio of the load that is passed onto the raft and piles.

Similar studies were conducted by Fedorovskii et al. [9], who, using the PLAXIS 2D software package, modeled PRF (low-profile raft) under conditions of the axisymmetric and plane problems, including the core of a raft of unlimited dimensions in plan. In all cases, the compressible layer below the tips of the piles was restricted to the same arbitrarily selected depth of 10 m. Other depths were not permitted. Fedorovskii's results [9] are presented in the form of numerical tables, on the basis of which qualitative conclusions are drawn. The question that arises as to what will the distribution of the load be between the raft and piles, if a compressible layer of another thickness is selected? Well then, the thicknesses of the raft and piles are brought together when this thickness is increased, but, if this thickness is reduced, the stiffness of the piles will be appreciably increased as compared with that of the raft.

It is therefore possible to distinguish the results obtained in [9] from those in [7, 8], and the formula cited above for K_{pr} from [8] will actually "work."

The need to account for "punching-through" of piles, which is of particular import for "rare" pile-pedestals, which are supported on a rigid sublayer, is emphasized in [9]. As is indicated below, it is better to evaluate the distribution of the load between raft and piles differently than in [9]: separate the entire compressible stratum into a comparatively thin layer beneath the ends of the piles - "sensitive" to their punch-through, and the underlying layer, at the depth of which the load due to the PRF is close to a smooth distributed load.

Consideration of "punch-through" will *significantly change the load distribution* on the piles in a group, especially on the *edge and corner* piles, which are overloaded, and may therefore "fall-through" - subside with no increase in their load, i.e., go over into the limiting state with respect to the soil, if they do not fail with respect to material.

These piles are called "creep" piles. The "creep" of the edge and corner piles alters *quite significantly* the load distribution on all piles under the raft. This can be considered only under conditions of the three-dimensional problem, for example, using the PLAXIS 3D software, which is extremely labor-intensive. A comparatively simple method that makes it possible to solve this three-dimensional problem for high- and low-profile rafts with allowance for the load distribution on the raft and piles, "punch-through" of the piles, and the limiting state of the edge and corner piles is described below.

Note that foundations in which some or all of the piles are "creep"piles may be the optimal solution that permits maximum reduction in the number of piles, using them for partial unloading of the compressible soil bed beneath PRF. Buildings constructed on such foundations are known [10].

Analysis of Load Distribution on Pile Foundation with Consideration of Mutual Effect of the Piles One on the Other

It is known that a *loaded pile* will affect an unloaded pile, causing its settlement S_0, which diminishes with increasing distance r between piles. This effect can be represented as being dependent on r, solving the elastic three-dimensional (3D) problem for the two cases, which are reduced to the axisymmetric (2D) problem, if it is permitted that displacement field $S(r, z)$ of the soil around a single loaded pile within the limits of its length can be described as "telescopic shear." This means that the vertical displacements of the soil $S(r, z)$ at points lying on a cylindrical surface coaxial with the piles ($r = R = \text{const}$) are similar over the length of the pile. The assumption concerning "telescopic shear" is approximate, but for analysis of the pile foundation as a whole, the error generated is completely acceptable with appropriate calibration.

The hypothesis concerning "telescopic shear" permits determination of the settlement of a pile field loaded by an arbitrary system of vertical forces as the sum of their mutual effect one with another through the soil. This approach was used in [11-15].

For a field of piles of similar length, which are arranged in a regular grid, summing can be replaced by integration over the area, and the pile field represented in the form of a three-parametric contact model of a PSG consisting of a two-parametric Pasternak and Filonenko-Borodich model covered by a Winkler layer ([16]); this substantially simplifies the analysis. This model is acceptable for a foundation with a high-profile raft of any planform. The following statement in [9] is therefore *incorrect*: "... the *method is good* for small pile groups, but does not work for large groups (fields) ... ."

This statement in [9] most likely follows from the incorrect recommendation in "Design guidelines for pile foundations" [17], which is based on Fedorovskii's logarithmic formula from [1] for the settlement of a single pile. When the mutual effect of the piles one on the other is determined on the basis of this formula, it is found that starting from a certain distance, the unloaded piles are "lifted" due to the effect of the loaded pile, whereupon this "uplift" increases without restriction with increasing distance between piles. In [17], therefore, the condition is also introduced: the piles will not influence one another, if the distance between them is such that the indicated logarithm is negative. This constraint does not, however, have a physical basis – the *logarithmic function cannot be calibrated* for approximation of the *mutual effect* of remote piles one from the other, and the *statement in [9] is incorrect*; this method, which is based on the hypothesis of telescopic shear "... does not work for large groups (fields) ... ."

In [16[, this logarithmic function was, even prior to the appearance of "Guidelines [17]," approximated by a MacDonald function, which attenuates exponentially to infinity; this made it possible to account for the mutual effect of the piles at any distances, and represent the pile field as a PSG contact model [16], the parameters of which depend on the parameters of the pile and soil.

For a *low-profile raft*, the assumption of telescopic shear is unacceptable, since the pressure of the raft on the interpile soil sharply distorts the pattern of telescopic shear of the interpile soils [9].

A pile foundation with a *low-profile raft* with allowance for the mutual effect of the piles through the soil can be analyzed differently. For this purpose, let us examine the behavior of a single pile, using the approximate Fedorovskii formula [5], which is based on *good* approximation of the function of the settlements of an elastic pile that cuts through the upper layer of soil, and is supported on the lower layer under an axial load.

This formula can be generalized in the case of a pile in a low-profile raft by introducing the condition whereby there are no tangential stresses along the side of the pile in its upper section, where it is deformed like a free rod under an axial load, similarly to what was done in [18, 19].

In this statement, the settlement of the pile under a unit axial load

$$w_0(t) = \frac{1}{L}\left[\frac{\beta(1-t)}{G_1} + \frac{t}{E_p F}\right], \tag{1}$$

where L is the length of the pile, E_p is the elastic modulus of the pile material, and F is the cross-sectional area of the pile.

When t = 0, formula (1) yields the settlement of a single pile, or a pile under a high-profile raft due to a unit load. And, for a pile under a low-profile raft, it is possible to adopt the widely used assumption [20]: t = 2/3.

The following assumptions are introduced from what has been indicated above:

1) in the comparatively "thin" layer being "punched-through" (PTL), the soil under the tips of the piles is deformed just as under single piles; and,

2) the piles influence one another primarily through the soil below the PTL, as is noted in many applications, for example, in [18-21].

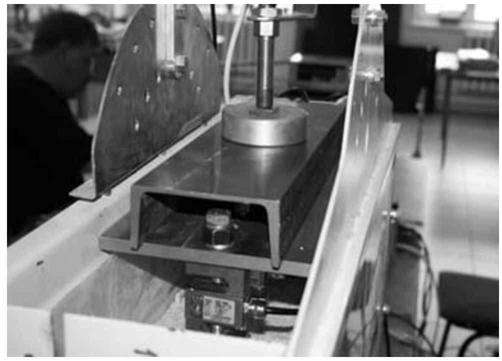

Fig. 1. Loading device of experimental unit.

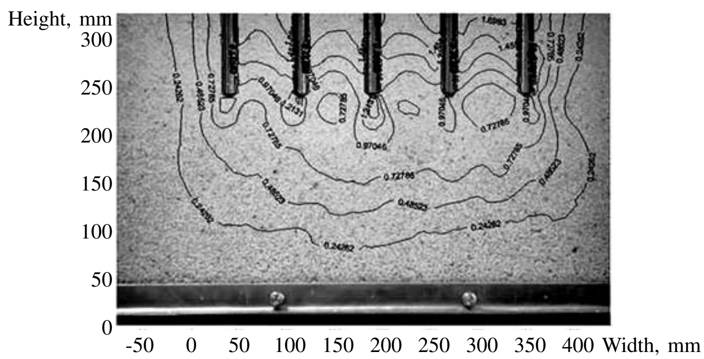

Fig. 2. Isolines of vertical soil displacements.

Physical and numerical experiments were performed to confirm these assumptions and quantitative assessment of the thickness of the "thin" layer.

Experiments in Sand Tray

The experiments were conducted in a 71×55×20-cm tray filled with pure quartz sand with a grain size ranging from 0.8 to 2.0 mm. One of the long vertical walls of the tray was transparent so as to permit freeze-frame photography of the displacements of the sand grains by the method of particle image velocimetry – PIV [20]. The piles were simulated by steel rods with a diameter d = 1 cm and length L = 20 cm, which were arranged in two rows with a spacing of 6 cm (6d). A load was transmitted through the model of a low-profile raft – steel raft, upon which a steel channel was placed to increase the bending stiffness of the raft (Fig. 1). A load of 1,440 N was applied centrally.

Computer processing of the photographs produced a digital field of sand-grain displacements in the form of fields of isolines, and/or a plot of the displacements of the soil at various depths below the tip of the piles (Figs. 2 and 3).

Despite inevitable scatter, qualitative trends are clearly visible in Figs. 2 and 3: the plot of soil displacements at a depth of 2d beneath the tips of the piles "senses" their effect, while at a depth of 4d and lower, this "sensitivity" drops off – the plots are smoothed (if the scatter is disregarded). At a depth of 2d, the settlements of the soil under the tips of the piles is 1.5-2.0 times higher than the displacements of the inter-pile soil, which remain virtually constant to a depth of 6d, and gradually attenuate only below this depth.

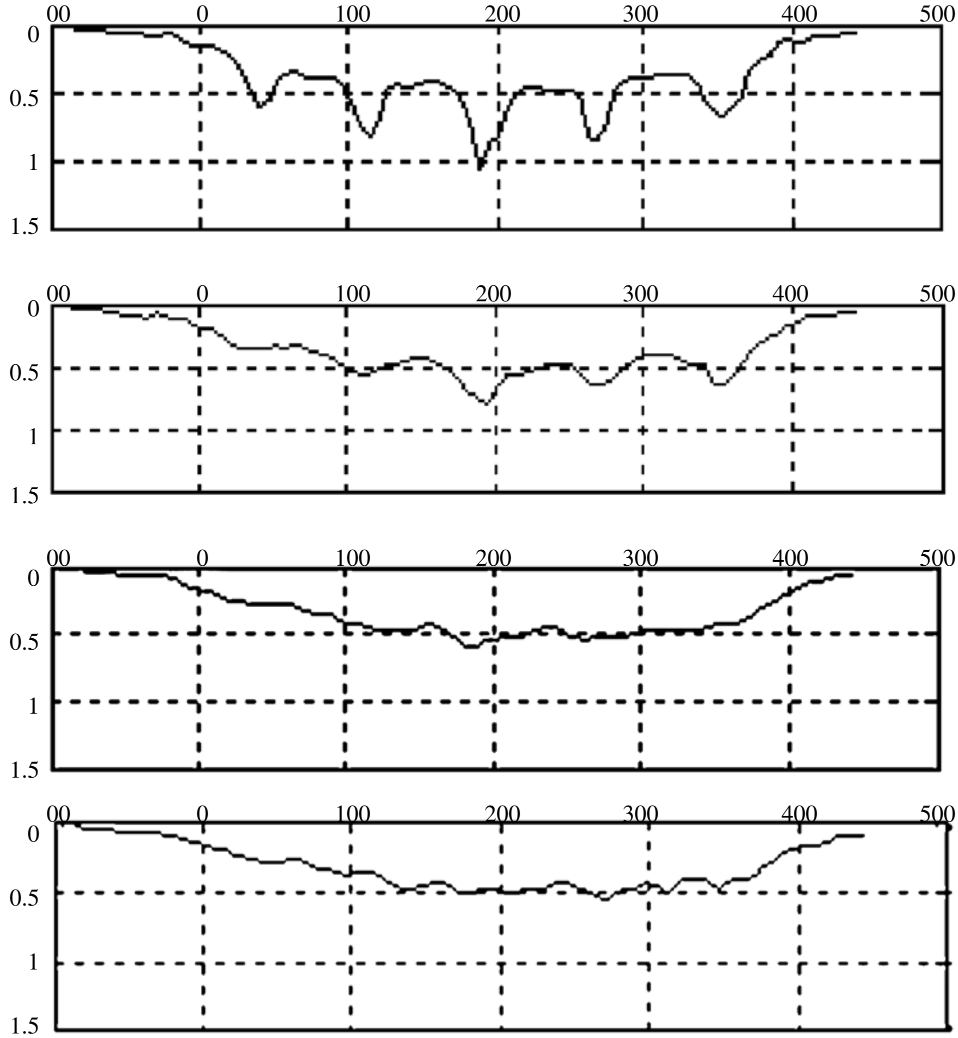

Fig. 3. Vertical displacements of soil (mm) beneath tips of piles at depths of 0, 2, 4, and 6 cm (0, 2, 4, and 6d)

One can assume that in the initial stages of loading, the model functions as a "high-profile raft - the piles punch-through the soil, and then as a "low-profile" raft. The central pile was loaded more heavily than the others. This can be explained by the fact that the raft has a finite stiffness (ergo, the deflection is in the center), and the applied load was close to the limiting value. The last conclusion agrees with results of investigations of the bearing capacity of pile foundations on small models [22].

In any case, it is apparent that "punch-through" is essentially completed at depths of 2d-4d.

Mathematical Modeling (MM) of "Punch-Through"

The goal of MM is to evaluate the depth of the upper "punch-through layer" (PTL) beneath the tips of the piles, which "senses" the discreteness of the piles. Below the PTL, the settlements of the soil are smoothed over, and they can be determined by replacing the effect of the piles by the effect of an equivalent system of concentrated forces applied to the surface of the bed; this corresponds to the St. Venant principle (1855): "*a balanced system of forces applied to some part of a solid induces in the latter stresses that diminish rapidly with distance from this part, and can be replaced by an equivalent system of forces*."

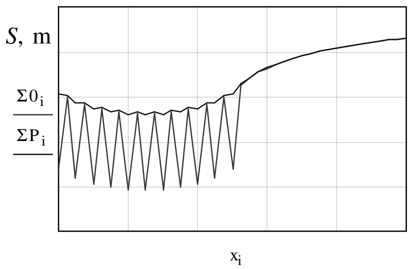

Fig. 4. Curves of vertical displacements of soils beneath ends of piles at different depths (MM was performed using MathCad software).

Let us denote the thickness of the compressible (linearly deformable) layer (CL) under the tips [1, 5] of the piles by H, and the depth of the PTL by $h < H$. The piles can be replaced by concentrated forces, and the settlement of the group of equally loaded piles will then be

$$S(x, y, h) = \sum_{j=1}^{N} P_j [s(x-\xi, y-\eta, h)- \\ -s(x-\xi, y-\eta, H)], \tag{2}$$

where $s(x - \xi_j, y - \eta_j, z)$ is the vertical displacement of the elastic bed at point (x, y, z) of the elastic half space due to a single unit concentrated force P_j applied at point $(\xi_j, \eta_j, 0)$, and N is the number of these forces. The depth of the PTL was determined from the condition whereby the settlement curve is virtually smooth.

Figure 4 shows two settlement curves beneath the central row of the system of concentrated 10 × 10 forces applied to the surface of an elastic half space with a spacing of 2 m. A curve with pronounced *fluctuations* was obtained at a depth of 0.4 m, and a *smooth curve* (variation of less than 2%) at 0.95 m. The curves coincide beyond the limits of loading. According to the St. Venant principle, therefore, settlements can be calculated at a depth of 0.95 m by replacing the system of forces with an equivalent uniform load. This implies that for arbitrary groups of piles with a diameter d, which are arranged with a spacing a, there is a depth h below which the settlement curves are smoothed.

Calibration with use of mathematical modeling yielded the approximate formula $h = 28d/17 - a/d)$ for groups of piles of any diameter d arranged with a uniform spacing a.

Method of Calculating Settlements of Pile Group

The settlements $S = S(x, y)$ of a group of N piles loaded by forces P_j ($j = 1 \dots N$) can be determined on the basis of results obtained by summing on the assumption that the mutual effect of the piles is linear

$$S(x, y, t) = \sum_{j=1}^{N} P_j \left[s(x-\xi_j, y-\eta_j, h)- \\ -s(x-\xi_j, y-\eta_j, H) + \frac{1}{K(t)} \right], \tag{3}$$

where $K(t) = K_p(t) + K_r$ is the stiffness of the combined PRF, $K_p(t) = 1/w_0(t)$ is the stiffness of the piles, $w_0(t)$ is the settlement of a the piles due to a unit load as calculated from formula (1), $K = \frac{E_1(1-\nu_1)}{L(1+\nu_1)(1-2\nu_1)}(A-F)$ is the stiffness of the raft, $0 \leqslant t < 1$ is a parameter, A and F are the areas of

TABLE 1

	0	1	2	3	4	5	6	7	8	9
0	1,647	1,311	1,225	1,191	1,173	1,164	1,158	1,154	1,152	1,151
1	1,311	941	856	825	811	803	799	796	795	794
2	1,225	856	768	737	723	715	711	708	707	706
3	1,191	825	737	705	690	683	678	675	674	673
4	1,173	811	723	690	676	668	663	660	659	658
5	1,164	803	715	683	668	660	655	652	650	650
6	1,158	799	711	678	663	655	650	647	646	645
7	1,154	796	708	675	660	652	647	644	643	642
8	1,152	795	707	674	659	650	646	643	641	640
9	1,151	794	706	673	658	650	645	642	640	...

the core occupied by the piles, and the cross-sectional area of a pile, respectively, L is the length of the piles, and $K(0)$ and $K(2/3)$ correspond to the stiffness of a piled foundation with a "high-" and"low-profile" raft. Other values of t can be determined from results of MM.

The problem of calculating the settlement S_0, tilts α and β of the rigid raft, and the load distribution P_j on the piles reduces to a system of $N + 3$ linear equations:

$$\sum_{j=1}^{N} P_j \left[s(x_i - \xi_j, y - \eta_j, h) - s(x_i - \xi_j, y - \eta_j, H) + \frac{1}{K(t)} \right] =$$
$$= S_0 + \alpha x_i + \beta y_i;$$
$$\sum_{j=1}^{N} P_j x_j = Q; \quad \sum_{j=1}^{N} P_j y_j = M_x; \quad \sum_{j=1}^{N} P_j y_j = M_x. \tag{4}$$

where Q, M_x, and M_y are the resultant force and moments applied to the rigid raft at the origin of the assigned coordinate system (x, y).

Solution (2) yields the load distribution on piles $(P_p)_j = P_j K_p(t)$ and soil $(P_r)_j = P_j K_r(t)$.

The solution of this problem was programmed, and the calculations were performed in the MathCad system. The computed load distribution Q_{pr} (kN) of the high-profile raft supported on a group ($20 \times 20 = 400$) piles is presented in Table 1. The raft is loaded uniformly by a distributed load, and the average load on a pile is 800 kN, i.e., a resultant external load of 320 MN. The cross section of the piles was 0.4×0.4 m, the spacing 1.6 m in a square grid, and the length 12 m. The piles are supported on a soil with a compression modulus $E_2 = 40$ MPa, and the compression modulus of the interpile soil $E_1 = 20$ MPa. The numbers of the rows and columns correspond to the spaced axes of the pile field (settlement of 6.6 cm, computing time of ~1 sec). The loads on the piles within the limits of *one-fourth* of thee rectangular pile field are presented in Table 1. It is apparent that the loads on the internal piles are similar to one another, and correspond to the values adopted in [1] on the corner and edge piles.

The MM indicated that an increase in the spacing of the piles reduces the load on the edge and corner piles, while the loads on the internal piles are increased.

When restrictions on the load Q_{pr} on the piles are accounted for by their limiting values $P_{\lim}$, their distribution is smooth; this is apparent from Table 2, where the load amounts to 1,150 kN. The calculation was performed by iterations to attainment of stable load values on the piles (author's "PRF" program in the MathCad system). For the case in question, two iterations were repeated for attainment of the solution.

TABLE 2

	1	2	3	4	5	6	7	8	9	10
0	1,149	1,149	1,149	1,149	1,149	1,149	1,149	1,149	1,149	1,149
1	1,053	912	862	838	825	817	813	810	809	809
2	912	793	753	736	726	721	718	716	715	715
3	862	753	716	700	691	686	683	681	681	681
4	838	736	700	684	676	671	668	666	665	665
5	825	726	691	676	667	662	659	657	657	657
6	817	721	686	671	662	657	654	652	651	651
7	813	718	683	668	659	654	651	649	648	648
8	810	716	681	666	657	652	649	647	646	...

TABLE 3

	1	2	3	4	5	6	7	8	9	10
0	1,131	1,078	1,057	1,047	1,041	1,037	1,035	1,034	1,033	1,033
1	902	844	823	813	808	805	803	802	801	801
2	844	783	761	751	746	743	741	739	739	739
3	823	761	739	728	723	719	717	716	715	715
4	813	751	728	718	712	708	706	705	704	704
5	808	746	723	712	706	702	700	699	698	698
6	805	743	719	708	702	698	696	695	694	694
7	803	741	717	706	700	696	694	692	692	692
8	802	739	716	705	699	695	692	691	690	...

TABLE 4

	2	3	4	5	6	7	8	9	10	11
0	947	929	920	915	911	909	908	908	908	908
1	741	723	715	710	707	705	704	704	704	704
2	688	669	660	655	652	651	650	649	649	650
3	669	649	640	635	632	630	629	628	628	629
4	660	640	630	625	622	620	619	619	619	619
5	655	635	625	620	617	615	614	613	613	614
6	652	632	622	617	614	611	610	610	610	610
7	651	630	620	615	611	609	608	608	608	608
8	650	629	619	614	610	608	607	606	606	607
9	649	628	619	613	610	608	606	606	606	606

The MM indicated that consideration of limiting-load values on the corner and edge piles is *more significant* than all other factors. In the low-profile raft, the portions of the load transferred onto the piles and interpile soil will depend on the stiffness of the interpile soils and piles with allowance for "punch-through" of the soil.

The load distribution on the interpile soils and piles beneath the low-profile raft are presented in Tables 3-5: Table 3 lists the total loads, Table 4 Q_p on the piles, and Table 5 Q_r on the interpile soil. The pile spacing is 7d (2.8 m).

TABLE 5

	0	1	2	3	4	5	6	7	8	9
0	162	137	131	128	127	126	126	126	126	126
1	137	110	103	100	99	98	98	98	97	97
2	131	103	95	92	91	91	90	90	90	90
3	128	100	92	90	88	88	87	87	87	87
4	127	99	91	88	87	86	86	86	86	86
5	126	98	91	88	86	86	85	85	85	85
6	126	98	90	87	86	85	85	85	84	84
7	126	98	90	87	86	85	85	84	84	84
8	126	97	90	87	86	85	84	84	84	...

It is apparent from Tables 3-5 that ~85% of the load is passed onto the piles. This distribution will depend heavily on the characteristics of the bed soils.

The "πRF" program, which is compiled in the MathCad system, makes it possible to calculate the settlements and tilts of PRF of arbitrary planform with a stiff subfoundation structure (with consideration of soil nonuniformity in plan and throughout the depth), and limiting loads on the piles based on data derived from static tests.

Using "πRF," it is possible to calculate the distribution of the equivalent coefficient of subgrade reaction for continued analysis of structures of finite stiffness on piled-raft foundations using the SCAD, Lira, and Mirco-Fe software packages. Barvashov [23] indicates that the distribution of the equivalent coefficient of subgrade reaction beneath a rigid foundation where "cut-through" of the soil is considered beneath the edges of the foundation, essentially coincides with this distribution under a structure having an actual finite stiffness. This conclusion enables us to simplify and accelerate considerably the iteration process of calculating the distribution of the coefficient of subgrade reaction, since involvement of the entire structure is not required in the iteration process.

Note that the software packages compiled in the MathCad system are highly graphical and universal, and their compilation and the insertion of corrections and deletions into the text is available to the engineer (and not just the programmer). This distinguishes them from commercial programs that have an extremely attractive "friendly" graphical user interface. Their compilation, debugging, and subsequent text changes are, however, highly time-consuming, and are usually possible only with the participation of their authors.

Conclusions

1. The *inaccuracy and ineffectiveness* of two of the three methods of analyzing pile foundations, which are recommended in [1], and the *paradox of the useful method* based on the principle of a conventional foundation are demonstrated.

2. Experimental and theoretical investigations indicated that a compressible stratum under the tips of piles can be separated into two layers: a comparatively thin upper "punch-through" layer, the deformations of which depend on the discrete effects of the individual piles, and a lower layer, the deformations of which will depend on the action of the piles and interpile soils as a distributed load.

3. Many factors are *insignificant* for the analysis of piled-raft foundations. The most *significant* is the limiting load on a pile, which must be considered for determination of the loads on the edge and corner piles beneath the raft.

4. The MathCad ("πRF") software is compiled for nonlinear analysis of stiff pile foundations of arbitrary planform with allowance for nonuniformity of the bed in plan and throughout the depth, and

the limiting loads on the piles. The computer time required for such a foundation formed from 1,000 piles is less than 1 min. This program can be used to compute the distribution of the coefficient of subgrade reaction beneath pile foundations of structures of arbitrary planform, and determination of the load on the piles and interpile soil.

REFERENCES

1. *Building Code 50-102-2003. Design and Installation of Pile Foundations* [in Russian].
2. George E. P. Box and Norman R. Draper, *Empirical Model-Building and Response Surfaces*, Wiley (1987).
3. V. A. Barvashov, "Sensitivity of 'bed-foundation-structure' systems," *Osn., Fundam. Mekh. Gruntov*, No. 3, 10-14 (2007).
4. Z. G. Ter-Martirosyan and A. Z. Ter-Martirosyan, "Modern problems of soil mechanics in high-rise construction," *Inzh. Geol.*, 33-41 (March 2007).
5. *Building Code 50-101-2004. Design and Installation of Beds and Foundations of Buildings and Structures* [in Russian].
6. *Construction Rule and Regulation 2.02.01-83. Beds of Buildings and Structures* [in Russian].
7. P. Clancy and M. F. Randolph, *Analysis and Design of Piled-Raft Foundations* (1993).
8. M. F. Randolph, Design methods for pile groups and piled rafts, *Thirteenth International Conference on Soil Mechanics and Foundation Engineering*, New Dehli, India (1994).
9. V. G. Fedorovskii, V. F. Aleksandrovich, S. V. Kurillo, and A. G. Skorokhodov, "Analysis of combined piled-raft foundations," *Novi Tekhnologii v Budivnitstevi*, No. 1(15), 59-72 (2008).
10. S. Hansbo and L. A. Jendeby, "A case study of two alternative foundation principles: conventional friction piling and creep piling," *Vag-och*, No. 7-8, 23-27 and 29-31 (1983).
11. V. A. Barvashov, Computer analysis of pile settlement under a vertical load and determination of soil displacements, *Proceedings of the Third Scientific-Technical Conference of Young Scientific Workers* [in Russian], Moscow (1967).
12. V. A. Barvashov, "Analysis of a high-profile rigid piled raft with allowance for pile bearing capacity," *Tr. Nauchno-Issled. Inst. Osn. Podzem. Sooruzh.*, No. 59 (1969).
13. V. A. Barvashov, Deformation-based methods of analyzing pile foundations: Author's abstract of dissertation of Candidate of Technical Sciences, Moscow (1969).
14. V. A. Barvashov and B. L. Fayans, "Analysis of pile foundations with a flexible raft and consideration of the group effect," *Osn., Fundam. Mekh. Gruntov*, No. 1, 35-37 (1969).
15. R. A. Frank, Etude theorique du comportement des piex sous charge verticale: Introduction de la dilatance, Rapport de Recherche, No. 46, LaboratorieCentrale des Ponts et Chausses, Paris (1974).
16. V. A. Barvashov and V. G. Fedorovskii, "Three-parametric model of a soil bed and pile field that accounts for irreversible structural deformations of the soil," *Osn., Fundam. Mekh. Gruntov*, No. 4, 17-20 (1978).
17. Design Guidelines for Pile Foundations, Nauchno-Issledovatel'skii Institut Osnovanii i Podzemykh Sooruzhenii, Gosstroya SSSR, Moscow (1980).
18. M. J. Tomlison, *Pile Design and Construction Practice*, 4th edition, E & FN SPON (1994).
19. A. B. Fadeev and E. È. Deval'tovskii, Pile-group performance, *Interuniversity Collection of Scientific Proceedings*, Voronezh (1988).
20. D. J. White, An investigation into behavior of pressed-in piles, Dissertation for PhD, Churchill College, University of Cambridge (2002).
21. D. A. Brown et al., Geotechnical Engineering Circular (GeO) No. 8, *Design and construction of continuous flight auger (CFA) piles*, Final (April 2007).
22. I. T. Mirsayapov and D. A. Artem'ev, *Experimental and Theoretical Investigations of Model Pile Foundations* [in Russian], KazGASU (2008).
23. V. A. Barashov, Method for determination of depth of soil-failure zones under the edges of a foundation with consideration of the natural stress state, *Collection of Scientific Proceedings Marking the 75th Anniversary of the N. M. Gersevanov Scientific-Research Institute of Foundations and Underground Structures* [in Russian], Izdatel'stvo EST (2006).