The geoid-to-quasigeoid difference using an arbitrary gravity reduction model

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ABSTRACT

Recently it was proved that the classical formula for computing the geoid to quasigeoid separation (GQS) by the Bouguer gravity anomaly needs a topographic correction. Here we generalize the modelling of the GQS not only to Bouguer types of anomalies, but also to arbitrary reductions of topographic gravity. Of particular interest for practical applications should be isostatic and Helmert types of reductions, which provide smaller and smoother components, more suitable for interpolation and calculation, than the Bouguer reduction.

Keywords: geoid, quasigeoid

1. INTRODUCTION

The classical formula to determine the geoid-to-quasigeoid separation (*GQS*) is given by $\Delta g_B H / \bar{\gamma}$, where Δg_B is the (refined) Bouguer gravity anomaly, H is the orthometric height and $\bar{\gamma}$ is mean normal gravity between the reference ellipsoid and normal height (e.g., *Heiskanen and Moritz, 1967, Sect. 8−13*). *Flury and Rummel (2009)* and *Sjöberg (2006, 2010)* showed that the classical formula need to be corrected in rugged terrain by a term related with topographic potential. (For more references, see e.g. the reference list of *Flury and Rummel, 2009*) The resulting formula implies that the topographic effect is removed from the gravity anomaly in the principle term ("direct effect") and restored as a potential correction in the added term ("indirect effect"). With this in mind, we assume that the Bouguer gravity anomaly has no unique status in this formula, but it can be generalized to an arbitrary gravity reduction, (e.g. to the isostatic or Helmert type of gravity anomaly). After the theoretical deduction in Section 2, Section 3 is devoted to a discussion on various choices of the reduction method.

2. THE GQS FOR AN ARBITRARY GRAVITY REDUCTION MODEL

Using the boundary condition of physical geodesy (*Heiskanen and Moritz, 1967, p. 86*)

$$\Delta g = -\frac{\partial T}{\partial h} + \frac{T}{\gamma}\frac{\partial \gamma}{\partial h}, \tag{1}$$

it follows that the *GQS* can be written (see also Appendix)

$$GQS = N - \zeta = \frac{T_g}{\gamma_0} - \frac{T_P}{\gamma_Q} = -\int_0^{H_P} \frac{\partial}{\partial h}\left(\frac{T}{\gamma_{Q'}}\right) \mathrm{d}h = \int_0^{H_P} \frac{\Delta g}{\gamma_{Q'}} \mathrm{d}h . \tag{2}$$

In these equations Δg is the (free-air) gravity anomaly, N and ζ are the geoid and quasigeoid heights, T_g and T_P are disturbing potentials at the geoid and surface point P, respectively, and γ is normal gravity (with sub indices 0, Q and Q' at the reference ellipsoid, the normal height corresponding to P and the normal height corresponding to the height in the integrand, respectively). Finally, h is geodetic height (reckoned along the ellipsoidal normal) and H_P is orthometric height at P.

We will now assume that the disturbing potential and gravity anomaly are reduced by the removal of the topographic masses (with or without restoration/compensation of these masses inside the geoid). This corresponds to that the disturbing potential is reduced by the reduction potential

$$\delta T^c = V^t - V^c , \tag{3}$$

where V^t and V^c are the topographic and compensation potentials, respectively. The *reduced disturbing potential* is thus given by

$$T^c = T - \delta T^c . \tag{4}$$

In the same way the reduced gravity anomaly becomes

$$\Delta g^c = \Delta g - \delta\Delta g^c , \tag{5a}$$

where

$$\delta\Delta g^c = -\frac{\partial\, \delta T^c}{\partial h} + \frac{\delta T^c}{\gamma}\frac{\partial \gamma}{\partial h} \tag{5b}$$

is the gravity anomaly reduction, and h is geodetic height (i.e. the height along the ellipsoidal normal). Then it follows from Eq.(2) that the *GQS* can be rewritten as

$$GQS = \int_0^{H_P} \frac{\Delta g^c + \delta\Delta g^c}{\gamma_{Q'}} \mathrm{d}h = \frac{\delta T_g^c}{\gamma_0} - \frac{\delta T_P^c}{\gamma_Q} + \int_0^{H_P} \frac{\Delta g^c}{\gamma_{Q'}} \mathrm{d}h , \tag{6}$$

or

$$GQS = \frac{\Delta g^c}{\bar{\gamma}} H_P + TC1 + GC1 , \tag{7a}$$

where

$$TC1 = \frac{\delta T_g^c}{\gamma_0} - \frac{\delta T_P^c}{\gamma_Q} \tag{7b}$$

and

$$GC1 = \int_0^{H_P} \frac{\Delta g^c}{\gamma} \mathrm{d}h - \frac{\Delta g^c}{\bar{\gamma}} H_P \,. \tag{7c}$$

Eqs.(7a)–(7c) are the main results of our derivations. They allow the *GQS* to be computed with an arbitrary type of topographic gravity reduction. As in *Sjöberg (2010)* we call *TC*1 and *GC*1 the topographic and gravimetric corrections, respectively.

3. DISCUSSION

In principle, from a theoretical point of view each gravity reduction procedure should result in the same *GQS*, but in practice the result may differ due to various approximations in the numerical performance. This will be considered in the examples that follow.

Let us first choose the extreme gravity reduction $\delta T^c = V^t$, implying that the topographic potential is removed without any compensation. This implies that Eqs.(7a)–(7c) become

$$GQS = \frac{\Delta g^{nt}}{\bar{\gamma}} H_P + TC1 + GC1 \,, \tag{8a}$$

where

$$TC1 = \frac{V_g^t}{\gamma_0} - \frac{V_P^t}{\gamma_Q} \tag{8b}$$

and

$$GC1 = \int_0^{H_P} \frac{\Delta g^{nt}}{\gamma} \mathrm{d}h - \frac{\Delta g^{nt}}{\bar{\gamma}} H_P \,. \tag{8c}$$

Here Δg^{nt} is the so-called "no-topography gravity anomaly" (*Sjöberg, 2010, Eq.(8)*), implying that all the effect of topography on the gravity anomaly has been removed. In fact, this gravity anomaly is practically the same as the refined Bouguer gravity anomaly, and the formulas are those of *Sjöberg (2010)*, and the major difference to *Flury and Rummel (2009)* is the small term *GC*1.

Another extreme choice of reduction is to put $\delta T^c = 0$, implying no topographic reduction. This yields

$$GQS = \frac{\Delta g}{\bar{\gamma}} H_P + GC1 , \tag{9a}$$

where

$$GC1 = \int_0^{H_P} \frac{\Delta g}{\gamma} \mathrm{d}h - \frac{\Delta g}{\bar{\gamma}} H_P . \tag{9b}$$

From a practical point of view, the first term of Eq.(9a) is very simple, as it includes the free-air gravity anomaly (without any reduction), and there is also no topographic correction (*TC*1). However, the remaining gravimetric correction (*GC*1) can be expected to be most significant, and there is no obvious practical way to estimate it properly.

Next, let us choose δT^c as the isostatically compensated topographic potential, i.e.

$$\delta T^c = \delta T^{ti} = V^t - V^i , \tag{10}$$

where V^i is the isostatic compensation potential. Using the notation Δg^I for the isostatic gravity anomaly, this leads to the following expression for the geoid-quasigeoid difference:

$$GQS = \frac{\Delta g^I}{\bar{\gamma}} H_P + TC1 + GC1 , \tag{11a}$$

where

$$TC1 = \frac{\delta T_g^{ti}}{\gamma_0} - \frac{\delta T_P^{ti}}{\gamma_Q} \tag{11b}$$

and

$$GC1 = \int_0^{H_P} \frac{\Delta g^I}{\gamma} \mathrm{d}h - \frac{\Delta g^I}{\bar{\gamma}} H_P . \tag{11c}$$

If the Earth is in isostatic balance, the isostatic gravity anomaly and disturbing potentials are all close to zero. Thus we can expect all terms of the *GQS* representation to be fairly small, and as they are smooth and easy to interpolate, they are the most suitable anomalies for practical use in the *GQS* calculations. Also Helmert types of anomalies are suitable, as they are easy to calculate and smaller/smoother than Bouguer anomalies.

4. CONCLUDING REMARKS

The *GQS* modelled by Eqs.(7a)–(7c) are valid for any method of gravity reduction. Of particular interest should be the isostatic and Helmert type of reductions, which methods provide small and smooth quantities of the components vs. the Bouguer reduction. Also, the extra workload to compute them is negligible with modern computers.

The extreme case with no topographic reduction at all is not recommended, as the main component, using the free-air gravity anomaly, varies considerably from point to point, and the needed gravimetric correction term (*GC*1) is not easy to estimate.

APPENDIX
PROOF OF EQ.(2)

From Eq.(1) follows:

$$\int_0^{H_P} \frac{\Delta g}{\gamma_{Q'}} \mathrm{d}h = -\int_0^{H_P} \frac{\partial}{\partial h}\left(\frac{T}{\gamma_{Q'}}\right) \mathrm{d}h = \left[\frac{T}{\gamma_{Q'}}\right]_{H=H_P}^{H=0} = \frac{T_g}{\gamma_0} - \frac{T_P}{\gamma_Q} = N - \xi \, . \qquad \text{(A.1)}$$

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