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Publication, cooperation and productivity measures in scientific research

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The literature on publication counting demonstrates the use of various terminologies and methods. In many scientific publications, no information at all is given about the counting methods used. There is a lack of knowledge and agreement about the sort of information provided by the various methods, about the theoretical and technical limitations for the different methods and about the size of the differences obtained by using various methods. The need for precise definitions and terminology has been expressed repeatedly but with no success.

Counting methods for publications are defined and analysed with the use of set and measure theory. The analysis depends on definitions of basic units for analysis (three chosen for examination), objects of study (three chosen for examination) and score functions (five chosen for examination). The score functions define five classes of counting methods. However, in a number of cases different combinations of basic units of analysis, objects of study and score functions give identical results. Therefore, the result is the characterization of 19 counting methods, five complete counting methods, five complete-normalized counting methods, two whole counting methods, two whole-normalized counting methods, and five straight counting methods.

When scores for objects of study are added, the value obtained can be identical with or higher than the score for the union of the objects of study. Therefore, some classes of counting methods, including the classes of complete, complete-normalized and straight counting methods, are *additive*, others, including the classes of whole and whole-normalized counting methods, are *non-additive*.

An analysis of the differences between scores obtained by different score functions and therefore the differences obtained by different counting methods is presented. In this analysis we introduce a new kind of objects of study, the class of cumulative-turnout networks for objects of study, containing full information on cooperation. Cumulative-turnout networks are all authors,

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institutions or countries contributing to the publications of an author, an institute or a country. The analysis leads to an interpretation of the results of score functions and to the definition of new indicators for scientific cooperation.

We also define a number of other networks, internal cumulative-turnout networks, external cumulative-turnout networks, underlying networks, internal underlying networks and external underlying networks. The networks open new opportunities for quantitative studies of scientific cooperation.

1. Introduction

The literature on publication counting displays the use of various terminologies and methods. Different terms are used for the same method and many methods used in the literature are not precisely defined. This was one of the reasons for the statement in 1994 that scientometrics/bibliometrics was a field in crisis (GLÄNZEL & SCHOEPFLIN, 1994). The problems also led to the convening in 1995 of a workshop in River Forest to discuss the need for the development of both conceptual and technical standards in bibliometric research and technology. The proceedings of the workshop were published in *Scientometrics* (GLÄNZEL et al., 1996).

The participants in the workshop agreed that ambiguous terms and definitions should be avoided. All bibliometric publications should include adequate information about methodology. The general opinion was that standardization should not be enforced but encouraged. Exchange of experience and sharing of knowledge would lead to standards.

The importance of the *unit of analysis* in publication counting was stressed. "Variously called "sampling units," "sampling categories," "objects of study," "tokens", "cohorts," or "items about which inferences are made," the unit of analysis is inconsistently defined, identified or not even mentioned in published studies" (MCGRATH, 1996). The need for a systematic terminology was stressed and a methodology grid proposed (BOURKE & BUTLER, 1996).

In 2004 the need for standardisation was stated: "Although basic indicators like [shares and averages of] absolute publication and citation counts are widely accepted as useful tools in measuring research performance, their uncritical use can result in incorrect interpretations. A proper normalization of standard measures and the use of relative indicators are, therefore, indispensable in trend analyses or medium- or long-term studies [in order to] guarantee the validity of conclusions drawn from bibliometric results." (PERSSON & DANELL, 2004). But no common terminology, precise definitions, or standardization of methods for counting publications and citations have been reached. There is not sufficient knowledge and agreement on the theoretical and technical limitations connected with the different methods. In many scientific publications, no information at all is given about the counting methods used. Even if more sophisticated scientometric approaches and methods are introduced, counting of publications and citations will not be obsolete. On the contrary, publication and citation numbers are the basis for many other indicators.

A literature survey has shown that the problem of counting methods for publications and citations is neglected. Thus in a recent review on bibliometrics the problem is treated in one sentence: "Higher rates of collaboration are usually associated with higher productivity, although counts will vary based on the method of allocating authorship (one credit for each publication vs. partial credit based on number of authors, etc.)" (BORGMAN & FURNER, 2002). In a comprehensive *Handbook of Quantitative Science and Technology Research. The Use of Publication and Patent Statistics in Studies of S&T Systems* from 2004 (MOED et al., 2004) counting methods and the problems associated with counting methods are not mentioned. In a recent book on citation analysis in research evaluation (MOED, 2005), the problems of counting methods are briefly discussed. Most of the discussion is based on unpublished material. The conclusion is that "an analysis *per publishing author* … is most useful". However, this conclusion is only for studies assessing the trend in a single country's publication output and it is not supported by references to the literature.

There is close to consensus about three main counting methods, *whole counting*, *fractional counting*, and *first author counting*. In whole counting, all unique countries, institutions or authors contributing to a publication receive one credit. In fractional counting, one credit is shared between the unique countries, institutions or authors with equal fractions to each participant. In first author or first address counting one credit is given to the country to which the first author or institutional address belongs, to the institution first listed among the institutional addresses or to which the first author belongs, or to the first author. In 2000 it was stated that total counting is the de facto standard method used for scientometric evaluations (EGGHE et al., 2000). However, the total counting method described is an example of complete counting and not identical with the whole counting method mostly used. In complete counting all basic units of analysis (for example all authors, all institutions or all countries) contribute with one credit each to the objects of study (for example institutions or countries). It is evident from the literature that complete counting is not the de facto method for evaluation of countries and institutions. The Center for Science and Technology Studies (CEST) produced for several years results based on complete counting at the level of institutions, sectors and countries (CEST, 2002, 2003, 2004 b, 2004 c). Only complete counting gives the real number of acts of cooperations at the national, international, inter-institutional and intra-institutional levels.

Different counting methods for publications give different results (GAUFFRIAU & LARSEN, 2005). The cause of the differences is national and international research cooperation. If all scientific publications had only one author, all counting methods would give equal results. Already in 1988 it was recognized that whole scores cause problems in determining *world shares for countries* (ANDERSON et al., 1988). Comparisons of the European Union (EU) with the USA using whole counting are misleading.

In this paper we present a formal analysis of counting methods. We propose a terminology and a classification for counting methods for publications and on this basis display the different properties of different counting methods. We differentiate between rank-dependent and rank-independent, additive and non-additive and normalized and non-normalized counting methods.

When the different counting methods give different results it is not because some methods are correct, others wrong. The methods just provide answers to different questions. Therefore, we interpret the results of the different counting methods and the information on scientific cooperation and productivity available by combining the different counting methods.

During our analysis we have defined two new classes of objects of study, cumulative-turnout and underlying networks. We depend on relational databases (CODD, 1970; ZITT & TEIXEIRA 1996), set and measure theory (HALMOS, 1950) and other tools from fundamental mathematics.

2. Terminology and notation

We differentiate between *counting methods* and *classes of counting methods*. A class of counting methods is for example all counting methods depending on the same score function. For our purpose five score functions are necessary:

The terms *counting methods* and *crediting schemes* have often been used indiscriminately. We reserve the term *counting method* for the series of steps from publications to scores for objects.

The term *crediting* has been used for attribution of credits to the basic units as well as to the objects. We reserve crediting for the first purpose. In consequence we use the term *crediting scheme* for the functions attributing credits to the basic units, whereas we use the term *score function* for the functions attributing scores to the objects. We use the term *scoring* for the production of the results of implementing a counting method and the term *scores* for the results obtained. The terms score and scoring were introduced by EGGHE (1999) and EGGHE et al. (2000). Scoring and scores are preferred to the commonly used but imprecise terms *counting* and *counts*.

We use the terms *normalized, complete-normalized* and *whole-normalized* scores instead of *fractional* scores and a number of other terms. Fractional counting is a term often found in the literature. But fractional counting is a term used for two different classes of counting methods, complete-normalized and whole-normalized counting.

A *counting method* is defined by the choice of basic unit, object and score function.

We use the term *whole* rather than *total* because total counting has been used both for whole scores (BRAUN et al., 1989) and complete scores (EGGHE, 1999; EGGHE et al., 2000).

We use *straight scores* as a common term for *first author scores, first address scores* and a number of other terms.

We use *rank-dependent methods* for methods in which basic units with different ranks receive different credits.

We use *additive methods* for methods where the sum of scores for a set of objects is equal to the score for the union of the objects in the set.

Record number of publication, *R*
Basic unit of analysis - *basic unit credited* – for short *basic unit*, *B*
Classes of basic units:
  Basic unit author, *A*
  Basic unit institution, *I*
  Basic unit country, *C*
Rank of basic unit in a publication, *r*
Notation for basic units: B_{Rr}, where *B* indicate the basic unit, the first subscript, *R*, the record number, and the second subscript, *r*, the rank number
B_R denotes the number of basic units in publication *R*.
Object of study - *object of study scoring the credits* – for short *object*, *o*
Classes of objects
  Object author, *A*
  Object institution, *I*
  Object country, *C*
X denotes the whole world of a class of objects.
$B_R(o)$ denotes the number of basic units in publication *R* coded to *o*
o_R denotes the number of different objects to each of which at least one basic unit from a publication *R* is coded (for online counting for countries o_R denotes the number of different countries in the institution list of publication *R*).
Counting method - the series of steps leading from publications to scores for objects
Score functions, *SF* – the set functions attributing scores to the objects
Classes of counting method, *CL* – the five classes of counting methods based on the five score functions considered
  Complete counting methods, *C*
  Complete-normalized counting methods, *CN*
  Whole counting methods, *W*
  Whole-normalized counting methods, *WN*
  Straight counting methods, *S*, including first-author counting, first-address counting (only the first basic unit in a publication gets a credit of 1) and other methods described in the literature.
Notation for score functions: SF_{CL}, where *CL* indicates the class of counting method.
Normalization - the credited basic units from a publication share 1 credit (including fractionation).
Normalized score functions, SF_N
Additive score functions, SF_A
Crediting schemes, *CS*, - functions attributing credits to the basic units

Notation for crediting schemes: $CS_{Rr}(B)$, where *CS* indicate the crediting scheme, the first subscript the record number, the second subscript the rank and *B* the basic unit.
Notation for counting methods: CL_{oB}, where *CL* indicates the class of counting method, the first subscript, *o*, indicates the object and the second subscript, *B*, indicate the basic unit.
Cumulative-turnout network for objects, *CTN*(*o*)
Internal cumulative-turnout network, *ICTN*(*o*)
External cumulative-turnout network, *ECTN*(*o*)
Underlying network, *UN*(*o*)
Internal underlying network, *IUN*(*o*)
External underlying network, *EUN*(*o*)

3. The substance of counting methods

3.1. Scientific publications

The basis for scientometric measurements of productivity and citations are the individual publications. We restrict the term publications to *articles*, *notes, reviews* and *letters* in scientific journals and so exclude books, patents, collections and other media used for publication. A scientific publication gives the name of the journal, the volume and page numbers and the year of publication. The scientific publication contains lists of authors, addresses and references. Publications provide information about the relationship between authors and addresses but this relationship is not standardized and therefore not easily amenable to computerization. Normally the addresses include the institutions and countries from which the authors come (SCHUBERT et al., 1989; TIJSSEN & VAN LEEUWEN, 2003; CEST, 2004 a).

3.2. Databases and basic units of analysis

A database is a set of data derived from publications. The data from the individual publications are the elements of the set. They are called records and each record consists of fields for the *record number R*, the journal title, the list of authors, the list of addresses and the list of references and further information. Two or more publications may have the same author names and/or addresses but a different record number.

Scientific publications must be linked with objects (Section 3.3). The necessary linkage is provided by basic units. Therefore new databases can be created containing the basic units chosen. They are produced by coding and are of the form:

1. (Record number, author rank, author) = *basic unit author*
2. (Record number, institution rank, institution) = *basic unit institution*
3. (Record number, country rank, country) = *basic unit country*

In the following analysis, we will include only these three triplets but of course many other basic units can be constructed from the data in the databases.

If the record number is called *R*, the rank of the basic unit in the publication is called *r* and the author, institution, or country is abbreviated *B*, the triplet of a basic unit can be denoted as B_{Rr}.

The choice of the basic unit depends on the problems addressed. The basic units are elements in a finite set *X* derived from the set of publications in the databases as follows: *X* is the (disjoint) union of all the sets/lists of basic units from all publications.

The new databases contain codes for institutions and countries instead of addresses (SCHUBERT et al., 1989). The explicit and precise construction of these databases can be complicated, time consuming, expensive etc., but in principle, it can be done.

In the major databases there is no link between the individual authors and their addresses. Therefore the assignment of nationality to authors cannot be computerized and, as a consequence counting methods based on the numbers of countries or the number of institutions from the various countries, can be computerized on the basis of the addresses, whereas methods based on the number of authors from institutions or countries are difficult to use. One study has claimed that the coding of authors to countries has been computerised. However, it was stated as having been done using a number of plausible assumptions and decision rules with no further information. No details are available and the results are only approximate (MOED, 2005, page 275).

A special problem is that authors may be connected with more than one address. In a study covering 121,432 internationally co-authored publications it was shown that at least 5 pct. had more addresses than authors (WAGNER & LEYDESDORFF, 2005). This however only gives a lower limit for the number of authors giving more than one address. It is of course possible that an author can have more than one address also in a publication where the number of authors is equal to or larger than the number of addresses.

About 20 mill. addresses are listed in SCI (Science Citation Index), SSCI (Social Science Citation Index) and AHCI (Arts & Humanities Citation Index) 1981-2002 (CEST, 2004 a). This is contrasted with an estimated number of only about 50,000 different institutions. The addresses on which coding for basic unit institutions are based often contain not only the name of one institution but for example the name of both a department and a university. The address may also contain only the name of the department or the university.

Experience at CEST has shown that larger institutions are often connected with several thousand different address-writings. The large number of addresses causes a technical problem: The coding of the addresses cannot be computerized and several man-years of work are needed to code the addresses manually. It is also necessary to make more or less arbitrary decisions about what should count as two separate institutions and what should be treated as one institution.

In most cases countries can be unambiguously coded from the addresses in the publications, but there are problems. Countries can appear under different names and sometimes can only be inferred from institutions or cities. Of course for a publication there will be just as many basic unit countries as basic units institutions.

CEST databases of codes suitable for indicators and suitable for empirical comparisons of counting methods have been constructed by coding the information given in the publications. This means that the resulting databases contain unambiguous records of institutions and countries (CEST, 2003). Author names will always remain ambiguous because of homonyms (MOED, 2005).

3.3. The classes of objects of study

Objects, variously described as *cohorts, items about which inferences are made, sampling categories, sampling units, scoring units, tokens or units of analysis* (MCGRATH, 1996), are the items finally represented in tables, graphs etc. displaying scientometric results.

Various objects are and have been used in scientometric studies: *individual authors, institutions, countries, the world, journals, fields* and *subfields* as well as individual *publications, regions, groups of countries, research sectors etc.*

All the various objects mentioned above have one common property: each of them defines a set of basic units which can be identified with the object and each of these sets is a subset of X:

An object country defines the set of all basic units coded to the country.

An object author defines the set of basic units coded to the author. (This set is often called *the author's publication list*)

A publication as an object defines its (ranked) set of basic units.

etc.

For a given database and a given set of basic units there is a unique subset of X coded to each object. Therefore it is tempting (but not precise) to give identical names to the objects and to the subsets defined by them. It is convenient to follow HALMOS (1950) and define *classes* of objects (*classes of subsets*). The class of all subsets of X is called $\boldsymbol{R}$. The *classical* objects in existing scientometric studies can be denoted as subclasses of $\boldsymbol{R}$:

The class of all countries (about 200 elements)

The class of all institutions

The class of all publications (about 1 mill. per year for SCI, SSCI and AHCI as the databases)

The class of all authors (the number of elements is difficult to estimate due to homonyms).

The class of all journals (about 9,500 for SCI, SSCI and AHCI as the databases)

The class of member countries of OECD (30 elements)

The class of member countries of the European Union EU (27 elements)

The subsets defined by the objects above are disjoint if they belong to the same class. So, for example, two distinct countries define disjoint sets.

In Section 4 we will restrict ourselves to the following classes of objects:

Authors, *A*

Institutions, *I*

Countries, *C*

3.4. Score functions

Given an object, we can define the score for the object. Examples of vanishing scores, for example from straight scoring, are given by TRUEBA & GUERRERO (2004, Table 6 on page 196).

For all counting methods in the literature the scores for a subclass of objects can be arranged in a *score function* (set functions):

$$SF \Rightarrow \text{nonnegative real numbers} \tag{1}$$

The basic units and the classes of objects can be used as a basis for a huge set of possible score functions and therefore for counting methods (see Section 3.10). In order to illustrate and explain the differences between counting methods and classes of counting methods it is convenient, first to subdivide/classify the set of all possible score functions into smaller sets with properties corresponding to the different properties of counting methods described in the literature:

One subset of score functions defined only for a proper subclass of ***R***.

One subset of score functions (not) based on crediting schemes for the basic units.

One subset of *additive* (*non-additive*) score functions.

One subset of *rank-dependent (rank-independent)* crediting schemes (and counting methods based on them).

One subset of *normalized (non-normalized)* score functions.

3.5. Score functions defined for all objects/defined for a proper subclass of objects

This property is decisive for a *discussion of a score function*. More precisely, a score function defined just on a proper subclass of ***R*** can reduce the opportunities for discussion. For example whole-normalized scores were originally defined only for the proper subclass ***C*** of all countries (NEDERHOFF & MOED, 1993). Therefore, in this case it cannot be decided whether whole-normalized counting methods are additive or non-additive. In Section 4.5 we show how to decide if whole-normalized counting is additive or not.

3.6. Score functions based on fixed crediting schemes

In Section 3.4 we defined the score function giving scores to the objects. LINDSEY (1980) emphasizes that the basic units are the ones "who did the work" and therefore are to be credited. If a score function is based on a fixed crediting scheme, the basic units are credited and the objects (countries, institutions, authors) collect (score) the credits of their basic units. If a score function is not based on a fixed crediting scheme, the object scores the credits as in the first case, but the credits can only be assigned to the basic units when the object has been chosen.

In the literature we find a number of counting methods based on fixed crediting schemes for the basic units. We define a fixed crediting scheme, *CS*, as a function:

$$CS{:}X \Rightarrow \text{nonnegative real numbers} \tag{2}$$

and thus

$$CS{:}B_{Rr} \Rightarrow C_{Rr} \tag{3}$$

An example of such a crediting scheme is complete crediting, where every basic unit is credited with a full credit of 1 (used in complete counting methods). Another example is straight crediting (used in first author counting, first address counting) where the basic unit ranked first is credited 1 and all other basic units are credited 0. Whole and whole-normalized score functions are not based on fixed crediting schemes for the basic units (see Sections 4.4 and 4.5).

Given a crediting scheme an object receives scores from all basic units coded to the object. This can be denoted by defining and using the characteristic function χ_o of an object o. The characteristic χ_o function is defined for all basic units and has value 1 for the basic units coded to o and value 0 for the other basic units (χ_o: $X \Rightarrow \{0,1\}$):

$$\chi_o(B_{Rr}) = d_{Rro} \tag{4}$$

where d_{Rro} equals 1 if the basic unit B_{Rr} is coded to o and equals 0 otherwise.

With this, any score function based on a fixed crediting scheme *CS* can be written as:

$$SF_{CS}\,(o) = \sum_R \sum_r d_{Rro}\, C_{Rr} \tag{5}$$

Every score function *SF* can (usually in many ways) be written as a sum of (other) functions:

$$w_{Rr}\text{: } \boldsymbol{R} \Rightarrow \textit{nonnegative real numbers} \tag{6}$$

such that

$$SF\,(o) = \sum_R \sum_r d_{Rro}\, w_{Rr}\,(o)\ \forall\ o \tag{7}$$

For a given object o the values $w_{Rr}(o)$ are called *transferred credits*. For score functions based on a fixed credit scheme, the transferred credits are *constant functions* i.e.

$$w_{Rr}(o) = C_{Rr} \; \forall \; R,r,o \tag{8}$$

whereas for all score functions not based on fixed crediting schemes w_{Rr} are *non-constant functions*.

3.7. Additive, non-additive and sub-additive score functions

The distinction between additive and non-additive score functions is scientometrically the distinction between functions giving scores that can be consolidated (ZITT & TEIXEIRA, 1996) (for example so that the sum of scores for the institutions in a country equals the score of the country or that the sum of scores for all countries equals the score of the world). The intuitive motivation for a definition of additive score functions is that e.g. the definition of the world-share for a country or the country-share for an institution is much easier, if the sum of scores for all countries equals the score for the world and if the sum of scores for all institutions in a country equals the score for the country. For a non-additive score function, ANDERSON et al. (1988) proposed to determine adequate shares by replacing the score for the world with the sum of scores for all countries. However, this is an unsatisfactory solution, because of the big differences in the level of international cooperation in different countries. We will therefore determine the properties and the information and insights obtainable from both additive and non-additive score functions.

In Section 4.4 about whole counting methods we will show how non-additive and especially sub-additive counting methods can be used to create new and valuable scientometric indicators.

In order to define additive and non-additive score functions, we utilize the concept of disjoint unions of objects. We define therefore a score function as additive, if for two disjoint objects o and p the score of the disjoint union of o and p equals the sum of the scores of o and p. A counting method has an additive score function, SF_A, if for all disjoint pairs of objects, o and p ($o \cap p = \emptyset$) and for all sets of publications the following condition is valid:

$$SF_A(o \cup p) = SF_A(o) + SF_A(p) \tag{9}$$

All other counting methods are called non-additive. We call a score function, SF, *sub-additive* if for all pairs of disjoint objects o and p and for all sets of publications:

$$SF(o \cup p) \leq SF(o) + SF(p) \tag{10}$$

The choice of *disjoint* sets o and p is *a precondition* for a successful definition of *additive* score functions: To demonstrate this, we consider the non-disjoint sets o and p. The union of o and p can be divided into three disjoint subsets, giving $o \cup p = (o - p) \cup (o \cap p) \cup (p - o)$. Similarly, we can divide o and p into disjoint subsets $o = (o - p) \cup (o \cap p)$ and $p = (p - o) \cup (o \cap p)$. Then we have:

$$SF_A (o) + SF_A (p) - SF_A (o \cup p) =$$

$$SF_A (o - p) + SF_A (o \cap p) + SF_A (p - o) + SF_A (o \cap p) - SF_A (o - p) - SF_A (o \cap p) - SF_A (p - o) =$$

$$SF_A (o \cap p) \tag{11}$$

Since o and p are not disjoint the last term is larger than 0 and the score function is non-additive.

For score functions only defined on the proper subclasses (authors, institutions, countries) of ***R*** it cannot be decided, whether they are additive or not, because the unions of two elements of these subclasses are not elements of these subclasses. We will come back to this problem in Section 4.5 on whole-normalized counting methods.

We will show that all score functions based on a fixed crediting scheme are additive. To this end we use the expression for a score function based on a fixed crediting scheme given in Section 3.6, formula (5). Then for two disjoint objects o and p:

$$d_{Rr(o \cup p)} = d_{Rro} + d_{Rrp} \tag{12}$$

Therefore, for any pair of disjoint objects o and p, we have:

$$SF_B (o \cup p) = \sum_R \sum_r d_{Rro \cup v} C_{Rr} = \sum_R \sum_r d_{Rro}\, C_{Rr} + \sum_R \sum_r d_{Rrp}\, C_{Rr} = SF_B (o) + SF_B (p) \tag{13}$$

For any two objects o and p, the characteristic function has a second property:

$$d_{Rr(o \cap p)} = d_{Rro}\, d_{Rrp} \tag{14}$$

Therefore for $o = p$:

$$d_{Rro} = d_{Rro}\, d_{Rro} \tag{15}$$

Remembering formula (2) we see that the special transferred credit functions of the form:

$$W_{Rr} (o) = C\, d_{Rro} \tag{16}$$

are multiples of complete scores based on a fixed crediting scheme:

$$\sum_R \sum_r d_{Rro}\, w_{Rr} (o) = \sum_R \sum_r cd_{Rro}\, d_{Rro} = \sum_R \sum_r d_{Rro} = SF_C(o) \tag{17}$$

Therefore, score functions based on formula (16) are additive.

3.8. Normalized and non-normalized score functions

The scientometric motivation for attempting to define normalized/non-normalized score functions is the argument in the literature that all publications should be treated equally, i.e. should give equal scores.

We first define normalized (non-normalized) crediting schemes for score functions based on a fixed crediting scheme.

A crediting scheme (and the corresponding score function) is called normalized, if the sum of the credits of the basic units of any publication R equals 1:

$$\sum_r C_{Rr} = 1 \ \forall \ R \tag{18}$$

A crediting scheme is called non-normalized if for at least one publication:

$$\sum_r C_{Rr} \neq 1 \tag{19}$$

However, a more general definition of normalized and non-normalized score functions also covering score functions not based on a fixed crediting scheme is desirable. The *key* to such a generalized definition is the common property of the classes of objects (all authors, all institutions, all countries and all publications).

1. The elements (objects) in each class are pairwise disjoint
2. The union of the objects in each class equals (*covers*) the *world* (X)

In other words, each of these classes of objects forms a *disjoint cover* (also called *tiling*) *of the world X of all basic units*. For any tiling of the world (there is a huge but finite set ***T*** of tilings) we can choose *fixed transferred credits for all basic units for any score function*.

For whole scores, basic unit authors, the present publication and tilings based on all authors, institutions, countries or publications we *can* obtain the *transferred* credits shown in Table 1.

Table 1. Tilings of the basic unit authors in the present publication using the whole score function

	tiling			
Class of objects	all authors	all institutions	all countries	all publications
Marianne	1	1	1/2	1/5
Peder	1	1	1/2	1/5
Anne	1	1/3	1/3	1/5
Isabelle	1	1/3	1/3	1/5
Markus	1	1/3	1/3	1/5

For score functions not based on a fixed crediting scheme we must choose a fixed tiling and define the resulting normalized score function only for the objects of the tiling. This was done originally for the tiling with countries by NEDERHOFF & MOED (1993).

For these fixed transferred credits determined by the tiling chosen we can with the old definition decide that all are non-normalized. Therefore we can define normalized score functions:
A score function is called normalized, if for every tiling (disjoint cover) of the world X and for every publication R, the sum for all ranks r of the transferred credits equals 1.

For any score function based on a fixed crediting scheme, this definition breaks down to the old definition, because all the transferred credits are equal.

3.9. Rank-independent and rank-dependent counting methods

The literature contains arguments for giving all basic units in a publication the same credit but also for giving some basic units more credit than others (TRUEBA & GUERRERO, 2004). The discussion has resulted in the largest set of score functions in the literature, the set of additive, normalized and rank-dependent score functions based on fixed crediting schemes and defined for all objects.

A crediting scheme is rank-independent, if for each publication *R*:

$$CS_{Rr} = CS_{Rs} \ \forall \ ranks\ r,s \tag{20}$$

Similarly a crediting scheme is rank-dependent, if for at least one publication *R* and two ranks *r,s*:

$$CS_{Rr} \neq CS_{Rs} \tag{21}$$

If the underlying crediting scheme is rank-(in)dependent, the corresponding score function is called rank-(in)dependent. In contrast to the concept of normalized scores, the concept of rank-(in)dependent counting cannot be extended to score functions not based on fixed crediting schemes.

3.10. What have we obtained?

All the classification criteria above are yes/no decisions. The number of different classes can therefore be displayed in a "decision tree" (Table 2).

There are 8 possible classes of counting methods. For three of these classes there are no examples in the literature and no score functions are presented for these classes in Section 4.

Table 2. The decision tree for the different score functions and classes of counting methods

Defined for all objects	Based on a fixed crediting scheme	Additive	Rank-independent	Normalized	In Section	Classes of counting methods described in the literature
Yes	Yes		Yes	No	4.1.	**Complete**
				Yes	4.2.	**Complete normalized**
			No	No		
				Yes	4.3	**Straight**
	No	Yes	Not applicable	No		
				Yes		
		No		No	4.4.	**Whole**
				Yes		
No	No	No	Not applicable	No		
				Yes	4.5.	**Whole normalized**

4. Classes of counting methods in the literature and their score functions

In the following subsections we describe the five classes of counting methods described in the literature and give formulas for the score functions for these classes.

For some score functions we use Kroneckers δ-function:

$$\delta_{r\,s} \text{ equals 1 if } r = s \text{ and zero if } r \neq s \tag{22}$$

For some score functions we use the Θ-function. The Θ-function is defined for all real numbers and has the value 1 for positive numbers and 0 otherwise.

4.1. The class, based on fixed crediting schemes (additive), non-normalized and rank-independent

From this class only *complete counting methods* are described in the literature. Complete counting method gives a credit to each basic unit. The score function is:

$$SF_C\,(o) = \sum_R B_R\,(o) = \sum_R \sum_r d_{Rro} \tag{23}$$

From the second term, we observe that with complete scores an object scores as many credits from a publication as there are basic units coded to the object. From the third term, we observe that complete scores are based on a fixed crediting scheme for the basic units where every basic unit is credited 1 (C_{Rr} = 1 for all R,r). Therefore complete scores give additive results. From the third term, we also observe that complete scores are rank-independent. Because

$$\sum_r C_{Rr} = B_R \geq 1 \tag{24}$$

we observe that complete scores are non-normalized.

In Section 6.1 we will discuss the use of complete counting in measuring cooperation inside objects.

4.2. The class based on fixed crediting schemes (additive), normalized and rank-independent

From this class only *complete-normalized counting methods* are described in the literature. In complete-normalized counting methods a credit of $1/n$ is given to each basic unit where n is the number of basic units in the publication. The score function is:

$$SF_{CN}(o) = \sum_{R} \frac{B_R(o)}{B_R} = \sum_{R}\sum_{r} d_{Rro} \frac{1}{B_R} \tag{25}$$

From the second term, we observe that an object o scores the share of its basic units in all basic units of the publication. The score function is therefore normalized. From the third term, we observe that complete-normalized counting is based on a fixed crediting scheme.

4.3. The class based on fixed crediting schemes (additive), normalized and rank-dependent

Most of the counting methods in the literature fall into this class. It is characterized by the demand that all publications have identical scores and that some ranks of basic units deserve special scores (TRUEBA & GUERRERO, 2004). Here we will only consider *first author and first address counting (straight counting methods)*, (COLE & COLE, 1973). In these methods a credit of 1 is given to the basic unit at rank 1. A credit of 0 is given to all other basic units. The score function is:

$$SF_S(o) = \sum_{R}\sum_{r} d_{Rro}\, \delta_{1r} \tag{26}$$

This counting method is based on the assumption that the first author (first basic unit) is the person with the highest merits. This is however not always the case. First author counting is often in reality performed as first address counting.

4.4. The class not based on a fixed crediting scheme, non-additive and non-normalized

From this class, only *whole counting methods* are described in the literature. Whole counting is often described as giving a credit of 1 to each country with at least one occurrence in the institution list. This definition is familiar to users of online-methods or online-search routines and corresponds to the second term in the score function:

$$SF_W(o) = \sum_R \theta(B_R(o)) = \sum_R \sum_r d_{Rro} \begin{Bmatrix} 0 \; if \; d_{Rro} = 0 \\ 1/B_R(o) \; if \; d_{Rro} = 1 \end{Bmatrix} \tag{27}$$

From the second term, we observe the *classic* definition of whole counting based on objects: An object scores 1 from a publication if it has at least one basic unit in the publication's list of basic units.

From the second term, we also see that whole counting methods are non-normalized.

4.4.1. SF_W is not based on a fixed crediting scheme. Whole counting can be described in two equivalent ways:

1. In each publication, the basic units coded to an object and occurring first in the list of basic units obtain a credit of 1. All other basic units obtain a credit of zero (the second term in (27)).
2. For each publication, a list of the different objects (o) is prepared. For each entry in this list, the number of basic units coded to d is calculated, $n(d)$. Each basic unit is assigned a credit of $1/n(d)$ according to its coding to one of the objects d (the third term in 27).

The third term in formula (27) can be regarded as an unsuccessful attempt to define a crediting scheme for whole scores (the expression in brackets {}). This is not possible: A basic unit will receive different credits if different objects are considered; (explicitly $1/B_R(o)$ has different values for different objects o; in general $1/B_R(o)$ will have small values for large objects and large values approaching 1 for small objects). It makes no sense to question whether the crediting scheme is rank-dependent or rank-independent.

The present publication illustrates that whole scoring is not based on a fixed credit scheme. Let the score function be whole scores and authors be the basic units. We have 5 basic units (Marianne, Peder, Anne, Isabelle and Markus). If the objects are institutions, then Marianne and Peder will each transfer a credit of 1 to their respective institutions whereas the three basic units Anne, Isabelle and Markus will have to share (in whatever way) in transferring a credit of 1 to their institution. In total, Marianne and Peder dispose of 2 credits. But if instead the objects are countries, then Marianne and Peder will have to share (in whatever way) in transferring a credit of 1 to Denmark whereas the three basic units Anne, Isabelle and Markus again will have to share (in whatever way) in transferring a credit of 1 to Switzerland (Table 1).

4.4.2. SF_W is non-normalized. The present publication also illustrates that whole scores are non-normalized. If the objects are countries, the score for the publication is 2, 3 if the objects are institutions, 5 if the objects are authors, and 1 if the object is publications.

We call $o_R(t)$ the number of objects occurring in the list of basic units of publication R for tiling t and p_R the object publication R. With this terminology we obtain the following:

$$SF_W(p_R) = \sum_{o \in t} \sum_r d_{Rro} \left\{ \begin{array}{c} 0 \; if \; d_{Rro} = 0 \\ 1 \big/ B_R(o) \; if \; d_{Rro} = 1 \end{array} \right\} = \sum_{o \in t} \theta(B_R(o)) = o_R(t) \qquad (28)$$

We call c_R, i_R, and a_R the number of different countries, institutions and authors in publication R. The tilings $\boldsymbol{C}$ (tiling with countries), $\boldsymbol{I}$ (tiling with institutions) and $\boldsymbol{A}$ (tiling with authors) can also be defined as the underlying network for publication P.

The conclusion of this subsection is:

A. Whole scores are non-normalized.

B. Whole scores measure the number of countries, institutions or authors producing publication P.

4.4.3. SF_W is non-additive but subadditive. For two disjoint objects, o and p, the class of all publications, X, in the database can be split into the following (pairwise disjoint) subsets:

1. A set E containing all publications with no basic units attributed to o or p (containing $X(E)$ publications).
2. A set A_1 containing all publications with at least one basic unit attributed to o but none to p (containing $X(A_1)$ publications).
3. A set A_2 containing all publications with at least one basic unit attributed to p but none to o (containing $X(A_2)$ publications).
4. A set $I_{1,2}$ containing all publications with at least one basic unit attributed to o and at least one basic unit attributed to p (containing $X(I_{1,2})$ publications).

The set $I_{1,2}$ can also be denoted as the set of publications having *inter-object cooperation* between o and p. If o and p are countries the formulation is *having international cooperation* between o and p.

Because the four sets of publications are pairwise disjoint, we can split the sum of scores for all publications (formula (1)) to four sums for the four sets of publications. Thus we obtain:

$$SF_W(o) = X(A_1) + X(I_{1,2}) \qquad (29)$$

$$SF_W(p) = X(A_2) + X(I_{1,2}) \qquad (30)$$

$$SF_{WF}(o \cup p) = X(A_1) + X(A_2) + X(I_{1,2}) \qquad (31)$$

Combining these three formulas we obtain:

$$SF_W(o) + SF_W(p) - SF_W(o \cup p) = X(I_{1,2}) \qquad (32)$$

According to the definition of the set $I_{1,2}$ this difference equals the number of publications with inter-object cooperation between o and p.

Because $(I_{1,2}) \geq 0$, whole scores are non-additive and more precisely sub-additive:

$$SF_W(o \cup p) \leq SF_W(o) + SF_W(p) \tag{33}$$

The two values are equal if and only if $X(I_{1,2}) = 0$, i.e. if and only if there is no publication with inter-object cooperation between o and p.

This conclusion we will expand by considering classes of disjoint objects, $\{o_1, \ldots o_n\}$, where n is a whole, positive integer > 1. Here the set of all publications X in the database can be split into the following subsets:

1. A set E containing all publications with no basic units attributed to $\{o_1, \ldots o_n\}$ (containing $X(E)$ publications).
2. A class of all sets A_i containing all publications with at least one basic unit attributed to an object o_i but none to any other unit in $\{o_1, \ldots o_n\}$ (containing $X(A_i)$ publications).
3. A class of all the sets I_{ij} containing at least one basic unit from an object o_i and one basic unit from an object o_j (containing $X(I_{ij})$ publications).

Then we obtain:

$$\sum_i SF(o_i) - SF(o_1 \cup \ldots \cup o_n) = f(X(I_{ij})) \tag{34}$$

The scores for sets of type A_i will be counted by our formula (26) exactly once in $SF_W(o_1 \cup \ldots \cup o_n)$ and exactly once in $\Sigma SF_W(o_i)$ and therefore make no contribution to the difference. On the other hand, the scores for sets of type I_{ij} will be counted at least twice in $\Sigma\, SF_W(o_i)$ and exactly once in $SF_W(o_1 \cup \ldots \cup o_n)$. Therefore, only the sets of type I_{ij} make contributions to the desired difference. These sets contain exactly the publications having inter-object-cooperation for at least two objects in $\{o_1, \ldots, o_n\}$.

This has a number of consequences:

1. When sums of whole scores of several objects $\{o_1, \ldots, o_n\}$ are considered, *publications* containing basic units from more than one object are counted multiply, in other words *publications with inter-object cooperation* are counted multiply. This means that whole scores *give quantitative information about inter-object cooperation*. In contrast, additive score functions cannot measure inter-object cooperation, because for such score functions SF_A the difference $SF_A(o_1) + \ldots + SF_A(o_n) - SF_A(o_1 \cup \ldots \cup o_n)$ vanishes also for sets of type I_{ij}.
2. The difference $SF_W(o_1) + \ldots + SF_W(o_n) - SF_W(o_1 \cup \ldots \cup o_n)$ is a function of *just* the sets of type I_{ij} having *inter-object cooperation*:

$$\sum_{i=1}^{n} SF_W(o) - SF_w(o_1 \cup \ldots \cup o_2) = f(X(I_{1,2}), \ldots, X(I_{n,,n-1}) \geq 0 \tag{35}$$

and the right hand side of this formula vanishes if and only if there is no inter-object cooperation between the objects $\{o_1, \ldots, o_n\}$.

3. The result presented in formula (34) can also be expressed in the form:

$$SF_W(o_1 \cup \ldots \cup o_n) \leq \sum_{i=1}^{n} S_W(o_i) \tag{36}$$

emphasizing that whole scores are non-additive (more precisely sub-additive).

The conclusion of this subsection is:

C. Score functions based on a fixed crediting scheme cannot be used to measure inter-object cooperation.

D. Whole scores are non-additive but sub-additive.

E. Whole score functions can be used to measure inter-object cooperation.

The example given in Table 3 shows that at least in one case a whole counting method gives a non-additive result. Therefore the analysis above is not necessary for proving that whole counting methods are non-additive. But the purpose of the analysis was not only to prove that whole scores are non-additive but also that they are sub-additive and most important that they provide quantitative information on inter-object cooperation. This will provide more insight into sub-additive score functions and into the scientometric content of outer measures (HALMOS, 1950) induced by a subring of ***R***.

Table 3. Scores for a database containing one publication with authors and institutions from three countries and with one basic unit (authors or institutions) attributed to the first country, two basic units to the second country, and three basic units to the third country

Country, union of countries or group of countries	Basic units (numbers of authors or institutions	Scores			
		C (Complete counting)	CN (Complete-normalized counting)	W (Whole counting)	WN (Whole-normalized counting)
Three countries, u_1, u_2 and u_3					
u_1	1	1	1/6	1	1/3
u_2	2	2	1/3	1	1/3
u_3	3	3	½	1	1/3
$u_1 + u_2$	3	3	½	2	2/3
An union of two countries, u_1 and u_2, and a single country, u_3					
$u_1 \cup u_2$	3	3	½	1	½
u_3	3	3	½	1	½
Conclusion		Additive	Additive	Non-additive	Non-additive

4.4.4. SF_W is non-subtractive. Formula (32) can be rearranged to:

$$SF_W(o) - (SF_W(o \cup p) - SF_W(p)) = N(I_{12}) \tag{37}$$

o is a subset of $o \cup p$. Therefore, whenever an object o is a subset of an object p:

$$SF_W(p\text{-}o) - (SF_W(p) - SF_W(o)) = N \geq 0 \tag{38}$$

where N is the number of publications with contributions both from o and from $p - o$.

This demonstrates that *whole-scores are non-subtractive* (HALMOS, 1950). If and only if there is no cooperation between o and the rest of p, are whole scores subtractive.

This result has two consequences:

F. Whole scores *are non-subtractive*.

G. Whole scores *can be used to obtain new indicators*.

For example the cooperation between an institution in a country and the rest of country can be obtained from the left hand side of (37).

4.4.5. For the world as object, whole scores measure the number of publications in the database. If we calculate the whole score for the world w as object, we obtain:

$$S_W(w) = \sum_R \sum_r d_{Rrw} \begin{Bmatrix} 0 \; if \; d_{Rrw} = 0 \\ 1/B_R(w) \; if \; d_{Rrw} = 1 \end{Bmatrix} = \sum_R \sum_r 1/B_R = \sum_R 1 = number\ of\ publications \tag{39}$$

where we used the fact that $B_R(w) = B_R$ = number of basic units in publication R and that:

$$d_{RrW} = 1 \ \forall \ R,r \tag{40}$$

*4.4.6. SF_W is defined for all objects in **R**.* ***R*** as the class of all subsets of X forms a ring, meaning that for any two elements in ***R*** also their union and the difference between them are elements in ***R***. This means, for example, that if the 15 member countries of EU-15 are elements in ***R*** also their union (EU-15) is an element of R.

4.5. A proper subclass of sets, not based on a fixed crediting scheme, non-additive, normalized

From this subclass, only *whole-normalized counting methods* are described in the literature. The score function is:

$$SF_{WN}(o) = \sum_R \frac{\theta(B_R(o))}{o_R} = \sum_R \sum_r d_{Rro} \begin{Bmatrix} 0 \; if \; d_{Rro} = 0 \\ 1/o_R B_R(o) \; if \; d_{Rro} = 1 \end{Bmatrix} \tag{41}$$

Normalization was first defined for score functions based on fixed credit schemes. In Section 3.8 the definition of normalized score functions was expanded to cover all score functions based on a fixed tiling ***T*** of the world. This definition covers the whole-normalized score function.

However, the definition is based on the tiling ***T*** and defines whole-normalized scores only for objects of the tiling ***T***. If the tiling is the class of all countries, C, and if we want to study also unions of countries as for example EU-15, then the whole-normalized score of EU-15 or other unions is not defined. Therefore, we need an additional prescription for the whole-normalized score for e.g. the union EU-15. From Section 3.8 we see that we need a tiling containing the union EU-15 – but there are many such tilings. A glance at the third term of formula (18) shows that for different tilings containing EU-15 the resulting score for EU-15 is different.

We therefore use an additional prescription for *chosing* a tiling $\boldsymbol{E}$ containing EU-15:

1. We remove all member-objects (of the union) from the tiling $\boldsymbol{T}$

2. We add the union (e.g. EU-15) and obtain the new tiling $\boldsymbol{E}$.

With this tiling $\boldsymbol{E}$ we obtain an unambiguous definition of a whole-normalized score for the union. Now we can compare the sum of the whole-normalized scores of the member objects (based on $\boldsymbol{T}$) and the whole-normalized score of the union (based on $\boldsymbol{E}$). In other words, with the additional prescription we can decide whether whole-normalized scores are additive or not. The example given in Table 3 shows that at least in one case whole-normalized counting gives non-additive results. Therefore whole-normalized counting methods are non-additive. Therefore the scores for countries obtained by whole-normalized counting (WN_{CA} (WN_{CC})) cannot be added to give scores obtained by whole-normalized counting for unions of countries.

We emphasize that the choice of the prescription above is decisive. It is unknown whether there are prescriptions leading to additive whole-normalized scores.

Whole-normalized counting for countries was introduced by NEDERHOFF & MOED (1993). The choice of method was partially determined by the opportunities for online counting but also by the knowledge that for large classes of publications with small numbers of basic units (< 5) scores from whole-normalized counting are very near to scores from complete-normalized counting.

In (41) the second term contains a new notion, o_R, the number of different objects with non-zero basic units in publication R. o_R takes different values for different classes of objects. Therefore the classes of objects o_R with unique values for all publications R must be identified.

5. Counting methods

In Sections 4 we treated all the three classes of basic units and all the objects simultaneously for the four classes of counting methods. The only exception is whole-normalized counting methods.

This could lead to the wrong impression that the results (and their interpretation) of the above procedures only depend weakly on the choice of a class of basic units (3 at disposal) or a class of objects (3 chosen for analysis). This is not the case. Therefore we have also to classify the set of results (set of credits/scores) for the above general score functions according to the classes of basic units and classes of objects. A counting method is defined, if a score function, a class of basic units and a class of objects is given, in other words a counting method is a triplet:

counting method = (score function, class of objects, class of basic units).

The triplets can be denoted as SF_{oB} where SF indicates the score function (for example C for complete, CN for complete normalized), the first subscript, o, indicates the class of objects and the second subscript, B, indicates the basic unit of analysis. The resulting counting methods include all the elements described in the subsections.

We restrict ourselves to three basic units, basic unit author, basic unit institution and basic unit country and to three objects, authors, institutions and countries. We restrict ourselves to the counting methods in the five classes described in Section 4.

5.1. Counting methods

With five classes of scoring functions, three classes of objects and three classes of basic units we could expect 5 x 3 x 3 = 45 *counting methods*. But two factors reduce the number of counting methods. Not all basic units can be applied to all classes of objects and some of the counting methods give identical results.

It does not make sense to use institutions as basic units for attribution of scores to authors, or to use countries as basic units for attribution of scores to authors or institutions. Therefore the number of possible counting methods is reduced to (5 x 3) + (5 x 2) + (5 x 1) = 30.

The complete set of counting methods is described in Table 4. For each counting method it is indicated whether it is based on a fixed crediting scheme or not and therefore whether it is additive or non-additive, normalized or non-normalized and rank-dependent or rank-independent. The Table also provides information about methods giving identical scores. The Table provides notations for all the counting methods.

5.2. Identical scores from different methods

Some objects may obtain equal scores using different counting methods and therefore different combinations of score functions, basic units and objects. However, there are 19 possible combinations giving different results for at least some objects.

Because authors appear only once in the author list of a publication $C_{AA} \equiv W_{AA}$ and $CN_{AA} \equiv WN_{AA}$.

In whole counting, a country will get the score of 1 if at least one basic unit, author, institution or country, is attributed to the country ($B_R > 1$). In whole-normalized counting for all three methods there is the same number of countries. A parallel argumentation can be used for institutions. Therefore $W_{CA} \equiv W_{CI} \equiv W_{CC}$, $WN_{CA} \equiv WN_{CI} \equiv WN_{CC}$, $W_{IA} \equiv W_{II}$ and $WN_{IA} \equiv WN_{II}$.

The rank for countries is determined by the rank for institutions and therefore $S_{CI} \equiv S_{CC}$.

Table 4. Description and properties of 30 counting methods covering the three basic units of analysis, the five score functions and the three objects of study. The 19 distinct methods are in bold face whereas redundant types are in normal face. C Complete, CN Complete-normalized, S Straight, W Whole, WN Whole-normalized, A Author, I Institution, C Country, 1. Subscript: Object of study, 2. Subscript: Basic unit of analysis

Underlying score function and properties	Basic unit of analysis	Object of study	Basic unit receiving credit. For whole scores as well as whole-normalized scores the score function is not based on a fixed crediting scheme but for the classes of objects chosen the corresponding counting methods can be based on a transferred crediting scheme.	Rule for attributing scores	Code for counting method	Identical scores with
Straight scores Based on a fixed crediting scheme Additive Normalized Rank-dependent	Authors	Authors	First author	1 score to first author	$\mathbf{S_{AA}}$	
		Institutions		1 score to the institution to which the first author is coded	$\mathbf{S_{IA}}$	
		Countries		1 score to the country to which the first author is coded	$\mathbf{S_{CA}}$	
	Institutions	Institutions	First institution	1 score to the first institution	$\mathbf{S_{I\,I}}$	
		Countries		1 score to the first country	$\mathbf{S_{CI}}$	S_{CC}
	Countries	Countries	First country	1 score to the country to which the first institution is coded	S_{CC}	$\mathbf{S_{CI}}$
Complete scores Based on a fixed crediting scheme Additive Non-normalized Rank-independent	Authors	Authors	All authors	1 score to each author	$\mathbf{C_{AA}}$	W_{AA}
		Institutions		1 score for each author from the institution	$\mathbf{C_{IA}}$	
		Countries		1 score for each author from the country	$\mathbf{C_{CA}}$	
	Institutions	Institutions	All institutions	1 score to each institution	$\mathbf{C_{I\,I}}$	
		Countries		1 score for each institution from the country	$\mathbf{C_{CI}}$	C_{CC}
	Countries	Countries	All countries	1 score to each country	C_{CC}	$\mathbf{C_{C\,I}}$

Table 4. (cont.)

Complete-normalized scores Based on a fixed crediting scheme Additive Normalized Rank-independent	Authors	Authors	All authors	1/n score to each author, where n is the number of authors	**CN_{AA}**	WN_{AA}
		Institutions		1/n score for each author from the institute, where n is the number of authors	**CN_{IA}**	
		Countries		1/n score for each author from the country, where n is the number of authors	**CN_{CA}**	
	Institutions	Institutions	All institutions	1/p score to each institution, where p is the number of institutions	**CN_{II}**	
		Countries		1/m score for each institution from the country, where m is the number of institutions	**CN_{CI}**	CN_{CC}
	Countries	Countries	All countries	1/m score to each country, where m is the number of countries	CN_{CC}	**CN_{CI}**
Whole scores Not based on a fixed crediting scheme Non-additive Non-normalized Rank-independent	Authors	Authors	All authors	1 score to each author	W_{AA}	**C_{AA}**
		Institutions	Those authors named first in the list of authors coded to a unique institution*	1 score to an institution if at least one author is from this institution	**W_{IA}**	W_{II}
		Countries	Those authors named first in the list of authors coded to a unique country*	1 score to a country if at least one author is from this country	**W_{CA}**	W_{CI} W_{CC}
	Institutions	Institutions	All unique institutions	1 score to each unique institution	W_{II}	**W_{IA}**
		Countries	Those institutions named first in the list of institutions coded to a unique country*	1 score to a country if at least one institution is coded to this country	W_{CI}	**W_{CA}** W_{CC}
	Countries	Countries	All unique countries	1 score to each unique country	W_{CC}	**W_{CA}** W_{CI}
Whole-normalized scores Not based on a fixed crediting scheme Non-additive Normalized Rank-independent	Authors	Authors	All authors	1/n score to an author, where n is the number of authors	WN_{AA}	**CN_{AA}**
		Institutions	Those authors named first in the list of authors coded to a unique institution*	1/m score to an institution, where m is the number of unique institutions	**WN_{IA}**	WN_{II}
		Countries	Those authors named first in the list of authors coded to a unique country*	1/p score to a country, where p is the number of unique countries	**WN_{CA}**	WN_{CI} WN_{CC}
	Institution s	Institutions	All unique institutions	1/m score to an institution, where m is the number of unique institutions	WN_{II}	**WN_{IA}**
		Countries	Those institutions named first in the list of institu-tions coded to a unique country*	1/p score to a country, where p is the number of unique countries	WN_{CI}	**WN_{CA}** WN_{CC}
	Countries	Countries	All unique countries	1/p score to a country where p is the number of unique countries	WN_{CC}	**WN_{CA}** WN_{CI}

* In the table we write that only the first basic unit of those coded to the object is credited. An alternative is to let all the basic units coded to the object share the credit of 1 equally (see Section 4.4.1.)

The number of basic units for a country is identical with the number of basic unit institutions to this country. Therefore $C_{CI} \equiv C_{CC}$ and $CN_{CI} \equiv CN_{CC}$.

Straight counting with basic unit authors and with basic unit institutions give very similar results. However, for example, in publications in *Physical Review Letters* with several hundred authors and many institutions, both the authors and the institutions are alphabetized. Therefore there is no identity between S_{IA} and S_{II}. The identities described above are mathematically precise. In Section 8 we will briefly discuss the near identities of some of the 19 different counting methods.

6. Cumulative-turnout and underlying networks

The objects of study do not contain the full information on cooperation found in publications. We therefore construct a new object, the cumulative-turnout network, containing all information about cooperation and at the same time related to the object originally studied.

The cumulative-turnout network for an object *o*, *CTN*(*o*), is constructed in three steps:

1. For the object, *o*, we choose the class of basic units (authors, addresses or countries)
2. We construct the set of all publications in our database containing at least one basic unit from the object.
3. We build the set of the added lists of basic units chosen at step 1 for the publications found in 2.

The set generated in step 3 is an accumulation of the lists from the publications generated in step 2. This is the reason for the choice of the name cumulative-turnout network. Because the full lists of basic units are included, all available information on cooperation is retained in *CTN*(*o*). The object *o* itself is a subset of *CTN*(*o*).

Scientific cooperation is not precisely defined but is an intuitive concept. Here we will use the term cooperation in two ways:

1. A publication is *in cooperation* if there are at least two basic units in the list of basic units for the publication.
2. A publication is with *inter-object cooperation* if at least two different objects defined as sets each contain at least one basic unit from the publication.

6.1. The concepts of size and productivity

The size of an object is the cardinality of the corresponding set of basic units or in other words the *number of basic units in the corresponding set*.

Because $CTN(o)$ is an accumulation from complete lists of basic units in publications, we can without any problem define the *productivity of the cumulative turnout network* $CTN(o)$ as the number of publications used for the construction of $CTN(o)$. This number can be easily calculated using the definition of the whole score function and we obtain:

$$\text{Productivity of } CTN(o) = SF_W(o) \tag{42}$$

The productivity of $CTN(o)$ can therefore be obtained without prior construction of $CTN(o)$.

We can also use the definition of normalized score functions SF_N and obtain:

$$\text{Productivity of } CTN(o) = SF_N(CTN(o)) \tag{43}$$

The CTN can be divided into the internal cumulative-turnout network, $ICTN$, and the external cumulative-turnout network, $ECTN$. $ICTN(o)$ comprises the set of basic units in $CTN(o)$ internal to o and therefore the size of $ICTN(o)$ is equal to $SF_C(o)$. $ECTN(o)$ denotes the set of basic units in $CTN(o)$ external to o.

6.2. Properties of cumulative-turnout networks

Based on the definition of a cumulative-turnout network, we can also define the *class of cumulative-turnout networks*, $\boldsymbol{N}$, constructed from all objects of $\boldsymbol{R}$.

Here we mention only properties of the objects in $\boldsymbol{N}$ necessary for our discussion and for comparison of counting methods:

1. For an object o the cumulative-turnout network contains the names in the underlying network at least once, but often more than once.
2. The world X is a cumulative-turnout network.
3. The class $\boldsymbol{N}$ can be constructed in a different way: We chose an arbitrary subset of publications and build the disjoint union of these object publications. So there are as many cumulative-turnout networks, as there are subsets of the set of all publications.
4. The union of two cumulative-turnout networks is again a cumulative-turnout network. Similarly the difference between two cumulative-turnout networks is a cumulative-turnout network. Therefore $\boldsymbol{N}$ is a proper, non-hereditary subring of $\boldsymbol{R}$.
5. If o is a subset of p then also $CTN(o)$ is a subset of $CTN(p)$
6. The cumulative-turnout network of the union of two objects is the union of the two cumulative-turnout networks defined by o and p respectively:

$$CTN(o \cup p) = CTN(o) \cup CTN(p) \tag{44}$$

7. For any object o its cumulative-turnout network contains o as a subset, i.e.

$$\mathrm{ICTN}(o) \subset CTN\,(o).$$

6.3. The sizes of cumulative-turnout networks are non-additive.

Using the same method as for whole counting it is easy to show that the sizes for cumulative-turnout networks for objects are non-additive, i.e. cannot be added to give the size for unions of the objects.

Complete counting methods measure the size of an object o, the number of basic units assigned to the object. Therefore the size of $CTN\,(o)$ is $SF_C\,(CTN(o))$, the size of $ICTN(o)$ is $SF_C(ICTN(o))$ and the size of $ECTN(o)$ is $SF_C(ECTN(o))$. Based on the fact that cumulative-turnout networks are unions of objects as sets derived from the publications in our database this leads us to *a score function* for the object o:

$$SF_C\,(CTN(o)) = \sum_R B_R\,(CTN(o)) = \sum_R \theta(B_R(o))B_R \tag{45}$$

where B is a basic unit, R a publication, B_R the number of basic units in publication R and $\theta\,(B_R(o))$ has the value 1 for positive numbers and 0 otherwise.

The size of the cumulative-turnout network defined here must not be confounded or mixed with the (much smaller) size of an underlying network, i.e. the number of object names in an underlying network (see Section 6.4).

When studying the properties of this new score function $SF_C\,(CTN(o))$, we first recognize that even for only two disjoint objects o and p, the corresponding cumulative-turnout networks in general are not disjoint. For example if o = Denmark and p = Switzerland, then the present publication (respectively its list of basic units) lies in the intersection of the corresponding cumulative-turnout networks.

In the following discussion we use a subdivision of a cumulative-turnout network:

$$CTN\,(o) = CTN(o) \cap ICTN(o) \cup CTN(o) \cap (X - ICTN(o)) = ICTN(o) \cup ECTN(\mathrm{o}) \tag{46}$$

If o is a country then *ICTN(o) is the national part of the cumulative-turnout network* and similarly *ECTN(o)* is the *international part of the cumulative-turnout network.*

Complete scores are additive and therefore:

$$SF_C(CTN(o)) = SF_C(ICTN(o) \cup ECTN(o)) = SF_C(ICTN(o)) + SF_C(ECTN(o)) = \\ SF_C(o) + SF_C(ECTN(o)) \tag{47}$$

This formula may seem trivial. It says essentially that the size of the internal part of the object network and the size of the external part of the object network add up to the size of the object network. But it gives us the possibility of insight into the relative strength of national and international cooperation.

To this end, we divide the first and the last term in (47) by the size of the cumulative-turnout network:

$$1 = \frac{SF_C(o)}{SF_C(CTN(o))} + \frac{SF_C(ECTN(o))}{SF_C(CTN(o))} \tag{48}$$

From this we obtain the *extent of autonomy or of internal/national cooperation:*

$$\frac{SF_C(o)}{SF_C(CTN(o))} \tag{49}$$

and the *extent of external/international dependence or of external/international cooperation*:

$$\frac{SF_C(ECTN(o))}{SF_C(CTN(o))} \tag{50}$$

With no external cooperation in the object o, the two score-functions $SF_C\ (CTN(o))$ and $SF_C(o)$ (and their corresponding sets of counting methods) coincide. The only factor causing differences between these two score functions is (a type of) cooperation. In other words the difference between SF_C(CTN(o)) and SF_C (o) indicates cooperation:

$$SF_C\ (CTN(o)) - SF_C(o) = \sum_R \theta\ (B_R\ (o))B_R - \sum_R B_R(o)\ = \sum_R \theta(B_R(o))(B_R - B_R(o)) \tag{51}$$

The difference defines *a new score function* and the third term indicates the properties of this function. Only publications with at least one basic unit from the object are counted. However, only basic units not from the object are counted. Therefore the difference is an indicator of the strength of the external cooperation. We recognize that this indicator of cooperation gives positive scores only for publications with at least two basic units. This is in agreement with the intuitive definition of cooperation. They coincide if and only if the object is not involved in external cooperation.

(51) also gives the inequality:

$$SF_C(CTN(o)) \geq SF_C(o)\ \forall\ (o) \tag{52}$$

Whole scores for the object o measure the productivity of the cumulative-turnout network $CTN(o)$. We combine (42), (43) and (47) and obtain for all normalized and additive score functions SF_{NA}:

$$SF_W(o) = SF_{NA}(CTN(o)) = SF_{NA}\ (SF_C\ (o)) + SF_{NA}\ (ECTN(o)) \tag{53}$$

Therefore, we find that any normalized and additive score function subdivides the productivity of the cumulative-turnout network into the *internal productivity of the object* and the *productivity of the external part of the cumulative-turnout network* without a change in the definition of a publication.

With the choice of complete-normalized scores we can give an illustration of the rather abstract object *ECTN(o)* by calculating the difference between whole scores and complete-normalized scores:

$$SF_{CN}(ECTN(o)) = SF_W(o) - SF_{CN}(o) = \sum_R \theta(B_R(o)) - \sum_R \frac{B_R(o)}{B_R} = \sum_R \theta(B_R(o)) \frac{B_R - B_R(o)}{B_R} \quad (54)$$

This again is *a new score function*.

We can interpret the last expression in (54) as the productivity of the external part *ECTN*(o) of the *CTN*(o). The quotient in the last term is just the percentage of basic units in *CTN*(o) but external to o.

With (53) we can define further indicators:

$$\frac{SF_{NA}(SF_C(o))}{SF_W(o)} \quad (55)$$

which is the degree of internal productivity.

Similarly:

$$\frac{SF_{NA}(ECTN(o))}{SF_W(o)} \quad (56)$$

is the degree of external productivity.

From (53) we see that with no external production, *ECTN(o)* is empty. In this case whole scores and all normalized/additive score functions and the corresponding counting methods provide identical results. Therefore with the use of the interpretation in (53) it is again shown that the only factor causing differences between these two score functions is cooperation.

From the score functions for *W* and *SCTN* we obtain:

$$SF_C(CTN(o)) - SF_W(o) = \sum_R \theta(B_R(o))\, B_R - \sum_R \theta(B_R(o)) = \sum_R \theta(B_R(o))(B_R - 1) \quad (57)$$

The difference defines *a new score function* and the third term indicates the properties of this function. Only publications with at least one basic unit from the object and with at least two basic units and therefore based on cooperation are counted. Therefore the difference is an indicator of the strength of total (internal and external) cooperation.

(57) gives the inequality:

$$SF_C(CTN(\mathrm{o})) \geq SF_W(o) \quad (58)$$

Cumulative-turnout network scores and whole scores coincide if and only if there is no cooperation in the object.

6.4. Underlying networks

The cumulative-turnout networks are different from the notion of scientific networks found in the literature. Therefore, we define a new type of score functions and describe the class of underlying networks, quantifiable and in accord with the general concept of scientific networks.

Underlying networks for objects, $UN(o)$, are the authors, institutions or countries behind the cumulative-turnout networks. This means that an author will be counted once and only once if he or she is contributing to one or more of the publications in the set of publications produced by the cumulative-turnout network. Underlying networks can be divided into internal underlying networks, $IUN(o)$, and external underlying networks, $EUN(o)$. $IUN(o)$ can for example be the authors from the same institution or the same country contributing to the publications published by an $UN(o)$. Then $EUN(o)$ is the authors from all other institutions or all other countries included in the underlying network.

The sizes of $UN(o)$, $IUN(o)$ and $EUN(o)$ are determined by counting all unique occurrences of authors, institutions or countries in the corresponding $CTN(o)$, $SF_C\,(o)$ and $ECTN(o)$. The sizes are non-additive for $UN(o)$ and $EUN(o)$, additive for $IUN(o)$. The size of $IUN(o)$ is equal to $SF_W(o)$.

6.5. The usability of cumulative-turnout networks and underlying networks

The construction of cumulative-turnout and underlying networks will give a connection between counting methods and productivity and cooperation measures and give a quantitative interpretation of the abstract term *cooperation*. This will give new insight into and opportunities for quantifying the various aspects of cooperation.

The networks can be constructed with authors, institutions or countries as objects. For most of these objects the basic unit authors, institutions or countries can be used. Therefore the networks will give opportunities for answering many interesting questions about scientific cooperation.

Among these are:

With how many authors is an author cooperating?
How many of these authors belong to the same institution as the author?
How many of these authors belong to the same country as the author?
How many of these authors belong to other institutions in the author's country?
How many of these authors belong to other countries?
What is the number of countries in which an author has cooperators?
With how many institutions is an institution cooperating?
How many of these institutions belong to the same country as the institution?
How many of these institutions belong to other countries than the institution?

What is the number of countries in which an institution has cooperators?
What is the number of authors in a country?
What is the number of authors in other countries with which authors in a country are cooperating?
What is the number of institutions in a country?
What is the number of countries with which authors from a country are cooperating?

Answers to these questions can be used to provide fractions and time series. They can also be restricted to scientific field. Therefore the networks proposed and their connection with publication counting give new opportunities in scientometrics.

It must be stressed that in our analysis we are distinguishing between production and cooperation. Measurements of production are based on assigning 1 credit to a publication. Production can be measured for *CTN*, *ICT*, *ECT*, *UN*, *IUN* and *EUN*. Cooperation measures the number of authors, institutions or countries behind production.

7. Combination of scores to provide indicators for size, productivity and cooperation

7.1. Indicators for productivity

We mentioned above (Section 6.3, (53)) that any normalized and additive score function subdivides the productivity of the cumulative-turnout network into the *internal productivity of the object* and the *productivity of the external part of the cumulative-turnout network* without a change in the definition of a publication.

7.2. What can we learn from comparisons of score functions?

In (57) the comparison between the size of the cumulative-turnout network $SF_C(CTN(o))$ and the object $SF_W(o)$ informs us about the extent of external cooperation.

7.3. Combination of whole scores and complete-normalized scores

In Section 6.3, formula (54) we defined a new score function.

The last term in (54) is an indicator for cooperation. The last expression in (54) is the productivity of the external part $ECTN\,(o)$ of the $CTN(o)$.

In Section 6.3, formulae (55) and (56) we defined the indicators for the degree of internal productivity and the degree of external productivity.

The indicators can also be interpreted as an indicator for the extent of autonomy, whereas (56) can be interpreted as an indicator for the extent of external dependence (dependence from abroad if *o* is a country). (55) can also be interpreted as an indicator for the extent of external networking (international networking for countries).

For countries the last two interpretations of (56) are two aspects of the same issue: If the international networking is strong, this will necessarily result in a large dependence from abroad for the production. If the dependence is large, this also indicates that international networking is strong. For institutions (55) can be interpreted as an indicator for visibility and (56) as an indicator for networking activity.

The quotients in (55) and (56) therefore can be used to answer a couple of possible research questions. In this way they show that several questions can be equivalent (can be answered by the same method). On the other hand the questions fix the counting methods to be used for the answer.

From (54) we see that with no external cooperation $ECTN(o)$ is empty. In this case whole scores and all normalized/additive score functions and the corresponding counting methods provide identical results. Therefore with the use of the interpretation in (54) it is again shown that the only factor making differences between these two score functions is cooperation.

Of course $SF_{CN}(o)$ is an element of the class of all normalized and additive score functions $SF_{NA}(o)$. Therefore (54) gives us the inequality:

$$SF_W(o) \geq SF_{NA}(o) \tag{59}$$

7.4. Combination of complete and whole scores

From the definitions of the score functions we obtain:

$$SF_C(o) - SF_W(o) = \sum_R B_R(o) - \sum_R \theta(B_R(o)) = \sum_R \theta(B_R(o))(B_R(o) - 1) \tag{60}$$

Again the last term gives positive scores only for publications with more than one basic unit and is an indicator for cooperation. The term ($B_R(o)$ - 1) indicates that it is an indicator of *internal cooperation*.

The difference defines *a new score function* and the third term indicates the properties of this function. The only publications contributing to this score function are publications with $B_R \geq 2$, i.e. publications *with internal cooperation* (*national/domestic cooperation* when *o* is a country). The difference between complete and whole scores therefore is an indicator for the strength of internal (national/domestic) cooperation.

(60) gives the inequality:

$$SF_C(o) \geq SF_W(o) \tag{61}$$

Complete and whole scores coincide if and only if there is no internal cooperation in the object.

7.5. Combination of cumulative-turnout network sizes and whole scores

The difference in formula (51) in Section 6.3 defined *a new score function* giving an indicator for the strength of total (internal and external) cooperation.

(51) gives the inequality:

$$SF_C(CTN(\mathrm{o})) \geq SF_W(o) \tag{62}$$

Cumulative-turnout network scores and whole scores coincide if and only if there is no cooperation in the object.

7.6. Combination of the size of an object with the productivity of an object and therefore combination of complete scores and complete-normalized scores

In (53) we recognized that in principle any normalized additive score measures productivity. However, based on the principle of equal treatment of internal and external basic units (the principle of fairness) complete-normalized scores must be chosen. This leads us to the comparison:

$$SF_C(o) - SF_{CN}(o) = \sum_R B_R(o) - \sum_R \frac{B_R(o)}{B_R} = \sum_R \frac{B_R(o)}{B_R} (B_R - 1) \tag{63}$$

Comparison of (51) with (63) shows that the third terms look very similar. This is not surprising since whole scores measure the productivity of the underlying network. In both formulas sizes and productivities are compared. In (63) the factor (B_R-1) selects the publications with at least two basic units and which are thus based on cooperation. The factor $B_R(o) / B_R$ weights the publications proportional to the fraction of basic units from the object.

(63) gives the inequality:

$$SF_C(o) \geq SF_{CN}(o) \tag{64}$$

Complete and complete-normalized scores coincide if and only if there is no cooperation in the object.

7.7. Equality of and difference between scores

The comparisons in Sections 6.3 (52) and (58), 7.4 (61), 7.5 (62) and 7.6 (64) can be summarized:

$$SF_C(CTN(o)) \geq SF_C(o) \geq SF_W(o) \geq SF_{CN}(o) \tag{65}$$

If there was no cooperation at all, then all these score functions would give identical results.

The indicators described in Sections 7.5 and 7.6 are indicators for the total cooperation. This however, does not mean that the indicators give identical values. They are indicators for the same thing but they give different answers.

8. Identities

If there was no cooperation, all counting methods would produce the same results.

For the following classes of publications the scores can be identical for different counting methods:

1. Publications with only one basic unit (author, institution or country)
2. Publications containing basic units from only one scoring object ($B_R(o)$=B_R and o_R = 1)
3. Publications with n basic units from n scoring objects
4. Publications, P, for which o_P $B_P(o)$=B_P, including the classes described in 1–3.

In 1993 only a few publications belonged to other classes but this is no longer the case. Therefore, today scores from whole-normalized counting differ significantly from scores from complete-normalized counting.

The exact difference between complete counting and whole counting contains the factor (B_{Ro} –1). This factor is zero whenever there is no internal cooperation but it is independent of how many external partners there are. Whereas it is known that the larger objects tend to more internal cooperation, the contrary is also true: smaller and smaller objects in the series "large countries/regions", "medium countries", "small countries", "institutions", "sub-institutions", "local groups", and finally authors tend to less and less internal cooperation and for authors the difference is exactly zero. This means that for small institutions and below whole counting is a fairly good approximation of complete counting. For not too large institutions the difference between complete and whole counting is only a few per cent.

Similarly the difference between whole counting and complete-normalized counting is only boosted by publications with external cooperation (B_R –B_{Ro}). So for the largest object, the world, whole counting and complete-normalized counting coincide. In general, due to increased external cooperation, whole counting can be taken as an approximation for complete-normalized counting only for objects with very similar external cooperation.

The replacement of complete-normalized counting by straight counting makes only small changes for large objects. The changes become larger and larger for small objects (TRUEBA & GUERRERO, 2004; LANGE, 2001). The reason is that in countries, the distribution of author names in the alphabet are very similar whereas one author has a definite place in the alphabet.

9. Conclusion and recommendations

The meaning of scores from the counting methods discussed can be derived from the exact definitions of the score functions and the methods based on these schemes. On this basis the exact relations between the scores from counting methods and between different counting methods and their relation to scientific cooperation have been proved mathematically. In consequence it has been shown how the combinations can be used to provide valuable scientometric information. These results are summarized in Table 5.

We have shown that the construction of cumulative-turnout networks can be used to provide a connection between counting methods, productivity and cooperation and to give a quantitative interpretation of the abstract term *cooperation*. This analysis has also given new insight into and opportunities for quantification of the various aspects of cooperation. A new type of interactive score functions measuring the interactions (acts of cooperation) and therefore comprising pairs of objects have been found. We have also defined several other networks and shown that they can open possibilities for quantitative studies of scientific cooperation.

In our discussion we have restricted ourselves to defined basic units (authors, institutions, countries) and objects (authors, institutions, countries). There are other possible objects such as subdivisions of institutions or countries, sectors and regions. There can be studies covering all scientific publications and subfields of science.

The different methods give different results, sometimes widely different results. As an example it is obvious that some countries are winners, other countries losers, in rankings based on whole counting. Among the winners are countries with a small science base and/or with a high level of international cooperation. Both small and large countries are represented here. Among the losers are countries with a low level of international cooperation, including Japan and USA. There is a disregard of the dominating position of USA in the world of science. USA has a comparatively low level of international cooperation but this is not because scientists there do not cooperate. Cooperation between different institutions in USA substitutes for international cooperation.

Even if the different counting methods for publications give different results there is only limited knowledge about the size, and the change through time, of the differences. Some information is available about the situation for countries but virtually no information is available about institutions and individuals.

Table 5. Score functions, sizes for cumulative-turnout networks, counting methods and combinations of sizes of networks with scores from counting methods as scientometric indicators

Score function. References in brackets are the formulas for score functions defined in the present publication	**Network size and publication scores**	**Indicator**	**Purport**
Complete scores	C_{AA} C_{IA} C_{CA} C_{CI}	Score for an object	The size of the internal network in which the object of study is an element. Complete scores measure the size (number of basic units) of objects including the special objects networks.
Complete-normalized scores	CN_{AA} CN_{IA} CN_{CA} CN_{CI}	Score for an object	The productivity of the object of study appreciating external contributions
Size of the cumulative-turnout networks (45)	CTN_{AA} CTN_{IA} CTN_{CA} CTN_{II} CTN_{CI} CTN_{CC}	Size for the network of all basic units connected with the object	The present state and growth of cooperation in the network to which the object of study belongs
Straight scores	S_{AA} S_{IA} S_{CA} S_{II} S_{CI}	Score for an object	
Whole scores	W_{IA} W_{CA}	Score for an object	The productivity of the object of study disregarding external contributions *or* the productivity of the network in which the object of study is an element (Whole counts measure contribution)
Whole-normalized scores	WN_{IA} WN_{CA}	Score for an object	
Complete scores versus whole scores (60)	C_{IA} versus W_{IA} C_{CA} versus W_{CA}	Difference between scores	The extent of the internal cooperation. If the difference is 0 there is no internal cooperation.
Size of cumulative-turnout networks versus complete scores (51)	$SCTN_{AA}$ versus C_{AA} $SCTN_{IA}$ versus C_{IA} $SCTN_{CA}$ versus C_{CA}	Difference between scores	The extent of the external cooperation. If the difference is 0 there is no external cooperation.
Whole scores versus complete-normalized scores (54)	W_{IA} versus CN_{IA} W_{CA} versus CN_{CA}	Difference between scores	The extent of the external cooperation. If the difference is 0 there is no external cooperation.
Size of cumulative-turnout networks versus whole scores (57)	$SCTN_{IA}$ versus W_{IA} $SCTN_{CA}$ versus W_{CA}	Difference between scores	The extent of the total cooperation. If the difference is 0 there is no cooperation.
Complete scores versus complete-normalized scores (63)	C_{IA} versus CN_{IA} C_{CA} versus CN_{CA} C_{AA} versus CN_{AA} C_{CI} versus CN_{CI} C_{II} versus CN_{II}	Difference between scores	The extent of the total cooperation. If the difference is 0 there is no cooperation.

There is an urgent need for information about the differences obtained by using different counting methods. Here it is also necessary to study the differences between different scientific fields, an area not included in the present work.

There is a special problem of making time series which partly depend on older analyses. This is because counting methods are often insufficiently described if at all, and because counting methods have changed.

The major reasons for the problems are the lack of clear definitions and a clear and consistent terminology. In order to reach a common understanding of counting methods, these deficiencies must be rectified. Clear definitions are also a prerequisite for using the mathematical tools in set and measure theory for an exact analysis of the properties of counting methods.

Definitions, common terminology, models, theoretical predictions and empirical tests are needed. However, the problems of common terminology can only be solved by a joint effort involving scientists, those responsible for national and international reports on R&D statistics and science indicators, and those responsible for the databases providing the basis for the work.

We are not recommending standardisation. Different counting methods measure different indicators. Therefore the question is: Which counting methods for which indicators?

When it comes to citation counting there is a nearly complete lack of knowledge on the effects of different counting methods. The problems in citation counting are even larger than for publication counting. It is well known that multiauthored publications, publications coming from more than one institution, and publications coming from more than one country on average are cited more than single-authored publications (BUTLER, 2003; GLÄNZEL, 2000, 2001; GLÄNZEL & SCHUBERT, 2001; LINDSEY, 1980; NARIN et al., 1991; VAN RAAN, 1997). It is also known that international co-authorship results in publications with higher citation rates than purely domestic publications (GLÄNZEL, 2000, 2001; NARIN & WHITLOW, 1990; NARIN et al., 1991; PERSSON et al., 2004; PERSSON & DANELL, 2004). Different counting method therefore will give citation counts with larger differences than those for publication counts.

The choice of counting methods must depend on the questions addressed and on knowledge of the different results. The combined use of counting methods provides valuable information on the extent and character of scientific cooperation between scientists, institutions, and countries. In conclusion we note that the following questions must be addressed:

1. Is the difference between scores from different counting methods important or trivial? In short: Does it matter?
2. Are the differences constant through the years or are they changing because of the ever-increasing national and international cooperation in science?

3. If the differences are changing with time are they then making the choice of counting method more important?

We are convinced that it does matter and that the differences are increasing with the ever-increasing cooperation in science. Therefore it is important to choose the right method(s) and to exploit the information provided by the differences between different methods. But further theoretical and empirical studies will have to be carried out to provide more precise answer to the questions above. Furthermore the results must be used to give a better understanding of and quantitative information about scientific cooperation and scientific networks.

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