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ARTICLE

Guiding Physics Teachers by Following in Galileo's Footsteps

Maayan Schvartzer[1] · Michael Elazar[2] · Shulamit Kapon[1]

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Abstract

Many physics learners take the specific mathematical representations they are using as part of their learning and doing physics for granted. The paper addresses this problem by highlighting two goals. The first is to show how a historical investigation from history of science can be transformed into a concrete lesson plan in physics, in a physics teacher education program. The second is to explore the role of mathematical representations in scientific inquiry and discuss the educational affordances of historical case studies in explicating this role in preservice and in-service physics teacher education. The historical artifact that formed the basis of the lesson is a page cataloged as "folio 116v" that contains Galileo's authentic laboratory notes and calculations written at the time he made his revolutionary discoveries on freely falling objects and projectile motion. To understand and reproduce Galileo's authentic notes, students must first become explicitly aware that the mathematical tools and representations available in his time were radically different from the tools and representations available to physics learners now. The activity thus sparked discussions and reflections on the meanings and implications of mathematical representations in learning and doing physics.

1 Introduction

We present a reflection on the design and teaching of a lesson plan that was implemented in a physics teacher education program at a research university. The lesson plan emerged from the collaboration between two science educators (1st and 3rd authors) and a historian of science (2nd author) in a project supported by the Israeli Ministry of Education aimed at suggesting instructional models that creatively incorporate the history of science into the instruction of physics. This article illustrates how an investigation of a history of science can be transformed into a concrete lesson plan in physics in a physics teacher education program. It then reflects

✉ Shulamit Kapon
skapon@technion.ac.il

[1] Faculty of Education in Science and Technology, Technion – Israel Institute of Technology, Haifa, Israel

[2] The Cohn Institute for the History and Philosophy of Science, Tel Aviv University, Tel Aviv, Israel

on the role of representations in scientific inquiry elicited by this design, and the affordances of discussing this issue with preservice and in-service teachers.

The activity (~ 180 min) was designed and implemented in a professional development course for in-service physics teachers that focused on utilizing the history of science in science education, and in a course for preservice physics teachers on key topics in high school physics curricula from different perspectives. The activity positioned the students as scientists in the role of assistants to a historian of science. It engaged them in deciphering the meaning and implications of (almost) authentic laboratory notes written by Galileo Galilei (1564–1642). The students had to solve a problem that has preoccupied historians of science for many years, namely, whether Galileo primarily relied on theories and pure thought experiments or whether experimental studies played an important part in his scientific work (and if so to what extent). To understand Galileo's authentic notes, the students had to become explicitly aware that the mathematical tools and representations that were available in his time were radically different from the tools and representations available to current day physics learners. Our reflection focuses on the affordances of these experiences for future and in-service physics teachers.

2 The Role of Representations in Scientific Inquiry

Uhden et al. (2012) differentiated between two approaches to using mathematics in physics teaching. The first, termed "technical," involves an instrumental (tool-like) use of mathematics and is mainly found in instruction that focuses on end-of-chapter problem solving. The second, termed "structural," focuses on reasoning about the physical world mathematically. Uhden et al. called for physics instruction that explicates the strong conceptual relations between mathematics and physics as embodied in the second approach. Karam (2014) describes a case study of an outstanding university instructor teaching a course on electromagnetism. The analysis broke down how the instructor explicated the process of mathematizing, i.e., "constructing a mathematical representation for a physical situation" (p. 5) to his students. To be able to "translate" successfully students need to learn to "see the world through mathematical lenses" (p. 6). "Seeing" involves making simplifying assumptions about the phenomena in question, and then finding an appropriate "mathematical structure" to represent them. Karam discusses the ways in which these processes were embedded in the instructor's discourse and his instructional moves. Other moves were aimed at depicting the reverse process, i.e., inferring a physical interpretation from a mathematical representation. This back and forth between the mathematical representation, and the phenomenon it represents has also been documented in the context of students' reasoning during authentic problem solving sessions (Bing and Redish 2007; Hu and Rebello 2013a, 2013b; Sherin 2001; Tuminaro and Redish 2007).

One underlying implicit assumption that appears to be shared by all the studies cited above is that the specific mathematical representations the students work with are "simply" there. Studies of real-world cognition suggest that it recruits external resources and achieves its goals through a complex interaction with people, artifacts, and representations in the physical environment (Hutchins 1995, 2006). Other studies have noted that representations are culturally bound (Saxe and Esmonde 2005, 2012). As clearly evidenced by the history of science and mathematics, new representational forms can entail radical intellectual consequences. Consider for example the affordances of the transition from Roman numerals to our current Hindu–Arabic numerals. Try to multiply 1977 by itself as opposed to doing the same for

MCMLXXVII. Hindu–Arabic notation makes this exercise something an average upper elementary school pupil can solve (diSessa 2018). Cognitive scientists who compared the cognitive processes involved in simple arithmetic calculations with Hindu–Arabic vs. Roman numerals claim that the latter entails "greater cost in terms of the number of necessary elementary information processes, especially those using external resources" (Schlimm and Neth 2008, p. 2102). Similarly, in his *Changing Minds*, diSessa (2000) discusses how not having algebraic representations impacted Galileo's work in simple kinematics (Galilei 1989). diSessa retrospectively describes his amazement at the complexity and length of Galileo's proofs which are expressed in text-based representations: "I was baffled by Galileo's proofs because algebraic reasoning was so natural and simple to me. Indeed, the properties of representations and their cultural power are not commonly appreciated" (diSessa 2018, p. 6). Similar arguments are often made with respect to the affordances of computational-based representations in scientific inquiry (diSessa 2000; Wilensky and Papert 2010).

Awareness of the role played by the mathematical tools and representations of the time in shaping the nature and products of scientific inquiry is a crucial aspect of teachers' professional knowledge (Grossman 1990; Kapon and Merzel 2019). This awareness can lead to a better understanding of the importance of the "structural role" of mathematics in physics in physics instruction (Uhden et al. 2012).

3 The Historical Context

3.1 Galileo's Experimental Studies

The historical artifact (see Fig. 1a) constituting the basis of the activity we designed is a page cataloged as "folio 116v" in volume 72 of Galileo's manuscripts, also known as "Codex 72." This folio contains authentic laboratory notes and calculations related to projectile motion. It was written by Galileo between 1604 and 1610, thus exactly the time when he was making his revolutionary discoveries on freely falling objects and projectile motion (Naylor 1977). Folio 116v has a fascinating history of its own (Drake 1973; Hill 1986, 1988; Naylor 1976, 1977; Teichmann 2015; Wisan 1984). Along with many other original working notes by Galileo, it was not included in the official edition of Galileo's writings published by Antonio Favaro between 1890 and 1909, which was supposed to include all Galileo's published works, letters written by him or to him, and unpublished manuscripts and other documents (Galilei 1890–1909). Folio 116v was in fact one of the many drafts reposing in the National Central Library of Florence (*Biblioteca Nazionale Centrale di Firenze*) waiting to be rescued from oblivion. During the 1970s they were rediscovered and have profoundly influenced how historians now describe Galileo's route to his most important achievements. One example is the law of falling bodies, namely, the direct proportion of the distance fallen to the square of the times elapsed (Naylor 1977).

Historians have known that Galileo owed his success to his unique combination of skills and methods. Specifically, Galileo used mathematical deductions, formulated thought experiments, conducted real experiments (notably the famous "inclined plane experiments"), and also skillfully relied on the work of his predecessors (for example, fourteenth century mathematicians, who had worked out important uniform acceleration theorems that Galileo later developed, but had failed to relate them to falling bodies (Clagett 1948, p. 39).

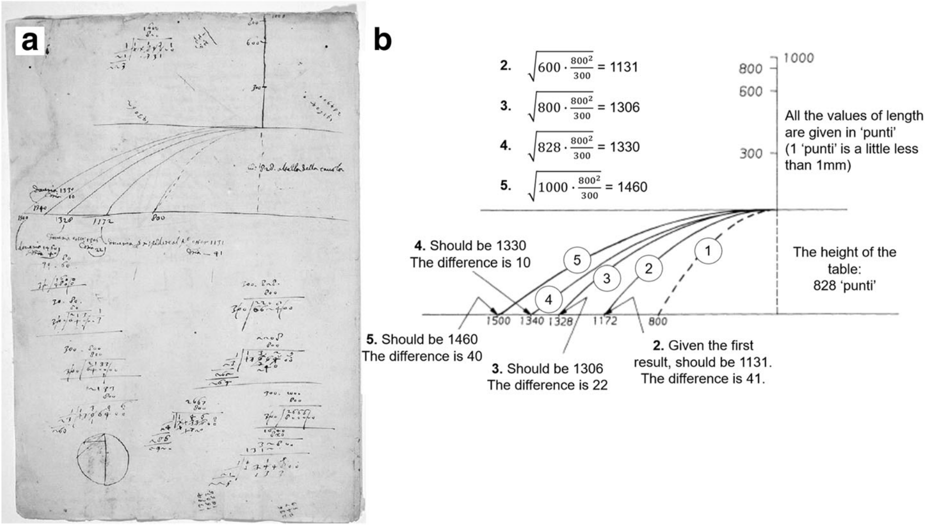

Fig. 1 **a** The original page from Galileo's laboratory notes, Galileo's folio 116v from the National Library, Florence (appears in Hill 1986, p. 285; Teichmann 2015, p. 37). **b** A translation of the page, adapted for educational purposes

The debate, however, centers on the relative importance of each of these components. Galileo never tired of alluding to the numerous experiments he conducted, as did his first biographer, Vincenzo Viviani, whose reliability is often viewed with doubt. Nineteenth and early twentieth century historians became convinced that Galileo's experiments were a highly important ingredient in his work and overall scientific outlook (Segre 1998, pp. 390–393). This is why Galileo came to be depicted as "the father of experimental physics."

However, around 1940, the French historian Alexandre Koyré literally revolutionized this field. Koyré (1943) challenged many of Galileo's accounts of his own experiments, especially those published in *Two New Sciences* (Galilei 1989, originally printed in 1638), his book describing his most important contributions to physics. Koyré claimed that Galileo's reports of his own experiments were unreliable. He argued that if Galileo had really performed the experiments he mentioned, he could not have ended up with the results he described. Koyré reached the (highly influential) conclusion that the main (and perhaps the only important) component of Galileo's toolbox of methods was theoretical, i.e., Galileo's thought experiments, mathematical deductions, and philosophical considerations. Koyré ascribed them to the influence of Plato, which thus contrasted with the Aristotelian inclinations that characterized most of Galileo's opponents (Koyré 1943).

The tide began to turn again in 1961, when the historian David Settle (1961) successfully reconstructed Galileo's inclined plane experiments. Settle showed that contrary to Koyré's claims, Galileo indeed could have conducted (with satisfactory accuracy) the key experiments described in *Two New Sciences* (Galilei 1989). During the 1970s, the discovery of 160 folio pages (among them folio 116v), i.e., Codex 72, further corroborated Settle's conclusion that the empirical side of Galileo's work was far more important than Koyré ever knew.

Codex 72, which does not appear at all in Favaro's edition (Galilei 1890–1909), was carefully examined by the historian Stillman Drake. It became immediately apparent that these pages were actually Galileo's laboratory reports depicting many of the experiments described in his published works (Drake 1973). There is now a general consensus among all historians that Galileo's methods and scientific mindset cannot possibly be understood without thoroughly studying Galileo's working notes, including folio 116v.

However, beyond the obvious problem that these working notes are not easy to decipher, understand, or interpret (see Fig. 1a), the unfortunate fact is that the Codex 72 folios are not dated. What can safely be assumed is that they must have been written between 1604 and 1610 (Naylor 1977, pp. 365–366). Thus, historians are forced to order them chronologically as they see fit. The result is that Galileo's route to his achievements is still being debated today. In truth, Galileo's imagined trajectories to his theorems are almost as numerous as the number of historians who have pondered or expounded on them (Heilbron 2010, pp. 126–128; Renn et al. 2000).

3.2 The Mathematical Tools and Representations Available at Galileo's Time

During Galileo's time, algebra as we know it today did not exist, since René Descartes' *La Géométrie*—which was destined to invent modern algebra—was only printed in 1637. At the time when Galileo wrote his *Two New Sciences* the typical way of expressing the relations between two variables was not through equations but in terms of proportionality expressed in texts and drawings. This method was based on the theory of proportion described in Euclid's *Elements* (c. 300 BC) which was invented by Eudoxus of Cnidus (c. 390—c. 337 BC) (Drake 1974). Moreover, when modeling motion at that time, proportionality mainly described relations between directly measured variables such as displacement and time. Relatively complex variables such as instantaneous velocity and acceleration were impossible to express (directly, at least) by these representations. For example, today we use the equation $x = \frac{1}{2}at^2$ to describe the displacement x of a body that moves (from rest) in a straight line at a constant acceleration a, over a period of time t. In Galileo's era this relation would have been described by a simple proportionality between the displacement (x) and the square of the time elapsed (t^2), a proportionality that can be parsimoniously described today by algebraic representations such as $x \propto t^2$. In Galileo's time this proportionality was described in words (i.e., text-based representations): "If a moveable descends from rest in uniformly accelerated motion, the spaces run through in any times whatever are to each other as the duplicate ratio of their times; that is, are as the squares of those times" (Galilei 1989, p. 166).

Galileo's understanding that free fall involves constant acceleration was expressed (in the same book) by what he saw as a kind of axiom: "I say that motion is equably or uniformly accelerated which, abandoning rest, adds on to itself equal momenta of swiftness [i.e., velocity] in equal times". This "axiom" amounts to $v \propto t$ in an algebraic representation. In fact, Galileo's failure to adequately prove this exposed him to criticism. In the same manner, today we mathematically model motion at a constant velocity with the equation $x = vt$. In Galileo's time this relation would have been formulated in a lengthy textual representation that could be easily represented today by $x \propto t$. Moreover, Galileo's insight that the projectile's velocity only changes in the vertical direction was not yet accepted by many of his contemporaries, although Galileo could have inferred this consequence from an ancient text. This text, attributed then to Aristotle but actually written by one of his disciples, was rediscovered in the mid-sixteenth century and was well known in Galileo's time (Elazar 2013, pp. 190–192). As is well known, the concept of energy was introduced much later, so that it comes as no surprise that Galileo did not employ energy considerations.

4 Transforming a Historical Conundrum into a Physics Lesson for Preservice and in-Service Physics Teachers

The activity discussed in this section consists of four lessons (~ 180 min). The required prior knowledge includes an understanding of proportionality, quadratic equations, and the kinematics of motion with constant acceleration in one and two dimensions. Although the concept of energy was unknown during Galileo’s time, a qualitative notion of energy transformations is required for the final stage of the students’ analysis.

4.1 Introducing the Activity

The activity starts with a fictional letter from Luigi Baron (imaginary character), the chief historian of the Florence National Central Library in Italy (see Fig. 2). The imagery “chief historian” Luigi Baron (Fig. 2) asks the students (“members of the scientific community”) for assistance in deciphering the meaning of the figures and calculations in an attached copy of a page from Galileo’s original laboratory notes (Fig. 1). The students start working in groups on this assignment guided by a worksheet. The instructors circulate among the groups, probing, problematizing, and assisting, and often interrupt the group work to hold whole class discussions. In principle, we could have chosen to present the activity directly (i.e., an in-depth examination of Galileo’s original writings). We chose to frame the activity in an entertaining way that is explicitly related to the content, which aimed to encourage the students to engage deeply in the activity (Wanzer et al. 2010). The role of scientists that assist a historian of science reinforced the kind of engagement we were hoping to foster.

4.2 Guided Deciphering of Authentic Scientific Text

The following sub-section describes how the students’ group work related to deciphering the meaning of the text was scaffolded by a specific worksheet and classroom discussions. The

Addressing the scientific community from the Florence National Central Library

Dear members of the international scientific community,

My name is Luigi Baron and I am the chief historian of the Biblioteca Nazionale Centrale di Firenze (The National Central Library of Florence) in Italy. Our collection has many science and history books and writings, but yesterday, an exciting, unprecedented event occurred in our library. During her evening routine inspection, our librarian, Ms. Martha Albergo, found a page on the floor. As she returned to her post, she noticed that the page was very old and contained a graph and calculations: a copy of the page is attached to this letter.

Since Ms. Albergo suspected the page was of historic value, she contacted me. I examined the page carefully and consulted with my colleagues at the University of Florence. We believe it was written by Galileo Galilei at the beginning of the 17th century. We marked the page as folio 116v from volume 72 (“Codex 72”) of Galileo’s manuscripts.

Despite our vast experience with scientific and historical texts, we failed to understand the meaning of the graph and calculations on the page. The page appears to describe an experiment, but we cannot figure out its goal, the discovery it led to, or the theory it intended to establish. Therefore, we are contacting you – distinguished members of the international scientific community – in the hope that you can solve the mystery of the experiment depicted on this page. For your convenience, our translation team has translated the page from Italian to English.

Sincerely,
Luigi Baron, Chief Historian
The National Central Library of Florence, Italy

Fig. 2 The (fictional) letter the students were given to read first assignment on the worksheet was to carefully examine Fig. 1b, pay attention to the axes and the different curved lines on the graph, notice the documented data (e.g., the height of the table), and then identify the phenomenon Galileo was studying. The students struggled a bit, but most groups were able to figure out that Galileo was studying projectile motion, that the ball was horizontally thrown from the edge of the table at different velocities, and that the curved lines on the graph represented the trajectories of projectiles with different initial velocities.

In the second assignment the students were asked what they thought Galileo's experimental apparatus looked like and were then requested to describe the related experimental process. The students quickly came to the conclusion that Galileo had placed an inclined plane on a table, which ended at the edge of the table, and had then released a ball from different heights on the track, so that each release ended with the ball moving horizontally from the edge of the table at a different velocity. The height of the table remained constant so that the height of the free fall did not change throughout the experiment. A schematic diagram that represents what the students were expected to draw appears in Fig. 3. A photograph of a realistic restoration of Galileo's experimental apparatus can be found elsewhere (Teichmann 2015, p. 38).

In the third assignment the students were requested to identify the magnitudes represented by numbers on the horizontal and vertical axes on the graph in Fig. 1b, and then the dependent variable and the independent variables. At this stage, with occasional help from the instructors who circulated among the groups, the students were able to determine that the numbers on the vertical axis in Fig. 1b represented the height of the release point (i.e., H in Fig. 3) and that the numbers on the horizontal axis represented the horizontal displacement of the ball from the edge of the table, when it reached the floor (i.e., D in Fig. 3). Since Galileo could control the height of the release point (H), and measured the resulting horizontal displacement (D), H is the independent variable and D is the dependent variable.

This exercise laid the groundwork for the mathematical modeling required to make sense of Galileo's calculations. To truly follow Galileo's path of reasoning, the students would have had to derive the proportion $H \propto D^2$ (see Fig. 3) without using equations (i.e., using proportionality relations). They would not have been allowed to use Newton's laws of motion—published only in 1687)—or energy considerations, which were not formulated until the nineteenth century (Elkana 1974).

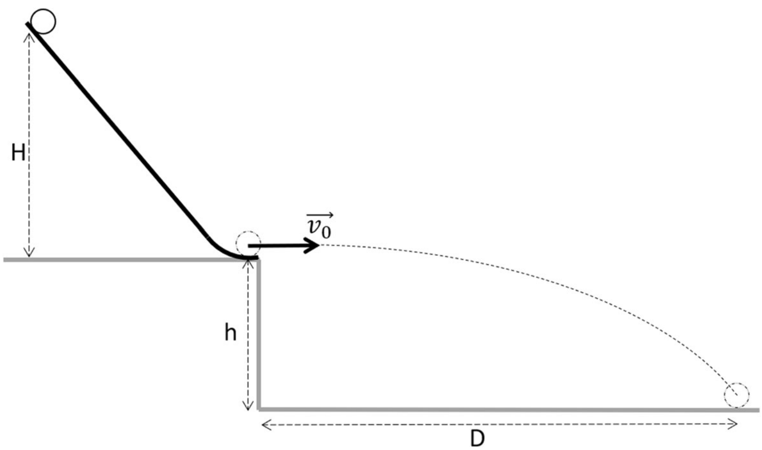

Fig. 3 The schematic diagram of Galileo's experimental apparatus that we expected the students to draw

Given the above constraints, the background of our students, and time management considerations, we made a didactical decision. Our students were used to working with equations rather than proportionalities. Thus, we decided to let them use the algebra they were familiar with and employ Newton's second law of motion to analyze the projectile motion in order to prove the central proportionality relation that Galileo had employed to find the velocity of the ball at the bottom of the inclined plane. The instructions in the worksheet guided them to derive an expression that would immediately allow them to determine the proportionality $H \propto D^2$ that was tested in the experiment. We told them about the different (and rather limited) tools (proportionality) and representations (text and drawings) that were available to Galileo and discussed the rationale in class behind the specific form of calculations in his authentic notes. Hence, when we asked the students to model the motion, we told them that they could use the kinematics of free fall, which was known to Galileo (though not in the same mathematical representation), but not energy considerations, since the concept of energy was not known at the time.

We scaffolded the modeling as follows. First, we told the students that they should take note of the fact that the motion of the ball is a combination of two separate motions, and asked them to divide it into its two components (i.e., motion on the inclined plane followed by projectile motion; i.e., free fall while the ball is in the air).[1] Second, we asked the students to derive an expression that described the relation between the speed (as opposed to the velocity) of the ball at the bottom of the inclined plane (v_0) and the initial height above the table of the release point (H—see Fig. 3). As mentioned above, we told the students that energy considerations could not be used. Instead, we suggested that they use an empirically based "rule" formulated by Galileo at the time and known in the history of mechanics as "Galileo's postulate." This "postulate" asserts (correctly of course, from our point of view) that if a body is released from rest and moves down along a (smooth) inclined plane, its speed at the bottom of the inclined plane does not depend on the inclination of the plane but only on the initial height of the release point (Galilei 1989, p. 162).[2] Hence, to find the relation between the speed v_0 and the height of the release point H, the students were faced with a much simpler problem: finding the speed of an object that is released from rest and freely falls a distance H along a straight line. Using the kinematic equations of motion in constant acceleration in one dimension, which the students had already learned, they ended up with expression $v_0 = \sqrt{2gH}$.

At this point the students were asked to derive kinematic equations for the motion of the ball after it leaves the table and express the horizontal and vertical displacements (D and h, respectively, see Figure 3) when the ball hits the floor ($D = v_o\, t_h; h = \frac{1}{2} g t_h^2$, while t_h is the time elapsed during the projectile motion). We then asked the students to use these two expressions to derive a relation between D and v_o only using the variables that were constant throughout the experiment; namely, g is the gravitational acceleration and h is the height of the table ($D = \sqrt{\frac{2hv_0^2}{g}}$). Since the students had already found that $v_0 = \sqrt{2gH}$, they could substitute it into the previous equation to get the equation $D = \sqrt{4hH}$, in which the proportionality Galileo was studying ($D \propto \sqrt{H}$) becomes transparent.

[1] It might be a good idea at this stage to make sure the students realize that the ball encounters air resistance during its motion, but that this does not significantly affect either its motion on the inclined plane or its projectile motion (free fall).

[2] As was the case with $v \propto t$, Galileo's not entirely satisfactory attempt to prove his "postulate" elicited considerable criticism, for example, from Descartes.

After a class discussion on the differences between the algebraic tools and representations the students were using and the proportionality relations expressed by the texts that Galileo employed (see section 3.2 and the discussion above), the students returned to work on the next assignment in the worksheet. The students were now asked to reexamine the figure Galileo drew and focus on the dashed curve labeled 1 in Figure 1b. They were told to assume that this curve represented Galileo's first measurement (resulting in $D_1 = 800$ punti)[3] and were asked to try to deduce the height H_1 of the first release point from the graph. Since $D_1 = 800$ punti is the lowest horizontal displacement measured, the students correctly deduced that the ball was released from the lowest height that was documented on the graph, thus $H_1 = 300$ punti. In the next assignment they were requested to carefully read Galileo's notes opposite each measured horizontal displacement on the graph (translated in Figure 1b), make sense of the related calculations (see Figure 1b), and explain the content of the verbal notes and the calculations. For example, consider curve 2 in Figure 1b. Near the marking of the measured horizontal displacement $D_2 = 1172$, Galileo added a note: "given the first result, should be 1131, difference of 41" (Hill 1986, pp. 286–287). The result of the calculation is 1131 (calculation #2 in Figure 1b). Hence, $D_2 = 1131$ according to Galileo's prediction. Let us examine this calculation[4]: $\sqrt{600 \cdot \frac{800^2}{300}} = 1131$. Galileo's hypothesis was that $D \propto \sqrt{H}$. Hence, $\frac{D_2}{D_1} = \sqrt{\frac{H_2}{H_1}}$, and thus, $D_2 = \sqrt{H_2 \frac{D_1^2}{H_1}}$, where $D_2 = 1131$, $H_2 = 600$, $D_1 = 800$, $H_1 = 300$. Thus, Galileo's note stated that the measured horizontal displacement was 1172 punti, whereas the predicted one was 1131 punti, resulting in a difference of 41 punti between the measured and the predicted results. Today this difference would have been termed the measurement error (Joint Committee for Guides in Metrology 2012).

At this point the students were asked to examine the proportionality constant, which was empirically determined by the first measurement (i.e., $\frac{D_1^2}{H_1} = \frac{800^2}{300}$) and was used to calculate the other predictions (calculation # 2–5 in Figure 1b). We reminded the students that they derived the equation $D = \sqrt{4hH}$ and asked them to *calculate* a prediction for the first horizontal displacement D_1. Since Galileo wrote that the table height was $h = 828$ punti, $D_{1-\text{calculated}} = 997$ punti, whereas $D_{1-\text{measured}} = 800$ punti constituted an error of almost 25%.

While Galileo did not use the algebra that we implement today, he was aware of the discrepancy (Wisan 1984, pp. 280–281). We can see this in his decision to release the ball from an initial height that was equal to the table height (i.e., $D_4 = h = 828$ punti—see prediction #4 in Figure 1b). Galileo worked with proportionalities, and he formulated a rule based on the assumption of constant acceleration known as the "double distance law." This theorem was developed in the fourteenth century, but Galileo was the first scientist to apply it to falling bodies. This law postulates that if a body starts moving from rest, and moves along a straight line with a constant acceleration over a period of time t covering a distance x, then if it continues to move with the (constant) velocity it reached for the same amount of time (i.e., t again), it will cover an additional distance of 2x (Galilei 1989, p. 168). As mentioned earlier, Galileo inferred the speed of the ball at the edge of the table from the "postulate" described above: if a body is released from rest and moves down along (a smooth) inclined plane, its speed at the bottom of the inclined plane depends solely on the initial height of the release

[3] All the units are in punti. One punti equals about 0.95 mm.

[4] The calculation is a simplification of what Galileo essentially did. His actual division calculations can be found in Hill (1986, p. 287).

point and not on the inclination of the plane. According to this rule, for the purpose of calculating the speed at the bottom of the inclined plane, we can think of a simpler "equivalent" problem, a free fall from rest, in which the body falls from the same height as the height of the inclined plane. Galileo knew that whether the free fall starts from rest, or whether the body receives an initial velocity which is only horizontal, its movement along the vertical axis would be similar. Thus, if the ball is released from rest on the inclined plane at a height H above the table, and $H = h$, where h is the table height above the floor (see Figure 3), the combination of the first free fall and the following horizontal motion in the speed reached in the free fall can be seen as a unidimensional motion to which the double distance law can be applied. Thus, if the ball was released (from rest) at height $H = h = 828$ punti, the corresponding horizontal displacement according to this "rule" is $D_{4-\text{double distance}} = 2H = 1656$ punti. On the other hand, using the proportionality rule $D \propto \sqrt{H}$ with the proportionality constant that was empirically found from the first measurement we get $D_{4-\text{proportionality}} = 1330$ punti, which is much smaller and far closer to the measured horizontal displacement $D_{4-\text{measured}} = 1340$ punti.

There is no doubt that Galileo fully understood that friction between the ball and the inclined plane affected the empirical results, but he could not have known that the main reason for the discrepancy between the two predictions (proportionality using the empirically established constant vs. the double distance rule) was the rotation of the ball, which "absorbs two sevenths of the energy which in an ideal slide or fall would go into linear acceleration" (Hill 1986, p. 286). Even in his later writings Galileo did not consider the effect of the rotation (or "rolling") of the ball, i.e., rotational inertia, a concept that "remained beyond the scope even of Newtonian mechanics until the work of Euler in the second half of the eighteenth century" (Naylor 1976, p. 417). Today, we can explain this phenomenon using energy considerations by noting that the gravitational potential energy is transformed not only into displacement kinetic energy but also (about two sevenths of it) into rotational kinetic energy, due to the rolling of the ball. This results in lesser speed at the bottom of the inclined plane, which leads to the smaller horizontal displacement Galileo observed. Galileo's predictions were roughly accurate precisely because the constant he used for the proportionality-based calculations was empirically based, and not theoretically constructed.

The worksheet presented an explanation of the double-distance rule, and the students were requested to prove it based on their knowledge of kinematics in one dimension. Then they were asked to calculate the predicted horizontal displacement in the special case in which the ball is released from a height similar to the height of the table above the floor, and to try to explain the discrepancy. This was a challenging task, and we had to help the students. Before the lesson ended we were careful to summarize these insights with the entire class.

After this discussion we told the students that while working on the interpretation of Galileo's notes, some historians emphasized that in the fourth measurement, where the height of the release equals the height of the table ($H_4 = h = 828$ punti, $D_4 = 1340$ punti), Galileo presented the value of D_4 on the horizontal axis but did not present the value of H_4 on the vertical axis. It has been claimed that Galileo must have been very puzzled by the obvious discrepancy between this result and the prediction based on the double-distance law (Wisan 1984, pp. 280–281). We interpreted the fact that Galileo did not present H_4 by referring to the scale, since 828 is too close to 800, relative to the other measurements of height presented on the axis of the graph.

In the course for preservice physics teachers, in which two thirds of the students were recent graduates of physics and engineering departments, the instructor connected the discussion at this point to dealing with systematic errors in experimental physics (Joint Committee for Guides in Metrology 2012). Some of the students explicitly applauded Galileo's decision to use the empirical constant (i.e., "ingenious").

4.3 Reconstructing the Historical Experiment

This part of the activity can only be carried out if the instructor can devote 4 h to the whole activity. We only taught this part in the course for in-service teachers, since the preservice teachers had recently conducted a similar experiment in a different context. After analyzing and understanding Galileo's experiment, the in-service teachers were asked to design and construct the apparatus from ordinary materials available in any school laboratory or easily (and cheaply) purchased. Then they had to plan the experiment, conduct it, and compare their results to Galileo's. Figure 4 shows a group of in-service physics teachers taking measurements with the apparatus they designed and built. One key difference between the apparatus the teachers designed (based on the materials available in the lab) and Galileo's apparatus is the track. Whereas Galileo's original track was an inclined plane with a constant angle of inclination (see Teichmann 2015, p. 38) that only became smoothly horizontal in its lower part, the teachers made the track out of the cover of a flexible cable so that one end of the trough was attached to a retort stand and the other to the edge of the table. Thus, the angle of inclination of the hanging cable changed constantly before the ball left the table at a horizontal velocity. The teachers, who were aware of the conservation of mechanical energy in this experiment, did not consider this to have made a significant difference.

Teachers are familiar with experiments demonstrating projectile motion. Many experimental setups used in school utilize the height of the release point on an inclined plane to estimate the velocity of a ball as it leaves the plane by employing energy considerations. Since the Israeli national high school physics syllabus does not include the mechanics of rigid bodies, most teachers initially do not take rotational inertia in these setups into account. This leads to discrepancies between the theoretical predictions and the observed results. The origin of these discrepancies is similar to the one reported by Galileo. Many teachers *qualitatively* explain

Fig. 4 A group of in-service physics teachers taking measurements with the apparatus they designed and built. The black arrow on the upper left points to the track. A key difference between the apparatus the teachers designed (based on the materials available in the laboratory) and Galileo's apparatus is the shape of track (curved as opposed to straight inclination)

these discrepancies a-posteriori using rotational kinetic energy. Employing Galileo's method by determining the proportionality constant empirically reduces the observed discrepancies between the theoretical prediction and the observed results. This "correction" may result in a decrease in the systematic measurement error (Joint Committee for Guides in Metrology 2012). However, it does not account for the source of the discrepancy, so we discussed this issue thoroughly with our students.

5 Discussion

The activity described above engaged preservice and in-service physics teachers in mathematical modeling of the physical phenomena of projectile motion. However, to follow Galileo's reasoning, and decipher the nature of and meaning of his authentic experiment and calculations in folio 116v, the students (preservice and in-service physics teachers) had to realize that the mathematical representations and tools that they are familiar with did not exist in Galileo's time. To understand Galileo's reasoning and decisions, they had to "do the physics" with the representations available to him. What did this experience afford preservice and in-service physics teachers?

Arguments supporting the integration of the history of science into science teaching suggest that this type of integration can highlight "the contrast between thinking then, and now, bringing into a sharper focus the nature and achievement of our current conceptions" (Monk and Osborne 1997, p. 409), as well as illustrate the nature of scientific practices and knowledge (Matthews 1994). This experience sparked discussions and "bafflements" (diSessa 2018) among the students regarding Galileo's "awkward" data analysis. But when the students realized that the algebraic representations they took for granted did not exist in Galileo's time, they were struck by the pivotal role that mathematics plays in physics.

Some writers have argued that providing students with in-depth and accurate accounts of the history of science do not always cohere with the goals of traditional science education, and that textbooks often present "quasi" and "fictional" history for this very reason (Brush 1974; Kragh 1992). Kragh argued that traditional science education aims to present scientific content in an easy to follow, coherent, and logical manner, which is seldom reflected in the actual course of history; Brush argued that traditional science education emphasizes the ideal of the scientist as a neutral fact finder, whereas careful accounts of the history of science suggest that doing science cannot be divorced from metaphysical or esthetic considerations (Brush 1974). Nevertheless, these authors (Brush 1974; Kragh 1992) and others (Allchin et al. 2014; Branchetti et al. 2019; Chang 2011; Galili 2012) have also argued that case studies that familiarize students with messy and authentic historical accounts of scientific work also have educational affordances, since they highlight the cultural, social, epistemological, and non-static nature of science.

Hence, students who only recently studied about projectile motion, and thus only have a fragile understanding of the model and its applications, may be confused by the activity presented here. However, our students were in-service and preservice teachers who had at least 2 years of academic courses in physics and mathematics. They fully understood and could easily apply the model of projectile motion. The activity afforded learning of other ideas. Regardless of the intervention, all of them would have acknowledged, as would any university physics student, that physics is strongly related to mathematics if explicitly asked. However, they were only explicitly aware of the technical (as opposed to structural) role of mathematics in physics (Uhden et al. 2012). The activity explicated and illustrated the structural role of mathematics in physics by highlighting the influence of the available mathematical representations on the scientific inquiry in question. The students were highly impressed by Galileo's remarkable achievements with such "limited" representations. Although his analysis would have been considered incorrect by modern standards, since it ignored the rotational kinetic energy, they were very impressed by how he dealt with the discrepancies between theory and experiment, by empirically determining the reference value for the proportionality relation. Several students voiced the opinion that this was "a stroke of genius" in this context several times.

We argued above for the educational affordances of an in-depth detailed historical case study. However, we introduced "Galileo's postulate" and the "Double distance law" without a full historical discussion in the lesson. The rationale for this decision was pragmatic. As Kragh's stated: "In educational context, history will have to be incorporated in a pragmatic, more or less edited way. There is nothing illegitimate in such pragmatic use of historical data *so long as it does not serve ideological purposes or violates knowledge of what actually happened"* (Kragh 1992, p. 361; our emphasis). In our view, supplying more detailed historical background about these two rules would have distracted our students from the main line of inquiry that the activity aimed to foster, namely, appropriating the available mathematical representations at Galileo's time to make sense of his authentic laboratory work.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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