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Encountering Productive Forms of Complexity in Learning Modern Physics

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Abstract This paper aims at supporting the claim that some forms of hyper-simplification, by making physics seem easy, are at risk of dangerously distorting the content as well as the process of learning physics. The paper presents examples of dangerous simplifications in the teaching of quantum physics. Then, examples of productive forms of complexity are discussed, both as criteria for designing teaching proposals, and for realizing appropriate learning environments, namely properly complex territories. Empirical results, from a teaching/learning experiment on quantum physics at upper secondary school (grade 13), are reported. These results show examples of students' reactions to travelling through a complex territory, and allow us to argue that unavoidable difficulty in learning quantum physics can be transformed into cultural challenges within reach of secondary school students.

Keywords Physics education · Learning quantum physics · Complexity · Secondary school

1 Introduction

An examination of Italian physics textbooks shows the progressive simplification of their physics content over recent years, according to the belief that students' disaffection toward science is due to its complexity.[1] Research reports, at European level, show instead to what

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[1] Many examples of Italian textbooks could be mentioned. In their recent editions, the argumentative apparatus became progressively lighter in order to leave room to tables and pictures. The tables usually contain students' facilitations like exemplar solutions of exercises or lists of "inverse formulas" of a physical law.

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extent forms of simplification can be counterproductive: learning notions without any deeper understanding, instead of reaching a larger number of students, makes physics to be perceived by the students as boring and meaningless.[2]

According to these European reports, the research presented in this paper[3] intends to challenge the trend that follows simplification as the main strategy to involve students in scientific discourse. The research is indeed based on the assumption that a teaching/learning process is meaningful if it combines, in a productive way, three complex systems: the real world, the system of physics disciplinary knowledge and the cognitive system of learners. Because of the involved forms of complexities, hyper-simplified instructional descriptions and explanations, by making the material seem easy, can dangerously distort the learning process as well as the content. On the contrary, we assume that, once useless complications are avoided, physics should be made as simple as possible, but not so simple as to lose its cultural and educational value.

In this paper, the importance of embracing complexity and helping students manage it will be argued by focusing on teaching/learning quantum physics and by showing some empirical results from a teaching/learning experiment on quantum physics at upper secondary school (grade 13).

Following the presentation of examples of *dangerous simplifications* (Sect. 2), examples of *productive forms of complexity* will be discussed as criteria for designing teaching proposals potentially able to foster the creation of appropriate learning environments, namely *properly complex territories*[4] (Sect. 3). In the last section, students' reactions to travelling through a complex territory will be reported to show that unavoidable difficulty in learning quantum physics can be transformed into cultural challenges within the reach of secondary school students (Sect. 4).

2 Dangerous Simplifications

In secondary textbooks and popular science books, quantum physics often appears as a conceptually disconnected territory made up of fragments of information, usually supported by semi-classical models and often related to each other only by the chronological

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[2] Sjøberg, for example, in a very interesting report writes: "The implicit image of science conveyed by these curricula is that it is mainly a massive body of authoritative and unquestionable knowledge. Most curricula and textbooks are overloaded with facts and information at the expense of concentration on a few 'big ideas' and key principles. […] There is often repetition, with the same concepts and laws presented year after year. Such curricula and textbooks often lead to rote learning without any deeper understanding so that, unsurprisingly, many pupils become bored and develop a lasting aversion to science. Moreover, this textbook science is often criticized for its lack of *relevance* and deeper *meaning* for the learners and their daily life. The content is frequently presented without being related to social and human needs, either present or past, and the historical context of discoveries is reduced to biographical anecdotes. Moreover, the implicit philosophy of textbook science is considered by most scholars to be a simplistic and outdated form of empiricism." (Sjøberg 2002).

[3] This paper has evolved from a joint work on the role of complexity in learning through an AERA symposium (Levrini et al. 2006). In the symposium, the importance of embracing complexity was emphasized by presenting a set of examples taken from different research programs that refer to different scientific domains and demonstrate how epistemological complexity plays out in students' learning and sense making. The current paper focuses on one of these examples, learning modern physics. Another example, concerning learning harmonic motion, is developed in Parnafes (2010).

[4] The notion of *properly complex territory* was formulated in the joint work that followed the symposium "Why complexity is important for learning", presented at the AERA Conference in San Francisco, 2006 (Levrini et al. 2006).

order of their discovery. It is well-known in physics education research[5] that a problematic consequence of such conceptual fragmentation is that students, in their effort to fill the gaps in information, tend to assign classical properties to quantum systems (or relativistic ones) resulting in their arriving at unsatisfactory conclusions that reveal a deep skepticism about quantum physics itself. Typical reactions are[6]:

Matteo: In my opinion, everything was so clear before and now I'm confused… Is it possible that the laws change only because of scaling? Maybe we have to wait for another Newton who will make everything fall into a unique law, like for gravity.

Tiago: In my opinion it is time the scientists do something about it, because they have not yet discovered everything; so far they have created only a great mess, something is missing… this is the only possible explanation that we have still to discover something in order to explain what happens.

Reactions like these represent the result of a traditional approach in teaching modern topics that tends to make simple (single) connections to familiar ideas, when that connection is misleading and the students perceive its incomplete status. In particular, it is well-known in the literature that the result of such an approach is that:

> students, in their effort to reconcile the features of the analogue structure (classical physics) with those of the target (quantum mechanics), tend to assimilate the newly considered quantum mechanical concepts into categories and modes of thinking that are deeply rooted in classical physics worldview. (Hadzidaki 2006, p.2).

One example of a problem derived through the use of classical thinking (e.g. a Newtonian particle picture of objects) in interpreting quantum phenomena is that students deny that what is usually thought of as a particle can behave as a wave. They see it as a thing that may be split in two, but must be either "here" or "there".

Matteo: It is not possible that a particle is simultaneously at two different points and can interfere with itself; it is either on the right or on the left.

Alessandro: Still, there will be something that goes through the slits, either an enormous quantum or a quantum split in half.

The problems identified in physics education research lead us to claim that, in the traditional approach to modern physics, at least two kinds of dangerous or unproductive simplifications can be seen. Simplifications that, while they provide comfort in short-term, may be problematic or even disastrous in the long term. As we will discuss in the following subsections, they are: the "constitutive" *hypersimplicity* of semi-classical images that are commonly used; the linearity and "thinness" of the content sequences.

2.1 The "Constitutive" Hypersimplicity of Semi-Classical Images That are Commonly Used

The recourse to pictures in teaching/learning physics represents, in many cases, a productive simplification strategy, given their power of focusing on the relevant details of a physical system ("taking away the impedimenta", as Galileo said) and of synthesizing them in a coherent and intelligible (often somehow familiar, although abstract) whole. Think, for example, of images showing light rays and how images are constructed from

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[5] Examples of empirical studies that show this kind of behaviour among secondary and/or university students are: Giliberti and Marioni 1997; Ireson 1999; Kalkanis et al. 2003; Mashaldi 1996; Seifert and Fischler 1999.

[6] The following quotations are taken from Tarozzi (2005).

them that are used to illustrate the behaviour and the physical conceptualization of the phenomena of geometrical optics.

Such images, as well as good metaphors, take their power in bridging the gap between the rich world of experience and the formal physical reconstruction of such a world. The gap is in-principle bridgeable when the mathematical reconstruction of the world provides a "projection" of the world itself in a Euclidean space, i.e. when the properties conceptualized by the mathematical description can be synthesized in a representation that "lives" in a space somehow "isomorphic" to the space we experience.[7]

This game becomes increasingly problematic in modern physics, since the new mathematical description projects the real world in highly abstract, unfamiliar spaces, such as the Minkowskian space–time or the Hilbert space. These abstract spaces are intellectual constructions that cannot be related to "real world space" in an intuitive way.

The wave-particle dualism can be seen as an exemplar case of the loss of visualization in quantum physics: any attempt to synthesize the behaviour of a quantum system in only one familiar picture (either particle or a wave) unavoidably shows the inner partiality of such a description. Instead of providing a new synthesis, these familiar pictures return a contradictory image of the phenomenological behaviour of the quantum world. The synthesis requires changing discourse and acceptance that quantum systems do not admit any visualization (*Anschauung*) by means of familiar images such as an image representing the atom's planetary model. At best, they can be described by graphical representations showing mathematical properties ("visualizability", *Anschaulichkeit*) such as Feynman diagrams.[8]

The recourse to visualization as a simplification strategy should be done very carefully, as pictures are extremely partial, if not, in many cases, misleading. They risk short-circuiting (hiding) one of the crucial points of modern physics, such as the necessity of referring to the abstract spaces as new synthetic scenarios where the properties of physical systems can be coherently rearranged and integrated with each other.

From this perspective, the discomfort created by the tendency to rescue classical images that lead students to perceive quantum physics as a "mess" can be transformed into a productive form of complexity if the "visualization" issue is placed within an epistemological framework, as it will be shown in Sect. 4.2.1.

2.2 The Linearity and "Thinness" of The Content Sequences

Another form of unproductive simplification in the traditional approach to teaching physics (not only modern physics) is the linearity and what can be called the "thinness" of the content sequences: sequences where reasoning is forced to follow a single route according to pre-fixed inner step-by-step logic. Minsky says:

> An idea with a single sense can lead along only one track. Then, if anything goes wrong, it just gets stuck–a thought that sits there in your mind with nowhere to go. That's why, when someone learns something "by rote" –that is, with no sensible connections–we say that they "don't really understand." The secret of what anything means to us depends on how we've connected it to all the other

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[7] We are here supposing that the productive character of those pictures is intrinsically related to the epistemological and metacognitive competence of students to recognise the role and the meaning of modelling in physics. Indeed, even during the study of classical physics, these pictures can become empty *hypersimplifications* if students "read" them literally.

[8] The importance of the distinction between the words *Anschauung* and *Anschaulichkeit* in the development of quantum mechanics and in interpreting the debate between Heisenberg and Schrödinger has been acknowledged and investigated by a few authors, notably Miller (1978, 1984), de Regt (1997).

> things we know. That's why it's almost always wrong to seek the "real meaning" of anything. A thing with just one meaning has scarcely any meaning at all. (Minsky 1986, p. 64)

Because meaning itself is complex, the content organization, that somehow respects the complexity of meaning, has to foresee rich and well-connected meaning structures since:

> too many indiscriminate connections will turn your mind to mush. But well-connected meaning structures let you turn ideas around in your mind, to consider alternatives and envision things from many perspectives until you find one that works. And that's what we mean by thinking! (Minsky 1986, p. 64).

To respect the complexity of meaning, the educational value of a teaching proposal ought to be searched for three aspects: (1) its inner coherence in moving longitudinally from one step (concept) to another; (2) the richness of the transverse connections it is able to suggest and foster with other possible routes (the meaning structures); and (3) its thickness: a sort of third dimension represented by metacognitive, epistemological or philosophical discourse from which the physical contents can be looked at and also be well-connected.

In particular, our studies contribute to supporting the claim that problems, topics and arguments concerning the nature of physics can play a crucial role in understanding modern physics for at least two reasons:

- Entering modern physical thinking means recognizing how traumatic it has been to give up the classical image of the world, together with becoming aware of the revolutionary contribution of modern physics in redefining very basic thinking categories;
- Although some kinds of paradoxes generated by the limits of classical representation tools can be solved by modern formalism, other kinds can be pinpointed or synthesized away only within a sophisticated epistemological discourse, as the results of the study presented below will show.

3 The Learning Environment as Properly Complex Territory

The notion of the learning environment as *properly complex territory* includes some forms of productive complexity that, in our opinion, should be implemented in the design of instruction and instructional materials so as to pursue, during implementation, the general goal of enabling students to find out their own ways of pinpointing or solving the problems and puzzlements they perceive.

The main forms of productive complexity considered in the phase of designing the proposal on quantum physics are the following:

A. *Multi-perspectiveness*—the same physical contents (phenomenologies) are analyzed from different perspectives so as to encourage multiple connections among the content and conceptual routes;
B. *Multi-dimensionality*—the different perspectives and multiple connections are analyzed and compared also for their philosophical-epistemological peculiarities, as well as for their relations with experiments and formalism;
C. *Longitudinality*—The "game" of modelling quantum phenomena is systematically analysed and compared with the models already encountered by the students during the study of other physics topics (classical mechanics, special relativity and thermodynamics).

In order to implement the first two forms of complexity, the teaching tool we have employed involves the guided analyses of historical-epistemological debates.

The historical-epistemological debates were explicitly used both for allowing a variety of possible connections (the meaning structures) to be stressed according to the multiple perspectives supported by the physicists involved in the debates, and the development of the epistemological dimension (multi-dimensionality) to be promoted.[9]

The systematic comparison of quantum physics with classical theories (longitudinality) has been explicitly implemented in the choice of designing and developing the teaching proposal as a progressive reformulation of the following problematic question: *How does the concept of object change from classical to quantum physics?* More specifically, the proposal was articulated in an historical introduction and in two parts.

In the introduction, the necessity of rethinking physical objects was pointed out. In particular, problematic situations were presented, namely situations where the classical models of object present limits by themselves (e.g. photoelectric effect) or where different explanatory schemes clash with each other in examples of border problems (the black-body radiation and specific heat problems) (Tarsitani 2008).

The *first part* of the proposal was focused on the analysis of some important historical debates, with the aim of showing how many different positions can be found among physicists themselves addressing the conflicts opened by quantum physics. In particular, the students were guided to develop arguments for analysing the debates between:

- Heisenberg and Bohr about the interpretation of uncertainty and complementarity;
- Bohr and Einstein about determinism and the relationship between knowledge and reality;
- Heisenberg and Schrödinger about the visualization of quantum objects.

By reading and analysing excerpts of original texts of the physicists,[10] the students were guided, for example, to reflect on:

- The Heisenberg microscope to problematize the "disturbance" interpretation,[11] also in the light of Bohr's criticisms and of his interpretation of uncertainty on the basis of complementarity (Hadzidaki 2006);
- Bohr's view about both the limits of natural language (and classical images) for describing the quantum world, and the claimed necessity of classical language for describing the experimental (macro) apparatus;

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[9] The use of debates or "dialogues" about quantum physics issues is a topic already explored within the field of physics education research. In particular its role has been investigated in: i) promoting conceptual understanding (see, for example, Pospiech 2003, and also Hadzidaki 2006), and/or ii) fostering students' awareness of the relevance of philosophical interpretations in enhancing scientific "progress" (see, for example, Garritz 2012). According to the specific goal of our study, the role of debates is explicitly stressed in relation to their power of implementing multi-perspectiveness and multi-dimensionality.

[10] Examples of readings are taken from the Italian editions of the following papers and/or books: Bohr, N. (1949). Discussion with Einstein on Epistemological Problems in Atomic Physics, in Schilpp P. A. (ed.) (1949). *Albert Einstein. Philosopher-Scientist*. Evanston, Ill: Library of Living Philosophers; Heisenberg, W. (1927). Über den anschulichen Inhalt der quantentheoretischen Kinematik und Mechanik, *Z. Phys.* 43 (3–4), 172–198, doi:10.1007/BF01397280; Heisenberg, W. (1971). *Physics and Beyond: Encounters and Conversations*, Harper & Row; Heisenberg, W. (1958). *Physics and Philosophy: The Revolution in Modern Science*, New York: Harper & Brothers Publishers; Heisenberg, W., Born, M., Schrodinger, E. & Auger, P. (1961). *On Modern Physics*, New York: Clarkson N. Potter; Schrodinger, E. (1950). What is an elementary particle? *Endeavour*, 9, 109-116.

[11] The "disturbance" interpretation is still proposed as the main interpretation of uncertainty in secondary physics textbooks.

- Einstein's sharp attacks against uncertain and probabilistic physics descriptions of the world and how and why his criticisms moved from a supposed inner inconsistency of the theory to its incompleteness;
- Heisenberg's and Schrödinger's world views which led the former to support the need for accepting that quantum description does not admit any visualization, and the latter to support the need for a sort of visualization for making theory intelligible and aesthetically (physically) acceptable (de Regt 1997).

The *second part* of the proposal was introduced by a detailed analysis of experiments à la Stern and Gerlach (1922) and double slit experiments at low intensity to address the formalism when the concepts of *quantum state, state preparation, operator, eigenstate, eingenvalue, superposition principle, complementarity, uncertainty, measuring process* and *entanglement* (Pospiech 1999; 2000) had been already introduced through the analysis of historical-epistemological debates as well as of experimental situations. According to Pospiech, the formal structure was concretely developed by referring to the quantum property of spin and by using Pauli's matrices. This part of the proposal benefitted from collaboration between physics and maths teachers.[12]

4 Students Travelling Through a Complex Territory: Some Results

4.1 Context and Methods of Data Analysis

The teaching proposal under current discussion was implemented in parallel in two regular, grade thirteen classes (18–19 year old students) from a science-oriented high school in Rimini (Italy). "Class A" was comprised of 19 students (10 boys and 9 girls), "class B" of 20 students (11 boys and 9 girls). The teacher was the same (PF) and both implementations took about 25 h each.

Both classes take physics as a mandatory course through the 5 years of upper secondary school (grades 9–13). During the first 2 years the teacher guides students in their knowledge construction of pre-theory structures (natural phenomena and physical concepts) suitable for the phenomenological description of motion, heat, light and vision, waves. This approach implies emphasis on laboratory activities. Building upon that knowledge, the teaching turns to the expansion of the acquired knowledge as well as its rearrangement in terms of physical theories starting from the third year (grade 11). The "theories" studied by the students of these classes are: Newtonian mechanics (grade 11); special relativity and thermodynamics (grade 12), electromagnetism and quantum physics (grade 13).[13]

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[12] The teaching of quantum physics at upper secondary school touches the well-known problem of coordinating the physics and math curriculum. Physics and math are, in some cases, taught, in Italy, by two different teachers. In this particular context, the math teacher, in agreement with the physics teacher, developed the topic of linear algebra, just before the students started the quantum physics proposal. The study of linear algebra is not out of reach of students who are attending a "Liceo Scientifico" and who are required to study mathematical analysis. In spite of that, such a topic is not officially foreseen in the math curriculum. The official curriculum for secondary school in Italy provides however general indications about the contents and the timetable. The teachers are asked to take the responsibility for detailed programming and the approach to follow. In our experiment, such freedom played a crucial role, because of the productive collaboration between the two teachers.

[13] Teachers have a certain degree of freedom to plan the order of the topics as well as the time devoted to each of them, according to their educational goals. For examples, most of the teachers prefer to devote more time to classical physics and to teach both special relativity and quantum physics during the second semester of grade 13.

The teacher was familiar with the students, since she had been teaching them physics from the second year and the students had already experienced a multidimensional and multiperspective approach when studying special relativity. Special relativity was taught according to the approach designed by the research group, built around the epistemological debate between Einstein and Minkwoski (Levrini 2002; Levrini 2013). During their studies, the students gained and refined epistemological competencies also thanks to the course of "History of Philosophy" that is compulsory from grade 11.

In each implementation on quantum physics the following data were collected:

- Answers to an initial questionnaire on "the classical models" of objects;
- Intermediate written tasks (qualitative and quantitative problems) on the topics addressed, respectively, in the first and in the second part of the proposal;
- Transcripts of videotaped classroom discussions (at the end of the activities);
- Answers to a final questionnaire about students' reactions to the experience.

Throughout the teaching/learning activities, weekly meetings occurred between the teacher and members of the university team to triangulate the various points of view and to create a common narrative around what was happening in the class and what issues arose. The first result was that the students appeared to make progress not only of a conceptual nature, but also in developing epistemological competences. In a previous study (Levrini et al. 2008), the transcripts of the final videotaped discussions and the final questionnaire were selected and analyzed to extract general keys for describing the classroom discourses. More specifically, the aim of the previous study was to describe and to interpret the following observations:

A. *The shift of learning difficulties from intelligibility to acceptability of quantum physics* (in both classes). The students did not react against the formal challenges of the proposal. That part proved to be somewhat intelligible for all the students, although not easy for many of them: "The problem was not understanding but accepting the consequences of the theory" (Michele); "I found the work very stimulating and interesting (really!); easy ONLY[14] from a technical point of view (calculation, exercises), since the theoretical part requires a lot of reflection and, I think, personal interpretation." (Francesco).

B. *The different roles attached to the formalism by the students of two classes*. In particular, all the students of class A wrote in the final questionnaire that formalism has been fundamental for understanding, whereas most of the class B students who considered it not particularly useful.

The study reported in this paper represents a further level of analysis carried out to answer the following research questions:

RQ1: How did the students cope with the problems known in the literature, which we interpreted as consequences of dangerous simplifications?

RQ2: Did the complexity of the path allow different students to find different ways of pinpointing or solving the puzzlements they perceived?

According to the research questions, the transcripts of the final discussions were reconsidered and specific moments of students' discourses were selected in order to zoom in on *how* students collectively coped with the two puzzling problems of visualization and determinism. Narratives were chosen for describing and interpreting what was happening.

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[14] The capital letters are in the original answer to the final questionnaire. They are not added.

4.2 Findings

4.2.1 The Crisis of Visualization

As noted previously, the students of class A shared the idea that quantum formalism was fundamental for understanding. In particular all of them seemed to agree that: (1) formalism was the key for "entering a new mechanism" (Stefano); (2) "the part concerning the formalism, although complicated, has been useful to establish a clear-cut detachment from the classical view of reality" (Giacomo); (3) "formalism helps you see in a different way and it makes you free from objects, […] from a materialistic [object-like] representation" (Francesca).

The students also agreed that a clear-cut detachment from the classical view was represented by the crisis of visualization, emphasized by the conflict between representations provided by familiar pictures. Indeed, this topic stimulated a lively discussion among the students.

In face of the crisis of visualization, the discussion yielded two different main positions among the students: one position led by Luca and Federico and the other by Jessica and Silvia.

Luca: The picture of microscopic reality, in this case, is sufficiently supplied by the mathematical formalism. Therefore, in my opinion, to have a graphical representation is not important for scientific progress: What's the use of the graphical representation? It may help in explaining the object as it is to children. But mathematics already explains it. […] In my opinion anyway, the picture of microscopic reality is already described well enough by mathematics. It is enough to have the tools for comprehending it and it seems to me that everyone can do so…

The quotation shows that Luca assumed a quite sharp position in which a "formalism that works" represents a convincing and final argument for accepting the detachment from the classical view of reality, while looking at pictures was seen as "stuff for children".

The attitude of Luca toward formalism is interesting since it is representative of those students who find the formalism a *simplification*, even when, as in this case, that formalism copes with non-trivial concepts, such as matrices, eigenvalues and eingenstates.

Federico said very clearly *why* formalism was a simplification: it allowed him to see *where* and *how* quantum mechanics assumes a familiar aspect.

Federico: I think that the concept of superposition of states, we have formalized, helps a lot. […] It is moreover linked to the concept of vector that is familiar. So it helps a lot to understand. […] Then there is still a deterministic part that is the one concerning the Schrödinger equation: It says that something changes continuously as a function of time. This is another concept that is very similar to what we studied in classical mechanics. So I think the formalism drives well.

Because of such features of formalism, the use of pictures become, for Federico, unnecessary and useless to create a bridge between classical and quantum physics.

With respect to such a position, other students in the class, led by Jessica and Silvia, expressed the important perplexities that arrived at outlining that well-working formalism was necessary but not sufficient to *have the feeling* of understanding: comprehension requires the "formal mechanism" to be interpreted also in terms of (smooth) links to ordinary language and classical description.

Jessica: But for me it [visualization] is necessary in order to understand…

Luca: Ah, but what if you can't do it…

Jessica: Because it is impossible to talk about something without trying to have a picture of what we are talking about, even unconsciously. It may help, in my opinion, also to give a meaning to formulas, because otherwise, even if we say that it is nonsense to represent the microscopic object, we make a picture anyway… I think so, although we decide not to draw it because we don't want to give a model that… […] it helps me, it helps me to remember. […] honestly I can explain the Compton effect by keeping in mind the drawing. […] we know that to be untrue but…

Federico: OK, but it is just an icon, you could draw a little star to make a photon.

Jessica: Yes, exactly.

The excerpt shows that, unlike Luca and Federico, pictures are—although conventional—necessary for Jessica. The arguments used by Jessica for advocating the relevance of pictures refer explicitly to a cognitive need she felt: the need of being able "to talk about something", "to give a meaning to formulas". Pictures can be untrue, conventional, creative: also a little star can be used if it helps to talk about photons. Jessica stressed three times that pictures should not act as representations (models) of reality ("although we decide not to draw it because we don't want to give a model…", "even if we say that it is nonsense to represent the microscopic object", "we know that [the drawing] to be untrue"). Nevertheless, without pictures she could not talk, give meaning to formulas or remember: she could not understand.

The association of visualizability with understanding has been the subject of historical and philosophical studies aimed at interpreting Schrödinger's view:

> The association of visualizability with understanding rather than with realism may be elucidated by considering the German word *Anschaulichkeit*, which is the term Schrödinger used in his writings. This word does not only mean 'visualizability' but also 'intelligibility'. (de Regt 1997, p.461).

Jessica, with her own words, seems to have stressed exactly this point.

Silvia is another girl who advocated the insufficiency of formalism for understanding and the helpful role of pictures. Silvia managed to formulate her challenge in terms of the cognitive need for finding out a plausible criterion to move from the quantum language to the ordinary and classical ways of thinking:

Silvia: In relativity it was different […] there you have a demarcation line. If you apply our velocity in formulas, you re-find our formulas. [In relativity] the two things are compatible, here not. […] In relativity, in my opinion, there was a greater compatibility with reality.

In the quotation, Silvia is referring to the need to feel that the formalisms are compatible with "reality", where by "reality" she seems to mean the space–time framework we usually make our visual experiences. In relativity, the existence of a demarcation line between the relativistic world and the ordinary one provided a criterion of "comparability": the value of the velocity, with respect to the speed of light, represented an intelligible and effective criterion for guiding imagination toward abstract spaces defined by formalism. Without such a criterion of demarcation, the quantum formalism risks becoming nothing but a "*mechanism*", "*a mentality*" (Silvia) to jump into, lacking what she felt to be a way for making the worlds comparable. Silvia's position is interesting because her attention focused on the demarcation line reveals that she was not compelling the impossibility of projecting classical images on the quantum world. She was instead manifesting the need of making the two "worlds", however different, comparable, where comparability includes also the knowledge of where one fades into the other.

These examples show that the students who expressed the need to refer to classical pictures, unlike the students cited at the beginning of the paper (Sect. 2), did not manifest the will of reading the pictures literally and did not arrive, because of visualization problems, at perceiving quantum physics as "a mess". Jessica and Silvia showed they were aware of the peculiarities of the quantum description with respect to the classical one. They were able to pinpoint consciously their puzzlement concerning visualization/visualizability in terms that still represent real problems for physicists and philosophers: the problem of interpreting the images used in quantum physics and the problem of establishing a line of demarcation between the classical and quantum world.

To return to the research questions, the explicit reference to pictures fostered by the activities (not for simplifying but for confronting directly the problem of visualization) not only did not prevent students from entering the quantum description, but it also helped them to conceptualize their comprehension problems without arriving at skeptical and defeatist positions. The idiosyncratic, genuine and emotion-laden words used by the students in confronting each other make the voices of Heisenberg and Schrödinger to be, if present, nothing but echoes. Our guess is that multi-perspectiveness and, in particular, the analysis of the debate between Heisenberg and Schrödinger about visualization opened a space for legitimating multiple attitudes towards the problem: a space that the students experienced for interpreting their personal cognitive needs and finding out the words for making them explicit.

4.2.2 The Crisis of Classical Determinism

Most of the students in class B agreed that the quantum formalism was, as the following quotation shows, intelligible, relevant for supporting learning, but not particularly helpful for understanding since it is clear only in its internal logic:

> *Simone:* The part on formalism helped me from a human point of view, since it is always comforting to count on equations, calculations and mathematical laws. Still, in spite of its inner coherence, mathematics did not contribute to giving me a more convincing idea about the quantum object than the [first] physical part. […] Of course: it was not a complication, since the formalism has been easy to memorize and understand, but, I mean, only internally clear.

Simone, in the last questionnaire, wrote very explicitly that mathematics can represent, for some students, a safe and comfortable anchor to thinking. However the main point is, in our opinion, the emphasis that he put on math's "closure". Continuing with his reasoning, Simone explained why the formalism is, in his opinion, only internally clear: it does not provide effective contributions to answer the main question raised by quantum physics, i.e. the question he called "the knowledge problem" which arose in the fall of determinism.

> *Simone:* The hardest point to understand has been giving up classical determinism […] Deterministic physics was an exact science, at least at a theoretical level. Quantum mechanics is upsetting since it requires facing the knowledge problem, it makes you ask if what we observe is really what it is.

In the face of questions like this, the mathematical formalism shows its limits, while a refined epistemological (philosophical) language is needed to address the strong emotional impact:

> *Simone:* Until such questions do not emerge, everything goes on for the best but, when they come to light and we have to answer that we cannot say or foresee what we want to observe, the emotional impact is very strong.

This critical issue of the fall of determinism, raised by Simone, went in resonance with the widespread need to discuss it. A lively discussion, led by Luigi and Michele, developed around the following questions: Should "real" be synonymous with "determined", "known in all detail", "known with certainty"? Why should a description based on uncertainty, on a "non-epistemic probability" be less realistic than a classical one?

Luigi: I think that realism is not lost. I mean that what we are talking about is something real and is not metaphysical, therefore realism is not excluded. We are talking about something that, so to say, is undeterminable because it has a non-epistemic probability. Realism is not excluded anyway; on the contrary it is defined in another way… let's say on the basis of its probability instead of its certainty. Mathematics (in this case) allows us to explain the superposition principle, the principle of uncertainty. And that's what I found somehow difficult to understand: how mathematics gave us an explanation of how nature is not something exact but is instead undeterminable.

Michele: I think Luigi pointed out that mathematics has never been associated with the concept of realism, it has always been abstraction. Mathematics has always given us certainty, something certain and computable. So mathematics providing here a concept of probability and uncertainty can be a little disconcerting. But when has mathematics ever been associated with realism? It has always been abstraction, model.

The previous statements show the students' involvement in dealing with demanding issues and their need to reorganize the relations between maths and reality: "Realism is not excluded anyway; on the contrary it is defined in another way", said Luigi, using an important word "realism" just for stressing, in our opinion, the link to reality, without specific references to its philosophical nuances. A reorganization of the relations between maths and reality is said to be needed for accepting what sounded particularly difficult to understand: "how mathematics gave us an explanation of how nature is not something exact but is instead undeterminable" (Luigi).

Michele pointed out clearly the main point of discomfort expressed by Luigi: classical physics could lead someone to make an implicit and undue association between math's certainty and "realism", i.e. a mathematical description could be accepted as real because of the certainty it seems to guarantee. Instead, "when has mathematics ever been associated with realism? It has always been abstraction, model" (Michele). The detachment of maths from a strict and trivial link to reality led Luigi and Michele to find (even if with a little discomfort) a new space of freedom for allowing maths to embody probability and uncertainty and to problematize the relationship between determinism (certainty) and realism.

At the end of the discussion, Michele arrived at a quite surprising position:

Michele: The problem is that macroscopic properties are real to us because we are macroscopic. But if we think about it, it is the contrary indeed because the macroscopic is nothing but a composition of microscopic objects. What is microscopic is more real.

"We are a composition of microscopic objects…," Michele said. But his strongest argument to support that a world description based on determinism is less real than a description based on a non-epistemic probability was the following: "In my opinion, determinism à la Laplace creates science-fiction worlds… I understood this when I watched Minority Report.[15]" (Michele). So, paradoxically, the fall of determinism became for some students the way to accept the quantum description as "realistic": a description where every phenomenon is perfectly determined does not fit in with our reality.

To return to the research questions (and in particular to our second research question), this example demonstrates that perceived complexity becomes resolved into simplicities (a new synthesis) moving the reflection along a new discourse: The complexity of quantum

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[15] Michele refers to the science fiction movie directed by Steven Spielberg, loosely based on the Philip K. Dick short story. The movie is set in the year 2054, where a special police department called "pre-crime" apprehends criminals based on foreknowledge, provided by three psychics (the "pre-cogs") able to foresee deterministically into the future.

physics (its subverting classical categories and, mainly, showing them as apparently paradoxical) gets organized by an epistemological analysis.

These results support the idea that multidimensionality and, in particular, epistemological debates can possibly facilitate and scaffold the organization of knowledge by privileging certain types of information (e.g., by highlighting new connections between formalism, reality and thinking categories). However, as the example demonstrates, the process itself is not simple and should not be simplified away by short-circuiting challenges that strongly characterize modern physics thinking itself. The epistemological complexities, for the most part, do not put students off, but instead, provide an interesting ground for personal engagement and perspective.

5 Concluding Remarks

Learning quantum physics presents, with respect to classical physics, specific sources of difficulty. Some of them cannot be avoided if teaching aims to lead students to grasp the sense of the radical transformations imposed by modern physics on thinking. In particular, sound teaching cannot neglect, in our opinion, emphasizing the extent to which modern physics requires:

- redefining basic physics concepts, overcoming, for example, the classical notion of object;
- pushing imagination toward worlds where objects are neither particles nor waves;
- using sophisticated formalisms in order to design the abstract spaces where the "physical objects live", i.e. where they find their constitutive properties (abstract Hilbert space);
- developing a new epistemological paradigm, by revising classical categories, such as causality, determinism, and the separability of subsystems.

By discussing empirical results, this paper provided arguments to support the claim that these kinds of difficulty can be transformed into cultural challenges, provided that some forms of complexity are exploited in the design of a learning environment. The forms of complexity which were revealed to be productive are: naming and confronting fundamental problems (for example, visualization) which make quantum physics different from classical physics (*longitudinality*); the multiple connections and the multiple dimensions stimulated by the analysis of historical debates and by the comparison of multiple interpretations of the same content knowledge (*multi-perspectiveness* and *multi-dimensionality*).

The data show that the students, immersed in such a complex learning environment, experienced the opportunity to reflect on the intricacies of physics thinking. Their voices sound deeply authentic: they were confronting each other's ideas and their personal learning challenges. No attempts to search for the teacher's approval or to refer to the authority of scientists were noticed.

The students were instead encouraged to find out productive syntheses (simplifications) by themselves. For some of them, quantum formalism was a form of productive simplification: it was the necessary and sufficient key for accepting the results of a "theory that works" and having the feeling of appropriating its essential meaning. For other students a well-working formalism was necessary but not sufficient to have the feeling of understanding; comprehension would have required the "formal mechanism" to be interpreted also in terms of (smoothed) links to ordinary language and classical description. For still other students, the formalism was necessary but not sufficient for accepting a theory and appropriating its meanings; acceptance required also a philosophical analysis of the theory implications for knowledge in general.[16]

These encouraging results led us to design more specific studies, now in progress:

A. empirical studies aimed at collecting new data for investigating both the peculiarities of the individual learning processes of students immersed in properly complex territories, and the teaching mediation actions. These studies are focused on pointing out how, when and why the features of this kind of learning environment trigger and support personal appropriation of physics and how, when and why good practices can be disseminated (Levrini et al. 2011).
B. theoretical studies aimed at drawing a model of educational reconstruction of physical content knowledge where productive forms of complexities are progressively implemented in the whole physics curriculum for upper secondary school. So far we have designed teaching proposals only for the last years: as well as the proposal on quantum physics, a proposal on special relativity (Levrini 2002)[17] and a proposal on thermodynamics (Levrini et al. 2010; Levrini et al. 2011; Levrini et al. 2013). Our final goal is however to design a longitudinal curriculum where students are progressively guided to manage more and more sophisticated forms of complexities up to the ones implemented in the proposal on quantum physics.

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[16] A positive outcome that gave us some indications about the quality of students' understanding arrived, by chance, the year after the teaching/learning experiment on quantum physics: 5 students from these classes matriculated in Engineering and all of them addressed the examination of the General Physics Course at the Degree Course in Engineering in the first semester of the first academic year. Such an exam is very selective. In that case, only 30 students passed the written task out of over 100 students who tried, and only 11 students passed the oral task (and definitively passed the examination) out of the 30 admitted. Among the 11 students who passed the examination there were all the 5 students coming out from "our" classes who matriculated in Engineering. This result does not prove, of course, that the intervention was the only, or even the main, reason for the students' success. Nevertheless, it was, for us, a surprising, promising and insightful piece of evidence that provides some indications that something important happened in these classes: not only were the students shown to have acquired intellectual autonomy in discussing about physics but also to have an outstanding potential for developing technical and formal abilities. Our hypothesis is that the success is somehow related to the students' habit of dealing with the complexity of the disciplinary knowledge since it provided students with the opportunity not only to learn physics and about physics, but also to learn, through physics, both to manage consciously one's own potentialities and *to think*.

[17] The materials designed on Special Relativity have been used both in classes of secondary school students (Levrini and diSessa 2008) and in contexts of teacher education (De Ambrosis and Levrini 2010). In both the cases the emphasis on multiple perspectives and dimensions revealed to be productive for learning.

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