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The Model-Based View of Scientific Theories and the Structuring of School Science Programmes

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Abstract. Model theory in contemporary philosophy of science interprets scientific theories as sets of models, and contributes significantly to the understanding of the relation between theories, models, and the real world. The clarification of this relation is fundamental for the understanding of the nature of scientific methods and scientific knowledge and can contribute to the shaping of epistemologically pertinent educational models in science education. We initially present a reconstruction of the most important model-based contributions concerning mainly the nature, construction and the functions of theoretical models. Our interest focuses particularly on the theory structure scheme of the model-based view, which Ronald Giere explicitly formulated using as a paradigm the structuring of classical mechanical models, and which we extend to become a base for a structuring of basic quantum mechanical models. We consider this scheme to provide an adequate basis for the structuring of school science contents; and further propose in outline a way of structuring typical physics content, in which the topics are presented as theoretical models of the same theory, together with the modelling rules that led to their construction, and also some examples of their empirical application.

Key words: Nature of science, Model based view, Theoretical models, Theory structure, Theories, Models and real world, Classical mechanics and quantum mechanics, Science education

1. Introduction - Views of Scientific Theories in Contemporary Philosophy of Science

In the past few decades, understanding of the nature of science has been introduced into science curricula worldwide as a basic goal of science education. It is held that an understanding of the nature of science and of its historical and social context contributes, *inter alia*, to a more substantive understanding of scientific knowledge, encourages students to take an active interest in science, and generally raises the educational and cultural dimension of science courses (Matthews 1994). Science education seeks to shape valid conceptions about scientific methods and knowledge, and is therefore interested in and studies analyses of the philosophy of science. Scientific theories are the basic vehicle of scientific knowledge, and understanding their nature and structure is the most basic issue in the philosophy of science.

According to the original version of the statement view of scientific theories, which was formulated at the beginning of the 20th century and became broadly accepted in philosophy, scientific theories are axiomatic systems of theoretical statements (axioms), which express the relation between the theoretical terms of a theory, and of the correspondence rules that interpret those terms empirically (Losee 1990; Suppe 1977). In the statement view, which was basically developed by logical empiricism, the axioms of a theory are essentially understood as immediate descriptions of the real world and as statements of universal scope, verifiable through logical and empirical proofs, the experience of the senses is thus posited both source and ultimate verification of scientific knowledge. These principles, however, led to dead ends, as shown for instance by the failed trial to develop an inductive logic (Stegmüller 1971) and the insuperable difficulty to connect all theoretical terms with observable terms (Suppe 1977; Götschl 1980).

The separation of the language of scientific theories into theoretical terms and empirical terms, and the resulting dichotomy in the constituents of the world, based on human sensory capacity, are characteristics of the statement view that have been most severely criticised by its opponents (Götschl 1980; Suppe 1977). The critiques were based on the vagueness of the definition of empirical observation (understood e.g. as only with naked eye, or also with experimentation-devices, etc.) and on well-founded analyses of the theory ladeness, and thereby the non-objectivity of observations and interpretations of observed data (e.g. Popper 1989; Suppe 1977; Devitt 1991; Götschl 1980).

Criticism was also levelled at the separation of the end content of the scientific theory and its justification methods (context of justification), which in the view of the Logical Empiricists is an issue of the philosophy of science, from the discovery process of scientific knowledge (context of discovery), which ought to concern the psychology, or the history and sociology, of science (e.g. Popper 1989, p. 6; Suppe 1977, p. 125). It was also noted that, with the exception of certain theories, chiefly in the field of physics, and assuming science to embrace sociology and psychology, not all scientific theories are susceptible to the axiomatic reconstruction required by the statement view (Suppe 1977, pp. 65, 111).

In the 1960s, the classical view was vigorously criticised by a series of philosophers, and principally historians and sociologists of science, who based their arguments on the common view that the development and choice of theories takes place under the deciding influence of concrete world views such as a mechanistic world view for classical mechanics (Toulmin, Kuhn, Feyerabend, Hanson, Bohm, and others, see in Suppe 1977). This means that the observations, the concepts, and even the empirical data underlying the assessment of the appropriateness of a given theory, are dependent upon and shaped by the world view of the pre-existing theories and the related scientific theoretical tradition that has been developed in the scientific community (e.g. Boyd 1983; Giere 1988).

Kuhn's position in particular (1989), that acceptance of a new theory cannot occur (only) logically and in accordance with pre-determined rules but will be based on psychological, sociological and institutional criteria (dissemination, personal ambitions, influence, authority), excited the greatest reaction, and it was criticised for unbounded relativism. If the methodological standards of science (selection of problems and experiments, interpretation of data, etc.) were dependent on the scientific tradition created on each occasion by the corresponding theory, and if the choice of a theory were a matter not of logic but of personal judgement and social commitment, scientific knowledge would be unable to raise any serious claim to objectivity and validity. The counter-argument was that the 'incommensurability' between the different methodological rules developed in the several frameworks of the various theories precludes the choice of the best theories on the basis of unambiguous logical rules assumed to govern the relation between empirical data and the acceptance or rejection of a theory (Kuhn 1989; Popper 1974; Kuhn 1974a, 1974b; Feyerabend 1981; Lakatos 1974).

In particular, sociological approaches argued that the determinant factors in the choice of theories, and therefore of their content, are not rationalism and empirical evidence, but inner-disciplinary conventions with respect to the interpretation of the experimental data, the virtues of theories, and so on, as well as personal, professional and social bargaining. More radical approaches claimed that scientific theories are wholly constituted on a foundation of personal interests and human interaction and are thus merely social constructs.

These historically oriented philosophical and sociological approaches significantly contributed to a better understanding of scientific activity and how science actually works, but they were also strongly criticised as promoting relativism, and that they –among other weaknesses - contributed nothing of importance to the analysis of the structure of the scientific theories (Suppe 1977, p. 220; Giere 1999, p. 35).

Despite truly substantial improvements made to it (Suppe 1977, p. 52; Götschl 1980, p. 640), the statement view, named also received view because of its broad acceptance in the 1920–1950 philosophy, was finally considered inadequate as a general theory for the analysis of scientific theories (not least when including psychological and sociological theories in addition to some specific theories of physics), although none of the alternative approaches put forward at the time achieved any broad acceptance. The most important of the alternative views that emerged at the beginning of the 1970s as a specific philosophical tendency in the analysis of scientific theories was the model-based view. The various other names attached to this view, such as 'non-statement view', 'semantic view' (in contrast to the linguistic-syntactic approach to theories in the statement view), 'predicate view', 'model-theoretic view', reflect the fact that it unifies different philosophical model based accounts, all of them sharing the common characteristic of ascribing particular importance to the concept of the model, understood as a basic structural element of the theories and as a mediator between theory and reality (e.g. Suppe 1977; Giere 1999).

The model-based view has attracted the lively interest of contemporary philosophical analysis, as well as that of cognitive psychology and science education (Grandy 2003). More specifically, science education sees in this new view, and primarily in the cognitive approach of Giere (1988, 1999), a satisfactory epistemological foundation for an understanding of the nature of science and its didactic transformation, for an innovative planning of analytical programmes, and for the further education of teachers and a reconsideration of their role (Aduriz-Bravo 2005).

In this paper, we initially present a reconstruction of the most important model-based contributions concerning mainly the nature, the construction and the functions of theoretical models. We are particularly interested in the model-based analyses about the structure of scientific theories that Giere formulated as a concrete theory structure scheme, using as a paradigm the structuring of basic theoretical models of classical mechanics. We use this scheme and apply it to quantum mechanics, where we structure in an analogous way some simple basic quantum-mechanical models, thus also testing the scope of the scheme in the realm of quantum mechanics. We believe that this scheme, applied in education, can contribute to a cohesively structured and didactically successful organisation of the course content of a scientific theory, and, by extension, to the effective planning of its instructional process.

We further propose a specific structure for certain typical elements of course content in the field of mechanics, in which the topics are presented as theoretical models of Newtonian theory, together with the modelling rules that led to their construction and also with some of their areas of application. We regard that in this way the topics are not only interrelated but also reveal the structure and functionality of the theory to which they belong, thus adding more structure and meaning to the learning of scientific knowledge.

We clarify that in this article we concentrate on the relation of theoretical models with theories, and we just briefly refer to the problem of the empirical testing and the choice of theoretical models, a likewise fundamental issue for the understanding of the scientific activity and its cognitive results. Moreover because the literature relating to models contains some uncertainties and omissions concerning the definition of the term 'model', the distinction between the different kinds of models and the distinction between models and theories, we clearly state that what we mean by 'models' in this article are mainly theoretical or mathematical models, and not, for example, physical scale models, visual models or diagrammatic representations.

2. The Model-Based View

The model-based view bases its analysis on the concept of the theoretical model. Exponents of model-based approaches believe that a re-interpretation of scientific theories as sets of models can be shown to be more successful for an understanding of scientific theorising (Suppe 1977, p. 221), and that it can give a more satisfactory picture of the relation between scientific theories and the real world through the mediating role of the models (Giere 1999, p. 73). Model-based analyses differ among themselves, yet they also develop and become more comprehensive. The pioneering works of Suppes, Beth, Suppe, and van Fraasen (see in Suppe 1977), the structural analyses of Sneed, Stegmüller, Moulines, and others (see in Giere 1999, p. 251), and the cognitive approach of Giere (1988, 1999) are held to be some of the key expressions of this view.

In the next section, we present a unifying reconstruction of the most basic model-based contributions, focusing on the views about the nature and the derivation of the theoretical models and their relation with the theories, and especially about the related model-based scheme of the structure of scientific theories, which we will later use for educational purposes as a frame for a meaningful structuring of didactical content in science education.

2.1. NATURE, FUNCTIONS AND CONSTRUCTION RULES OF THEORETICAL MODELS - RELATION OF THEORIES AND THEORETICAL MODELS

The modelling of objects, and more generally the modelling of structures and of the behaviour of systems of objects, is related to the method – characteristic of physics – of abstraction and idealisation that was developed and decisively advanced by Galileo (concise accounts of Galileo's methodological paradigm are given in Matthews (2005) and Nola (2004)). The modelling of objects is based on the abstraction and idealisation of specific properties of the objects, in particular of those that are not considered essential in the framework of the theory that studies them. Moreover, in the modelling of structures or of the behaviour of systems of objects, the interactions are also idealised. One defines, for instance, stars with point masses, homogeneous fields of forces, the existence of only gravitational interaction, and so on. The laws of physics are, consequently, also idealised, they are valid with absolute precision for no object or system of objects in the real world (e.g. Stöckler 1995; Nola 2004). In this sense, a theoretical model represents, according to Nola (2004, p. 60), a set of idealised objects that have idealised properties and interactions and obey idealised laws. It is worth noting that, despite this idealisation, the models reveal characteristics of their empirical prototypes and hidden relations in the phenomena that we could not possibly perceive empirically.

The theoretical model is also described, as '... a conceptual system mapped, within the context of a specific theory, onto a specific pattern in the structure and/or behaviour of a set of physical systems ...' (Halloun 2004, p. 24), a pattern which these systems seems to exhibit on a global scale. Theoretical models are creative ideas formulated by scientists about physical realities, which are refined and further specialised on the basis of empirical interactions with the real world, although they can also be developed solely in the world of pure logic (Nola 2004; Halloun 2004, p. 31).

Systematic definitions of models and a central role for models in the analysis of theories started to appear and to be developed in the semantic view of theories, so named in contrast to the linguistic conception of theories in the statement view and its purely syntactic treatment of the elements and the logical internal structure of theories. It was broadly accepted by philosophers of science (between 1920 and 1950) that a *scientific* theory can be reconstructed as an axiomatic system of theoretical statements (axioms), formulated in a specific language of mathematical logic, appropriate for the philosophical purpose of a rational reconstruction of the content of the theory (Suppe 1977, p. 12; Giere 1988, p. 47). The semantic view, on the other hand, understands the theories as sets of models, where model is defined as 'any structure in which the axioms of a theory are true' (see below), thus basing its analysis on semantic notions like model, structure and truth, and focusing on how science makes sense to people and gives meaning to the world (Suppe 1977).

In this framework, the structural theory gives a more systematic and formalistic definition for theoretical models. The basic common assumption in these accounts is that the axioms and fundamental principles of the theories are not, as in the statement view, universal statements, but provide the basis and the rules for the construction of 'theoretical' or 'mathematical' models, which mediate the application of the theories to complex physical systems. The theories are not understood, as in the statement view, as linguistic entities, as sets of sentences that directly describe physical phenomena, but, as a class of structures of sets of elements, or as a class of models, 'which could also contain the things the theory "is talking about"' (Sneed 1980, p. 649). A theoretical model is formed by every mathematical structure of sets of elements to which certain characteristics are attributed, and which fulfils the axioms of a theory (e.g. $F = ma$) and some additional specific mathematical functions (e.g. $F = -Dx$). These specific functions differentiate between the different theoretical models and are determined by the class of systems that the model intends to represent and by the idealisations that the model assumes for these systems. The axioms and the basic principles of a theory are consequently true, although in the sense of truth by definition, only for its models: they do not describe with the same precision the behaviour of the real systems that a model represents (Giere 1999).

Characteristic of the structural analysis is that these mathematical structures, the theoretical models, are initially defined in a completely abstract way. Some such structures contain only real numbers as constituent elements, others contain physical objects as well (Grandy 1992, p. 220). The abstract structures of the theoretical models are related to physical systems through the interpretation of their mathematical symbols as physical quantities, so that they also contain structures of physical systems in the examples of their applications. The real systems that a theory interprets at any given time are only some of those, real or otherwise, to which the theory can be applied; and in this sense the content of a theory is much more than its empirical application (Stegmüller 1985). With these mathematical structures, the theoretical models, scientists aim on one hand to successfully achieve an at least partial representation of real systems, and on the other to create more models by further specialising the general and abstract structure of the theory through the addition of further laws, thus broadening the scope of the empirical applications of the theory to real systems (Sneed 1980; Grandy 1992).

In the spirit of the above analysis, Stöckler essentially identifies theoretical models with small specific theories, such as for instance the theory of ideal gases, which he discerns from the general theories of, for example, classical mechanics, electrodynamics, or quantum mechanics. The theoretical models (the special theories in Stöckler's sense) are formed, as we have described above, through the introduction of specific functions into the general theory, implied from the class of phenomena that a models aims to interpret and from the idealising assumptions made for these systems. Stöckler terms these idealising assumptions model object, so that a theoretical model is yielded according to the scheme:

$$\text{General theory} + \text{model object (idealisations)} \rightarrow \text{theoretical model}$$

(See Stöckler 1995 and Bunge 1970 for an elaboration of this scheme)

For instance, classical mechanics, intending the application to gases, together with the assumption that gases consist of perfectly elastic particles, yields the theoretical model that is the special theory of ideal gases. The creation of theoretical models is, first of all, necessary for reasons of mathematical simplification. Because of the idealising assumptions that they make for physical systems, theoretical models facilitate the empirical application of general theories, whose direct application to physical systems usually becomes uncontrollably difficult from the mathematical point of view (Stöckler 1995). The models can also change or be (further) improved independently of the general theory to which they belong. By becoming more concrete, i.e. by reducing the idealisations they introduce (and thus at the same time becoming mathematically more complicated), they better represent their referent physical systems (Nola 2004; Stöckler 1995). For example, the specific law of the ideal gas, $(p + a/V^2)(V-b) = RT$, compared to the version $pV = RT$, has two less idealisations, the term a/V^2 takes into account the attractive forces between the particles and the term b the volume of the particles.

Moreover, the models can fulfil not only interpretive and predictive functions, but also inventive functions (Halloun 2004), in the sense that they work '... as an analogue for the construction of new applications' (Adúriz-Bravo 2005, p. 35; Halloun 2004, pp. 24, 62).

Van Fraasen (1980) made it clear that in the semantic view it is models, rather than axioms, that occupy centre stage in the theories, and stated that models (understood as extralinguistic entities) may be described or characterised in different languages, according to the field in which they are constructed and applied. In this way he rejected the requirement for a unique formal language for the philosophical reconstruction of theories, thus – according to Giere – liberating 'the philosophical study of science from the linguistic shackles of its logical empiricist predecessor' (1988, p. 48).

Characteristic in van Fraasens' account is the distinction of the structure of a theoretical model into two parts, the theoretical structure and the empirical substructure, which model, respectively, the unobservable and observable aspects of real systems (Giere 1988, 1999). This distinction is analogous to that made in classical empiricism between the empirical and the theoretical terms of the theories, assigning to the latter only instrumental functions. Van Fraasen represents the constructive empiricist viewpoint, which holds scientific models to be logical constructs, in which only the empirical substructure needs to match the observable phenomena for the model to be empirically adequate. Theoretical entities and their properties (such as electron and spin), postulated for the modeling of unobservable areas of the world, could in fact really exist, but that is something we cannot know with empirical certainty, nor is science committed to prove their existence. The purpose of science is not to discover the true story of the world, but to formulate empirically adequate theories, that means theories that correctly describe phenomena, a view different from that of the scientific realism, which argues that, at least sometimes, theories can refer to and discover hidden causal structures and unobservable entities of the world (Boyd 1983; Giere 1999, p. 150).

Giere's account expanded and made explicit many aspects of the nature and functions of theoretical models. He calls a theoretical model a (highly) abstract representation of real systems, constructed with well defined rules and accompanied with a well defined formal structure in the frame of a concrete theory. We note two basic characteristics that he attributes to theoretical models: that they are the basic structural components and the basic functional units of scientific theories and scientific activity, and that, although imaginary entities, they can nonetheless be representational, succeeding at least sometimes in representing various aspects of the world (1999, p. 54).

Giere reconceptualises crucial terms of the traditional analytical philosophy of science, such as scientific rationality and truth: Rationality is to be understood as an 'effective goal-directed action' (1988, p. 9), since the choice of a model, especially between other rivalling ones, can in practice not be a matter of 'pure reasoning or logical inference', but is rather a matter of making a personal decision (1999, p. 6). The literal, semantic concept of truth, which implied the understanding of axioms as universal laws of nature and was accompanied by the problematic notion of correspondence and the correspondence theories of truth, can in the model-based view be abandoned, since understanding axioms as definition resources for models (models are structures in which the axioms are true) reduces their truth to the sense of truth by definition (Giere 1988, p. 82; Giere 1999, pp. 6, 24).

Giere focuses on the truth of theoretical hypotheses (such as e.g. the claim 'the Earth-Moon system forms an empirical example of the two-particle Newtonian model'), and thereby on the sort of relationship they assert to exist between models and real systems. It is not, according to Giere, a relationship of truth or even of isomorphism that might most appropriately capture the relationship between models and real systems, but one of similarity or of fit, always of course meant in some respects and to some degrees of accuracy (1988, pp. 80–81).

The need to determine the respects and the degrees of accuracy that determine the acceptability of a model, and the related question of the nature of the consensus required for their determination, since these are dictated neither by the theories nor by the real systems (1988, p. 108), leads to different realistic or anti-realistic views on the nature of the scientific knowledge. Scientific realism, in its most developed version, recognises the constructive element of scientific activity, which in some cases can allow the promotion of personal or professional goals to affect the final choice among competing models and, therefore, even allows the imposition of cultural interests and values on the content of a scientific theory, especially when experimental data are not sufficient enough for a shared decision of the scientists. While Giere also accepts the constructive dimension of science, in his acceptance of the regulative role that the human capacity to engage in a causal cognitive interaction with the world plays in the scientific method, he adheres to a constructive or perspectival realism (1999, p. 150). This view designates models as constructs that manage to represent some aspects of the real systems, thus stressing the perspectival character of the scientific knowledge. (For the arguments of the scientific realism against social constructivistic arguments and overgeneralisations, see e.g. Boyd 1983 and Giere 1988, 1999).

Summarising at this point, we can say that the model-based view understands scientific theories as sets of theoretical models, and it holds that the modelling of the physical world forms the core of the scientific method that science has developed in the attempt to understand this world. A theoretical model is a mathematical structure of a set of elements to which concrete properties are attributed and which fulfil the axioms of the theory, and additional specific functions imposed by the class of phenomena to be represented by the model and the idealisations that were made during the construction of the model. Models provide mathematical simplifications, they guide and broaden the applications of the general theory in the complex states of real systems and phenomena, and they fulfil interpretative, predictive and inventive functions. A theoretical model represents and deals with only one phenomenon or one class of phenomena, while a general theory, like classical mechanics or electrodynamics or quantum mechanics, interprets in a unifying way several and various areas of phenomena within the field of its scope.

2.2. MODELS AND REAL SYSTEMS – EMPIRICAL TESTING OF MODELS

The acceptance or rejection of a theoretical model is made on the basis of its predictions, that is, on the basis of the empirically observable statements that are yielded by its equations. More generally, the improvement or acceptance of theoretical models is based on a process of interaction between empirical tests and theorising. The application of a theoretical model to yield predictions presupposes first of all the connection of its mathematical symbols with some characteristics of a real system, for instance of the symbol m_1 with the mass of the Earth or of r with the distance between the centres of mass of the Earth and the Moon, and then the use of further auxiliary assumptions, the choice of parameters, the determination of initial conditions, and so on. The equations of the model can then be used for predictions, whose numerical data should agree, within the limits of allowable error, with the values obtained from experimental measurements of the physical phenomena themselves, if the model represents them satisfactorily.

In the event of satisfactory agreement between the predictions of the model and the observed or experimental data of the real system, one can only conclude that the whole combination of the theory, the model and the other auxiliary assumptions that were used to produce the predictions, was successful. In the event of disagreement, however, it is not possible to determine in which of these elements – the theory, the model or the auxiliary assumptions – the error is situated. There are essentially no determinant experiments or unique unambiguous logical rules that allow the identification of wrong assumptions, theories and so on (Duhem 1978; Lakatos 1974; Kuhn 1989). Usually, in the case of negative results, the model is checked first, because of its possibly large number of idealisations, and every effort is made to improve the model and its predictions before the theory is re-examined for errors (see Nola 2004). The possibility of objective empirical verification of models and theories, however, is a separate issue, and one that is related to the more general philosophical problem of how closely the theories and their idealised laws are related to the real world or whether they refer only to a constructed world (see Section 2.1), issues that will not be dealt with further in this article.

Yet, even if the hypothesis that a physical system belongs to the area of applications of a model is shown to be correct, no more can be claimed for the model than similarity of a limited kind and degree with the physical system. Giere clarifies that the relation between the models and reality is not truth, or isomorphism, but similarity, concerning only certain aspects and only to a limited degree. The representation is incomplete and partial, although this does not prevent the models from providing us with a deeper understanding of how the world operates (Giere 1999, pp. 6, 92).

3. Model-Based Scheme for the Structure of Scientific Theories

According to Giere, who argues for a cognitive philosophy of science (1999, p. 54), scientific cognitive activity (the construction and interpretation of models) belongs to the more general cognitive processes of individuals representing their environmental world. He refers to the views of contemporary cognitive psycology: ‘The ability to construct models of complex and often remote aspects of the world is a deliberate and self-conscious extension of the evolved cognitive capacities for “mapping” their environment which humans share with many animals, particularly other mammals’ (1999, p.54). He tries to show that the model-based analysis of theory structure is consistent with the investigations of the cognitive sciences about concepts, categories, and classification of individuals: ‘... adopting a model-based approach makes it possible to apply Rosch’s analysis in a way that usefully increases our understanding of how theories are structured in scientific practice. And that is a reason for preferring a model-based account.’ (1999, p. 108).

In this section, we focus on the model based scheme for the structuring of theoretical models in the frame of the theory to which they belong. Giere gave a concrete example of a model-based conception of theory structure, on the basis of the inter-relation of basic classical mechanical models, which we present in Figure 1 (1999, pp. 106–111). Finding this scheme illuminating and satisfying, we apply it to quantum mechanics, testing thus also the applicability of the scheme to contemporary physics. In Figure 2,

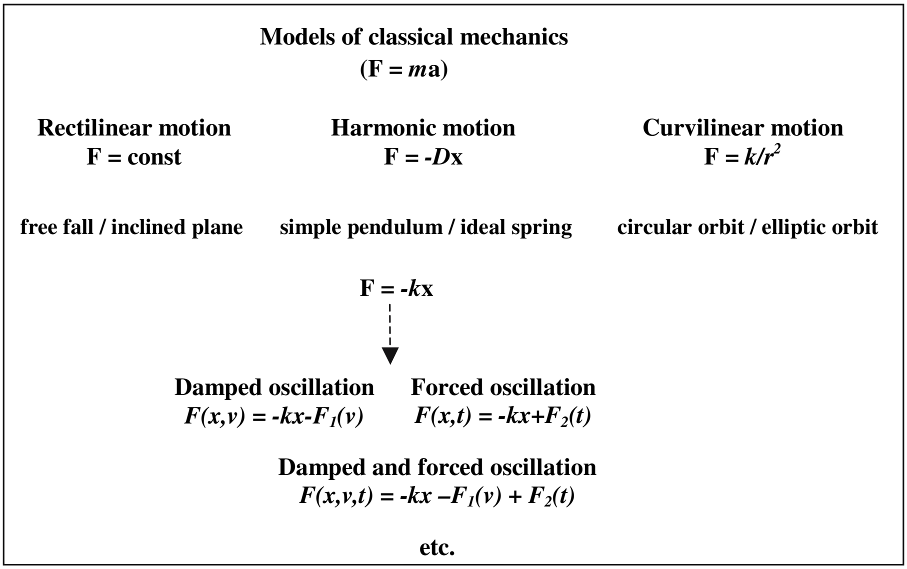

Figure 1. Inter-relation of models of classical mechanics (see Giere 1999).

Quantum mechanical models of a particle with constant energy

Schrödinger equation $(-\frac{h^2}{2m}\frac{d^2\Psi}{d^2x} - (E-U)\Psi = 0)$

Potential functions:

$U = 0$	$U = 0,\ 0 \leq x \leq L$ $U = \infty,\ x < 0,\ x > L$	$U = 0,\ 0 \leq x \leq L$ $U = U_0,\ x < 0, x > L$	$U = \frac{1}{2} m \omega^2 x^2$	$U = -\frac{e^2}{4\pi\varepsilon_0 r}$
free particle	particle in potential well with infinitely high walls	particle in potential well with walls of finite height	quantum mechanical oscillator	model of the hydrogen atom ↓ model for systems with several electrons, for molecules, etc

Figure 2. Structuring of quantum mechanical models in the framework of the model based view.

we present, following the model based structure scheme of theories, several basic models of quantum mechanics, and comment on them in the light of the model-based view. We believe that these examples can serve as a guide for content-structuring in science education in a way that makes clear the structure and interpretative function of a theory and adds structure and meaning to the subjects to be taught.

3.1. INTER-RELATION OF THEORETICAL MODELS IN CLASSICAL MECHANICS

According to Giere (1988, 1999), the theories consist of theoretical models and of hypotheses about the connection between the models and related real systems (e.g. the Earth-Moon system could be a two-body Newtonian gravitational system). In contrast to the view that the axioms of a theory are statements of universal scope, Giere believes that the axioms provide the basis, the general rules for the construction of theoretical models. Based also on the examination of the presentation of classical e.g. mechanics in modern textbooks, he describes a theory as a population of models consisting of related families of models: ‘The various families are constructed by combining Newton’s laws of motion, particularly the second law, with various force functions – linear functions, inverse square functions, and so on.’ (Giere 1988, p. 82).

More concretely (see Fig 1), the structure of elements that in addition to the Newtonian axioms, and specifically the equation $F = ma$, also satisfy, for example, the linear force function $F = -Dx$, determines the theoretical model of the harmonic oscillator. From the very abstract model of the harmonic oscillator a family of more specified models arises, according to the further specification of the force function that must be satisfied, which broadens the empirical applications of the theory. While the harmonic oscillator is a system with only one linear restoring force ($F = -Dx$), the damped oscillator presupposes one additional damping force, the driven oscillator one additional driving force, and the damped and driven oscillator forces of both types.

On the other hand, combining the Newtonian axioms with forces proportional to $1/r^2$ ($F = k/r^2$) yields a family of models for curved orbits, some of them closed, while combining them with a constant force ($F = const$) yields models for uniformly accelerated motion.

3.2. INTER-RELATION OF MODELS IN QUANTUM MECHANICS

The interpretative and prognostic limits of classical physics in the range of atomic and sub-atomic systems, led, *inter alia*, to the creation of quantum mechanics. While quantum mechanics was very successful in the interpretation and prediction of the behaviour of microcosmic systems, it demanded fundamental changes in the deterministic way of approaching phenomena in classical physics, and it seemed to conflict with natural human intuition.

The distinguishing characteristics of quantum mechanics, such as the statistical interpretation of microscopic processes and the recognition that they principally permit statements only in terms of probabilities, the uncertainty principle, and the highly formal-mathematical presentation and treatment of quantum physical ideas and concepts and the related problem of their interpretation, were problematic and very challenging issues for physicists and philosophers of science, as well as for science educators.

We now extend the application and trial of the theses of the model based view, particularly its theory structure scheme, to introductory quantum mechanics, on the basis of simple models of single particles with constant energy states. By analogy with classical mechanics, where the different families of models arise from the combination of the fundamental dynamic equation with different force functions, the quantum mechanical models arise from the combination of the fundamental equation of quantum mechanics, the Schrödinger equation, here in time-independent form, with different functions for the potential energy (or the potential, as it is usually called in quantum mechanics).

Combining the Schrödinger equation with a zero potential ($U = 0$) defines the model of the free particle, which is a very strong idealisation, whereas combining it with more specific potential functions, in our case with a potential that is a function of position, yields the set of models of, *inter alia*, bound particles, and specifically the models of a particle bound in a potential well, the model of the quantum mechanical oscillator, the model of the hydrogen atom, and so on (see Figure 2).

The less abstract a model is, that is, the more it satisfies more specialised potential functions in addition to the fundamental equation of the theory, taking into account more spatial dimensions and other parameters of the system so that it becomes mathematically more complex, the better it achieves its interpretative function of representing and interpreting the states of the real world. Already the strongly idealised model of the bound particle in a potential well with infinitely high 'walls' gives, through the solution of the Schrödinger equation for the boundary conditions of the model, the discrete energy states of the particle ($E_n = (h^2/8\,mL^2)n^2$, $n = 1,2,3,...$), that is, it is able to explain the characteristic phenomenon of energy quantisation of bound quantum mechanical particles.

The model of a particle bound in a one-dimensional potential well with walls of finite height can explain the quantum mechanical tunnel effect. In the model of the quantum mechanical oscillator, the solutions of the Schrödinger equation (for the wave function Ψ) determine its corresponding energy values ($E_n = (n + 1/2)\hbar\omega$, for $n = 0,1,2,...$), with difference in energy between them $\Delta E = \hbar\omega = hf$. The quantisation of energy, which Planck had axiomatically introduced to explain the interaction of matter with radiation, thus arises naturally from the solution of the Schrödinger equation.

From the solutions of the Schrödinger equation for the model of the hydrogen atom (Coulomb potential of the core, $U(r) = -e^2/4\pi\varepsilon_0 r$, three-dimensional form of the Schrödinger equation), all the orbital and energy states ($E_n = E_1/n^2$, $n = 1,2,3,...$) of the electron of the hydrogen atom can be mathematically calculated.

Further reduction of the idealisations of this model, that is, increasing the specification of the potential function, could in principal yield more complex models for systems with several electrons, for molecules, and so on. If, however, one extends the nucleus-electron system by the addition of just one more electron, and takes all the internal interactions of the system into account (i.e. a sum of terms of the form $U(r) = -e^2/4\pi\varepsilon_0 r$), then the complexity of the quantum mechanical equations increases so much that they can no longer be solved by the now known analytical methods.

The construction of models for atoms with two or more electrons proceeded thus by means of several kinds of approximations. In the perturbation method of approximation, for example, the influence of the second electron is treated as a perturbation on the state of the rest, nucleus-electron system, permitting the problem to be reduced to the already known and analytically solvable problem of the nucleus-electron system. The perturbation method itself is a particular mathematical treatment of the system equations that starts from the analytically known properties (e.g. energy states) of the simpler system and yields corrections to them, to different degrees of accuracy.

Other kinds of approximations are also used, e.g. for the modelling of atoms with many electrons, for which an appropriate approximate form of the potential function can be defined (e.g. as a suitably determined average of the electron-electron interaction terms), always with the intent to make the equations of the model analytically, or at least numerically, solvable.

To extend the models to more complex real systems, scientists need to use approximations, in addition to the initial idealisations that are inherent to the nature of the models. It is worth noting here that, with the use of modern computers, numerical methods are a helpful tool in modern science for the construction of more concrete – that is, less idealised – models, with more complex equations that are still at least numerically solvable.

4. A Proposal for Physics Content-Structuring Based on the Theory Structure-Scheme of the Model-Based View

In the structural scheme presented in the previous section, the different models of classical mechanics and quantum mechanics are organised and interrelated in the frame of the theory to which they belong, showing at the same time the theory's structure and its capacity to unify and interpret completely different phenomena, such as, for instance in the case of classical mechanics, the fall of bodies, the oscillations of pendulums, or the motion of planets.

The structure and unifying potential of a scientific theory usually get lost when the theory is presented in conventional physics textbooks or taught in conventional physics courses. Topics are usually presented fragmentarily and disconnectedly, and concepts are formulated only as mathematical, formalistic definitions, thus depriving physics of its historical-philosophical and social context and obscuring its relations with aspects of everyday life.

The physics-textbook planning philosophy of Greece's Pedagogical Institute has only recently evinced a conscious and clear reference to the concept of the model and to modelling as a fundamental element of scientific activity, which should consequently be present in the subject matter to be taught at all levels. This opens the possibility of making the content of physics textbooks self-consistent in a way that reflects the structure and functional power of the scientific theories and, more generally, the unifying tendency of scientific knowledge. We believe that the model-based theory structure scheme that we presented in the previous section can, when adapted to the educational context, provide a guideline for the meaning-giving structuring of the content of physics in science curricula and in physics textbooks, and by extension for the didactical planning of physics teaching.

In Figure 4, we give in outline a concrete example of the application of the theory-structure scheme as a basis for the structuring of typical physics contents at the junior and senior high school level, for example rectilinear and curvilinear motions, oscillations of different kinds, rotational motion, etc. These contents are presented in Figure 4 as theoretical models of Newtonian theory, together with the way in which they were derived from the theory and with some examples of their empirical application and some of their predictions. The individual topics are on the one hand interrelated as theoretical models that are derived from the fundamental equations of the same theory, but differing by the force functions and the scope of their applications, while on the other hand they add meaning to the theory, revealing its structure and its interpretative scope.

The derivation of the theoretical models in Figure 4 is based on the view that we formed on the basis of the different model-based contributions, namely that the theoretical models of a theory are constructed through the combination of the fundamental equations of the theory with certain additional functions, determined in each instance according to the class of systems/phenomena that a given model aims to represent, and the idealisations assumed for these systems during the construction of the model. This view is illustrated in Figure 3.

The theoretical models have different degrees of abstraction. We distinguish the most abstract theoretical models of the different families of models, such as that of the harmonic oscillator ($F = -Dx$), from the less abstract, particular theoretical models, such as the theoretical model of the simple pendulum (Figure 3), where the restoring force is a function of the gravitational force and the length of the string, or the theoretical model of

General theory Fundamental equations and principles of the theory e.g.	+	**Specific functions** according with the phenomena to be represented and the idealising assumptions about them	→	**Theoretical model** (specific theory)
$\Sigma \mathbf{F} = m\mathbf{a}$	+	$\Sigma \mathbf{F} = -D\mathbf{x}$ oscillating bodies idealisations: point masses, no frictions ↓ simple pendulum ($F=\frac{mg}{l}x$)	→	**Harmonic oscillator** ($s = s_m \sin \omega t$, $\omega = \sqrt{D/m}$, etc.)

Figure 3. Derivation of theoretical models through the combination of the fundamental equations of the general theory with specific functions.

the ideal spring-mass system, where the restoring force is identified with the restoring force of the spring due to its elasticity (Giere 1988, p. 80). The latter theoretical models refer more directly to the different oscillating and suspended real world systems, which they intend to include in the empirical applications of the harmonic oscillator. It is also easier to present them with sketches or pictures, interpreted as visual models of the theoretical model. Generally, theoretical models are abstract mathematical structures and are to be distinguished from the 'visual models', presented with sketches or pictures, and also from the physical scale models or the diagrammatic representations.

As can be seen from Figure 4, the different models all fulfil the fundamental equations of the theory ($F = ma$), but differ in the additional force functions ($F = k/r^2$, e.g. Newton's gravitational law, or $F = -Dx$, e.g. Hooke's law, etc), depending on the class of physical systems to be represented and the idealising assumptions made for them at the time of modelling.

In Figure 4, which shows the theoretical models for translational motion, other models of classical mechanics could also be included, such as the models for rotational motions of solid bodies, which are derived analogously through the combination of the basic equation $\mathrm{T}\tau = \mathrm{I}\alpha$ with the specific functions $\mathrm{T}\tau = 0$, $\mathrm{T}\tau =$ const, etc. Also the physics content for uniformly accelerated circular motion, for projectile motion, for the rolling of solid bodies, can be included in Figure 4 as composed models that are formed through the combination of the simpler basic models, for example the model of uniformly accelerated circular motion from the combination of the model of uniform circular motion and the model of uniformly accelerated motion, the model of the rolling solid body as the combination of the models of translational and rotational motion of a solid body, and so on.

By analogy with the contents of classical mechanics, topics of quantum mechanics at high school or college levels, understood as elementary basic quantum mechanical models derived from the fundamental equation of quantum mechanics, the Schrödinger equation, together with various potential functions, can also be similarly structured on the basis of the structural scheme that we presented in the previous section (see Figure 2).

5. Concluding Remarks and Educational Implications

Achieving science educational goals depends *inter alia* on the adoption of a philosophically valid theory of science that will serve as a basis for the conceptualisation of science in the educational context. In the philosophy of science, the most prevalent and promising view of scientific theories is

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School physics contents as theoretical models of classical mechanics
Newtonian theory ($\mathbf{F} = m\mathbf{a}$)
Introductory concepts: e.g.: system of reference, position, time, system, interaction

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Derivation of models

Axioms + specific force functions		**Theoretical models**		**Applications; Predictions**
$\mathbf{F} = m\mathbf{a}$ **and:**				
$\mathbf{F} = 0$ IP: physical objects with const. velocity in rectilinear motion IDL: point masses, e.g. without interactions	→	**Rectilinear uniform motion (or at rest)** (v = const. or $v = 0$, $x = vt$)	↔	e.g. objects at rest, or in free space
$\mathbf{F} = \mathbf{c}$ IP: physical objects with acceleration, in linear motion IDL: point masses, e.g. in homogeneous gravit. field	→	**Rectilinear motion with constant acceleration** ($\Delta v = at$, $\Delta s = v_0 t + 1/2at^2$, etc.)	↔	e.g. fall of objects, vertical or on inclined planes
$\mathbf{F} = b/\mathbf{r}^2$ IP: physical objects in orbital motion IDL: e.g. 2 point masses, with only their mutual gravitational interaction	→	**Uniform circular (or elliptical) motion** ($a_k = v^2/r$, $T = 2\pi R/v$, etc.)	↔	e.g. System Earth-Sun, or Earth-Moon etc. Pred.: satellites, escape speed, new planets
$\mathbf{F} = -D\mathbf{x}$ IP: oscillating physical objects IDL: point masses, no friction	→	**Harmonic motion or Harmonic oscillator** ($s = s_m \sin\omega t$, etc.)	↔	physical pendulums and springs, bars etc. Pred.: const. period, watches

↓

Family of models of the harmonic oscillator

↓

Damped oscillation
$F(x,v) = -kx - F_1(v)$

Forced oscillation
$F(x,t) = -kx + F_2(t)$

Damped and forced oscillation
$F(x,v,t) = -kx - F_1(v) + F_2(t)$

coupled pendulums,
chaotic pendulums and so on

Composed models (e.g. **projectile motion**: combination of models of rectilinear and accelerated motion)

Figure 4. School contents of physics as theoretical models of classical mechanics (IP: intended phenomena; IDL: idealizations; Pred.: predictions).

now the model-based view, enhanced by knowledge of the cognitive sciences for human cognitive and representational activities.

The model-based view has a great impact on science education, or the didactics of science, as it is called in many countries of continental Europe. (The didactics of science is focused on the teaching of the natural sciences, situated within the more general context of educational research, and is usually organised in specific didactics, e.g. the didactics of physics, of chemistry or biology. A comparison of the two terms, science education and didactics of science, is made in Adúriz-Bravo & Izquierdo (2005, pp. 32–33)). The re-interpretation of the scientific method and knowledge in the model-based view has led the science education community to a consensus on the need to develop an expanded notion for the scientific method, including the recognition of the role of modelling and of epistemic values and social processes and contexts in the development, evaluation and communication of scientific knowledge (Grandy & Dusch 2005). The impact of the model-based view is also reflected in arguments over the potential of the model-based view to contribute to the achievement of the aims of science education, '... since it now refers to *school science*', to the re-conceptualisation of science education, '... seen now as a process of modelling', or to the planning of a 'school scientific activity according to the current demands of science literacy for all' (Izquierdo & Adúriz-Bravo 2001, pp. 2–3).

The science education community is particularly interested in Giere's cognitive approach to science (Adúriz-Bravo & Izquierdo 2005), since it is believed that this account, being consistent with current contributions from cognitive psychology, sociology and linguistics, opens the promising prospect of combining 'philosophy of science and cognitive developmental psychology in the service of science education' (Grandy 2003, p. 773). Giere's cognitive account can be related with, e.g., the science educational concept of learner's ideas, beliefs or misconceptions and of conceptual change.

The model based view, to our opinion, describes scientific methods adequately and pragmatically, and it reflects the working climate of scientific researchers. Adopting the model-based view as a foundation for the science education conceptualisation of science, however, presupposes a clear understanding of the notion of the model and modelling. The basic aims of science education, namely students' acquisition of scientific knowledge and skills and of an understanding of the nature of science, then correspondingly imply knowledge of basic scientific models, familiarity with the modelling method, and an understanding of the nature and role of models (Justi & Gilbert 2003, p. 1369).

In this paper, we present a reconstruction of influential philosophical accounts on models, ranging from the original concepts of the model in the semantic view to its later developments in the cognitive approach of science, and use them to attempt an approach to Newtonian mechanics and introductory quantum mechanics from a model perspective. We focus primarily on two basic issues and their educational implications. The first concerns the concept of the theoretical model and the methods for constructing such models, which might contribute to the shaping of the theoretical background, necessary for a model-based teaching of theories and for the professional development of science teachers. For example, understanding the nature of the theoretical model provides a basis for distinguishing between the several kinds of models, while an understanding of the 'construction rules' provides a basis for distinguishing between theoretical models and the underlying general theories.

Our second focus is on the model-based scheme for theory structure, as exemplified by Giere in the case of classical mechanics, which we applied to the case of quantum mechanics, thus also testing the possibility of extending it to other theories of contemporary physics. We believe that this constitutes an appropriate basis for the structuring of the school science contents, in a way that adds more meaning to the acquisition of scientific knowledge. We give an example of the structuring of typical school physics contents in the field of classical mechanics (rectilinear and curvilinear motion, oscillations of different kinds, rotational motion, etc.), where the topics are presented as models of the same theory, together with the way they are formed and the range of their applications, such that on one hand they are interrelated and on the other they demonstrate the structure, the meaning and the interpretative power of the root theory. This approach could be used for a similar structuring of the introductory quantum mechanics contents at the high school and college levels. We believe that in this way school science contents, which are usually presented in a disconnected and fragmentary manner, can be both structured coherently and rendered more easily accessible to the students. This concrete example of content-structuring and other ones in the spirit of the model-based view suggest a scheme for a meaningful summarising and consistent overview of the contents of a theory that concerns curricula development and textbook writing. Of course, it implies specific changes in the way science topics are presented and developed in science instruction. For example, models are introduced successively, with the aim of interpreting new phenomena, that is, broadening the applications of the theory to new areas of physical phenomena, while new concepts of physics (velocity, acceleration, momentum) are introduced when and as they appear necessary as tools for the modelling of these new areas of phenomena (cf. Halloun 2004).

Suggestions and contributions regarding the realisation of model-centred teaching in practice already exist or are being developed (e.g. Halloun 1998; Halloun 2004; Izquierdo 2001; Grandy 2003). Proposals include, for example, the development of simple basic models by the students themselves, starting from familiar natural phenomena or case studies or software, with interventions from the teacher, especially for the further development of more complex models (Halloun 1998, 2004). Another possibility, in our opinion perhaps more suitable for the more advanced topics treated at high school and college levels, would be the presentation of the topic contents as models of a theory, with the emphasis on the students becoming familiar with the interpretive, predictive and inventive functions of the various models and on the selection of the appropriate model for problem solving.

Conditions for the success of model-based teaching have also been identified, including the need for a re-conceptualisation and restructuring of science education, and a philosophically valid presentation of the models and the modelling process in the textbooks, as well as the development of valid teacher views and practices with regard to the nature of the model. Justi & Gilbert (2003), however, report that the results of research on teachers' views show that teachers do not sufficiently understand the nature and functions of models, and they note that the use of the concept of the model and of modelling in science education curricula, especially in the older curriculas, is still completely confused and devoid of any clear terminology. Indeed, a non-systematic or fragmental study of the model-based analysis, which certainly in its broader framework displays differences and also is continuously further developed and enriched, can easily introduce ambiguity and non-completeness into science education literature and practice. In Justi and Gilbert's (2003) research, the teachers' views were elicited and assessed primarily on the basis of visual and concrete models.

This paper focuses on theoretical models, which are not only the most fundamental scientific models but are also quintessentially the models of scientific research. In Section 2.1, we give a definition for theoretical models that includes also the basic modelling rules for their construction. We described a theoretical model as a mathematical structure of a set of elements that fulfils the axioms of the theory and additional specific functions imposed by the class of phenomena to be represented by the model and the related idealisations that were made during its construction. According to Giere, theoretical models are to be considered 'as abstract entities having all and only the properties ascribed to them in the standard textbooks' (Giere 1988, p. 78). The properties ascribed to a theoretical model are represented in mathematical form: the mathematical representation of the simple harmonic oscillator, for example, is basically the equations $F = ma$ and $F = -kx$ (this can, of course, be more complex in other cases).This formulation, however, hides a series of auxiliary assumptions and principles of the general theory in the framework of which the theoretical model was constructed. It is the mathematical equations of a theoretical model that are treated to yield properties and behaviours of the model (e.g. equations of motion, oscillation periods, etc), which then are expected to appear (often by means of technical intervention) also in the intended real systems, if the model represents them satisfyingly.

The properties of a model can also be represented in a verbal or visual mode, in the latter case yielding visual models (physical scale models, sketches, pictures, diagrams). Scientists often make use of visual models to communicate their theoretical conceptions between them and to others, exploiting them for visual explanation and reasoning. Visual models can present in visual form some of the properties, or some aspects of the properties, of the theoretical models. Conversely, visual models may be conceived at the beginning of the process of creation of a theoretical model, and more generally at the outset of the approach to a problem, where they serve the derivation of the mathematical representations and theoretical hypotheses that contribute to the development and shaping of a new conceptual system or theory (see e.g. Nersessians' (2005) discussion on Faraday's and Maxwell's visual models). In other cases, a visual model may be part of a set of visual models that taken together constitute the content of a broader theoretical account (see Giere 1999, chapter 7).

Visual models are widely used in science teaching, because of their explanatory power in the case of classical physics, but also for pedagogic reasons, because of their effective contribution to the understanding and learning process, especially for younger pupils. Fischler & Lichtfeld (1992), however, pointed out the inadequacy, or rather the inappropriateness, of the trials to visualise quantum mechanics, especially by means of visual models drawing on the methods and concepts of classical physics, such as the planetary model of electron orbits. They argued that such models create stabilised students' misconceptions that hamper the assimilation of further quantum mechanical concepts, and they suggest instead more abstract forms of presentation as more appropriate for the quantum mechanical understanding of subatomic particles. A related observation is made by Grandy (2003), who notes that, since physics models are usually mathematical, more time and attention need to be devoted to the co-ordination of science (physics) and mathematics curricula.

We have related some aspects of the model-based analysis with the educational aims of giving science instruction more structure and meaning, fundamental preconditions for the acquisition of essential and robust knowledge. Further, a science education that rests upon a model-based epistemology might, considering that the model-based view is apt and pragmatic in its approach to scientific activity, convey to the class, and, by extension, to society, some conceptions of the scientific methods and knowledge that are compatible with those of active scientists.

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