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The Relation between Idealisation and Approximation in Scientific Model Construction

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Abstract. The notions of 'idealisation' and 'approximation' are strongly linked to the question of 'how our theories represent the phenomena in their scope'. Although there is no consensus amongst Philosophers on the nature of the process of idealisation and how it affects theoretical representation, at the level of science education much can be gained from the insights of existing philosophical analyses. Traditionally, teaching methodologies treat the observed divergence between theoretical predictions and experimental data by appealing to the more common-sensical notion of 'approximation'. The use of the latter notion, however, to explicate discrepancies between theory and experiment obscures the theory/experiment relation. It does so, I argue, because from the viewpoint of scientific modelling 'approximation' either depends upon or piggybacks on 'idealisation'.

1. Introduction

It is relatively clear that some conceptual skills are expedient to the learning of science (e.g. competence in mathematical reasoning), but what is not so clear is what meta-scientific conceptual ingredients are important for enhancing the ability of students to think scientifically, despite the recognition that among the central objectives of science education is to contribute to this goal. In this paper I aim to motivate the thesis that understanding the nature of science is a key meta-scientific ingredient in improving thinking scientifically. Of course, this thesis is widely held and argued for by many science educators (e.g. Lederman 1992; Abd-El-Khalick & Lederman 2000; Lederman et al. 2002; Leach et al. 2000; Hogan 2000), but as is well-recognised by several science educators (e.g. Abd-El-Khalick & Lederman 2000; Lederman et al. 2002; Leach et al. 2000), 'the nature of science' is a general notion which is used to refer '... to the epistemology of science as a way of knowing, or the values and beliefs inherent to the development of scientific knowledge' (Abd-El-Khalick & Lederman 2000, p. 666) and as such it is vague and hence not clear how to define. Hence, instead of attempting the rather impossible task of defining it I shall instead concentrate on, what in my view is, one of the important constituent parts of the general notion of 'the nature of science', namely 'how scientific theories relate to experiment', or simply the theory/experiment relation. So what I shall motivate in this paper is the more focused version of the above thesis: that understanding the theory/experiment relation is a key meta-scientific ingredient in enhancing the ability to think scientifically.

That theoretical predictions and experimental measurements in science do not exactly match each other is commonplace. Had they matched there would not be much room for doubt that our theories gave us the truth about particular aspects of the world. The fact that they do not has led to philosophical debate on several related issues concerning our scientific enquiry. Some examples are: 'Is it the approximate truth or the empirical adequacy of our theories that could be rationally justified?', 'Do we have rational criteria of choice between competing theories?', 'Given that our theories do not exactly mirror the world, how is it that they represent it?', etc. These debates among philosophers of science, which are at the root of the view expressed in Lederman et al. (2002) that there is no singular nature of science, were governed—in much of the twentieth century—by the assumption that theory could be directly related to observation. One such example is the view associated with Logical Positivism that the deductive consequences of a theory could be stretched all the way down to observation statements and reports. The last three decades philosophers have recognised that the theory/experiment relation is far more complex and most importantly that the role of scientific models is far more crucial in how theories are applied to phenomena than the interpretative role attributed to them by some thinkers of the Logical Positivist tradition, *viz*. Nagel (1979).

In the philosophical terrain two views on scientific models have emerged. The first, the Semantic View of scientific theories understands models as mathematical structures that are the constitutive parts of theories (e.g. van Fraassen 1980; Suppe 1989; da Costa & French 2003), and the second understands models as mediating conceptual instruments between theory and evidence (e.g. Morgan & Morrison 1999). Parallel to philosophical work on the analysis of scientific models, science educators increasingly recognise the value of scientific modelling in science learning. This is demonstrated in the search for a general modelling approach in education (e.g. Constantinou 1999; Hestenes 1987; Halloun & Hestenes 1987; Wells et al. 1995). But also in the more specific attempt to analyse and understand the characteristics of scientific modelling through the in-depth study of important models of the history of science, such as the pendulum, and their contribution to science education (e.g. Matthews et al. 2004; Newburgh 2004; Nola 2004). The basic idea behind the modelling approach in education is the organisation and orientation of a science course around a small number of basic models, e.g. the simple harmonic oscillator or the two body Newtonian system. Students are familiarised with the structure of these models and use them to explore the relevant features of physical systems, to invent new concepts that help characterise features of the physical systems that are otherwise left unspecified in the models employed and to construct more complex models that are used for the representation and explanation of particular physical systems. Empirical studies that indicate the success of this approach in science education are given in Halloun & Hestenes (1987), in Wells et al. (1995) and in Constantinou (1999).

My focus in this paper is the notion of 'scientific model' as a theoretical entity that provides the link between theory and experiment. As such it is characterised by two important features: it is both an 'idealisation' and an 'approximation' of its target physical system. The processes of model construction are accordingly affected by the processes of idealisation and approximation inherent in our scientific theorising. The process of idealisation, that as I shall argue below is a primary aspect of scientific discourse, has been investigated by a number of authors. Matthews (2004) and Nola (2004), both belonging to this list of authors, go to pains to point out that Galileo's methods, and subsequently the methods of modern science, are not simply inductive (i.e. the conception of a theory or model does not result from mere inspection of experience) but that they involve reasoning (i.e. the conceptual processing of ideas directly related to experience). 'The role of reason [in science] is two-fold; in the first place to 'construct' idealisations or models, and then to make inferences from the models about possible observations that might only fit our experience to some degree of approximation' (Nola 2004, p. 350). As Matthews (2004) explains, 'Galileo abstracted from 'impediments' and 'accidents' ... in order to get a mathematical formulation of the principal causal relationships' (p. 702). I concur with these general views of the two authors and in fact I will (Section 4) try to demonstrate the process of idealisation as employed in modelling the pendulum.

However, such a conceptual process gives rise to the question of how our theoretical constructs can be justifiably considered *representations* of the world. Matthews (2004) has an answer to this, 'through experimental manipulation, elimination of impediments, and progressive approximations, [Galileo] tried to have the real world mirror his ideal' (p. 702). My argument reverses the process of establishing the theory/experiment relation from the one implied in this suggestion. It is the idealised model, I claim, that through progressive de-idealisations is modified to an approximate representation of the target physical system. In this claim the notions of idealisation and approximation are intertwined. These two notions are features of the more general process of theoretical representation of physical systems and hence are strongly tied to the relation between theoretical statements and experimental reports. In fact, the discrepancy in the theory/experiment relation could be attributed in part to these two characteristics of scientific methodology in theory and model construction. My primary concern in this paper is to explore the relation between 'approximation' and 'idealisation', by this I mean how the two notions act together in the constructions of theoretical representations and furthermore in relating these to actual physical systems. Because my more general concern is the nature of the theory/experiment relation I explore these notions through the lens of scientific modelling.

What I have said so far can be sketched as follows. Understanding the nature of science is necessary for enhancing the ability of students to think scientifically. Because the nature of science is a broad and rather vague notion, I restrict my thesis to one of its components: understanding the nature of the theory/experiment relation is necessary for enhancing the ability to think scientifically. Because, in my view, the primary link between theory and experiment is achieved by scientific models, we cannot achieve a full understanding of the nature of the theory/experiment relation without studying the processes of construction of scientific models. Since scientific models involve idealisations and approximations of their target physical systems the notions of 'idealisation' and 'approximation' and their respective processes in model construction must be clarified and understood. In this paper I restrict myself to an argument that is based on an analysis of how the two notions operate together in the construction of scientific models that aim to bring theory closer to actual physical systems.

But there is more to my philosophical argument that is relevant to science education. By offering a way to understand the relation of idealisation and approximation in scientific model construction, and by arriving at the result that the processes of idealisation/de-idealisation are primary in modelling and upon them approximation processes piggyback, I also mean to urge that a focus on idealisation—involved in scientific model construction—would also facilitate teaching methodology in enhancing the ability to think scientifically. For one thing, it would bring more clarity to the vague notion of the 'approximation relation' in science.

In science and in philosophy, attention has been given primarily to the notion of approximation, possibly because of its significance for an 'inductivist' understanding of the nature of science, but also because it is widely recognised that it can be explicated and handled by the use of mathematical tools. This has led to conceptual confusions firstly because the concept of idealisation and the process of idealisation in science by and large have been attributed a lesser significance, despite their epistemological and methodological importance, but also because it became customary to use the concept of approximation as a surrogate to idealisation, thus hindering the recognition of those elements of scientific practice that are exclusively associated with the process of idealisation. That this indiscriminate use of approximation has created the impression that the two concepts could be used interchangeably is not, however, an argument for the synonymy of the two concepts. In Section 2 of this paper I shall argue that, and explain why, the two concepts are in fact distinct, and in Sections 3 and 4 I shall argue that their interdependence is such that if clarified it could illuminate the theory/experiment relation.

Aiming to minimise the epistemic effects of approximation, in science and mathematics much work is done in an attempt to minimize the theory/experiment discrepancy by the use of theories of systematic and random error of measurement. This, however, only attends to some of the causes of the discrepancy, namely those due to experiment. It does not, however, explicate the relation of approximation between theory and experiment, which is, it could be argued, a vague concept both in terms of its constitutive conceptual components and in regard to its impact on other semantic concepts like truth. Nevertheless, one thing is clear about the approximation relation; it presents a problem to the notion of truth. If theoretical proposition X is 'approximately true' of observation Y then strictly speaking X is false. Yet 'approximation' has been exploited in ways as to associate it to the concept of truth in recent philosophical attempts (of realist inclination) that explicate the approximate nature of the theory/experiment relation by focusing, for instance, on the notion of degree of truth or truthlikeness or verisimilitude of scientific theories (e.g. Popper 1979, 1989; Niiniluoto 1987, 1999), despite the shortcomings of such an approach (see Psillos 1999). It is a trivial matter that science is not concerned with the strict arithmetical sense of the notion of approximation. That is, the statement that the number *a* approximates the number *b* does not have any scientific significance. If a statement of such form is to have scientific value then *a* and *b* must be values of physical quantities that refer to the actual world. Hence the approximation relation between *a* and *b* would take on a different character if it were stated in a scientific context, it would refer to the closeness of the values of two quantities. Not to any two unrelated quantities, but to quantities that purportedly refer to the same thing and which would be computed in different ways, the first via the conceptual resources of a theory and the second via the experimental apparatus and the theories of measurement and experiment. In short, the approximation relation refers to the closeness of theoretical predictions to experimental measurements, and this closeness has been traditionally interpreted by (realistically inclined) philosophers and scientists alike as closeness to truth, or truthlikeness of scientific theories. To explicate what it means for a theoretical statement to be approximately true of the world, however, has not proved to be an easy task partly because of the vagueness of the concept of approximation and partly because of its relation and dependence on the process of idealisation in scientific methodology.

In science—teaching methodologies in the classroom, at all levels of education, it is tradition to ignore the vagueness of the approximation relation. In addition, it is considered as a primitive notion that suffices for making sense of the approximate truth or adequacy of the theory, in deliberating upon the discrepancy in the theory/experiment relation. This practice not only does not clarify what it means for a certain theoretical prediction to approximate an experimental measurement, but it also obscures the theory/experiment relation and hence it leads the science student to an elliptic understanding of the nature of scientific theories and scientific models. I shall not attempt to offer a comprehensive explication of the concept of approximation but I will focus on the latter problem and argue that by hooking up approximation to idealisation we manage to maintain a more lucid view of the theory/experiment relation. In fact, I will motivate a much stronger thesis: that a clear understanding of the ways by which idealisation and approximation interrelate in model construction is necessary for explicating the theory/experiment relation.

2. Distinguishing Idealisation from Approximation

In order to explore the relation between 'approximation' and 'idealisation' it is important that a clear understanding of the two notions and their distinction is achieved.

As a first step in distinguishing the two concepts I follow Suppe (1989) and understand *idealisation* of the features of physical systems to involve two primary modes: either (a) distorting the characteristics of relevant features of physical systems in the theoretical description, e.g. assuming a projectile to be a *point—like* mass in estimating its trajectory, or (b) abstracting relevant features or properties of the physical systems from the theoretical description, e.g. ignoring the effects of friction in the description of the motion of a body on an incline plane. Several authors call the former mode 'idealisation' and the latter mode 'abstraction' (*viz*. Suppe 1989; Cartwright 1989; Morrison 1999; Nola 2004; Portides 2005).[1] The terminological distinction is made to emphasize, among other things, the fact that the process of abstraction is a necessary methodological aspect of science. In particular, it is made to stress the importance of abstraction in the scientific attempt to isolate the common features of different physical systems, which may otherwise differ amongst them in a number of conspicuous ways. Thus enabling the description and explication of these features in terms of a small number of variables and parameters (i.e. what we commonly refer to as the laws of a theory). Since the objective of this paper is not to analyse or to articulate a theory of the processes of idealisation and abstraction but only to conduct an investigation of how they are used in scientific modelling, for the purposes of this paper, the distinction between idealisation and abstraction will be suppressed and the latter will be assumed to be a particular mode of the process of idealisation.

Approximation of the features of physical systems by theoretical descriptions (e.g. models, theories) could also be divided into two modes; it is achieved either (a) by simplifying the relevant parts of the descriptions of individual features and properties of the physical systems in the overall theoretical descriptions, e.g. assuming the effect of the damping force due to air resistance to the motion of the pendulum to be a linear or quadratic function of velocity, or (b) by simplifying the theoretical description of the physical system as a whole in order to produce a description that is not exact but it is tractable and close enough, e.g. assuming that the magnitude of all the effects to the motion of a body due to influencing factors are small thus allowing us to ignore their mutual interactions and treat them as separate contributions that give rise to linearly independent tractable equations (later I shall demonstrate both of these modes of approximation with reference to the process of modelling the pendulum). Of course, by narrowing down the notion of approximation to its mathematical sense, as it is obvious from the above, I do not mean to deny that the concept is, or may be, used in a variety of other ways. My focus on mathematical approximation partly has to do with the fact that the prevailing mode of approximation in science is mathematical and partly with the fact that approximation in this sense is clearly distinct from the notion of idealisation. It is not clear to me, however, if other uses of the concept are in fact anything distinct from idealisation, but I shall return to this idea in Section 3.

The two concepts of idealisation and approximation turn out to be clearly distinct after inspecting their logical properties. We generally understand idealisation, but not approximation, as a directional process. This intuition is captured by the logical property of symmetry. Idealisation is not a symmetric concept (in fact, it could be claimed that it is asymmetric) whereas approximation is. That is to say, thinking of idealisation and approximation as relations in which two statements enter (e.g. one deriving from theory, X, and the other from experimental reports, Y), if ‘X is an idealised description of Y’ is true then it is not true that ‘Y is an idealised description of X’, whilst if ‘X is an approximate description of Y’ is true then it is also true that ‘Y is an approximate description of X’. For example, the simple harmonic oscillator exemplifies what we would consider to be an idealised description of a target physical system like the motion of the pendulum in the lab; however, a description of the pendulum that accounts for all factors influencing the motion of the bob is not an idealised description of the simple harmonic oscillator. On the other hand, if the simple harmonic oscillator predicts that the earth’s acceleration g is equal to 9.8 m/s^2 and this is accepted to approximate the value of 9.81 m/s^2 that results from measurements dictated by more reliable procedures, it makes equally good sense to claim the converse that 9.81 m/s^2 approximates the prediction of 9.8 m/s^2.

In addition to being asymmetric idealisation is transitive, i.e. if ‘X is an idealised description of Y’ and ‘Y is an idealised description of Z’ then ‘X is an idealised description of Z’. This property captures our intuition that idealisation is a scalable concept, which is demonstrated by examples like the following: if the simple harmonic oscillator is an idealised description of the damped harmonic oscillator and the latter is an idealised description of the pendulum then the simple harmonic oscillator is also an idealised description of the pendulum.[2] Approximation on the other hand is not—unconditionally—transitive since it is a pragmatic—and context-dependent—issue whether, if ‘X is an approximate description of Y’ and ‘Y is an approximate description of Z’ then ‘X is an approximate description of Z’, and hence the conditional is not true for all X, Y and Z. This logical difference is in a sense indicative of the fact that the two concepts differ in their pragmatics (e.g. their role in heuristics), an issue which is central to section 4 of this paper. Furthermore, another characteristic that distinguishes the two concepts is that approximation is understood as having end-point limits, whereas idealisation cannot have clearly-cut limits. This intuition is partially captured by the property of reflexivity. Idealisation is not reflexive (and it could also be argued that it is irreflexive), i.e. it makes no sense to say that X is an idealisation of itself unless the concept of idealisation is trivialized, whereas approximation is reflexive, i.e. the statement ‘X is equal to itself’ makes sense as an approximation claim because the relation of identity can be understood to mean the limiting case of the approximation relation.

With these logical properties of the two concepts in mind it is clear that aphorisms like ‘all idealisations are forms of approximation’ or ‘all approximations are forms of idealisation’, which one encounters in philosophical as well as scientific literature, either distort our intuitions of the two notions or require careful qualification if they are to shed light on the theory/experiment relation. For the science educator the problem is twofold, on the one hand there are two concepts that are related in not very obvious ways and whose distinction is subtle and not easily comprehensible to the science neophyte. On the other hand, there are two methodological processes in science, that are equally important for the best possible understanding of science and its nature, and which if not discerned properly the theory/experiment relation is obscured; moreover, despite the fact that the concepts of idealisation and approximation are logically distinct, mere inspection of actual science reveals that the scientific processes in which they are employed are closely interconnected. The model construction process analysed in Section 4 aims to show, among other things, that a modelling approach in education can be used to clarify the idealisation and approximation processes, thus illuminating the theory/experimental relation and demonstrating that epistemological characteristics of science could be highlighted in the process of teaching the scientific subject matter and not necessarily as a separate teaching activity. In addition, it aims to show—and epistemologically justify—that modelling approaches in education (e.g. Hestenes 1987; Constantinou 1999) can gain by focussing on the specifics of idealisation and approximation processes.

Looking at idealisation and approximation from a methodological perspective another difference can be discerned that can be located in how the two concepts are employed in scientific representation. One of the functions of idealisation is to reach a level of generality in our representations of phenomena. When it is claimed that ‘X is an idealised description of Y’ it is meant that Y is not necessarily a description of a specific physical system. The simple harmonic oscillator, for instance, is an idealised description of the type ‘pendulum’ and not just of particular token pendulums. The same also holds for different levels (or degrees) of idealisation. The somewhat de-idealised version of the simple harmonic oscillator, known as the damped harmonic oscillator, also represents in a general way even though the class of physical systems it represents is not necessarily the same as that of its more idealised predecessor model. The rationale behind representation by the use of the idealisation/de-idealisation process is that the more idealised the representational model is the less properties and features of the type target systems it represents. By de-idealising a model we do not merely restrict the class of representations, but we add more of the relevant features in the general representation of the type target system. The process of idealisation could therefore be used for general as well as for particular representation. This characteristic of representation is not something that is present in approximation claims. It is not sound, for instance, to claim that the simple harmonic oscillator represents pendulums in general *because it approximates* the type pendulum, since there are actual pendulums with very large damping forces that make them anything but approximate to the model. In short, approximation is a feature of representation that concerns the specific cases of target systems. This is another reason why not all idealisations are approximations, although the converse is not ruled out by this argument. More importantly, however, in order for an approximation claim, i.e. 'X approximates Y', to be useful in the explication of the theory/experiment relation, X must be such as to refer to a particular physical system that involves a large enough number of features and properties that are present in Y. Otherwise X and Y would not be referring to the same thing. In other words, the statement 'X approximates Y' is useful for explicating the theory/experiment relation if X is sufficiently de-idealised so that it can be regarded as a genuine representation of Y. Another way of saying this is that X must be sufficiently de-idealised so that its reference is no longer a class of ideal-type systems, but types that can be actualised in the world. Because of this I claim that the process of idealisation is primary and upon it the pragmatics of the process of approximation depend (e.g. the role and use of approximation in the representation of physical systems, or the factors that are responsible for the successful use of approximation in prediction and explanation, etc.).

My argument is backed by an analysis of the well-known model of the simple harmonic oscillator (of classical mechanics) and the process by which it is used in order to construct derivative models that can be proposed for the representation of target physical systems such as the simple pendulum or the torsion pendulum. I employ this analysis of the process of construction of representational models to demonstrate that idealisation, and its converse process of de-idealisation, is present at every level of scientific theorising whereas the concept of approximation becomes methodologically valuable, and epistemically significant, either when a tractable mathematical description of a de-idealising factor is needed or after a certain point in the process is reached when a given theoretical construct (i.e. a scientific model) may be proposed for the representation of a physical system. Thus I explicate the dependence of approximation on idealisation on pragmatic grounds. In other words, although both concepts are epistemic in nature scientific methodology requires that a process of de-idealisation takes place before we can usefully and meaningfully employ the notion of approximation. Hence idealisation is a primary process in our scientific methodology and approximation piggybacks on it. Thus, if science education could accommodate this result in the analysis of the theory/experiment relation then an elucidation of the latter for the science student could be better achieved. But before I proceed with an analysis of this argument let me try to clarify what exact use of the notion of approximation (out of the plethora of different uses) is, in my view, clearly distinct from idealisation and whose relation to the latter is important for explicating the theory/experiment relation.

3. Linking Approximation to Idealisation

Although it is not the purpose of this paper to offer a theory of approximation that would clarify the notion, some aspects of the use of the concept are worth clarifying since my concern here is the use of the concept in illuminating the theory/experiment relation and hence it is crucial that its relation with idealisation is understood.

The ambiguity present in the statement that our theories approximate the world partly has to do with what the statement refers to and at what level of discourse it is used. Sometimes the notion of approximation is used at a level of discourse that we could call 'meta-meta-scientific'. Such is the case when it is claimed that the idealised description of the simple harmonic oscillator approximates the motion of the torsion pendulum, or more generally when it is claimed that *idealised descriptions* of physical systems *approximate* their actual target physical systems. In philosophical discussions of idealisation, this use of the notion of approximation is in a sense present in the belief that good idealisations are distinguished from bad ones if their claims better approximate the world. In such uses, the reference of approximation is either to the degree or to the kind of idealisation and not to the actual relation between the theoretical and experimental statements. A meta-meta-scientific use of the concept is employed to *qualify* characterisations of how theoretical statements relate to experiment hence it must be discerned from its meta-scientific use. Meta-scientific use means that approximation itself *is* a characterisation of how theoretical statements relate to experiment. The focus in this paper is to the latter use of approximation, which I believe to be the epistemically important use of the concept. At the meta-scientific level of discourse we could make either of two kinds of approximation claims. We could claim that X approximates Y, when X and Y describe properties and processes and the descriptions of X closely resemble those of Y. We could also claim that X approximates Y when X and Y are real-valued functions and the value of X is close to the value of Y for particular values of their arguments. Or we could claim that approximation refers to a combination of both of the above.

The first kind of approximation claim, which for instance could be understood to mean what Giere (1988) calls 'similarity in respects and degrees', is indistinct from an idealisation claim of either of the two modes mentioned in Section 2, i.e. if the properties and relations described in X closely resemble (i.e. approximate in this particular sense) those of Y, then X is an idealised description of the properties and processes of Y, either because relevant features of Y have been abstracted in X or because the characteristics of relevant features of Y have been distorted in X. The identification of approximation in this sense with idealisation is warranted because when we identify approximation with closeness of resemblance of the properties and processes in two descriptions, it is either because some of the characteristics of Y are absent from X or because some of the characteristics of Y have been changed or distorted in X or because of both reasons. In this sense approximation is either used as a surrogate to idealisation (and coincides with the latter's meaning) and adds nothing more to the content of the characterisation of the relation between the statements X and Y that idealisation would not, or it is a concept that can be broken down and analysed in terms of more primitive concepts such as idealisation. Because approximation in this sense is understood as being proportional, so to speak, to the number of features that have been abstracted or distorted in the theoretical description, often one is led to the view that a description X approximates Y better than Z does only if it is less idealised than Z; but this way of linking approximation to idealisation is unnecessary since it does not add anything instructive to the relation between the two concepts because in this sense all approximations may be understood to be specific forms of idealisation (i.e. they obey all the logical properties and methodological characteristics of idealisation, pointed out in Section 2 and not those attributed to approximation).

The second kind of approximation claim, which highlights the mathematical use of the concept, presents a more complicated problem. Clearly this kind of approximation claim is distinct from the notion of idealisation and not easy to relate to the latter. Because it is a direct consequence of representing theoretical descriptions in mathematical languages, approximation in this sense seems to be a concept that could be explicated exclusively by mathematical considerations. Indeed some philosophers of science (e.g. Laymon 1980, 1985, 1987), guided by the close links between idealisation and approximation, have attempted to explicate idealisation primarily in terms of mathematical considerations. Such attempts, however, fail to achieve a full explication of the process of idealisation in science because as a conceptual process idealisation is not a characteristic restricted to mathematical languages alone.

I shall herein concentrate on yet another problem with the view that mathematical considerations alone can help clarify the theory/experiment relation. If we do understand the statement that 'our theories approximate the world' as referring to a relation that can be explicated exclusively by mathematical considerations then we are faced with the following two possible points of view of approximation pointed out by Redhead (1980). The first is approximate solutions to exact equations. Consider his example: 'for the equation $dy/dx - \lambda y = 0$ we might expand our solution as a perturbation series in λ, the nth order approximation being just $y_n = 1 + \lambda x + \lambda^2 x^2/2! + \cdots + \lambda^{n-1} x^{n-1}/(n-1)!$, if we consider the boundary condition $y = 1$ at $x = 0$' (Redhead 1980, p. 150).

The second view of approximation that Redhead calls to mind is when we look for exact solutions to approximate or simplified equations. In the example above, y_n is an exact solution to the equation, $\mathrm{d}y/\mathrm{d}x - \lambda y + \lambda^n x^{n-1}/(n-1)! = 0$, which for small λ is approximately the same as the original equation above. It is easy to prove, as Redhead points out, that the two views are equivalent since, '...if we consider an approximate solution y_n for an exact [equation] ...we can always specify [another equation] ...which is 'approximately' the same as the first, for which y_n is an exact solution'. (Ibid, p. 150) Now, the number of logically possible approximate solutions to an exact equation is infinite and each of these is an exact solution to another equation, and each of these equations is an approximate (or simplified) version of the initial exact equation. Thus by viewing approximation only in a mathematical sense we run into the problem that different equally plausible approximating equations that purportedly represent the same target physical system will yield somewhat different solutions that will not be experimentally distinguishable. Hence it follows that we would have no non-arbitrary way of singling out one solution that approximates the data and a corresponding equation that represents the target physical system if we focus only on mathematical considerations.

There are various ways to see the consequences of this problem. One simple way, for instance, would be to suppose that we have a choice between two theoretical constructs (two distinct models) that are meant to represent a particular physical system. For the sake of the argument, let us suppose that the first model predicts a value for a physical quantity equal to a and the second predicts a value equal to b. Now suppose that the measurement of this quantity is such that it approximates both predictions without distinguishing between them (e.g. $(a+b)/2$). Based merely upon the criterion of approximation (understood in mathematical terms) in choosing the correct representation of the physical system means that we cannot choose between the two in a non-arbitrary way. This may seem as a very simple example which we can handle in familiar cases, when we know what factors influence the physical system, and hence we are tempted to say that we know the best choice for modelling the physical system and had experimental inaccuracies not been present experiment would have also distinguished the two. This response however is not convincing because we can imagine encountering the same problem in modelling a system that we are not familiar with, in which case we are not familiar with the factors that influence the physical quantity in question. With this in mind we are led to the conclusion that approximation of the experimental value by the theoretical prediction is not a sufficient condition for regarding the theoretical construct (e.g. a model) a *representation* of the target physical system (or if we are to adapt a realist jargon: that approximation of measurement by prediction is not sufficient condition for proximity to truth), but also neither is it the only condition by which scientists go about in choosing their theoretical representations.

This problem is about the epistemic value of our theoretical representations. If we rely entirely upon mathematical approximation of experiment by theory then we cannot explicate the theory/experiment relation and neither can we justify why our criterion of choice of our model's representational capacity is non-arbitrary. The way around this, I suggest, is to link approximation (understood in the mathematical sense explained here) to the process of idealisation on pragmatic and methodological grounds so that non-mathematical considerations also become part of our explication of the theory/experiment relation.

4. The Interplay between Idealisation and Approximation in Modelling the Pendulum

The process of idealisation enters at different levels of scientific theorising. Two principal levels could be identified that are useful to our understanding of how theories are formulated and applied. Assuming that we begin with the universe of discourse, the first level of idealisation that could be distinguished is that of selecting a small number of variables and parameters and using them to characterise the general laws of a theory. For example, in classical mechanics *position* and *momentum* are selected and used to establish a relation which we call Newton's 2nd law or Hamilton's equations. By abstracting a set of parameters we thus create a sub-domain of the universe of discourse in which the scope of the theory is confined and which we call the domain of a scientific theory. Thus, Newton's laws signify a conceptual object of study that we may call the domain of classical mechanics; similarly the Schrödinger equation signifies the domain of quantum theory, and so forth. Scientific domains, viewed from this perspective, are clearly distinct from physical domains, which they could represent only if they are expanded by or integrated with other conceptual resources. For instance, the dynamics of bodies may be influenced by factors that are related to electrical or heat phenomena that are not accounted by Newton's laws. In all the laws (which we may call idealised, in the sense that they are established by a small number of abstracted parameters) something is left unspecified: the force function in Newton's 2nd law, and the Hamiltonian operator in the Schrödinger equation. The specification of these is what would establish the link between the assertions of the theory and physical systems. The description I propose of this level of idealisation in scientific theorising is similar, if not identical, to Suppe's version of the Semantic View (Suppe 1989), where he maintains that by abstracting a small number of variables and parameters in order to characterise the general laws of a theory we thereby define a class of mathematical structures or models that may be used for the representation of phenomena.[3] It is apparent that without the specification of force functions or Hamiltonian operators etc., no prediction is made by the assertions of a theory, hence at this level of theorising the goal is not to approximate the world but to describe the common features of otherwise dissimilar physical systems by means of the laws of a theory. Thus the notion of approximation is redundant in understanding this level of theorising.[4]

The second principal level in which the process of idealisation enters in our scientific theorising is the process of specifying force functions or Hamiltonian operators etc., and it is effective in allowing us to bridge the assertions of the theory to physical systems. At this level, the process of idealisation is intertwined with that of approximation and in what follows I shall demonstrate this process and attempt to show the pragmatic nature of the relation between the two concepts by analysing how scientists model the simple pendulum.

Morrison (1999) has argued that in order for an idealised model, such as the simple harmonic oscillator, to accurately represent the respective physical system we cannot rely on theory alone but we must add several correction factors to the model. In order to analyse the process of constructing a representational model of the pendulum by blending a theoretical model with the relevant correction factors, it will be helpful if we work with a distinction between two kinds of model that I shall label the ideal model (model_I) and the concrete model (model_C). Let the class of ideal models be the class of theoretical models (as understood by the proponents of the Semantic View, e.g. Giere 1988; van Fraassen 1980, 1989; Suppe 1974, 1989; da Costa & French 1990, 2003).[5] Let the class of concrete models be the class of those models that are proposed by scientists for the theoretical representation of physical systems. Distinguishing between the class of models_I and that of models_C is not meant to mark a separation between theoretical and *a posteriori* models, but it is a working assumption that we can use to make sense of how the assertions of the theory are brought closer to experimental reports. Model_I is the theoretical model that we initially attempt to fit the physical system into, however its representational capacity is only—to say the most—suggestive. We could regard model_C, on the other hand, as the carrier of all the antecedent knowledge and physical intuitions that direct us to capture in concrete ways the attributes and features of a particular physical system. My thesis is similar to Morrison's (1999), that to turn a model_I into a representation of a physical system we must blend it with conceptual resources that extend beyond the conceptual confines of the theory and in the process the result is a distinct entity that

I call a model$_C$. The distinction is therefore not based on mathematical tractability but it is used primarily to emphasise the fact that the conceptual resources of model$_I$ are confined to the theory that gives rise to it, whereas those of model$_C$ extend beyond the theory.

Frequently in classical particle mechanics the initial stages of modelling a physical system involves the employment of one of the available models$_I$. The process by which the model$_I$ is chosen and employed has been analysed by Cartwright (1983) and dubbed as 'theory entry'. Fitting of facts to equations, Cartwright suggests, is a process that involves firstly the preparation of an informal description of the phenomenon such as to '...present the phenomenon in a way that will bring it into the theory' (Cartwright 1983, p. 133). In this informal description we use our background knowledge and try to confine the description to those elements that will allow us to match an equation to the behaviour of the physical system. Secondly, it involves the scrutiny of the description through the prism of the theory in order to dictate the necessary equations, boundary conditions and approximation methods. The process of theory entry is important because it also suggests that in preparing a description of the phenomenon as to 'bring it into the theory' we most frequently distort some features of the phenomenon or abstract others. Theory entry thus opens up the scene for a third stage which is operative in theory–application: the informal descriptions of the phenomena act as guidelines for the corrections that should follow the process of theory entry so that the equations dictated by the theory are corrected in ways that would lead to representations of actual physical systems. This stage leads to the construction of a representational model of the target physical system and thus a relation between theory and experiment is established. The process involves the 'moulding' of the equations of the model$_I$ as to capture as many of the features of the physical system as possible and the result is a model$_C$.

The simple pendulum is of educational and historic importance. In the hands of Galileo it was demonstrated that it could be used as a timekeeper (see Matthews et al. 2004) and that although a simple system it involves extremely rich physics (see Matthews 2004). Until recently the plane pendulum was used for the measurement of the local gravity, and to such use it is put in most student laboratories. But what is of utmost importance to my argument is that the case of the simple pendulum exhibits clearly the methodological elements present in our attempt to relate theory to experiment. To achieve theory entry for the pendulum we begin with a highly idealised description of the phenomenon that would sanction the use of a model$_I$. By assuming a mass-point bob supported by a massless inextensible cord of length l performing infinitesimal oscillations θ about an equilibrium point, the equation of motion of the simple harmonic oscillator (i.e. a model$_{\mathrm{I}}$) can be used as the starting point for modelling a real pendulum and thus attempting to measure the acceleration due to the Earth's gravitational field:

$$\ddot{\theta} + (g/l)\theta = 0 \tag{1}$$

The solution of this equation yields a relation among the period T_o, the cord length l and the acceleration g due to the Earth's gravity: $g = 4\pi^2 l/T_o^2$.

The experimental problem of determining g, therefore comes down to the fundamental experimental problem of measuring l and T_o. The model itself does not result in a numerical prediction, but to what we could call an instruction to an experimental operation: measure the cord length and the period of oscillation of the pendulum and you can compute the acceleration due to gravity. But the idealised assumptions underlying the model equation (1) above, do not describe how the apparatus is in the world but they dictate a theoretical (and at this stage, *ideal*) description of the apparatus. Hence it is obvious to physicists that if a reasonably accurate measurement is demanded, the theoretical description of the experimental apparatus must be actualisable in the real world. And if higher accuracy is demanded of a measurement, the theoretical description of the apparatus must become more realistic. Hence in the scientific attempt to estimate local gravity a simple measurement of l and T_o in any actual apparatus is known not to suffice and physicists would first proceed to achieve consistency between the theoretical description of the apparatus and the characteristics of the actual apparatus before such a measurement is used for the computation of g.

This is not something peculiar to the pendulum but it is expected in the majority of cases of modelling physical systems. The reasons are clear in the pendulum example. It is known that the actual pendulum apparatus is subject to influences that are not accounted for in the idealised assumptions underlying equation (1). That is to say, the model$_{\mathrm{I}}$, expressed through equation (1), involves many abstractions and idealisations that minimise its representational capacity. In fact I encourage an even stronger claim: that the model$_{\mathrm{I}}$ does not refer to the class of actual pendulums but to a class of ideal types that cannot be actualised, i.e. the class of mass-point bobs supported by a massless inextensible cord performing infinitesimal oscillations about an equilibrium point. Hence to claim that equation (1) describes approximately the motion of the pendulum is to commit an error in the reference of the model, since it does not refer to the actual pendulum apparatus but to a class of ideal-types that may resemble in some respects the characteristics of actual pendulums.

Moreover, if we were to claim that equation (1) describes approximately the motion of the pendulum, it would lead us to the problem of non-arbitrary criterion for choice, explained earlier. That is, the solution $g = 4\pi^2 l/T_o^2$ of equation (1) is experimentally indistinguishable, through experimental measurements of l and T_o, from many other logically possible solutions of other equations of motion that are equally good approximations of g. Hence the relation of approximation on its own does not offer a non-arbitrary criterion of choosing the simple harmonic oscillator as the correct representation of the pendulum. But we must also recognise that the reason physicists expect that the relevant implications of the model$_I$ about an ideal apparatus and the actual experimental apparatus differ significantly is because they know that a large number of important influencing factors are not included in the theoretical description. My contention is that when the degree of idealisation is high such that the theoretical construct refers to a class of ideal-types, the concept of approximation cannot be employed in any scientifically instructive way; hence we must search elsewhere in order to illuminate the theory/experiment relation.[6]

In their attempt to construct a representational model of the pendulum, Nelson & Olsson (1986) give the following list of influencing factors, that model$_I$ does not account for: (i) finite amplitude, (ii) finite radius of bob, (iii) mass of ring, (iv) mass of cap, (v) mass of cap screw, (vi) mass of wire, (vii) flexibility of wire, (viii) rotation of bob, (ix) double pendulum, (x) buoyancy, (xi) linear damping, (xii) quadratic damping, (xiii) decay of finite amplitude, (xiv) added mass, (xv) stretching of wire, (xvi) motion of support. They proceed to show how the value T_o can be corrected by introducing the different correction factors into the equation of motion. In effect, they are attempting to show what is involved and how it is involved in the construction of a model$_C$ that can be used for the theoretical representation of the actual pendulum apparatus. Consider some of the examples analysed by Nelson and Olsson (1986):

(1) Since the pendulum experiment takes place in air, it is expected that by Archimedes' principle the weight of the bob will be reduced by the weight of the displaced air. Since under such circumstances the effective gravity is reduced, this increases the period. The correction factor is determined by accounting for the mass of the air displaced.
(2) The air resistance acts on the oscillating system (pendulum bob and wire) to cause the amplitude to decrease with time and to increase the period. The Reynolds number for each component of the system determines the law of force for that component. The drag force is hence expressed in terms of a dimensionless drag coefficient, which is a function of the Reynolds number. In the pendulum case it can be shown that a quadratic force law should apply for the pendulum bob, whereas a linear force law should apply for the pendulum wire. Hence, it makes sense to establish a damping force which is a combination of linear and quadratic velocity terms: $F = b|v| + cv^2$. To determine the physical damping constants b and c the work-energy theorem is employed, an appropriate velocity function $v = f(\theta_o,t)$ is assumed, and under the assumption of conservation

of energy they are matched to experimental results. They proceed to solve the resulting equation of motion and determine the correction factors.

(3) A real pendulum has a bob of finite size, a suspension wire of finite mass and in addition the wire connections to the bob and the support have structure. All these factors have some contribution to the oscillations. Their effects are incorporated into the physical pendulum equation: $T = 2\pi\sqrt{I/Mgh}$. Where, I is the total moment of inertia about the axis of rotation, M is the total mass and h is the distance between the axis and the centre of mass. Depending on the shape of the bob we could calculate its moment of inertia and thus compute its contribution to the period of oscillation. Nelson & Olsson (1986) assume that the bob is a perfect sphere of radius a and proceed to compute a correction to the period. In a similar manner the correction contributions due to the wire connections and the mass and flexibility of the wire are computed.

(4) The length of the pendulum is increased by stretching of the wire due to the weight of the bob. By Hooke's law, when the pendulum is suspended in a static position the increase is $\Delta l = mgl_o/ES$, where S is the cross-sectional area and E is the elastic modulus. The dynamic stretching when the pendulum is oscillating is due to the apparent centrifugal and Coriolis forces acting on the bob during the motion. This feature is modelled by analogy with the spring-pendulum system to the near stiff limit. The result is a system of coupled equations of motion, which when solved yields the correction factor for the period.

These examples indicate a number of complexities involved in the process of constructing model$_C$. The root of these could be traced in the attempt to relax or overcome the underlying idealisations and abstractions of model$_I$. This could be put in the language of physicists: when the goal is to model a physical system then the initial problem of starting with a law of force (i.e. Newton's 2nd law) and using it to find a model$_I$ for the description of the physical system does not suffice. In this quest, the general problem of finding each law of force that may be responsible for a particular constituent of the external force function in Newton's law, and which would reduce the degree of idealisation, is of equal importance. In order to determine the various force laws to be used in model$_C$ we utilise either the antecedently available empirical laws (such as Archimedes' principle, the Reynolds number and the drag force expression, and Hooke's law, for the case of the pendulum) or postulate novel physical mechanisms (as is often the case in applications of Quantum Mechanics). By employing the various force laws in the construction of the model$_C$ we are turning the model into a representation of the respective physical system because when these correction factors are added, the reference of the model is no longer a class of ideal-types but a class of actualisable systems. To recognize that a model$_C$ approximates the physical system is not only reasonable but also scientifically useful because all the correction factors that are constituent parts of model$_C$ are approximations to particular aspects of the target physical system. In other words, the empirical laws and experimental parameters used in the description of each of these influencing factors are approximations of the characteristics of the actual physical system.

The construction procedure of model$_C$ is conventional and not peculiar to the pendulum. The mathematical expressions representing each influencing factor are determined by the use of various empirical laws from disparate areas of physics and are inserted into the equation of motion in a cumulative manner. Because the influence of each of these factors on the system is small, it is assumed that the resulting equation of motion approximates a system of linearly independent differential equations, each involving a different influencing factor. Each of the equations is solved individually to determine the values of the individual effects and the total value of the correction is computed by adding all the effects linearly (see Nelson & Olsson 1986). The methodological process we are faced with is the blending of experimental parameters and empirically determined laws together with a theoretical model (model$_I$) to produce a model$_C$. The theoretical model is a pure derivative of the theory that we can turn into a *representation* of a physical system by blending it with these ingredients. This is done in an effort to extend the scope of application of the theory beyond the class of ideal-type systems (e.g. isolated point-masses and inelastic cords) to which the class of models$_I$ may be understood to refer. To achieve this we give a concrete and specific context to the force function (i.e. to the abstract concept of 'force') for each and every different influencing factor. It is important to note that de-idealisation is the process by which the model is turned into a representation of the physical system. Approximation is the process by which the equation of the model$_C$ is made tractable. Both processes are in constant interplay in trying to turn a model$_I$ into a model$_C$ but the epistemic significance of approximation depends upon the degree of de-idealisation achieved and this is the pragmatic aspect of the relation between the two concepts.

In the process of constructing model$_C$ above, the primary concern is to discover those correction factors that would bridge the gap between model$_I$ and the target physical system, at this stage only the de-idealisation process is operative. The two modes of the approximation process enter into the picture once the de-idealisation process begins. Each de-idealising step involves an approximation and the entire de-idealisation of model$_I$ also involves an approximation technique that would make the model equation tractable. When each correction factor, and the force law responsible for its behaviour, is discovered it is approximated by a mathematical expression that gives rise to a tractable equation of motion. This part of the process is an example of the first mode of approximation (discussed in Section 2) which clearly piggybacks on the de-idealisation process. Once all the correction factors are introduced into the equation of motion and the model$_C$ is constructed, the second mode of approximation is used. The assumption that the effects of all correction factors are small hence we could approximate the equation of motion with a system of linearly independent tractable equations also piggybacks on the de-idealisation process, in the sense that the de-idealising assumptions dictate the approximation technique to be used.

In modelling physical systems the starting point is often an idealised model, such as the harmonic oscillator, whose force function could be expressed through a general functional relation: $H(x) = f_1(g_1(x), \ldots, g_n(x))$. A first-step de-idealisation would be to expand the functional relation by accounting for an influencing factor that has been initially ignored. This results in a new general functional relation which in its most simplified logical form could be presented as follows: $H'(x) = f_1(g_1(x), \ldots, g_n(x)) + f_2(h_1(x), \ \ldots, h_m(x))$. Supplementing the function $H(x)$ with cumulative correction factors is a process that goes on until our conceptual resources and background knowledge are exhausted. The idealised model can be understood to relate to its derivative de-idealised relatives in the following general way: $\lim_{f_2 \to 0} (H'(x) = f_1(g_1(x), \ldots, g_n(x)) + f_2(h_1(x), \ldots, h_m(x))) = H(x)$. In other words, on this account 'idealisation' is the process by which we let factors that are influential to the physical system tend to zero. De-idealisation is the converse process of allowing these factors to take finite values. That is, idealisation ignores the influence of factors and de-idealisation reintroduces their effect into the model. Approximation enters into this picture because each f_i is represented via the mathematical language of the theory in an approximate way and because the final H^k is solved by an appropriate approximation technique.

This process could be misconceived to only mean that model$_{\mathrm{I}}$ is a description with some unspecified parameters and model$_{\mathrm{C}}$ is a description with those parameters specified, thus the latter is a structure-type nested in the former. In other words, by specifying parameters we effectively create a sequence of nested mathematical structures. The idealisation/de-idealisation process viewed from this perspective is no more than a partial ordering of structures. The criterion (i.e. relation) of this partial ordering is that of the restriction of the domain, i.e. two models, M_1 and M_2, are partially ordered if and only if the domain of M_2 is a restriction of the domain of M_1. We could think of the criterion for partial ordering as a transformation rule that requires the specification (or addition) of a parameter in the above functional relation. In this picture the reference of the sequence of models remains the same type of system, i.e. the reference of model$_{\mathrm{C}}$ would not be a different type of system from that of model$_{\mathrm{I}}$, other than the restriction of the domain. Hence in this understanding of idealisation the use of approximation is meaningful at any level. However, this view of idealisation involves a misconception, because by specifying a parameter we are not simply correcting our mathematical description $H(x)$ but we are bringing our theory in touch with the world, i.e. f_1 derives from the theory alone but f_2 derives from empirical laws and experimental parameters. So de-idealisation is not just a process by which we paste together different descriptions to create a more complex final description, but it is the process by which we supply a theoretical description, that refers to a class of ideal-type systems, with those conceptual ingredients that would make it refer to actualisable physical systems (i.e. in the words I have chosen to present it in this paper, it is the process of turning a model$_\mathrm{I}$ into a model$_\mathrm{C}$). If idealisation were understood in the former way then approximation would be suitable at every level. But if idealisation is understood as I suggest then approximation is useful only in the process of turning a model$_\mathrm{I}$ into a model$_\mathrm{C}$. This is what I mean when I claim that the pragmatics of approximation depend upon idealisation. Unless a model$_\mathrm{C}$ is established by means of de-idealising techniques and hence a plausible representation of a target physical system is constructed we cannot employ the notion of approximation without obscuring the theory/experiment relation.

5. Conclusion

The idea that theories do not represent the concrete circumstances in which naturally occurring physical systems are found was pointed out by several authors (e.g. Cartwright 1983, 1999; Shapere 1984; McMullin 1985; Laymon 1985; Suppe 1989; Morrison 1998, 1999; Matthews 2004; Nola 2004). Among them proponents of the Semantic View like Suppe (1989) understand well that theoretical models are abstract and idealised descriptions and as such, it could be claimed, they do not represent physical systems in any direct sense. My argument leads to the contention that it is only after they give rise to a model$_\mathrm{C}$, appropriate for the representation of a particular physical system, that they acquire a certain capacity of representation. We say that the linear harmonic oscillator approximately represents the pendulum system, only because we have managed to use it successfully to construct a model$_\mathrm{C}$. We would not claim that all conceivable theoretical models are representations of physical systems. But what is more important, in the context of my discussion, is that a model$_\mathrm{C}$ is a representation of a target physical system that involves a theory-derived description blended with empirical laws and other auxiliaries, and this is why it makes sense to call it an approximation of the corresponding physical system. Theoretical models refer to a class of ideal-types whose empirical content is supplied when they are used in the construction of a model$_\mathrm{C}$ by de-idealising them, thus changing the reference to actualisable physical systems and thus meaningfully employing the notion of approximation.

Given the picture of scientific modelling that I have drawn and considering the arguments that I have given of the relation between the concepts of idealisation and approximation, not only do I hold that a modelling view of science is better founded than a pure inductive view, but I believe that science education can also benefit from these results. By giving the necessary weight to the processes of idealisation and approximation in model building, the student can come to terms with the complexities of the theory/experiment relation and thus achieve a better understanding of the nature of science. Comprehending the entanglement of theoretical principles and empirical results, that is so evidently present in representational models, is an essential part of learning science and I have tried to accentuate this in my argument. But most importantly by having a science course organised around the analysis and the employment of a certain number of crucial scientific models, e.g. the simple harmonic oscillator, allows the student to develop an understanding of phenomena through the creative employment of the interpretative framework of the models.

In this paper it was clear throughout, although not stated explicitly, that a model building instructional strategy is not just essential for learning how to handle a particular scientific theory but also for understanding the nature of science. I have implied throughout that models are not just devices for problem solving, but are the conceptual means by which we can make sense of the theory/experiment relation, hence they are devices by which we can learn science and by which we can learn about science. In the study conducted by Abd-El-Khalick & Lederman (2000) the tentative conclusion is that 'explicit' approaches to improving science teachers' conceptions of the nature of science were more successful than 'implicit' approaches. Explicit approaches are those in which instruction of the subject matter is geared towards particular aspects of the nature of science. Focussing on model construction processes, such as the one presented above, such an explicit approach is achieved that allows the instructor to avoid digressions from the subject matter of a science course and at the same time to address questions concerning the nature of science. It is obvious that if it were the goal of this paper to give a theory of instruction of science then I would have to focus on the question of '*how* the essentials of the subject matter of a scientific course should be effectively taught'. I have herein addressed the question of '*what* essentials of the subject matter of a scientific course must be taught' and have given an argument to explicate and substantiate my claim that scientific models and the processes operative in their construction are essential to science learning and to understanding how theory relates to experiment.

Notes

[1] McMullin (1985) also calls something like the first mode 'formal idealisation' and something like the second mode 'material idealisation', both of which he places under the more general category of 'construct idealisation', where the latter is distinguished from 'causal idealisation'.

[2] Of course to attribute the logical relation of transitivity to idealisation we must look at the process of idealisation (and abstraction) as applied in scientific theorising and in particular in scientific model construction. It is obvious that we can use idealisation in other contexts where transitivity is not present. For instance, a stone on the ground could be used as an idealisation of a group of soldiers in a battle field, and in another context a list of names on a paper can be used as an idealisation of the same group of soldiers. But although the list of names and the stone may be idealised representations of the same group of soldiers, between them they do not have to relate in any interesting way. In scientific modelling, however, when I speak of transitivity of the idealisation relation, I am referring to the relation of two or more models that purportedly represent the same physical system, which are constructed with the use of the same scientific language and along the same chain of heuristics.

[3] Elsewhere (Portides forthcoming) I have disputed the contention that such models of the theory could in fact be used for the representation of phenomena without being integrated with conceptual resources that transcend the theory's apparatus, but in this paper my concern is different and I shall avoid this issue.

[4] A concern in Science and Philosophy of Science is the question whether two competing theories e.g. the Special Theory of Relativity and Newtonian Mechanics, stand in the relation of approximation to each other. Whether it is reasonable to view Newtonian Mechanics as an approximation to Relativity theory at some limit is an issue that concerns the relation between two mathematical calculi and their interpretation and not the theory/experiment relation in any direct sense. The intertheoretic use of approximation is an issue which of course enters at this level of theorising but does not concern the explication of the theory/experiment relation. In this paper I explore the notion of approximation as a relation between theory and experiment and not as an intertheoretic relation.

[5] According to the Semantic View, the class of *theoretical models* is a class of mathematical structures that could be defined by the laws of the theory. E.g. in classical mechanics by means of the *position* and *momentum* vectors we establish a relation: Newton's 2nd law. The specification of any force function would define a theoretical model. For instance, if the force function is specified as $F=-k\xi$ (for a position coordinate ξ and constant parameter k), then the 2nd law defines such a model (known as the linear harmonic oscillator) that is expressed by the equation of motion: $\xi'' + (k/m)\xi = 0$. If the force function is specified as $F = -k\xi + b\xi'$, then the 2nd law defines another such model (known as the damped harmonic oscillator) expressed through the equation of motion: $\xi'' - (b/m)\xi' + (k/m)\xi = 0$, and so on. The mathematical structure of the theory, defined by the position and momentum vectors related through Newton's 2nd law, thus lays down an indefinite number of possible theoretical models which are available, according to the Semantic View, for representing mechanical systems. Notice that, in my argument above, I draw a distinction between theoretical models and representational models and upon it part of my argument rests.

[6] This is one way to understand why when it is not possible to determine scalable de-idealised versions of a model$_{\mathrm{I}}$ physicists employ perturbation theory (particularly in Quantum Mechanics), which is roughly a way to represent the aggregate effect of the different factors that influence the system. In other words perturbation theory is a way to de-idealise and

approximate simultaneously by representing an aggregate effect rather than the individual effects of each influencing factor. But when perturbation theory is employed physicists are not suggesting that the highly idealised model$_I$ approximates a physical system, but that the model$_I$ supplemented by an approximate representation of the aggregate effect of influencing factors approximates the physical system. In other words by adding the perturbation term the reference of the model in assumed to have changed.

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