DOI 10.1007/s11182-021-02438-8
Russian Physics Journal, Vol. 64, No. 6, October, 2021 (Russian Original No. 6, June, 2021)

VOLUME OPERATOR

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UDC 530.24

Keywords: size, weight, particles, strings, prequarks, braids, fundamental particles, operators, volume.

Paul Dirac derived the equation of motion for heavy fermion materials: $\left(\gamma^{\mu}\partial_{\mu} + \frac{mc}{\hbar}\right)\Psi = 0$. Well-known is also the Weyl equation $\left(\gamma^{\mu}\partial_{\mu}\right)(1 \pm \gamma^{5})\Phi = 0$, a relativistic wave equation for describing massless spin-1/2 particles called Weyl fermions. The Weyl equation is based on the Pauli σ-matrices, *viz*. $\sigma^{\mu}\partial_{\mu}\Phi = 0$, where $\sigma^{\mu} = \left\{\pm I, \sigma^{1}, \sigma^{2}, \sigma^{3}\right\}$ or $\left(\pm I\,\partial_{0} + \sigma^{1}\partial_{1} + \sigma^{2}\partial_{2} + \sigma^{3}\partial_{3}\right)\Phi = 0$. It is believed that in nature, only one equation for a two-component neutrino can be realized, namely the equation with negative *I*.

By analogy with the Pauli σ-matrices, four equations can be developed:

$$\left(\pm I\,\partial_{0} + \sigma^{1}\partial_{1} + \sigma^{2}\partial_{2} + \sigma^{3} mc/\hbar\right)\Phi_{3} = 0,\ \left(\pm I\,\partial_{0} + \sigma^{1}\partial_{1} + \sigma^{2} mc/\hbar + \sigma^{3}\partial_{3}\right)\Phi_{2} = 0,$$

$$\left(\pm I\,\partial_{0} + \sigma^{1} mc/\hbar + \sigma^{2}\partial_{2} + \sigma^{3}\partial_{3}\right)\Phi_{1} = 0,\ \left(\pm I\, mc/\hbar + \sigma^{1}\partial_{1} + \sigma^{2}\partial_{2} + \sigma^{3}\partial_{3}\right)\Phi_{0} = 0.$$

These equations, however, disturb the relativistic invariance.

Let us consider the Dirac equation $\left(\gamma^{\mu}\partial_{\mu} + \frac{mc}{\hbar}\right)\Psi = 0$ supplemented by the relativistically invariant relation $\left(n_{3}^{\mu}\partial_{\mu}\right)\Psi = 0$, where $n_{3}^{\mu} = (0,0,0,1)$. In this case, in the reference configuration, in which $n_{3}^{\mu} = (0,0,0,1)$, we get $\left(\gamma^{\mu}\partial_{\mu} + \frac{mc}{\hbar}\right)(1 \pm \gamma^{5}\gamma^{3})\Phi_{3} = 0$ and $\left(n_{3}^{\mu}\partial_{\mu}\right)\Phi_{3} = 0$ at $n_{3}^{\mu} = (0,0,0,1)$. The Dirac equation is both covariant and invariant relative to the rotation in four-dimensional space, whereas the given equation is covariant only, similar to the equation of plane $\left(\overline{r} \cdot \overline{n}\right) = a$, and the circle equation $\left(\overline{r} \cdot \overline{r}\right) = a^{2}$ is invariant relative to the rotation in three-dimensional space [1]. At the same time, $\delta_{\mu\nu}\left(n_{\tau}^{\mu} \cdot n_{\rho}^{\nu}\right) = \delta_{\tau\rho}$, i.e., the vectors are orthogonal, which is invariant relative to the rotations.

It was supposed to develop the volume $\varepsilon_{\mu\nu\tau\rho}\, n_{1}^{\mu}\, n_{2}^{\nu}\, n_{3}^{\tau}\, n_{0}^{\rho} = \pm 1$ using four orthogonal vectors, which would be invariant relative to the rotations in four-dimensional space. But discovered neutrino oscillations and neutrino masses set aside the theory of two-component neutrino.

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1. ELECTRODYNAMIC ANALOGY

Any vector field can be represented as $\overline{C} = \overline{A} + \overline{B}$, where $\overline{A}$ is the curl-free or potential field, $\overline{B}$ is the solenoidal field. Therefore,

$$\operatorname{div}\overline{A} = f(\overline{r}) \quad \equiv \quad \left(\overline{\nabla}\cdot\overline{A}\right) = f(\overline{r}), \tag{1}$$

$$\operatorname{rot}\overline{A} = 0 \quad \equiv \quad \left[\overline{\nabla}\times\overline{A}\right] = 0, \tag{2}$$

$$\operatorname{div}\overline{B} = 0 \quad \equiv \quad \left(\overline{\nabla}\cdot\overline{B}\right) = 0, \tag{3}$$

$$\operatorname{rot}\overline{B} = \overline{\omega}(\overline{r}) \quad \equiv \quad \left[\overline{\nabla}\times\overline{B}\right] = \overline{\omega}(\overline{r}). \tag{4}$$

From Eq. (2) we derive: $\left[\overline{\nabla}\times\overline{A}\right] = 0 \;\Rightarrow\; \overline{A} = \overline{\nabla}\,\varphi(\overline{r})$, because $\left[\overline{\nabla}\times\overline{\nabla}\,\varphi(\overline{r})\right] \equiv 0$. From here we get $\left(\overline{\nabla}\cdot\overline{A}\right) = f(\overline{r}) \;\Rightarrow\; \left(\overline{\nabla}\cdot\overline{\nabla}\varphi(\overline{r})\right) = f(\overline{r}) \;\Rightarrow\; \Delta\varphi(\overline{r}) = f(\overline{r})$.

From Eq. (3) we derive:

$$\left(\overline{\nabla}\cdot\overline{B}\right) = 0 \;\Rightarrow\; \overline{B} = \left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right],$$

because $\left(\overline{\nabla}\left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right]\right) \equiv 0$. Therefore,

$$\left[\overline{\nabla}\times\overline{B}\right] = \overline{\omega}(\overline{r}) \;\Rightarrow\; \left[\overline{\nabla}\left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right]\right] = \overline{\omega}(\overline{r}).$$

However,

$$\left[\overline{\nabla}\left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right]\right] = \overline{\omega}(\overline{r}) \equiv \overline{\nabla}\left(\overline{\nabla}\cdot\overline{\phi}(\overline{r})\right) - \left(\overline{\nabla}\,\overline{\nabla}\right)\overline{\phi}(\overline{r}) = \overline{\omega}(\overline{r}).$$

Assuming that $\left(\overline{\nabla}\cdot\overline{\phi}(\overline{r})\right) = 0$, we get $\Delta\overline{\phi}(\overline{r}) = \overline{\omega}(\overline{r})$. It is possible to assume that $\left(\overline{\nabla}\cdot\overline{\phi}(\overline{r})\right) = 0$, since $\overline{B} = \left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right]$ is set to the accuracy of $\overline{\phi}(\overline{r}) = \overline{\phi}^*(\overline{r}) + \overline{\nabla}\zeta$, where ζ is the arbitrary function [2].

Let us consider the Eq. (3). The absence of the magnetic charge is equivalent to the possibility of representing $\overline{B}$ as $\overline{B} = \left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right]$, and, consequently, the identical equality to zero of the $\left(\overline{\nabla}\left[\overline{\nabla}\times\overline{\phi}(\overline{r})\right]\right) \equiv 0$ volume developed on the vector potential $\overline{\phi}(\overline{r})$ and two identical $\overline{\nabla}$ operator vectors. Therefore, this volume operator automatically equals zero, which is proven by the properties of the triple scalar product

2. VOLUME OPERATOR

1. Let us consider the expression $\varepsilon_{ijkp} D_i$, where $D_i = \partial_i - i\, g\, t^a\, W_i^a$. The index *i* equals 0, 1, 2, 3. As a result, we have the relativistically invariant representation of all the four components of the vector D_i.

2. Let us consider the expression $\varepsilon_{ijkp} D_i D_j$. After definite manipulations we obtain $\varepsilon_{ijkp}\left[D_i D_j\right]$, where $\left[D_i D_j\right] = D_i D_j - D_j D_i$. Since $D_\mu = \partial_\mu - i g t^a W_\mu^a$, we get $\left[D_i D_j\right] \to t^a F_{ij}^a = t^a \left(\partial_i W_j^a - \partial_j W_{i\mu}^a + g f^{abp} W_i^b W_j^p\right)$. As a result, we have $\varepsilon_{ijkp} t^b F_{ij}^b = \varepsilon_{ijkp} t^b F_{ij}^b$, i.e., the gauge field tensor configuration.

3. Let us consider the volume operator $\varepsilon_{ijkp} D_i D_j D_k$ and rearrange *i* and *j*: $\varepsilon_{jikp} D_j D_i D_k = -\varepsilon_{ijkp} D_j D_i D_k$ and then sum up. We obtain the volume operator, but with a cyclic rearrangement of *i*, *j*, *k* without the items with the negative sign in front of the Levi-Civita symbol, *viz.* $\varepsilon_{ijkp} D_i D_j D_k - \varepsilon_{ijkp} D_j D_i D_k = \varepsilon_{ijkp}\left[D_i D_j\right] D_k$.

It follows from $D_\mu = \partial_\mu - i g t^a W_\mu^a$ that $\left[D_i D_j\right] \to t^a F_{ij}^a = t^a \left(\partial_i W_j^a - \partial_j W_{i\mu}^a + g f^{abp} W_i^b W_j^p\right)$. We thus obtain the volume operator $\varepsilon_{ijkp} t^b F_{ij}^b D_k = \varepsilon_{ijkp} t^b F_{ij}^b \left(\partial_k - i g t^a W_k^a\right)$, which is expressed through the first-order derivative only.

4. Rearranging the latter two and summing up them with the initial relation we obtain $\varepsilon_{pijk} D_p D_i D_j D_k \to \varepsilon_{pijk} D_p D_i \left[D_j D_k\right] = \varepsilon_{pijk} D_p D_i t^a F_{jk}^a$.

Next, let us rearrange the former two in the obtained relation and sum up them. This results in $\varepsilon_{pijk} D_p D_i t^a F_{jk}^a \to \varepsilon_{pijk}\left[D_p D_i\right] t^a F_{jk}^a = \varepsilon_{pijk} t^b F_{pi}^b t^a F_{jk}^a$. Here one can see the construction from point 2, but configurated from two gauge field tensors.

CONCLUSIONS

According to point 1, in the case of one D_i operator, we obtained the relativistically invariant operator expression in the convolution product with the Levi-Civita symbol. The obtained operator expression affected the function Ψ, thereby creating the equation of motion. The same we had in point 3, but in points 2 and 4 the differential equation was not derived.

It was shown that in the case of the group *U*(1), it made no difference in what an order to unify the covariant derivatives in a tensor. This was finally reduced to the equations given above.

The case of the group *SU*(2) and so on, must be considered. According to point 4, we observed the following invariant equation:

$$\varepsilon_{ijkp} F^{ij} F^{kp} = \text{inv} \equiv {}^*F^{ij} F_{ij} = \text{inv} \Rightarrow \left(\bar{E} \cdot \bar{H}\right)$$

$$\varepsilon_{ijkp} F^{ij} F^{kp} = 4\partial^i \left(\varepsilon_{ijkp} A^j \partial^k A^p\right)$$

It should be noted that the pseudoscalar can be represented in 4-divergence [3].

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