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# ON THE QUANTUM BIRTH OF THE UNIVERSE WITH A BIANCHI TYPE-IX METRIC IN THE PRESENCE OF AN ANISOTROPIC FLUID, A SCALAR FIELD, AND PURE RADIATION

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*Within the framework of the general theory of relativity, a cosmological model with expansion and rotation with a Bianchi type-IX metric is constructed for the scenario in which the gravitation sources are an anisotropic fluid, a pure radiation field, and a scalar field. The Wheeler–De Witt equation is derived, and the possibility of minisuperspace quantization is investigated.*

**Keywords:** first stage of inflation, dark energy, quantum birth of the Universe.

## INTRODUCTION

Ideologically, quantum cosmology is one of the most complex branches of theoretical physics. This is due not only to such difficulties of the quantum theory of gravitation as the problem of the ultraviolet divergences, but also, first of all, to the fact that the very statement of the problem within the framework of quantum cosmology is completely nontrivial. The results of such studies often seem paradoxical, and a great deal of open-mindedness is required so as not to brush them off from the very outset [1].

At the present time, it is customary to assume that our Universe is homogeneous and isotropic. However, there are observational facts [2–5] demonstrating the possibility of deviations from isotropy in the observable Universe. Note that global anisotropy of the Universe can be due to cosmological rotation among other reasons. By virtue of the absence of a complete quantum theory of gravitation, calculations in quantum cosmology are realized by various approaches. Note should be made in this regard of supersymmetric quantization of gravitationally bound homogeneous systems, which serves as a basis to construct cosmological models with Kantowski–Sachs metrics and various types of Bianchi metrics; semiclassical paths to the elimination of the Big Bang singularity in homogeneous, anisotropic models; and the Wheeler–De Witt equation, which has served up to the present time as a basic working tool in quantum cosmology and is presently being applied both in standard cosmology and in multidimensional cosmology [6]. The wave function of the Universe is represented as $\psi(h_{ij}, \varphi)$, where $h_{ij}$ is a three-dimensional spatial metric and $\varphi$ is the matter field. The Wheeler–De Witt equation is, in essence, a Schrödinger equation in the wave function in the standard case $\partial\Psi/\partial t = 0$. In the present paper, we investigate the possibility of the quantum birth of a model Universe with a Bianchi type-IX metric.

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## THE BIANCHI IX MODEL

This paper considers a cosmological model with a Bianchi type-IX metric of the form

$$ds^2 = \eta_{\alpha\beta}\theta^{\alpha}\theta^{\beta}, \quad \alpha, \beta = \overline{0,3}, \tag{1}$$

where $\eta_{\alpha\beta}$ are elements of a diagonal Lorentzian matrix and $\theta^{\alpha}$ are orthonormal 1-forms, expressed as follows:

$$\theta^0 = dt - R\nu_A e^A, \quad \theta^1 = RK_1 e^1, \quad \theta^2 = RK_2 e^2, \quad \theta^3 = RK_3 e^3, \tag{2}$$

$R = R(t)$, $K_A$ and $\nu_A = \text{const}$, $K_A > 0$, and for $A$ = 1, 2, 3, the 1-forms $e^A$ have the form

$$e^1 = \cos y \cos z dx - \sin z dy,$$

$$e^2 = \cos y \sin z dx + \cos z dy, \tag{3}$$

$$e^3 = -\sin y dx + dz.$$

We considered the case corresponding to the conditions

$$\nu_1 \neq 0, \quad \nu_2 = \nu_3 = 0, \quad \nu_1^2 = K_1^2 - K_2^2, \quad K_3 = K_2.$$

We chose the parameters $c = 1$, $\hbar = 1$, and $8\pi G = 1$ all to be equal to 1, where $G$ is the Newtonian gravitational constant. Here, the gravitational sources are an anisotropic fluid which describes rotating dark energy, a pure radiation field, and a scalar field. For a Bianchi type-IX metric, we obtained an inflationary cosmological solution of Einstein's equations. The gravitational sources were an anisotropic fluid, a radiation field, and also a scalar field. The energy-momentum tensor of the anisotropic fluid has the form

$$T_{\alpha\beta}{}^{(1)} = (\pi + \rho)u_{\alpha}u_{\beta} + (\sigma - \pi)\chi_{\alpha}\chi_{\beta} - \pi\eta_{\alpha\beta}, \tag{4}$$

where $u_i u^i = 1$, $\chi_i \chi^i = -1$, $\chi^i u_i = 0$, $k_i k^i = 0$, $\rho > 0$, $w > 0$, $\sigma > \pi$, $\rho$ is the energy density of the fluid, $\pi$ and $\sigma$ are components of the anisotropic pressure, and $\chi^i$ is the anisotropy vector.

The energy-momentum tensor of the pure radiation has the form

$$T_{\alpha\beta}{}^{(2)} = w k_{\alpha} k_{\beta}, \; k_{\alpha} = (k_0, k_0, 0, 0). \tag{5}$$

The energy-momentum tensor of the scalar field is equal to

$$T_{\alpha\beta}{}^{(3)} = \varphi,_{\alpha} \varphi,_{\beta} - \left\{ \frac{1}{2}\varphi,_k \varphi,_l g^{kl} - U(\varphi) \right\} g_{\alpha\beta}. \tag{6}$$

Here, the scalar field satisfies the equation

$$\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g} g^{ik} \phi,_k) + \frac{dU}{d\phi} = 0, \tag{7}$$

and the potential has the Higgs form

$$U=-\frac{m^2\varphi^2}{2}+\frac{\lambda\varphi^4}{4}\,. \tag{8}$$

With the goal of solving the Einstein equations, calculations were carried out with the help of the tetrad formalism [7]. The system of Einstein's equations in tetradic form is given by

$$\begin{aligned}
&G_{00}=-\left(2\frac{R''}{R}+\frac{R'^2}{R^2}\right)\frac{v_1^2}{K_1^2}+3\frac{R'^2}{R^2}+\frac{2K_1^2+K_2^2}{4K_2^4R^2}=\rho+U+k_0^2w+\frac{\varphi'^2}{2}\left(1+\frac{v_1^2}{K_1^2}\right),\\
&G_{11}\equiv-\left(2\frac{R''}{R}+\frac{R'^2}{R^2}\right)+3\frac{R'^2}{R^2}\frac{v_1^2}{K_1^2}+\frac{2K_1^2-3K_2^2}{4K_2^4R^2}=\sigma-U+k_0^2w+\frac{\varphi'^2}{2}\left(1+\frac{v_1^2}{K_1^2}\right),\\
&G_{22}=G_{33}\equiv\left(2\frac{R''}{R}+\frac{R'^2}{R^2}\right)\left(\frac{v_1^2}{K_1^2}-1\right)-\frac{1}{4K_2^2R^2}=\pi-U+\frac{\varphi'^2}{2}\left(1-\frac{v_1^2}{K_1^2}\right),\\
&G_{01}\equiv2\left(-\frac{R''}{R}+\frac{R'^2}{R^2}\right)\frac{v_1}{K_1}+\frac{v_1K_1}{2K_2^4R^2}=k_0^2w+\frac{v_1\varphi'^2}{K_1}.
\end{aligned} \tag{9}$$

We require that, in the spirit of inflation, $\varphi=\varphi_0e^{-Ht}$. Then, the solutions of system of equations (9) are given by the following formulas:

$$\begin{aligned}
&R=R_0e^{\alpha(t)},\\
&\alpha(t)=\tilde{Q}t-\frac{K_1^2\lambda\varphi_0^2}{6H^2K_2^2e^{2Ht}},\\
&\tilde{Q}=\frac{1}{3}H-\frac{K_1^2}{K_2^2}\frac{m^2}{3H},\\
&wk_0^2=\frac{4K_1v_1\lambda\varphi_0^2}{3K_2^2e^{2Ht}}+\frac{K_1v_1}{2R_0^2e^{2\alpha(t)}}-\frac{\varphi_0^2H^2}{e^{2Ht}}\frac{v_1}{K_1},\\
&\pi=-3\left(\tilde{Q}+\frac{K_1^2\lambda\varphi_0^2}{3HK_2^2e^{2Ht}}\right)^2\frac{K_2^2}{K_1^2}+\frac{4\lambda\varphi_0^2}{3e^{2Ht}}-\frac{1}{4R_0^2e^{2\alpha(t)}K_2^2}-\frac{\varphi_0^2H^2}{2e^{2Ht}}\frac{K_2^2}{K_1^2}-\frac{m^2\varphi_0^2}{2e^{2Ht}}+\frac{\lambda\varphi_0^4}{4e^{4Ht}},
\end{aligned} \tag{10}$$

$$\sigma=-3\left(\tilde{Q}+\frac{K_1^2\lambda\varphi_0^2}{3HK_2^2e^{2Ht}}\right)^2\frac{K_2^2}{K_1^2}+\frac{4K_1\lambda\varphi_0^2(K_1-\nu_1)}{3K_2^2e^{2Ht}}+\frac{-K_2^2+2\nu_1^2-2K_1\nu_1}{4R_0^2e^{2\alpha(t)}K_2^4}-\frac{m^2\varphi_0^2}{2e^{2Ht}}$$

$$+\frac{\lambda\varphi_0^4}{4e^{4Ht}}-\frac{\varphi_0^2H^2}{2e^{2Ht}}\frac{(K_1-\nu_1)^2}{K_1^2},$$

$$\rho=3\left(\tilde{Q}+\frac{K_1^2\lambda\varphi_0^2}{3HK_2^2e^{2Ht}}\right)^2\frac{K_2^2}{K_1^2}-\frac{4\nu_1\lambda\varphi_0^2(K_1-\nu_1)}{3K_2^2e^{2Ht}}+\frac{2K_1^2+K_2^2-2K_1\nu_1}{4R_0^2e^{2\alpha(t)}K_2^4}-\frac{m^2\varphi_0^2}{2e^{2Ht}}$$

$$-\frac{\lambda\varphi_0^4}{4e^{4Ht}}-\frac{\varphi_0^2H^2}{2e^{2Ht}}\frac{(K_1-\nu_1)^2}{K_1^2}.$$

The kinematic parameters of the co-moving anisotropic fluid in the given model are as follows. The expansion parameter is given by the formula $\Theta=\frac{3\dot{R}}{R}$, the acceleration is equal to $a=\frac{\dot{R}\nu_1}{RK_1}$, the rotation parameter $\omega=\frac{\nu_1}{2RK_2{}^2}$, and shear is absent. The inflation stage was considered in [8]. Here we posit that the cosmological solution given by Eqs. (10) also corresponds to quantum birth.

## DERIVATION OF THE WHEELER–DE WITT EQUATION

Space-time with the metric prescribed by Eqs. (1)–(3) can be split into space and time according to the standard procedure [9]. Toward this end, the metric can be represented in the form

$$ds^2=-N^2dt^2+g_{ab}(dx^a+N^adt)(dx^b+N^bdt)\,, \tag{11}$$

where $N$ is the lapse function, $N^a$ is the shift vector, the normal basis on hypersurfaces of constant value of the parameter $t$ = const is determined by the triad of tangent vectors $e_a^\alpha$ ($a$ is the reference index and $\alpha$ are the coordinate indices), $e_a^0=0,\ e_a^b=\delta_a^b$ ($a, b=1, 2, 3$). The unit timelike normal vector to the three-dimensional spacelike hypersurface of constant value of the parameter $t$ = const has the form

$$n_\alpha=(-N,0,0,0),\quad \alpha=0,1,2,3\,. \tag{12}$$

As is well known, Ψ, the wave function of the Universe, satisfies the Wheeler–De Witt equation

$$T_\perp\Psi=0 \tag{13}$$

and the supermomentum equations

$$T_a\Psi=0\,. \tag{14}$$

According to [9], the coupling equations can be written thus:

$$T_\perp=-\sigma_0G_{abcd}\pi^{ab}\pi^{cd}-g^{1/2}\cdot{}^3R-2\sigma_0g^{1/2}\cdot T_{\perp\perp}=0\,, \tag{15}$$

$$T_a = -2g_{ac}\pi^{cd}{}_{|d} - 2g^{1/2}\cdot T_{\perp a} = 0\,. \tag{16}$$

Here we denote the scalar of the unit normal as $\sigma_0$, and the De Witt supermetric as

$$G_{abcd} = \frac{1}{2\sqrt{g}}\left(g_{ac}g_{bd} + g_{ad}g_{bc} - g_{ab}g_{cd}\right), \tag{17}$$

the momenta that are canonically conjugate to the components $g_{ab}$

$$\pi^{ab} = -g^{1/2}(K^{ab} - g^{ab}K)\,, \tag{18}$$

the exterior curvature

$$K_{ab} = -n_{a;b}\,, \tag{19}$$

and the projections of the energy-momentum tensor onto the normal basis vectors

$$T_{\perp\perp} = T_{\alpha\beta}n^{\alpha}n^{\beta}, \quad T_{a\perp} = \sigma_0 n^{\alpha}e_a^{\beta}T_{\beta\alpha}\,. \tag{20}$$

In the given metric we obtain

$$T_{\perp} = \frac{6R\cos y K_2{}^5}{K_1{}^2}\left(-\dot{R}^2 + R^2\left(\tilde{Q} + \frac{A}{e^{2Ht}}\right)^2\right), \tag{21}$$

where $A = \dfrac{K_1{}^2\lambda\phi_0{}^2}{3K_2{}^2}$,

$$T_1 = T_2 = T_3 = 0\,. \tag{22}$$

In our model, the Lagrangian of the gravitational field is given by

$$\Lambda_{GE} = -\frac{3(4R'^2K_2{}^4 - K_1{}^2)}{2R^2K_1{}^2K_2{}^2},$$

$$g^{1/2} = -R^3K_2{}^3\cos y\,,$$

$$N = \frac{K_1}{K_2}\,.$$

We have

$$L_{GE} = \frac{1}{2}\int d^3x(Ng^{1/2}\Lambda_{GE}) = -6\pi^2\,\frac{R\left\{4\dot{R}^2K_2{}^4 - K_1{}^2\right\}}{K_1}\,. \tag{23}$$

We define the canonical momentum in the spirit of minisuperspace quantization as

$$\pi_R = \frac{\partial L_{GE}}{\partial R'} = -\frac{48\pi^2 R\dot{R} K_2{}^4}{K_1}. \tag{24}$$

Thus,

$$T_\perp = \frac{6R\cos y K_2{}^5}{K_1{}^2}\left(-\frac{\pi_R^2 K_1^2}{\left(48\pi^2\right)^2 R^2 K_2^8} + R^2\left(\tilde{Q} + \frac{A\tilde{R}_0}{Re^{2H/(Q+A)}}\right)^2\right). \tag{25}$$

We perform the quantization

$$\pi_R = \frac{1}{i}\frac{d}{dR}, \quad \pi_R^2 = -\frac{d^2}{dR^2}. \tag{26}$$

After various transformations, we obtain

$$T_\perp = \frac{6R\cos y K_2{}^5}{K_1{}^2}\left(\frac{K_1^2}{\left(48\pi^2\right)^2 R^2 K_2^8}\frac{d^2}{dR^2} + R^2\left(\tilde{Q} + \frac{A\tilde{R}_0}{Re^{2H/(\tilde{Q}+A)}}\right)^2\right). \tag{27}$$

As the end result, the Wheeler–De Witt equation has the form

$$\left[\frac{d^2}{dR^2} + \frac{2304\pi^4 R^4 K_2^8}{K_1^2}\left(\tilde{Q} + \frac{A\tilde{R}_0}{Re^{2H/(Q+A)}}\right)^2\right]\Psi(R) = 0. \tag{28}$$

Equation (28) can be written as

$$\left[\frac{d^2}{dC^2} - U(R)\right]\Psi(R) = 0, \tag{29}$$

where

$$U(R) = -\frac{2304\pi^4 R^4 K_2^8}{K_1^2}\left(\tilde{Q} + \frac{A\tilde{R}_0}{Re^{2H/(Q+A)}}\right)^2. \tag{30}$$

From Eq. (30) it can be seen that $U(R)$ cannot be positive. Thus we arrive at the conclusion that minisuperspace quantization of our solution (Eqs. (10)) with metric prescribed by Eqs. (1)–(3) is impossible.

We remark on the fundamental significance of [10–13] for the development of quantum cosmology. Quantum birth of the Universe is mainly considered for isotropic models. For non-isotropic models there exist in this regard only individual results. In [10–13] quantum birth of new non-isotropic model Universes is investigated in an exact, not a numerical formulation. For an exact cosmological model with rotation, the rate of rotation of the early Universe can be significant, which shows up in the probability of quantum birth of such a model. In [10–13] it was established for the first time for exact models that in certain cases the presence of cosmological rotation can increase the probability of quantum birth of model Universes. In this regard, approximate numerical estimates of the role of rotation for the probability of quantum birth of the Universe with rotation, even particular ones, have less value than the approach based on exact cosmological models with rotation.

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