DOI 10.1007/s11182-021-02261-1
Russian Physics Journal, Vol. 63, No. 11, March, 2021 (Russian Original No. 11, November, 2020)

STUDY OF THE DYNAMICS OF THE ASTEROID KAMO`OALEWA

S. A. Gur'yanov and T. Yu. Galushina UDC 521.1, 521.182

The motion of the asteroid (469219) Kamo`oalewa, which moves in 1:1 mean motion resonance with the Earth, is investigated. Over the course of studying the dynamics of this object, we revealed close approaches to planets of the Solar System and mean motion and secular resonances and estimated the chaoticity of its orbit. A peculiarity of the asteroid motion is its regular transitions from the state of a quasi-satellite to the state of a horseshoe.

Keywords: asteroid 469219 Kamo`oalewa, secular resonances, mean motion resonances, orbital evolution, MEGNO.

INTRODUCTION

The asteroid 469219 Kamo`oalewa (preliminary designation 2016 HO3), discovered in 2016, moves in 1:1 mean motion resonance with the Earth and at the present time is one of the quasi-satellites of our planet, but its state switches from time to time between the configurations of a quasi-satellite and a *horseshoe*. Its current quasi-satellite state began around 100 years ago and will end roughly 300 years from now [1]. This object was discovered with the help of the Hawaiian automatic telescope Pan-STARRS 1 (PS1) of the Pan-STARRS system by the astronomer Paul Chodas from the Jet Propulsion Laboratory of NASA in Pasadena (USA). After this discovery, Chodas together with his colleagues continues to study the asteroid in order to determine its future existence from its composition.

The aim of the present work is to consider the dynamics of the asteroid 469219 Kamo`oalewa, in particular, to reveal its close approaches to planets, to estimate the chaoticity of its orbit, and to investigate its mean motion and secular resonances. The organization of this paper is as follows. Section 1 briefly discusses the stages of investigation of the object and lays out the methods of research. Section 2 considers results of the asteroid orbit fitting. Section 3 presents results of studying its probabilistic orbital evolution. Section 4 investigates the chaoticity of the orbit of this object, and Section 5 addresses secular resonances.

1. METHODS OF RESEARCH

Our study of the dynamics of the asteroid 469219 Kamo`oalewa included the following steps:

– Investigation and analysis of the influence of various perturbing factors on the asteroid motion,

– Fitting of the asteroid orbit,

– Study of the probabilistic orbital evolution, in particular, identification and investigation of close approaches and mean motion resonances,

– Investigation of the chaoticity of motion,

– Identification of secular resonances.

National Research Tomsk State University, Tomsk, Russia, e-mail: magicboys2000@gmail.com; volna@sibmail.com. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 11, pp. 141–147, November, 2020. Original article submitted May 22, 2020.

The first step in the study of the dynamics of the asteroids is an analysis of the influence of different perturbing factors on the asteroid motion [2]. In this paper, the set of perturbations to be estimated includes the influence of the large planets, Pluto, the Moon, Ceres, Pallas, and Vesta; the Sun, the Earth, and Jupiter oblateness; and relativistic effects of the Sun, the large planets, Pluto, and the Moon. Our investigations revealed that the most important influence on the dynamics of 2016 HO3 is that of the gravitational forces of the Earth, the Moon, the large planets, and relativistic effects of the Sun.

The evolution of the orbit of the asteroid Kamo`oalewa was examined by numerical integration of the differential equations of its motion. The time interval of this study was chosen on the basis of calculation precision, we studied the future and past periods of 8 000 years from now. We considered the evolution of the elements of its nominal orbit obtained by fitting of its orbit by the least squares method and of clones distributed according to a normal law within the limits of the initial confidence region. As characteristics of the mean motion resonances, we used the resonance argument β defining the longitude of conjunction of the asteroid and the planet [3],

$$\beta = \lambda_1 - \lambda_2, \tag{1}$$

and its time derivative α (called the resonance *band* [4])

$$\alpha \approx n_1 - n_2, \tag{2}$$

where n_1 and n_2 are mean motions and λ_1 and λ_2 are mean longitudes of the asteroid and the planet, respectively, ω_1 is the argument of the pericenter of the asteroid, and Ω_1 is the longitude of the ascending node of the asteroid.

To reveal the chaoticity of asteroid motion, we applied the MEGNO parameter, which is a time-weighted integral form of the Lyapunov characteristic number. If we write the system of equations of motion of the celestial body in the rectangular coordinates and velocities, $\boldsymbol{q} = \boldsymbol{x}$ and $\dot{\boldsymbol{x}}$, where $\boldsymbol{q}$ is the state vector of the investigated asteroid, then the equation of motion of the object will look like this:

$$\frac{d}{dt}\boldsymbol{q}(t) = \boldsymbol{f}\left(\boldsymbol{q}(t), \gamma\right), \tag{3}$$

where γ is the vector of parameters of the force model. Denoting the initial small deviation of the state vector $\boldsymbol{q}$ as $\boldsymbol{\delta}(t_0) = \boldsymbol{\delta}_0$, we write the Lyapunov characteristic number (LCN) in the form

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(t_0)\|}$$

and in integral form

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \int_0^t \frac{\dot{\delta}(s)}{\delta(s)} ds .$$

The MEGNO parameter $Y(t)$ is the time-weighted integral form of the LCN

$$Y(t) = \frac{2}{t} \int_0^t \frac{\dot{\delta}(s)}{\delta(s)} s ds ,$$

and its average value $\overline{Y}(t)$ is calculated as

TABLE 1. Types of Apsidal-Nodal Resonance Relations of Lower Orders

No.	Type of resonance relation	No.	Type of resonance relation	No.	Type of resonance relation
1	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+\dot{\omega}-\dot{\omega}'_j$	8	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-2\dot{\omega}-2\dot{\omega}'_j$	15	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-2\dot{\omega}'_j$
2	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-\dot{\omega}+\dot{\omega}'_j$	9	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+2\dot{\omega}$	16	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+2\dot{\omega}'_j$
3	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+2\dot{\omega}-2\dot{\omega}'_j$	10	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-2\dot{\omega}$	17	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)$
4	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-2\dot{\omega}+2\dot{\omega}'_j$	11	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+\dot{\omega}$	18	$\dot{\omega}-\dot{\omega}'_j$
5	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+\dot{\omega}+\dot{\omega}'_j$	12	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-\dot{\omega}$	19	$\dot{\omega}+\dot{\omega}'_j$
6	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-\dot{\omega}-\dot{\omega}'_j$	13	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+\dot{\omega}'_j$	20	$\dot{\omega}$
7	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)+2\dot{\omega}+2\dot{\omega}'_j$	14	$\left(\dot{\Omega}-\dot{\Omega}'_j\right)-\dot{\omega}'_j$		

$$\overline{Y}(t)=\frac{1}{t}\int_0^t Y(s)ds\,.$$

The evolution of $\overline{Y}(t)$ over time allows us to accurately separate the regular and chaotic regimes of motion [5]. Thus, for example, $\overline{Y}(t)>2$ for chaotic orbits with exponential divergence of close trajectories. For quasiperiodic (regular) orbits with linear divergence of close trajectories, $\overline{Y}(t)$ oscillates about 2.

We also performed calculations to identify the apsidal-nodal resonances for the investigated object. By the apsidal-nodal resonance is commonly understood the commensurability arising between the velocities of precession of the orbits of the asteroid and a planet ($\dot{\omega}$, $\dot{\omega}'$, $\dot{\Omega}$, $\dot{\Omega}'$) [6–8]. Representing the argument of the perturbing function for the doubly averaged restricted three-body problem in the form $\psi=\left(l-2p'\right)\omega'-\left(l-2p\right)\omega-m\left(\Omega-\Omega'\right)$, where ω and Ω are the argument of the pericenter and the longitude of the ascending node of the asteroid, and ω' and Ω' are the argument of the pericenter and the longitude of the ascending node of the third body, l', p', l, p, and m are integers, and t and t_0 are the current and initial times. The condition for the appearance of a resonance can be written in the form $\dot{\psi}=0$.

The secular accelerations in the motion of the asteroid are determined by the influence of the third body and are calculated by numerical integration using the formulas for the derivative of the Lagrange polynomial. A complete set of the apsidal-nodal resonance relations up to the sixth order is given in Table 1. All of the described algorithms were realized previously in the software package IDA [9], which was used to perform the calculations in this paper.

2. ORBIT FITTING

Figure 1 plots the projections of the orbit of 469219 Kamo`oalewa and the Earth onto the ecliptic plane in the coordinate frame rotating with the angular velocity of our planet (*a*) and in the fixed coordinate frame (*b*). It can be seen that the orbits and periods of the asteroid and the Earth are close; moreover, as can be seen from Fig. 1*b*, in the coordinate system rotating with the angular velocity of the Earth, the object under study moves along a quasi-satellite trajectory.

Results of the orbit fitting are presented in Table 2, where *N* is the number of observations, Δt is the interval of observations in days, t_0 is the initial epoch, σ is the root-mean-square error of observations in units of arcseconds, and

TABLE 2. Observational Data and Results of Fitting of the Kamo`oalewa Orbit

Parameter	Value
N	80(–4)
Δt	17.03.2004–10.06.2016
t_0	06.08.2015
σ, ″	0.208
Δr, ua	$2.66 \cdot 10^{-7}$
a, ua	1.0011132
e	0.1042788
i, deg	7.76820582

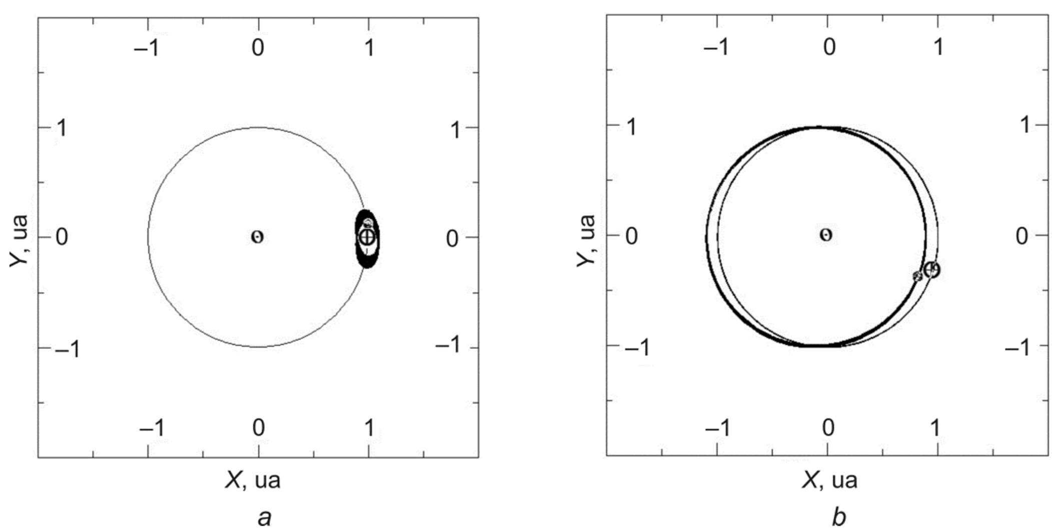

Fig. 1. Projection of the orbit of the asteroid 469219 Kamo`oalewa and the Earth onto the ecliptic plane in the coordinate frame rotating with the angular velocity of the Earth (*a*) and in the fixed coordinate frame (*b*).

Δr is the size of the initial confidence region. As a result of the orbit fitting, we excluded four observations from the total number according to the *three sigma* rule. Table 2 also gives elements of the orbit obtained by fitting: the semimajor axis a, the eccentricity e, and the inclination of the orbit plane to the ecliptic i. The obtained value of Δr shows that the orbit of the asteroid is well defined.

3. STUDY OF THE PROBABILISTIC ORBITAL EVOLUTION

To investigate the probabilistic orbital evolution, we modeled 10,000 clones distributed according to a normal law within the initial confidence region. The evolution of the orbit of each clone was investigated numerically with the influence of the Earth, the Moon, the large planets, and relativistic effects of the Sun taken into account. Figure 2 presents graphs of the close approaches of the asteroid to the Earth and the evolution of the resonance characteristics and elements of the orbit. In the graphs, it can be seen that the values of the elements vary insignificantly over the entire time interval of interest, which indicates regularity of the orbit of the object. It may also be remarked that the values of the elements of the orbit experience their greatest changes at the time of the closest approaches to the Earth.

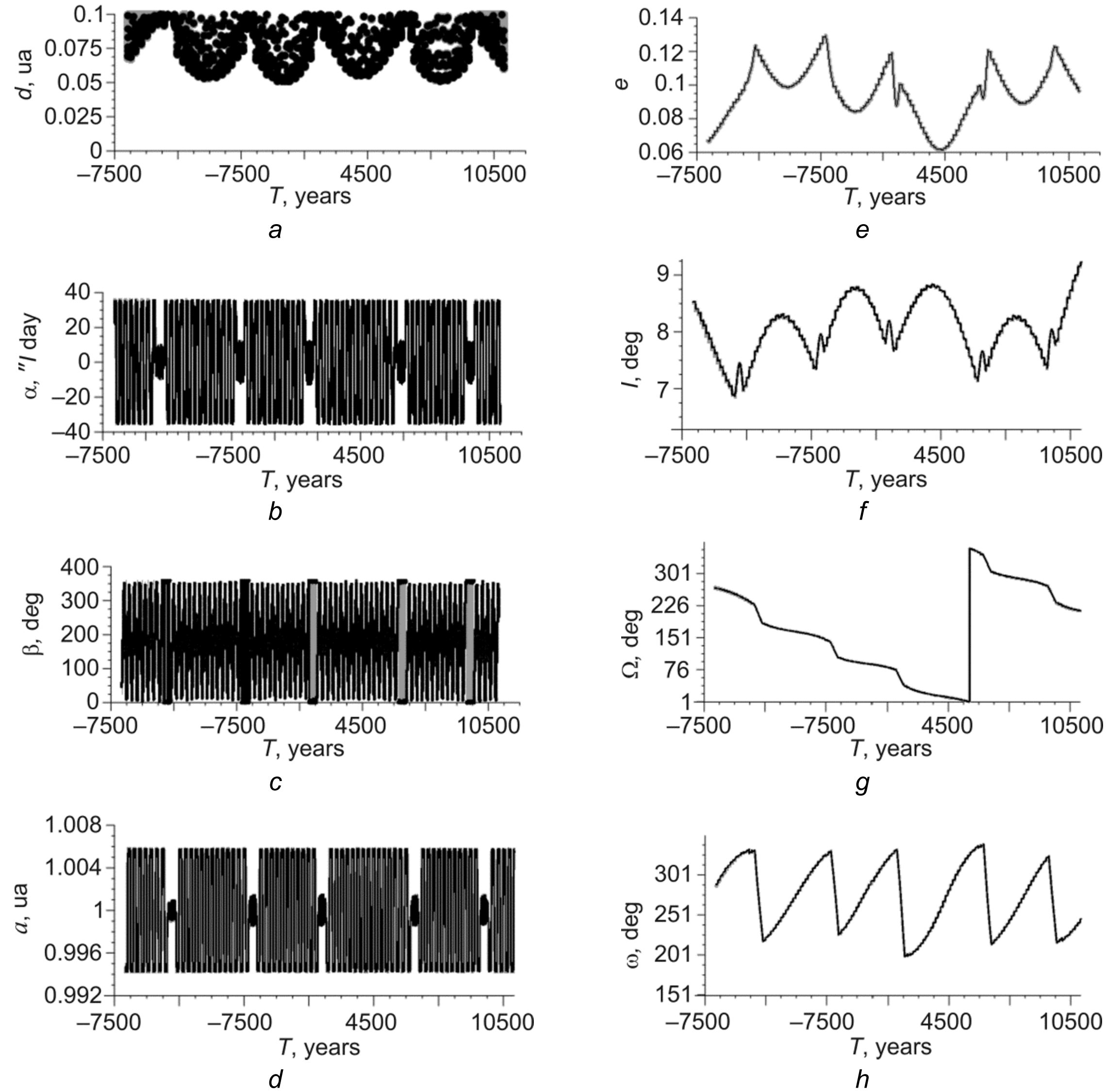

Fig. 2. Close approaches of the asteroid 469219 Kamo`oalewa to the Earth (*a*) (*d* is the distance from the investigated object to the center of the planet), evolution of the resonance band α (*b*), the resonance argument β (*c*), the semimajor axis *a* (*d*), the eccentricity *e* (*e*), the inclination of the orbit plane to the ecliptic *i* (*f*), the longitude of the ascending node Ω (*g*), and the argument of the pericenter ω (*h*). The gray background depicts the evolution of the clones, and the black background depicts the evolution of the nominal orbit.

Close approaches of the asteroid to the Earth are observed over the entire interval under study; however, the distance to the Earth varies. The resonance band oscillates about zero with small amplitude, but at very close approaches the amplitude increases. The resonance argument librates around zero, which says that the asteroid is presently a quasi-satellite of the Earth. It can also be seen in the graph that roughly 300 years in the future the value of the characteristic will change in the direction of an increase in the oscillations amplitude and a shift of their center, which will entail a transition of the object into the class of *horseshoes*. It can be seen from Fig. 2 that the given transitions of the object between a quasi-satellite and a *horseshoe* are regularly repeated over the entire interval under study. Moreover, for greater ease of visualization, we considered in more detail the evolution of the resonance argument

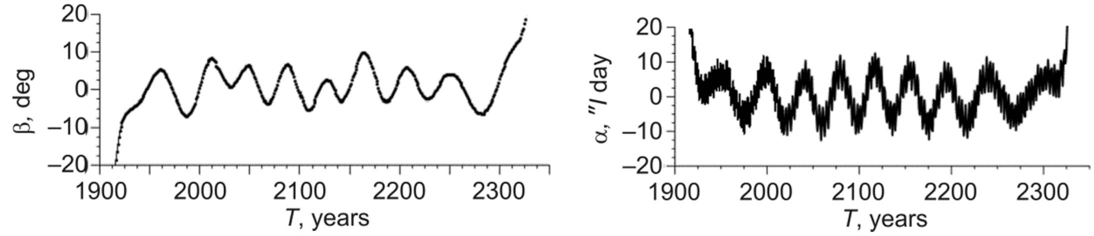

Fig. 3. Evolution of the resonance argument (*a*) and the resonance band (*b*) in the time interval from 1900 to 2350.

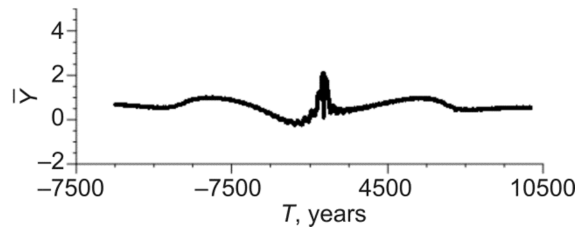

Fig. 4. Evolution of the MEGNO parameter for the asteroid 469219 Kamo`oalewa.

and the resonance band in the vicinity of the current epoch, during which time the asteroid is a quasi-satellite of the Earth. The results are plotted in Fig. 3.

In our study of the orbital evolution of the nominal orbit and clones, we found that the confidence region retains its configuration over time and that the divergence of the clones from the nominal orbit is practically imperceptible, which is indicative of stable behavior of the object.

4. INVESTIGATION OF CHAOTICITY

Figure 4 plots the evolution of the averaged MEGNO parameter. It can be seen that the MEGNO parameter varies insignificantly and is less than the value $\overline{Y}(t) = 2$ over practically the entire interval under study, i.e., the orbit can be considered regular.

5. INVESTIGATION OF SECULAR RESONANCES

We considered apsidal-nodal resonances with all the planets (Table 1); however, here we present examples of resonances only with the Earth since the given object presents a similar picture from the point of view of a search for secular resonances with planets of the inner group. Considering all 20 resonances over the investigated time interval, we can say that in some cases the argument completes librational motions (relations 13–19) while the remaining relations present circulation. All of the resonance relations oscillate about zero, but with different amplitude. During the closest approaches to the Earth, the amplitude of the resonance relation grows, but the resonance is preserved. On the basis of the given results, it is possible to conclude that the resonance is stable at the moment of libration of the resonance argument; however, in the case of circulation, a different picture is observed. Figure 5 presents only a few examples of the evolution of low-order apsidal-nodal resonance relations and the evolution of the corresponding resonance arguments for the asteroid Kamo`oalewa. Here, both circulation and libration are present.

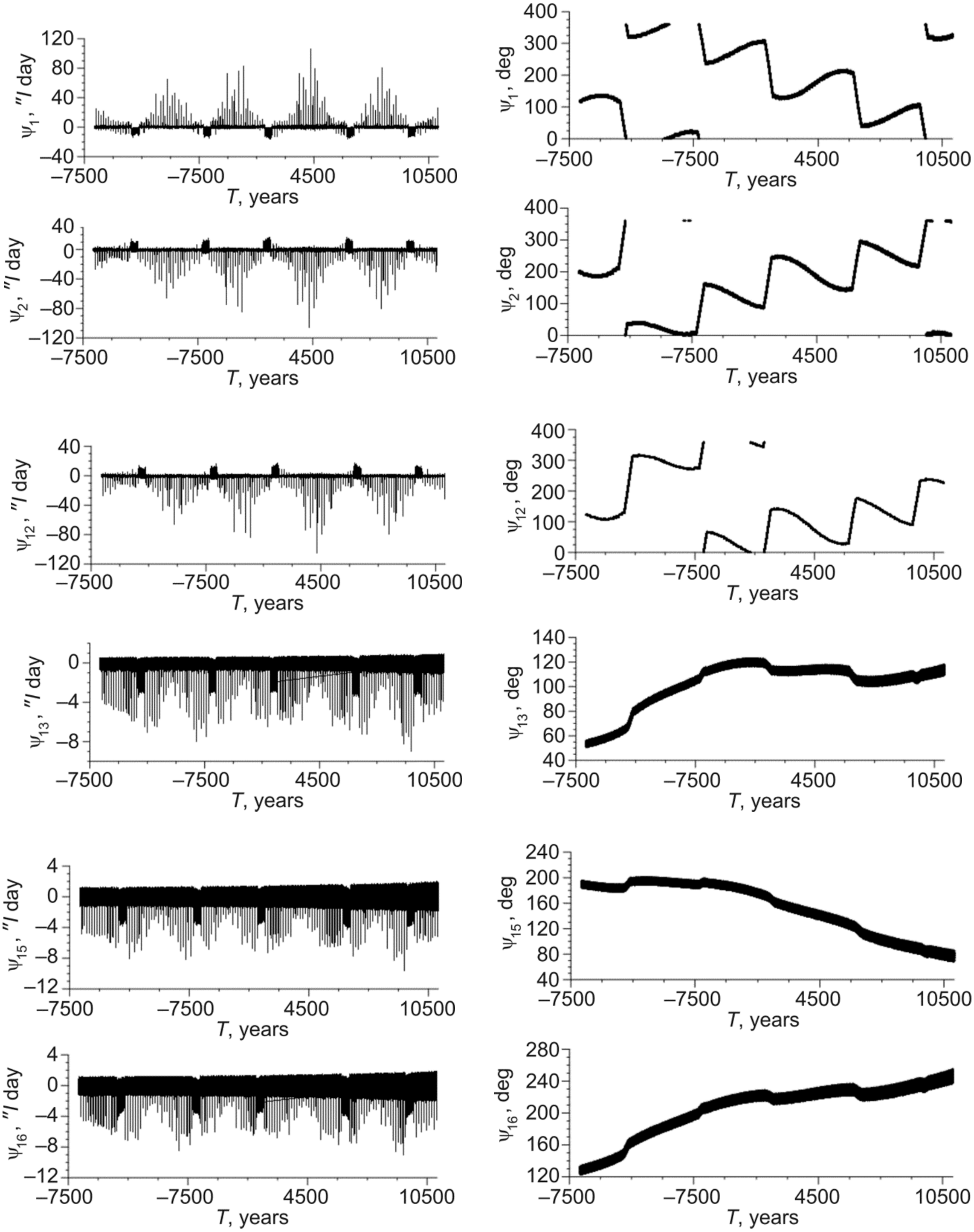

Fig. 5. Evolution of the resonance relations (*a*) and the corresponding resonance arguments (*b*) for the asteroid 469219 Kamo`oalewa.

CONCLUSIONS

We have considered the orbital evolution of the asteroid 469219 Kamo`oalewa, which at the present time is one of the quasi-satellites of our planet and on the basis of calculations, this state will continue for roughly another 300 years. Analyzing our study of the evolution of the parameters of the asteroid orbit and the chaoticity of its motion, we can say that the orbit of this object will remain stable over the course of at least another 8000 years. Our study has also showed the presence of close approaches of the asteroid to the Earth, but not such close ones as to consider this object to be dangerous for our planet. Considering the resonance characteristics of this object, we have confirmed the fact that the object does indeed move in 1:1 resonance with the Earth and is a quasi-satellite of the Earth. The resonance arguments for some low-order apsidal-nodal resonance relations librate in the time interval under consideration (relations 13–19 from Table 1), which is indicative of stability of the secular resonances for the investigated object; however, for the remaining relations circulation is observed, which speaks of the absence of resonance.

This work was performed within the scope of the State Assignment of the Ministry of Science and Higher Education of the Russian Federation (Theme No. 0721-2020-0049).

REFERENCES

1. C. de la Fuente Marcos and R. de la Fuente Marcos, Mon. Not. R. Astron. Soc., **462**, 3441– 3456 (2016).
2. P. V. Skripnichenko and T. Yu. Galushina, Izv. Yyssh. Uchebn. Zaved. Fiz., **56**, No. 6/3, 229–231 (2013).
3. C. D. Murray and S. F. Dermott, Solar System Dynamics, Cambridge University Press, Cambridge (2000).
4. E. A. Grebenikov and Yu. A. Ryabov, Resonances and Small Denominators in Celestial Mechanics [in Russian], Nauka, Moscow (1978).
5. P. M. Cincotta , C. M. Girdano , and C. Simo, Physica D, **182**, 151–178 (2003).
6. T. V. Bordovitsyna, I. V. Tomilova, and I. N. Chuvashov, Astron. Vestn., **46**, No. 5, 356–368 (2012).
7. T. Yu. Galushina and G. E. Sambarov, Planet. Space Sci., **142**, 38–47 (2017).
8. A. G. Aleksandrova, T. V. Bordovitsyna, N. A. Popandopulo, and I. V. Tomilova, Russ. Phys. J., **63**, No. 1, 64–70 (2020).
9. T. Galushina, L. Bykova, O. Letner, and A. Baturin, Astron. Comput., **29**, Article ID 100301 (2019).