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THE MODEL OF DYNAMIC STRESS RELAXATION OF ELASTOPLASTIC MATERIALS

P. V. Makarov

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The paper proposes the model of relaxation of the loaded elastoplastic material, with the dislocation kinetics of plastic shear. The proposed model includes two independent strain rates, namely the total strain rate describing the external load rate and the rate of the localized plastic response describing the medium ability to generate deformation defects. This model describes both the localized relaxation processes in the elastoplastic medium and the average stress relaxation in the loaded elastoplastic medium. The proposed model is in essence microscopic. All its parameters are obtained in the independent experiments concerning the evolution of the dislocation continuum during the material loading. The model describes well the observed dynamic effects of the material macroscopic response depending on the strain rate, i.e., the upper and lower yield points (sharp yield point and yield plateau), successive strain hardening, and cyclic and sign-variable loads, and the Bauschinger effect.

Keywords: dynamic plastic deformation, governing relaxation equation, dislocation kinetics, the Bauschinger effect, cyclic load.

INTRODUCTION

The idea of describing the dynamic loads in a loaded elastoplastic medium by the governing relaxation equations belongs to Sokolovskii and Malvern, who did it early in the 20th century [1, 2]. Over the last decades, many variants of models describing the relaxation processes have been proposed.

The new experimental data on mechanisms of irreversible plastic deformation of various materials require more and more conceptual models describing the inelastic deformation and structural changes in a wider range of external loads such as pressure, temperature, loading rates.

Merzhievskii [3] identified three groups of models of intensive dynamic loading, namely: 1) continuum or macroscopic models; 2) microstructural models; 3) molecular dynamics models and analyses. The first group included the traditional models of continuum mechanics, first of all, classical models of viscoelastic media and elastoplastic deformation as well as their numerous generalizations in the case of dynamic and shock loads. The second group was based on microstructural mechanisms of irreversible elastoplastic deformation and included the dislocation kinetics models of plastic deformation. The third group was based on the molecular dynamics approaches and techniques and the atomic interaction processes.

Emphasis is placed on the models of the first and second groups concerning the relaxation processes in the loaded materials. In the 1970–80s, the experimental and theoretical investigations of the physical mechanisms of shockwave-induced elastoplastic deformation of materials, first of all, metals, provide unique data on inelastic deformation mechanisms and the defect structure evolution in shock-loaded materials as well as the dependence of the latter on the loading parameters (amplitude, strain rate) [4–21]. These investigations are based on armor ballistics and

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Institute of Strength Physics and Materials Science of the Siberian Branch of the Russian Academy of Sciences, Tomsk, Russia, e-mail: pvm@ispms.tsc.ru. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 11, pp. 36–43, November, 2020. Original article submitted February 12, 2020.

high-speed collision of solid bodies due to aerospace problems of anti-meteorite protection of space objects. In terms of the fundamental problems of the physical theory of plasticity, the shockwave problem turns to be very productive for the generation of new knowledge. Specially organized experiments with planar shockwaves provide new important information about the physical mechanisms and defect structure evolution [5–8, 15, 17–19]. This results from the fact that such experiments proceed under strictly controlled conditions for amplitudes, strain rates, and inelastic deformation, including the processes in elastic precursors, at the shockwave front, and in unloading waves [5, 16, 22, 23]. The stress relaxation processes in shockwaves are very pronounced, which promotes the development of the stress relaxation models of elastoplastic deformation with dislocation mechanisms of plastic deformation [5–10, 23]. It is these relaxation models, that turn to be the most suitable for describing inelastic deformation of metals in shockwaves. No fundamentally new structures are observed under shock loads [8, 18, 19]. The governing relaxation equation with the dislocation kinetics of plastic shear, which describe elastoplastic deformation of metals in shockwaves, can be applied in modeling plastic deformation regardless of the load type in a wide range of strain rates, especially for the materials sensitive to the load rate.

The recent data, in particular on new processing technologies, arouse interest in developing the latest models of stress relaxation. Such models are proposed for new materials, glasses, amorphous systems [24], plastics, ceramics, and elastoplastic media of various rheology [25–30]. Stress relaxation models of the reduction in different stresses that occur in materials after mechanical and thermal processing, stand out of the other models. A number of works continue to develop new versions of the stress relaxation models based on the idea of Sokolovskii and Malvern and consider the evolution of the dislocation continuum as a relaxation mechanism [3, 22, 23, 30–32].

At present, the range of different dynamic models is very wide. However, most of the proposed empirical models are individual cases and capable of describing the dynamic response of a narrow range of materials [3–8], for which they are developed. This is because most of such models do not really describe the relaxation process, as it is not an essential parameter of plastic deformation. The latter is determined by the respective macro parameters obtained from experiments. In [25], Selyutina implements the interesting and promising idea of the incubation period criterion. This period must describe the duration of the structural rearrangement process of the material. This criterion introduced into the governing equations, provides the stress relaxation process. However, this and other similar models [26–29] are formulated in terms of the stress-strain state. Since plastic deformation is a process developing at each point of the loaded material with its specific velocity, the stress relaxation model must be formulated in terms of rates, which correspond to the idea of Sokolovskii and Malvern [1, 2]. The proposed model includes two independent strain rates, *viz*. the total strain rate $\dot{\varepsilon}_I^T$, which is the external load rate, and the rate $\dot{\varepsilon}_I^P$ of the localized plastic response describing the material ability to generate deformation defects.

STRESS RELAXATION MODEL OF ELASTOPLASTIC MEDIUM

In accordance with the idea of Sokolovskii and Malvern [1, 2], the one-dimensional model of tension-compression can be written in rates as follows:

$$\dot{\sigma}_1 = E\left(\dot{\varepsilon}_1^T - \dot{\varepsilon}_1^P\right). \tag{1}$$

In this model, the stress increment $\Delta\sigma_1 = E\dot{\varepsilon}_1^T \Delta t$ is always proportional to the total strain rate $\dot{\varepsilon}_1^T$. Stresses release with the development of plastic deformations at a rate $\dot{\varepsilon}_1^P$ characteristic to each particle, such that $\dot{\varepsilon}_1^T = \dot{\varepsilon}_1^P + \dot{\varepsilon}_1^e$. Equation (1) is Hooke's law, i.e., $\dot{\sigma}_1 = E\dot{\varepsilon}_1^e$. In Eq. (1), the total strain rate $\dot{\varepsilon}_1^T$ is defined by the external load rate, while the $\dot{\varepsilon}_1^P$ value is determined by the ability of the material to generate deformation defects at a given point and stress level, and provide relaxation of the localized stresses. Both at any point of the loaded material and for the σ~ε curve of the whole material, the stress-strain relation is a dynamic equilibrium between the applied load and the elastoplastic response of the material, i.e., $\dot{\varepsilon}_1^T \neq \dot{\varepsilon}_1^P$ for general case. At $\dot{\varepsilon}_1^T > \dot{\varepsilon}_1^P$ and $\dot{\varepsilon}_1^T < \dot{\varepsilon}_1^P$, stresses grow and release, respectively. It is clear that at $\dot{\varepsilon}_1^P = 0$, the material's response is only elastic. Equation (1) can be easily generalized for the three-dimensional model of deformation. In this case, the governing relaxation equation (1) can be written as

$$\dot{\sigma}_{ij} = \lambda\left(\dot{\theta}^T - \dot{\theta}^P\right)\delta^{ij} + 2\mu\left(\dot{\varepsilon}_{ij}^T - \dot{\varepsilon}_{ij}^P\right), \text{ where } \dot{\theta} = \dot{\varepsilon}_{ii} = \dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33}. \tag{2}$$

According to the theorem of plastic incompressibility of materials, $\dot{\theta}^P = 0$. Hence, $\dot{\theta}^T = \dot{\theta}^e$ in Eq. (2).

Let us assume that each stress tensor component in Eq. (2) releases in the similar way. This equation can then be rewritten *via* the stress intensity σ_i and the strain intensity ε_i:

$$\dot{\sigma}_i = 3\mu\left(\dot{\varepsilon}_i^T - \dot{\varepsilon}_i^P\right), \tag{3}$$

where $\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}$ и $\varepsilon_i = \frac{\sqrt{2}}{2}\sqrt{(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2}$.

After the relaxation process performed according to Eq. (3), each component of the current stresses σ_{ij}^{eff} is normalized to the value σ_{ij}^{rel} of the relaxed stress intensity:

$$\sigma_{ij}^{\text{rel}} = \sigma_{ij}^{\text{eff}}\left(\frac{\sigma_i^{\text{rel}}}{\sigma_i^{\text{eff}}}\right). \tag{4}$$

The plastic strain rate $\dot{\varepsilon}_i^P$ in Eq. (3) can be specified by any model. In our work, it is set in accordance with the Taylor–Orowan law and the modified dislocation model [33]. In our early research [23], this model modification is described in detail for shockwave-induced deformation of metals.

In the Taylor–Orowan relation for the intensity of plastic shears

$$\dot{\varepsilon}_i^P = gbNfv, \tag{5}$$

it is assumed that the continuum of similar dislocations develops in the stress field in accord with the following relations for the dislocation density N and the mobile part of dislocation f, both depending on the plastic strain rate ε_i^P, and dislocation slip rate v depending on the shear stress τ ($\tau = 1/2(\sigma_1-\sigma_2)$ for the two-dimensional model of deformation:

$$N = N^* + \left(N_0 - N^*\right)\exp\left(-\frac{A}{|g|b}\varepsilon_i^P\right), \tag{6}$$

$$f = f^* + \left(f_0 - f^*\right)\exp\left(-\frac{B}{|g|b}\varepsilon_i^P\right). \tag{7}$$

Our calculations show that it is advisable to use simple empirical dependences for the average dislocation slip rate, that are obtained in studying the defect mobility in the field of effective forces, for example:

$$v = v_0\exp\left(\frac{D + H\gamma^p}{\tau}\right), \quad v = \frac{\tau b}{\beta}, \tag{8}$$

$$v = v_0 \frac{ST^2}{1+ST^2}, \quad ST = \frac{(\tau - \tau_0)}{\beta_1}. \tag{9}$$

Our calculations are based on Eq. (9), which describes well the dynamic response of metals to the load, shockwaves, in particular [23].

The following notation are introduced in Eqs. (5)–(9): τ_0 is the averaged Peierls-Nabarro stress [34, 35] or stress relaxation to this value at quasi-static deformation; v_0 is the sound velocity of the shear wave; A and B are the parameters of the dislocation slip to the dislocation multiplication and pinning, respectively [23, 33]; N^* and f^* are the limit values of the dislocation density and the part of mobile dislocation N_m relative to their total amount ($N_m = Nf$), respectively.

Thus, the linear approximation in Eq. (6) results in a well-known empirical dependence $N = N_0 + N^* \frac{A}{|g|b} \varepsilon_i^P$ or $N = N_0 + \alpha\varepsilon_i^P$, $\alpha \approx N^* \frac{A}{|g|b}$, where α means the empirical coefficient of the dislocation multiplication, i.e. $|g| \approx 0.5$ [23, 33]. The internal oriented stresses appear during the material deformation, which can be approximately calculated as [34, 35]:

$$\tau_{bs} = \alpha\mu b\sqrt{N}. \tag{10}$$

These internal stresses generate effective stresses

$$\tau_{\text{eff}} = \tau - \tau_{bs}. \tag{11}$$

As the defect motion is induced by τ_{eff}, they determine the sign of the increment $\dot{\varepsilon}_i^P$ in Eq. (5). This is important when changing the load sign and cyclic loading. Equation (5) then takes the form

$$\dot{\varepsilon}_i^P = |g| bNfv(\tau_{\text{eff}}) \operatorname{sign}(\tau_{\text{eff}}). \tag{12}$$

In the regions of reversible plastic deformation, when the sign of the dislocation density N in Eq. (12) is changed, the ε_i^P value reduces according to the deformation rule. As a result, such nonphysical effects can be observed as the reduction in the dislocation density N in Eq. (6) and mobile dislocation f in Eq. (7). More adequate defect accumulation results in a physically correct accumulation of oriented residual stress τ_{bs} and effective stress τ_{eff} in Eqs. (10) and (11). That is why it is necessary to take irreversible plastic deformation in Eqs. (6) and (7) into consideration, when changing the sign of deformation and at a cyclic loading in Eq. (12). Irreversible plastic deformation accumulates independently of the load sign, and it is also necessary to replace the localized plastic deformation ε_i^P by the cumulative inelastic deformation ε_{ik}^P, which constantly grows:

$$\varepsilon_{ik}^P = \int_0^t \left|\dot{\varepsilon}_i^P\right| dt. \tag{13}$$

Some dislocations can detach in changing the sign of the effective stress τ_{eff}. This process must be also considered in Eq. (7) for mobile dislocations through, for example, the introduction of reversible plastic deformation ε_{ir}^P. The equation of reversible plastic deformation allows for the fact that detachment concerns only a part of dislocations, which can be calculated from

$$\varepsilon_{ir}^P = \varepsilon_{ik}^P \left(1 - N/N^*\right). \quad (14)$$

Taking into account these observations, the model of relaxation of elastoplastic deformation takes the form

$$\dot{\varepsilon}_i^P = |g| bNfv\left(\tau_{\text{eff}}\right)\operatorname{sign}\left(\tau_{\text{eff}}\right),\ \tau_{\text{eff}} = \tau - \tau_{bs},$$

$$\tau_{bs} = \alpha\mu b\sqrt{N},\ v = v_0 \frac{ST^2}{1+ST^2},\quad ST = \frac{\left(\tau - \tau_0 - \tau_{bs}\right)}{\beta_1},$$

$$N = N^* + \left(N_0 - N^*\right)\exp\left(-\frac{A}{|g|b}\varepsilon_{ik}^P\right),\ \varepsilon_{ik}^P = \int_0^t \left|\dot{\varepsilon}_i^P\right| dt, \quad (15)$$

$$f = f^* + \left(f_0 - f^*\right)\exp\left(-\frac{B}{|g|b}\left(\varepsilon_{ik}^P - \varepsilon_{ir}^P\right)\right),$$

$$\varepsilon_{ir}^P = \begin{cases} 0, \\ \varepsilon_{ik}^P\left(1 - \dfrac{N}{N^*}\right), & \text{in case of changing the sign } \left(\tau - \tau_{bs}\right). \end{cases}$$

In constructing the σ–ε curve, the loading is set as $\dot{\varepsilon}_i^T = \text{const}$ in the form

$$\dot{\varepsilon}_i^T = \dot{\varepsilon}_i^0 \cdot \operatorname{sign}\left(\sin\left(\omega t\right)\right) \text{ or } \dot{\sigma}_i = \dot{\sigma}_i^0 \cdot \sin\left(\omega t\right) \quad (16)$$

at cyclic and sign-variable loads. Here ω is the load rate.

Corrections of the initial model (Eqs. (3), (6), (7)) allow us not only to describe the material sensitivity to the load rate, sharp yield point and yield plateau (see Fig. 1), but also cyclic and sign-variable loads and the nonideal Bauschinger effect (see Figs 1*b*, 2 and 3). The model in Fig. 1*b* describes an often-observed effect of not adequately elastic relaxation in the initial *BC* region, because at this stress level, some of defects move, thereby increasing inelastic deformation. Here, the nonideal Bauschinger effect is well-pronounced and *CD* > *DE*. The mechanical responses of a polycrystalline material (low-carbon steel) to cyclic and sign-variable loads are presented Figs 2 and 3. The loading rule in these cases matches Eq. (16). According to Eq. (14), the correctness of allowing for only a fraction of the dislocation detachment is supported by the analysis of the sign-variable load shown in Fig. 4. When reversible plastic deformation ε_{kr}^P equals cumulative deformation ε_{ik}^P, that implies pinning of all dislocations, the stress-strain curve demonstrates plastic deformation with the stress relaxation, which does not satisfy the experimental data (Fig. 4, dashed line). With regard to Eqs. (10)–(14), Eqs. (15) are solved by using either the fourth-order Runge–Kutta method at a given load rate $\dot{\varepsilon}_i^P = \text{const}$ or Eq. (16) at cyclic and sign-variable loads.

As noted above, the parameters of the dislocation model are selected in accordance with the experimental data on the evolution of the dislocation structures. Figure 6 shows the dependence between the calculated internal oriented stress τ_{bs} and accumulated plastic deformation as compared to the experimental data [39]. In Fig. 7, one can see a comparison of the calculated dislocation density with the experimental data [17].

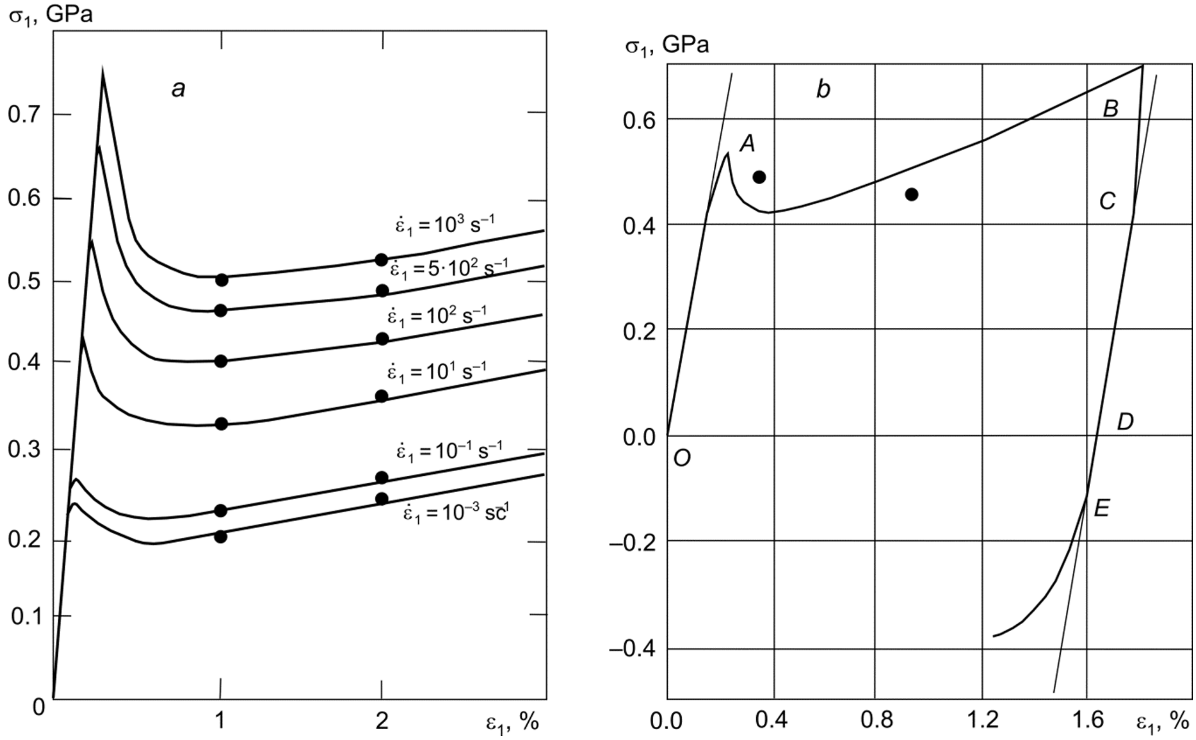

Fig. 1. Theoretical σ–ε curves for low-carbon steel: *a* – compared to experimental curves [36], $N_0 = 2\cdot10^8$ cm^{-1}, $\alpha = 10^{11}$ cm^{-2}, $b = 2.5\cdot10^{-8}$ cm, $v_0 = 3.25\cdot10^5$ cm/s; *b* – with nonideal Bauschinger effect. ● – experiments [37], strain rate $\dot{\varepsilon}_1 = 16$.

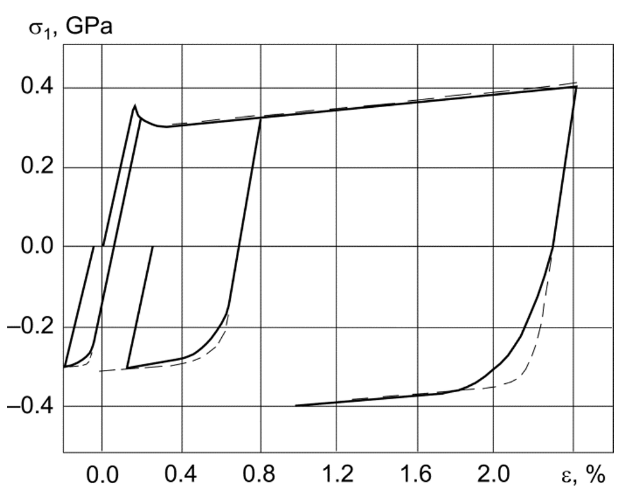

Fig. 2. Theoretical (dashed lines) and experimental [36] (solid lines) σ–ε curves of sign-variable load for low-carbon steel. Model parameters: $D = 2.02$ GPa in Eq. (8), $f_0 = 1$, $f^* = 10^{-3}$, $\dfrac{B}{|g|b} = 350$, $N_0 = 1.73\cdot10^8$ cm^{-2}, $N^* = 10^{12}$ cm^{-2}, $\dfrac{A}{|g|b} = 0.1$, $\dot{\varepsilon} = 1.6$ s^{-1}.

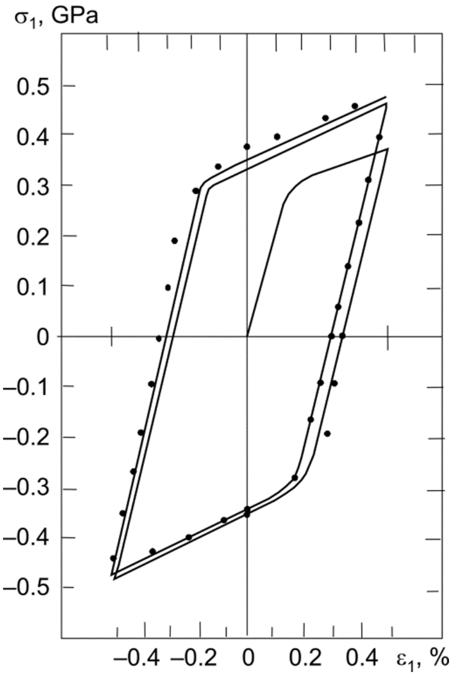

Fig. 3. Cyclic load of low-carbon steel. ● – experimental data [38].

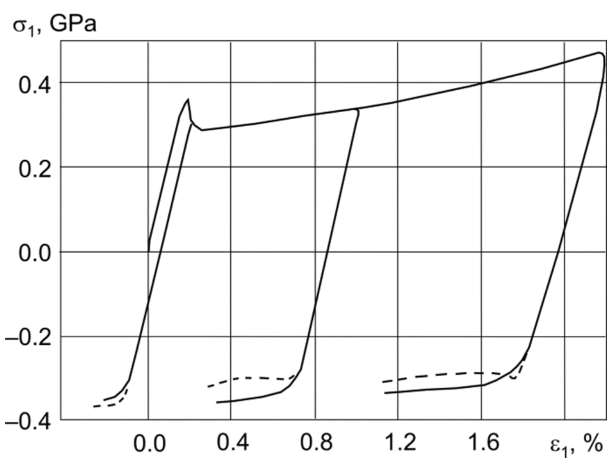

Fig. 4

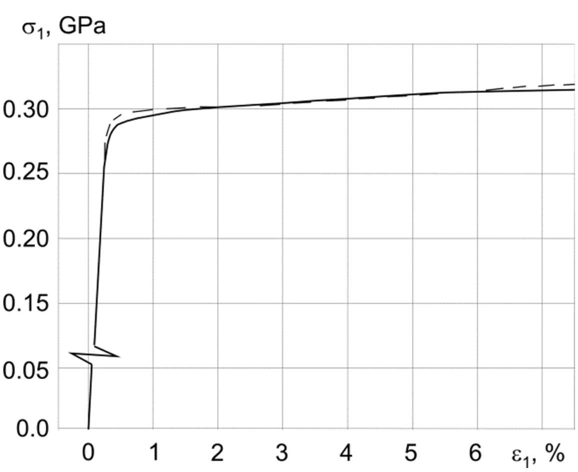

Fig. 5

Fig. 4. Theoretical σ–ε curves for low-carbon steel with nonideal Bauschinger effect. Dashed lines mean $\varepsilon_{ir}^{P} = \varepsilon_{ik}^{P}\left(1+\dfrac{N}{N^{*}}\right)$; solid lines mean $\varepsilon_{ir}^{P} = \varepsilon_{ik}^{P}$.

Fig. 5. Stress-strain curves for 6061-T6 aluminum alloy. Solid line indicates experimental data [4], dashed line indicates theoretical calculations for $\dot{\varepsilon}_1 = 10^{-2}\ \mathrm{s}^{-1}$, $\alpha \approx 0.5-0.6$, $\delta \approx 0.07+0.1\gamma^{p}$, $\beta = 0.2$ GPa.

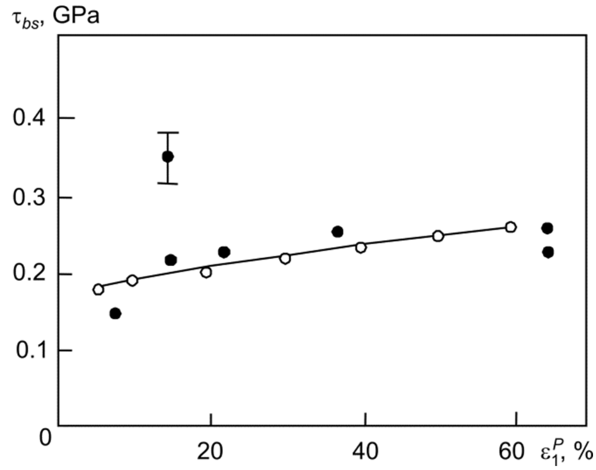

Fig. 6

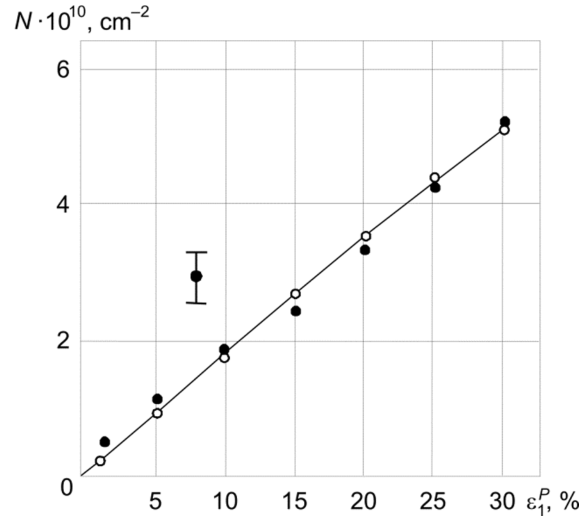

Fig. 7

Fig. 6. Internal oriented stress in steel 0.15C-1Cr-5Ni-1Mo-1V. ● – experimental data [39], ○ – our calculations. $N^* = 10^{12}$ cm^{-2}, $\alpha \approx 0.2 - 0.3$.

Fig. 7. Dislocation density under loading of steel 0.34C-1Cr-3Ni-1Mo-1V. ● – experimental data [17], ○ – our calculations.

CONCLUSIONS

The stress relaxation model with the dislocation kinetics of plastic shears was proposed for the mechanical response of the elastoplastic medium under load. The model included two independent strain rates and implemented the basic principle that at each time point, each particle of the loaded material and the whole material were in the dynamic equilibrium between the applied load and elastoplastic response of the material, its ability to generate deformation defects and release stresses. It was assumed that each stress tensor component changed in accordance with the stress intensity rule. The proposed model described well the material sensitivity to the load rate both at each point and the whole material. The theoretical stress-strain curves demonstrated the sharp yield point (upper), stress release to the lower yield point, and successive strain hardening. The stress relaxation was well-pronounced, including continuous plastic deformation at the initial stages of relaxation, when stresses were rather high. The proposed model described the nonideal Bauschinger effect and cyclic deformations in the minor cycle region. The model was tested in the region of low inelastic deformations of 10–15% until the formation of the deformation substructures (clusters, cells, striples). All the model parameters were selected in accord with the independent experimental data on observations of the dislocation structures. That is why the proposed model was in essence microscopic. Its macroscopic response to the load was fully determined by the evolution of the dislocation structure.

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