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COMPARATIVE ANALYSIS OF METHODS FOR OBTAINING THE YARKOVSKY EFFECT PARAMETER FROM OBSERVATIONS

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The paper presents the Yarkovsky effect parameter determined by fitting orbits of some asteroids with small perihelion distances using two methods based on minimization of the least square errors. A comparative analysis of the obtained values showed good agreement between the results of these methods.

Keywords: numerical simulation, asteroids with small perihelion distances, Yarkovsky's effect.

INTRODUCTION

Nowadays the Yarkovsky effect draws an increasing attention of experts in celestial mechanics [1, 2] because of the following factors: on the one hand, the recent accuracy of observations has significantly increased, and on the other hand, the requirements to the accuracy of predicting object motion, in particular, for asteroids dangerous to the Earth, have also increased. Among them are objects with small perihelion distance (less than 0.15 ua), because they can approach unnoticed to the Earth from the side of the Sun. Owing to a close passage of such objects near the Sun, the Yarkovsky effect can influence significantly on these bodies, since it gives to asteroids the additional acceleration due to thermal radiation of the surface heated in the daytime and cooled at night. However, the study of the Yarkovsky effect is a complex problem, since it depends on a number of physical properties of asteroids and parameters of their revolution known only for a very small number of objects.

The present work analyzes the methods of definition of the effect, their program implementations, and approbations for some objects with small perihelion distances. In Section 1, the research method is shortly described. Section 2 is devoted to the program implementation of the employed methods. In Section 3, the data on the asteroids chosen for our analysis of the methods under study are presented. Results of our analysis are given in Section 4.

1. METHODS OF DETERMINING THE YARKOVSKY EFFECT PARAMETER

As already mentioned above, the physical properties and the parameters of asteroid revolution necessary for the account of the Yarkovsky effect are known only for a small number of objects. For other small bodies, the only possibility is fitting of the parameters based on observations. Proceeding from the assumption that the Yarkovsky effect is inversely proportional to the squared distance from the asteroid to the Sun [1], we can write the perturbing acceleration caused by this effect in the following form:

$$a = \frac{A}{r^2},$$

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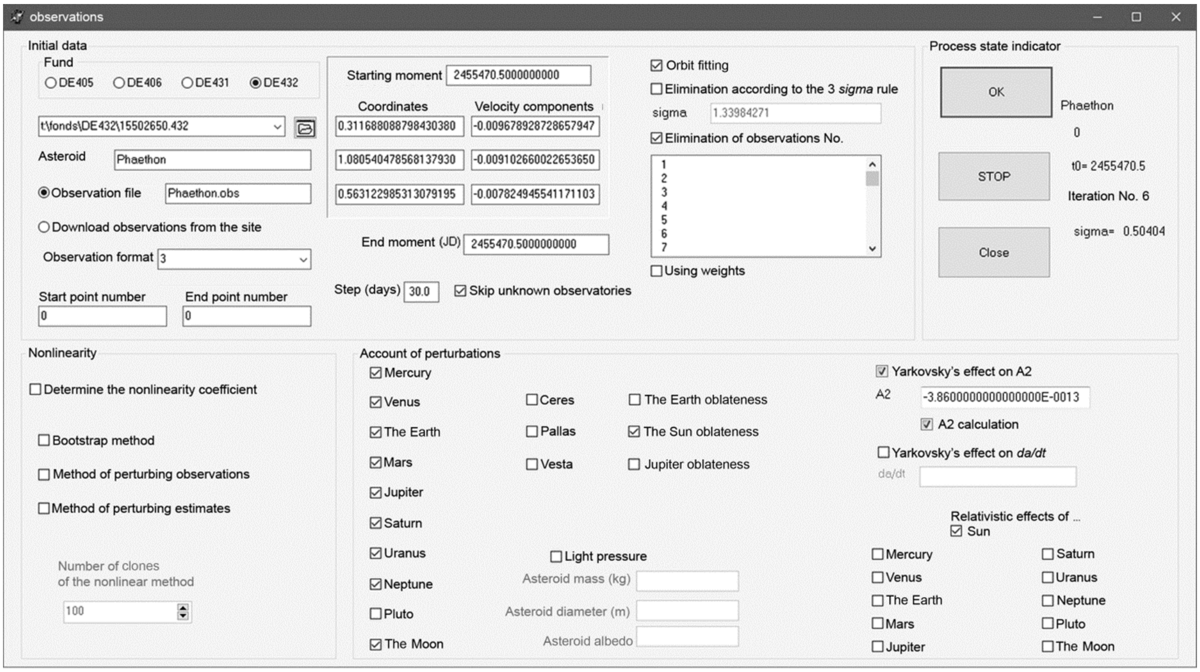

Fig. 1. Screenshot of the subsystem *Observations*.

where r is the heliocentric distance of the asteroid, and A is the Yarkovsky effect parameter. Unfortunately, with modern accuracy of observations, only the most significant acceleration component – transverse acceleration [2] designated by A_2 – can be found. The value of this parameter is determined by minimizing the errors of observation representation. Below we consider two methods of solving this problem.

In the first method (M1), we vary the parameter A_2 within the chosen interval with a preset step. For each parameter value, the least squares problem is solved, and the mean square error of observation representation σ [3] is calculated. The observations are preliminary rejected based on the *three sigma* rule for the model disregarding the Yarkovsky effect, and the obtained set of observations is then used to vary values of the parameter A_2. The value of the parameter of the transverse acceleration A_2 corresponding to the minimal σ is the desired parameter. A disadvantage of the method is that we are limited by the preset range of values and the discrete step. In addition, multiple application of the least squares method requires a long computational time.

The second method consists in determining the parameter A_2 by joint integration of the equations of motion, the equation for the parameter A_2, and the equations in partial derivatives [2]; in this case, the parameter A_2 is estimated together with the coordinates and the velocity components. An advantage of this method is not only the computation speed, but also the obtained error value. The second method is implemented in two ways. In the first way (M2.1), the object orbit is preliminary refined based on the model of motion disregarding the Yarkovsky effect. As a result, the observations are rejected based on the *three sigma* rule and then the Yarkovsky effect parameter A_2 is calculated for the obtained set of observations and the model of motion with account of the Yarkovsky effect. In the second way (M2.2), we obtain estimates of the coordinates, velocities, and parameter A_2 in the first step from all observations by joint integration of the equations of motion, the equations for the parameter A_2, and the equations in partial derivatives. Then on the basis of these estimates, we reject observations and find the refined value of the desired parameter A_2.

2. PROGRAM IMPLEMENTATION

Previously we developed the program complex *IDA* [4] which allows a comprehensive study of the asteroid dynamics to be performed. In the present work, we have added to this program complex the possibility of determining the parameter A_2 by the two above-described methods. As an example, Fig. 1 shows the screenshot of the modified

TABLE 1. Orbital Elements of the Asteroids

Object	a, ua	e	i, deg.	q, ua
431760 2008 HE	2.261150	0.950441	9.828476	0.112059
425755 2011 CP4	0.911391	0.870328	9.455386	0.118181
2017 AF5	2.479031	0.949752	20.90706	0.124566
2007 PR10	1.231907	0.892377	20.92459	0.132581
3200 Phaethon	1.271339	0.889879	22.26036	0.140001

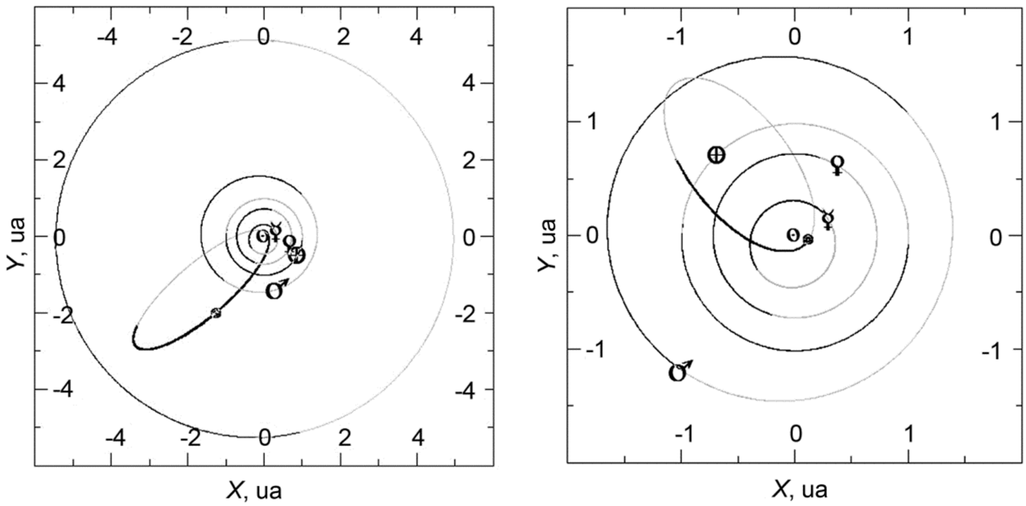

Fig. 2. Projections of the orbits of asteroids 431760 2008 HE (*a*) and 425755 2011 CP4 (*b*) on the ecliptic plane.

subsystem called *Observations* intended to refine the asteroid orbits based on positional observations, to consider the nonlinearity coefficient, and to construct the initial confidence regions using nonlinear methods.

3. DATA ON THE ASTEROIDS

To analyze the examined methods, some asteroids with small perihelion distances were chosen. The orbital elements of the examined objects given in Table 1 were borrowed from the Bowell catalogue [5]. Here a is the semi-major axis, e is the eccentricity, i is the inclination, and q is the perihelion distance. The data presented in Table 1 are sorted in the ascending order of the perihelion distance. Figure 2 shows the projections of orbits of asteroids 431760 2008 HE and 425755 2011 CP4 as well as of orbits of some planets on the ecliptic plane.

4. NUMERICAL RESULTS

The transverse acceleration parameters A_2 for the examined asteroids were obtained by different methods. The number of observations N and the measured interval ΔT are given in Table 2. Observations were borrowed from the website of Minor Planet Center (https://minorplanetcenter.net/). The period P of orbiting of each asteroid around the Sun is also given for a comparison. According to the methods M1 and M2.1, the observations were eliminated disregarding the Yarkovsky effect; therefore, the sets of observations coincided. In the method M2.2, measurements

TABLE 2. Main Parameters of the Objects

Object	ΔT, days	P, days	N (total)	N (M1)	N (M2.1)	N (M2.2)
431760 2008 HE	2560	1241.9	209	199	199	200
425755 2011 CP4	5869	317.8	140	134	134	134
2017 AF5	447	1425.8	289	288	288	288
2007 PR10	4020	499.4	54	53	53	53
3200 Phaethon	13085	523.5	5110	5044	5044	5042

TABLE 3. Results of Determining the Parameter A_2

Object	A_2, ua/day^2 (M1)	A_2, ua/day^2 (M2.1)	A_2, ua/day^2 (M2.2)	A_2, ua/day^2 (NASA website)
431760 2008 HE	$1.020 \cdot 10^{-13}$	$1.025 \cdot 10^{-13}$ $\pm 1.148 \cdot 10^{-13}$	$1.307 \cdot 10^{-13}$ $\pm 1.160 \cdot 10^{-13}$	–
425755 2011 CP4	$4.000 \cdot 10^{-14}$	$3.935 \cdot 10^{-14}$ $\pm 3.319 \cdot 10^{-14}$	$3.935 \cdot 10^{-14}$ $\pm 3.325 \cdot 10^{-14}$	$5.830 \cdot 10^{-14}$ $\pm 1.731 \cdot 10^{-14}$
2017 AF5	$3.362 \cdot 10^{-12}$	$3.364 \cdot 10^{-12}$ $\pm 7.649 \cdot 10^{-12}$	$3.364 \cdot 10^{-12}$ $\pm 7.649 \cdot 10^{-12}$	–
2007 PR10	$-3.860 \cdot 10^{-13}$	$-3.858 \cdot 10^{-13}$ $\pm 1.395 \cdot 10^{-13}$	$-3.858 \cdot 10^{-13}$ $\pm 1.402 \cdot 10^{-13}$	–
3200 Phaethon	$-1.100 \cdot 10^{-14}$	$-1.193 \cdot 10^{-14}$ $\pm 2.779 \cdot 10^{-15}$	$-8.432 \cdot 10^{-15}$ $\pm 2.797 \cdot 10^{-15}$	$-5.445 \cdot 10^{-15}$ $\pm 5.919 \cdot 10^{-16}$

were rejected taking into account the Yarkovsky effect; therefore, the sets of observations for asteroids 431760 2008 HE and 3200 Phaethon slightly differed.

Values of the transverse acceleration parameter A_2 determined by both methods for the examined asteroids are presented in Table 3. In the first method, we first considered the interval $[-10^{-12}, 10^{-12}]$ ua/day^2 with a step of 10^{-15} ua/day^2 for all asteroids, but we failed to identify a minimum for asteroid 2007PR10 during this period; therefore, we extended the interval under study to $[-10^{-12}, 10^{-11}]$ ua/day^2. The parameters A_2 obtained by the second method using the least squares technique are presented in Table 3 with their mean square errors.

As an example, Fig. 3 shows the plots illustrating changes in the mean squared errors of observation representation σ and the confidence interval Δr depending on the value of the parameter A_2 for asteroids 431760 2008 HE (*a*) and 425755 2011 CP4 (*b*). Account of the Yarkovsky effect can change sizes and positions of the initial confidence region. Figure 4 illustrates the initial confidence region (10 thousand clones) for asteroid 431760 2008 HE projected onto the ecliptic plane. Here the region constructed for the model of motion disregarding the Yarkovsky effect is shown in grey color, and the black color shows the corresponding region for the model of motion with account of the Yarkovsky effect. For descriptive reasons, the boundaries of the regions are schematically approximated by ellipses. The way of constructing the initial confidence region was described by us in [6]. From Fig. 4 it can be seen that the confidence region is significantly displaced under the influence of the Yarkovsky effect.

As can be seen from Table 3, the results obtained by the methods M1 and M2.1 practically coincide. The insignificant difference between the estimates obtained by the method M2.2 for asteroids 431760 2008 HE and 3200 Phaethon is explained by different sets of observations. However, it should be noted that these estimates are within the uncertainty limits obviously determined by the measurement interval. The observation interval for asteroid 2017AF5 is significantly shorter than the orbital period, thereby causing considerable uncertainty in the determination of the coefficient A_2. Asteroids 2007PR10 and 3200 Phaethon were observed during several revolutions; as a result, the uncertainty in the estimated Yarkovsky effect parameter was several times less than its value.

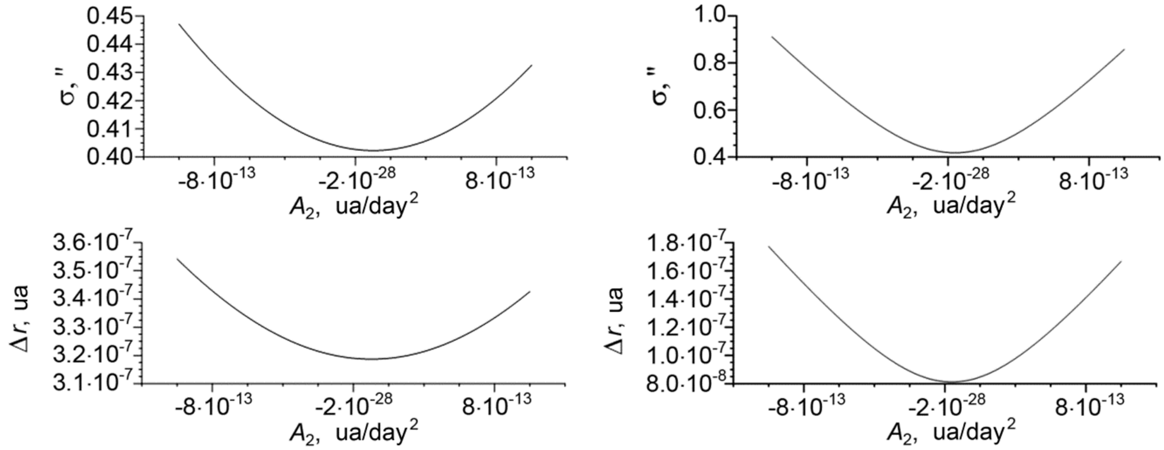

Fig. 3. Changes in the mean square error of observation representation σ and the confidence region size Δr depending on the parameter A_2 for asteroids 431760 2008 HE (*a*) and 425755 2011 CP4 (*b*).

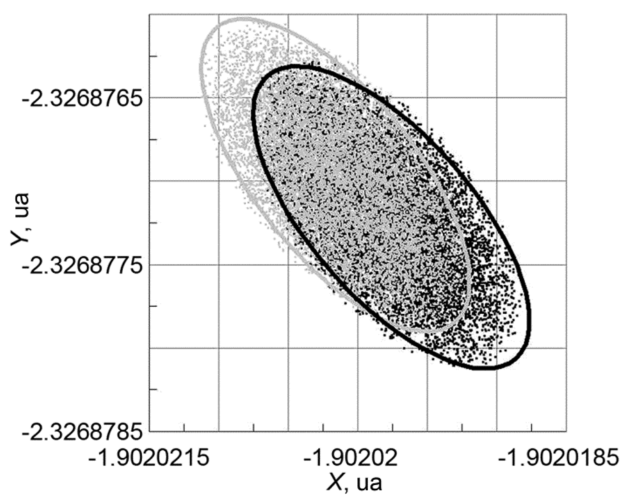

Fig. 4. Initial confidence regions for asteroids 431760 2008 HE with (black color) and without account (grey color) of the Yarkovsky effect.

On the NASA website (https://ssd.jpl.nasa.gov), values of the parameter A_2 for two objects 425755 2011 CP4 and 3200 Phaethon (see the last column in Table 3) are given. We note that results of radar observations are available only for these objects. Within the limits of the above-indicated uncertainty, the given values are in agreement with our data. The greatest difference is observed for asteroid 3200 Phaethon, which is explained by different numbers of object observations: 5044 and 5042 observations in our case and 4362 on the NASA website. For asteroid 425755 2011 CP4, our results were based on 134 observations, and those given on the NASA website were based on 144 observations. In addition, in our work we used only positional observations. In the future, we plan to upgrade the software to consider radar observations as well.

CONCLUSIONS

Thus, two methods of determining the parameter A_2 of the Yarkovsky effect have been considered in this work. In both cases, the parameter was determined by minimization of the mean square error of observation representation. In the first method, the problem was solved by enumeration of the A_2 values, and in the second method, this parameter was included in the number of estimated parameters. The proposed algorithms were previously implemented in the program complex *IDA*. A comparison of the methods demonstrated that they yield close results within the obtained uncertainty. For two objects, the satisfactory agreement was obtained with the NASA data. Based on the results obtained, we can conclude that the efficiency of the second method is higher. In the future we plan its application to the entire class of objects with small perihelion distances.

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REFERENCES

1. D. Farnocchia *et al*., Icarus, **224**, No. 1, 1–13 (2013).
2. A. I. Panasenko and Yu. A. Chernetenko, Trudy Inst. Prikl. Astron., No. 31, 59–65 (2014).
3. T. Yu. Galushina and O. N. Letner, Russ. Phys. J., **63**, No. 1 (2020).
4. T. Galushina, L. Bykova, O. Letner, and A. Baturin, Astron. Comput., **29** (2019).
5. E. Bowell, K. Muinonen, and L. H. Wasserman, in: Asteroids, Comets, Meteors, Kluwer, Dordrecht (1994), pp. 477–481.
6. O. M. Syusina, A. M. Chernitsov, and V. A. Tamarov, Astron. Vestn., **46**, No. 3, 209–222 (2012).