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QUASI-TWO-DIMENSIONAL ELECTRON-HOLE LIQUID IN A MAGNETIC FIELD

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An analytical expression is derived for the energy of a three-component electron-hole liquid (EHL) in a magnetic field. Results of calculations of the EHL properties in $Si/Si_{1-x}Ge_x/Si$ *quantum wells are presented. The influence of the magnetic field on the EHL properties and stability depending on the germanium concentration in the SiGe layer is revealed. It is found that the equilibrium electron-hole pair density strongly increases with the magnetic field. It is shown that the dependences of the filling factors of the Landau levels on the magnetic field are shaped as plateaus.*

Keywords: electron-hole liquid, equilibrium density, light and heavy holes.

INTRODUCTION

The electron-hole liquid (EHL) in semiconductors represents a unique system for studying a macroscopic quantum state. Properties and behavior of the three-dimensional EHL have been studied fairly well [1]. The three-dimensional EHL in a magnetic field was investigated experimentally in [2–6] in which it was demonstrated that the equilibrium electron-hole pair density strongly increases with the magnetic field strength. For example, the equilibrium concentration of electron-hole pairs in indium antimode more than doubled when the magnetic field increased from 2.5 to 5.5 T [6]. These experimental values of concentration are in good agreement with results of calculations presented in [6, 7]. We note that the EHL in InAs [6], AlGaAs [4], and severely deformed silicon [5] is formed only in the magnetic field.

The EHL properties are less studied in low-dimensional semiconductor structures. In [8] the possibility of forming the three-component EHL in $Si/Si_{1-x}Ge_x/Si$ was demonstrated. The three-component EHL contains both heavy and light holes, and with increasing germanium content in a quantum hole, the transition occurs to the two-component EHL with heavy holes. In [9] an analytical expression for the energy of quasi-two-dimensional EHL was obtained with allowance for holes of two types. Results of calculations of the properties of quasi-two-dimensional EHL in SiGe quantum wells for the model suggested in [9] agreed fairly well with experimental data presented in [8]. In the present work, this model is generalized for the EHL in a perpendicular magnetic field.

THEORETICAL MODEL

Let us consider the three-component EHL in the magnetic field B perpendicular to the liquid planes. Unlike the three-dimensional EHL, the charge carrier energy in the quasi-two-dimensional case has discrete values:

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$$E_k = E_0 + (k + 1/2)\hbar\omega_c\,, \tag{1}$$

where E_0 is the level of transverse quantization, ω_c is the cyclotron frequency, and k = 0, 1, 2, … For simplicity, we neglect spin and valley splitting; then the maximum carrier density on one Landau level is written as

$$N_{0,i} = 2g_i eB / h\,, \tag{2}$$

where g_i is the number of valleys for the charge carriers of the ith type and $i = e, hh, hl$. Below we take advantage of the exciton system of units in which the energy is measured in units of $Ry_{ex} = e^2/2\varepsilon a_{ex}$ and the length is measured in units of $a_{ex} = \varepsilon\hbar^2/\mu e^2$, where μ is the reduced mass and ε is the dielectric permittivity. In the exciton system of units, the cyclotron energy has the following form:

$$E_{c,i} = \frac{\mu}{m_{d,i}}\frac{2}{L^2}, \tag{3}$$

where $m_{d,i}$ is the mass of the state density, $L = 25.66/(\sqrt{B}a_{ex})$ is the magnetic length, and a_{ex} is the exciton radius, in nm. The longitudinal energy of the charge carriers of the ith type is written as

$$E_{c,i} = \frac{\mu}{m_{d,i}}\frac{1}{L^2}\sum_{k=0}^{k_{m,i}}(2k+1)N_{i,k}\,, \tag{4}$$

where $k_{m,i} = \mathrm{int}(N_i/N_{0,i})$, int($a$) is the whole part of the number a, $N_{i,k} = N_{0,i}$ for $k < k_{m,i}$, and $N_{i,k_m} = N_i - k_{m,i}N_{0,i}$.

We consider that the magnetic field influences only on the longitudinal motion of the charge carriers. Then to calculate the electron-hole pair energy, the results obtained in [9] can be used after replacement of the longitudinal kinetic energy of charge carriers by the energy of charge carriers in the magnetic field given by Eq. (4). Thus, the energy of one electron-hole pair is written in the form

$$\begin{aligned} E_{eh} = {} & -0.69K^{6/5}\left(\frac{m_{z,e}}{\mu}\right)^{1/5} N_e^{2/5} - 0.69K^{6/5}\left(\frac{m_{z,hh}}{\mu}\right)^{1/5}\frac{N_{hh}^{7/5}}{N_e} - 0.69K^{6/5}\left(\frac{m_{z,hl}}{\mu}\right)^{1/5}\frac{N_{hl}^{7/5}}{N_e} \\ & + \frac{\mu}{m_{d,e}}\frac{1}{N_e L^2}\sum_{k=0}^{k_{m,e}}(2k+1)N_{e,k} + \frac{\mu}{m_{d,hh}}\frac{1}{N_e L^2}\sum_{k=0}^{k_{m,hh}}(2k+1)N_{hh,k} + \frac{\mu}{m_{d,hl}}\frac{1}{N_e L^2}\sum_{k=0}^{k_{m,hl}}(2k+1)N_{hl,k}, \end{aligned} \tag{5}$$

where $K = 1.3$, $m_{z,i}$ is the transverse mass, N_i is the two-dimensional charge carrier density, and $N_e = N_{hh} + N_{hl}$. In Eq. (5) the first three terms were taken from [9], and each of them corresponded to the sum of the transverse kinetic and exchange energy of electrons and heavy and light holes, respectively.

To find the light and heavy hole densities, we take advantage of the condition

$$E_{hh} + \frac{\mu}{m_{d,hh}}\frac{2}{L^2}\left(\nu_{hh} + \frac{1}{2}\right) = E_{hl} + \frac{\mu}{m_{d,hl}}\frac{2}{L^2}\left(\nu_{hl} + \frac{1}{2}\right), \tag{6}$$

where $\nu_i = N_i \pi L^2$ is the filling factor of the Landau level for holes, and E_i is the energy level of transverse quantization. For the electrically neutral EHL, $N_e = N_{hh} + N_{hl} = N$, and from Eq. (6) we have

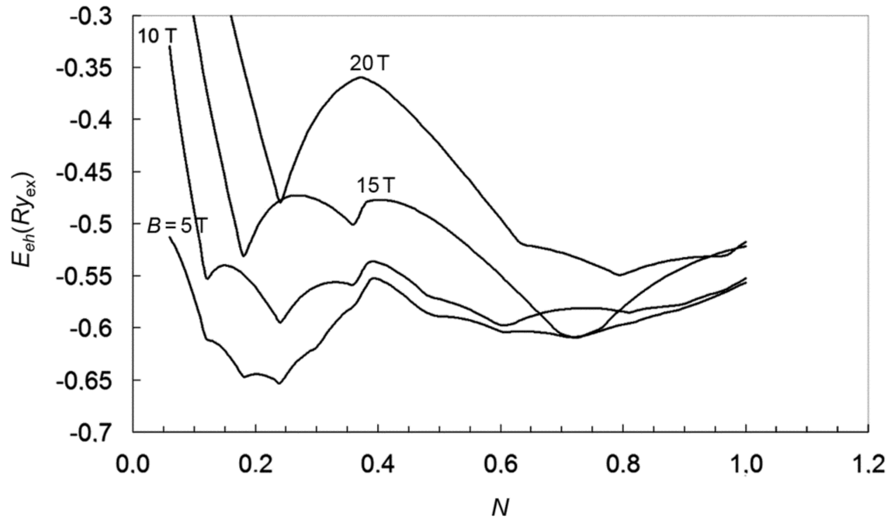

Fig. 1. Dependence of the energy per one electron-hole pair on the two-dimensional pair density at $x = 0.1$.

$$N_{hl} = N_e \frac{m_{d,hl}}{m_{d,hh} + m_{d,hl}} - \frac{m_{d,hh} m_{d,hl}}{m_{d,hh} + m_{d,hl}} \frac{E_{hl} - E_{hh}}{2\pi\mu} + \frac{1}{2\pi L^2} \frac{m_{d,hl} - m_{d,hh}}{m_{d,hh} + m_{d,hl}} . \tag{7}$$

If from Eq. (7) we obtain $N_{hl} < 0$, we must take $N_{hh} = N_e$ and $N_{hl} = 0$. We note that the first two terms in Eq. (7) coincide for the zero magnetic field [9], and the last term characterizes the difference between the cyclotron energies of light and heavy holes.

RESULTS OF INVESTIGATIONS

Calculations were performed for the $Si/Si_{1-x}Ge_x/Si$ structure. By analogy with [9], we used the following parameters: $g_e = 4$, $g_{hh} = g_{hl} = 1$, $m_{z,e} = 0.198m_0$, $m_{d,e} = 0.44m_0$, $m_{z,hh} = 0.28m_0$, $m_{d,hh} = 0.2m_0$, $m_{z,hl} = 0.18m_0$, and $m_{d,hl} = 0.25m_0$ (here m_0 is the free electron mass). The energy difference $E_{hl} - E_{hh}$ was calculated for splitting between light and heavy holes $\Delta_{hl} = 16.6x$. The results of calculations from formula (5) demonstrated that in the magnetic field $B < 1$ T, the dependence of the electron-hole pair energy on the charge carrier density has the same form as for the zero magnetic field presented in [9]. This result is due to the fact that in low magnetic fields, the number of the populated Landau levels is large, and the charge carriers behave as if they are almost free. As the magnetic field increases, its influence on the dependence $E_{eh}(N)$ starts to be manifested. Fractures in the energy curve seen in Fig. 1 are associated with transitions of charge carriers to the next energy level. For example, for $B = 5$ T, the energy minimum is reached at $N = 0.24$, and the neighboring breaks are spaced at $N_{0,hh} = 0.12$ which corresponds to the density of the heavy holes completely filling the Landau level. We note that the equilibrium electron-hole pair density in the magnetic field $B = 5$ T increases approximately by a factor of 1.3 in comparison with the density in the zero magnetic field.

In [9] it was shown that in the SiGe quantum wells with germanium concentration $x > 0.08$, the EHL with electrons and heavy holes is formed. The magnetic field can stabilize EHL with three types of charge carriers at $x > 0.08$. From the results shown in Fig. 1 it can be seen that at $B = 10$ T the curve of the dependence of the electron-hole liquid energy has two minima, and values of the minima are almost equal. The first minimum at smaller N value corresponds to the EHL with heavy holes, and the second minimum corresponds to the EHL with light and heavy holes.

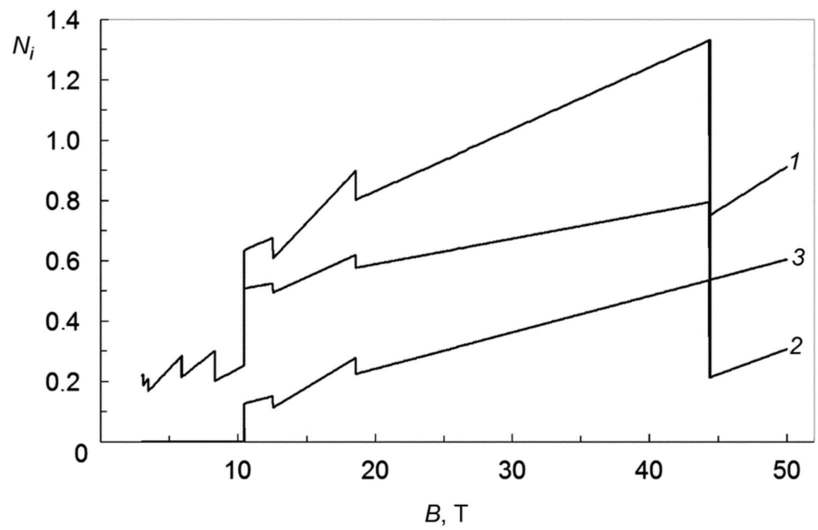

Fig. 2. Dependences of the densities of electrons (curve *1*) and light (curve *2*) and heavy (curve *3*) holes on the magnetic field at $x = 0.1$.

For $B > 10.3$ T, the EHL in the ground state consists of electrons and light and heavy holes, and the density of the electron-hole pairs increases with the magnetic field. We note that, changing the magnetic field, it is possible to control over the transition between the three- and two-component EHLs. The results of calculations for large x values demonstrated that the multicomponent EHL arises in high magnetic field. For example, at $x = 0.12$, the magnetic field B_c above which the three-component EHL becomes stable is equal to 15.1 T, and at $x = 0.14$, $B_c = 30.5$ T.

Figure 2 shows the dependence of the charge carrier density on the magnetic field. It can be seen that at low magnetic fields, the EHL consists of electrons and heavy holes, and with increasing magnetic field, light holes arise whose density is always lower than that of heavy holes. In addition to the occurrence of the transition from two- to three-component EHLs, a sharp growth of the equilibrium electron-hole pair density with increasing magnetic field should be noted. The charge carrier density changes linearly depending on the magnetic field. This phenomenon is visually illustrated by Fig. 3. The dependences of the Landau level filling factors on the magnetic field are shaped as plateaus. For low magnetic fields from 3.5 to 5.9 T, the ground state is the state with the completely filled Landau levels for electrons ($\nu_e = N_e \pi L^2 / g_e = 1$) and holes ($\nu_{hh} = 4$). The difference between the filling factors for electrons and holes is due to the fact that the valley splitting of electron energy levels is disregarded in this model. With increasing magnetic field, the state with completely filled energy level of holes and partially filled energy level of electrons becomes energetically favorable. As indicated above, the ground state at $x = 0.1$ and $B > 10.3$ T is the three-component EHL, and at the magnetic field from 10.3 to 12.5 T (Fig. 3),the Landau level appears completely filled by light holes. With further increase in the magnetic field, the Landau level for electrons is completely filled, and then the level of light holes is completely filled again. In the magnetic field close to 50 T, the Landau level for heavy holes in the ground state will be completely filled. Such alternation of complete filling of the Landau levels for light and heavy holes is due to the fact that the longitudinal masses of light and heavy holes are close to each other.

Figure 4 shows the dependence of the two-dimensional electron-hole pair density on the magnetic field for the indicated x values. At low magnetic fields, the density oscillations are seen caused by changes in the population of the Landau levels. Linear dependences of the magnetic field density are due to the fact that in the ground state, the Landau level completely filled with charge carriers can always be found. Sharp jumps in the density correspond to the transition to a new completely filled Landau level. For example, at $x = 0.04$, the jump in the density in the vicinity of 25 T corresponds to the transition from the state with the filling factor $\nu_{hh} = 2$ to the state with $\nu_{hl} = 1$. At $x = 0.5$, the EHL is two-component, and at $B > 14.5$ T, the electrons and heavy holes are in the lower Landau level. We note that in the

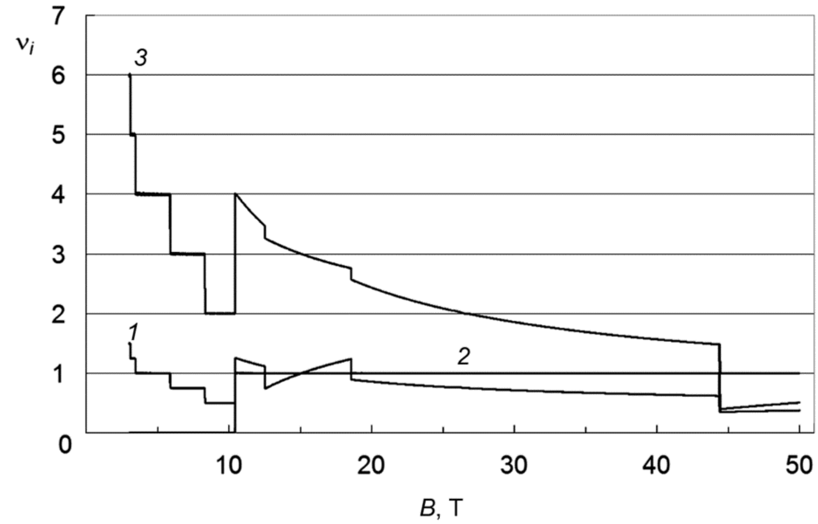

Fig. 3. Dependences of the Landau level filling factors for electrons (curve *1*) and light (curve *2*) and heavy (curve *3*) holes on the magnetic field at $x = 0$.

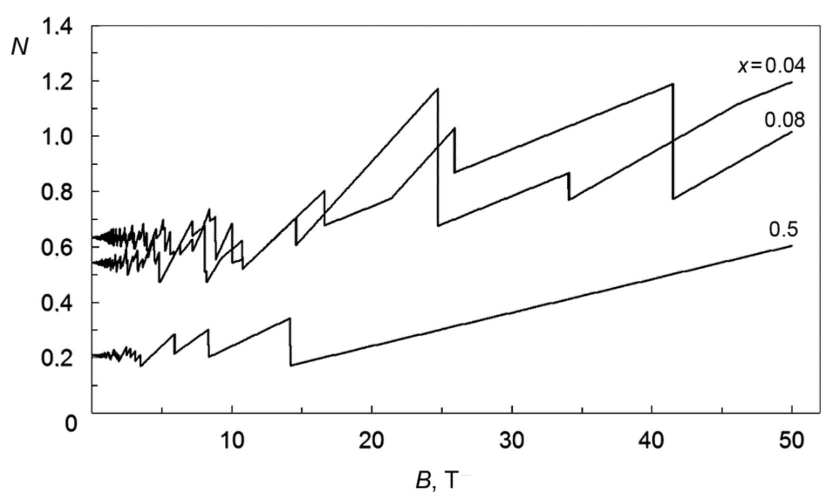

Fig. 4. Dependences of the electron-hole pair density on the magnetic field.

examined range of the magnetic fields, the electron-hole pair density doubles for the three-component EHL and increases almost three folds for the two-component EHL.

CONCLUSIONS

In this work, the analytical expression for the energy of the three-component EHL in a magnetic field has been obtained. The energy and the equilibrium density of the electron-hole pairs were calculated for the Si/SiGe/Si structure. It was demonstrated that with increasing magnetic field, the transition from the two- to three-component EHLs is observed, and the equilibrium density strongly increases. Changing the magnetic field, it is possible to control over the transition between the three-component and two-component EHLs. The dependences of the Landau level filling factors on the magnetic field were shaped as plateaus, i.e., at least for one type of charge carriers, a part of the Landau levels was completely filled. It was demonstrated that when magnetic fields increase from 0 to 50 T, the electron-hole pair density doubled for the three-component EHL and increased almost three folds for the two-component EHL.

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