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COMPOSITE PSEUDOCLASSICAL MODELS OF QUARKS

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Composite models of quarks are proposed, analogous to composite models of leptons. A model-based explanation of the appearance of generations of fundamental particles in the Standard Model is given. New empirical formulas are proposed for the quark masses, modifying Barut's well-known formula.

Keywords: quark generations, quark masses, Barut's formula, Grassmann algebra, supersymmetry, pseudoclassical mechanics.

The appearance of several generations of fundamental particles in the Standard Model within the framework of pseudoclassical mechanics (mechanics based on anticommuting variables) can be described as a result of the composite structure of higher generations of particles, not contradicting their point-like ($<10^{-19}$ m) sizes. This was demonstrated in [1] for leptons. The aim of the given paper is a generalization of the ideas presented in [1] to the case of quarks.

The action for a system of particles proposed in [1] has the form

$$S = \frac{1}{2}\int ds \int d\tau E^{kl} g_{\mu\nu} DX_k^{\mu} DDX_l^{\nu}\,, \tag{1}$$

where s is the even component and τ is the odd component of supertime, $D = -i\tau\partial_s + \partial_\tau$ is the covariant derivative,

$$X_k^{\mu}(s,\tau) = x_k^{\mu}(s) + i\theta_k^{\mu}(s)\tau \tag{2}$$

is the kth supercoordinate, and x_k^{μ} and $i\theta_k^{\mu}$ are its spatial component and spin component, respectively. The metric tensor $g_{\mu\nu}$ has the signature $(-+++)$, and the composition matrix E^{kl} is symmetric and has dimensions $N \times N$. Implicit summation is realized over both Greek and Latin indices. The Greek indices run through the values 0, 1, 2, 3, and the Latin indices run through the values 1, 2, … N. For $N = 1$ action (1) coincides with the action for a Di Vecchia–Ravndal particle [2]. In the case when the composition matrix is degenerate, i.e., its elements are chosen such that the Lagrangian density of the system of particles coincides with the Lagrangian density of one particle:

$$\mathcal{L} = E^{kl} g_{\mu\nu} DX_k^{\mu} DDX_l^{\nu} = g_{\mu\nu} DY^{\mu} DDY^{\nu}\,, \tag{3}$$

action (1) describes one free composite particle with spin 1/2, consisting of N copies of a Di Vecchia–Ravndal particle. The expression for the mass of such a particle obtained in [1] can be rewritten in the form

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$$m(N) = m_0(1 + aF(N)) . \tag{4}$$

Here m_0 and $aF(N)$ are free parameters of the model: m_0 is a parameter with units of mass, a is a dimensionless parameter, and $F(N)$ determines the elements of the composition matrix E_{kl} :

$$E = \left(E_{kl}\right) = \begin{pmatrix} 1 & a & (F(N)-1))a & \ldots \\ a & a^2 & (F(N)-1))a^2 & \ldots \\ (F(N)-1))a & (F(N)-1))a^2 & (F(N)-1))^2 a^2 & \ldots \\ \ldots. & \ldots & \ldots. & \ldots \end{pmatrix}. \tag{5}$$

In [1] formula (4) was identified with Barut's formula [3] for leptons

$$m(N) = m_e\left(1 + \frac{3}{2\alpha}\sum_{k=0}^{N-1} k^4\right), \tag{6}$$

which, as is well known [4], gives good agreement with the experimental data. Thus, in the language of pseudomechanics action (1) describes massive leptons if the values of the free parameters are the following:

$$m_0 = m_e\,,\ a = 3/2\alpha \approx 205.5\,,\ F(N) = \sum_{k=0}^{N-1} k^4\,, \tag{7}$$

where m_e is the mass of the electron and $\alpha \approx 1/137$ is the fine structure constant.

Since quarks are also fermions with spin 1/2, action (1) can be used to construct pseudomechanical models of the corresponding quantum particles, i.e., free quarks. The participation of quarks in electromagnetic and strong interactions requires the introduction of additional free parameters for them – charges: the electric charge and the color charge. The action in form (1) thus describes the motion of a free quark, which, as is well known, is unobservable due to confinement. Only colorless combinations of quarks, realized in the form of hadrons, are observed. Therefore, in contrast to leptons, when the color interaction is introduced into action (1), the region of direct application narrows to the classical dynamics of partons, which is usually not even discussed in connection with the obvious problematic nature of using a classical representation inside a hadron.

Let us discuss, nevertheless, some consequences of such a model, which draw it closer to the more realistic composite lepton model [1]. Since the composition matrix [5] is degenerate, the number of degrees of freedom of the entire composition remains as before: four boson and four fermion degrees, i.e., a composite particle, in contrast to a quark, does not possess internal dynamics. In fact, we are dealing here with one particle, but realized in the form of several copies distributed over supertime. The non-coincidence of *time-orderings* of copies is necessary by virtue of the Pauli principle. An electric charge and a color are assigned to every composition. To explain such a choice, let us consider the particle – world line correspondence. It is usually assumed that charges are an attribute of a particle which corresponds to its motion along a world line from the past to the future. The charges of an antiparticle are determined by its motion in the reverse direction – from the future into the past. This correspondence can be reversed and it can be assumed that the charges are attributes associated with the world line of the particle and its direction, so how many copies of a particle *live* on a world line is thus unimportant. For example, a *red b-antiquark* can be described as three *u*-quarks living on a *red world line running from the future to the past*.

To construct concrete composite models of quarks, it is necessary to assign the free parameters, analogous to the lepton parameters given by Eq. (7). Barut's empirical formula is also well-known for quarks. This is the same

TABLE 1. Experimental and Calculated Quark Masses (MeV)

Particle	$\underline{N}$	N	Experimental values	Values calculated according to formulas				
				(6)	(8)	(10)	(9)	(11)
u-quark	1	1	1.8–3.0	0.068	0.61	0.685	–	–
d-quark	2	1	4.5–5.3	14.1	–	–	5.65	6.46
s-quark	3	2	90–100	239	126	141	–	–
c-quark	4	2	1 250–1 300	1 378	–	–	1167	1 336
b-quark	5	3	4 630–4 690	4 978	2132	4646		
t-quark	6	3	172 500–173 920	13 766	–	–	19743	171 523

formula (formula (6)), where the electron mass m_e is replaced by some estimate of the mass of the *u*-quark: $m_u = 0.068$ MeV, and N is replaced by $\underline{N} = 1, ..., 6$. This formula gives only qualitative agreement with experiment [5] (see the table). Since the charges in our interpretation are an attribute that is common for all particles forming the generations, it is natural to construct its own family of generations for each charge. This gives two families for each color. One of them begins with a *u*-quark (electric charge $+2e/3$), and the other – with a *d*-quark (electric charge $-e/3$). Thus, it is logical to divide formula (6) into two series, and for each of them to choose its own estimate of the mass of a first-generation quark:

$$m(N) = m_u\left(1+\frac{3}{2\alpha}\sum_{k=0}^{N-1}k^4\right), \tag{8}$$

$$m(N) = m_d\left(1+\frac{3}{2\alpha}\sum_{k=0}^{N-1}k^4\right). \tag{9}$$

This improves the estimate of the masses of heavy quarks somewhat (see Table 1). It is possible to obtain a more accurate estimate if we modify empirical formulas (8) and (9) to

$$m(N) = m_u\left(1+\frac{3}{2\alpha}\sum_{k=0}^{N-1}k^5\right), \tag{10}$$

$$m(N) = m_d\left(1+\frac{3}{2\alpha}\sum_{k=0}^{N-1}k^7\right). \tag{11}$$

Choosing formulas (10) and (11) as more adequate, we obtain an estimate of the matrix elements and free parameters for the composite models of quarks:

$$m_0 = m_u = 0.685 \text{ MeV},\ m_0 = m_d = 6.46 \text{ MeV},\ a = 3/2\alpha \approx 205.5,$$

$$E_s = E_c = \begin{pmatrix} 1 & a \\ a & a^2 \end{pmatrix},\ E_b = \begin{pmatrix} 1 & a & 2^5 a \\ a & a^2 & 2^5 a^2 \\ 2^5 a & 2^5 a^2 & 2^{10} a^2 \end{pmatrix},\ E_t = \begin{pmatrix} 1 & a & 2^7 a \\ a & a^2 & 2^7 a^2 \\ 2^7 a & 2^7 a^2 & 2^{14} a^2 \end{pmatrix}.$$

Regardless of the specific choice of parameters, it is possible to assert that the mechanism for the appearance of generations is general, both for leptons and for quarks, i.e., in the language of pseudoclassical mechanics this implies a composite structure of the higher generations.

A pseudoclassical mechanism, occupying an intermediate position between classical and quantum mechanics, allows us to construct classical models of quantum phenomena, such as spin [6, 7], the spin-orbital interaction [8, 9], *quantum jitter* [2], etc. To such a list, it is now possible to add the presence of several generations of fundamental particles. The presence of non-interacting copies of particles united into a composite poses the question of their nature. Central here, apparently, is the question of the structure of supertime, on which the coordinates of the composite particle have been assigned. Since only the even component of super-time s is associated with the proper time of a composite particle, while the odd component τ is unobservable [10], it may be supposed that the copies of a particle associated with the odd component bear a virtual character. Virtual particles, being a commonplace attribute of quantum theory, possibly also have a reflection in pseudoclassical mechanics.

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