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RADIATIVE PROCESSES IN GRAPHENE AND SIMILAR NANOSTRUCTURES IN STRONG ELECTRIC FIELDS

S. P. Gavrilov[1,2] and D. M. Gitman[1,3,4]

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Low-energy single-electron dynamics in graphene monolayers and similar nanostructures is described by the Dirac model, being a 2+1 dimensional version of massless QED with the speed of light replaced by the Fermi velocity $v_F \simeq c/300$. Methods of strong-field QFT are relevant for the Dirac model, since any low-frequency electric field requires a nonperturbative treatment of massless carriers in the case it remains unchanged for a sufficiently long time interval. In this case, the effects of creation and annihilation of electron-hole pairs produced from vacuum by a slowly varying and small-gradient electric field are relevant, thereby substantially affecting the radiation pattern. For this reason, the standard QED text-book theory of photon emission cannot be of help. We construct the Fock-space representation of the Dirac model, which takes exact accounts of the effects of vacuum instability caused by external electric fields, and in which the interaction between electrons and photons is taken into account perturbatively, following the general theory (the generalized Furry representation). We consider the effective theory of photon emission in the first-order approximation and construct the corresponding total probabilities, taking into account the unitarity relation.

Keywords: quantum radiation theory, electric field, graphene.

INTRODUCTION

Low-energy single-electron dynamics in graphene monolayers at the charge neutrality point and similar nanostructures is described by the Dirac model, being a 2+1 dimensional version of massless QED with the Fermi velocity $v_F \simeq 10^6\,\text{m/s}$ playing the role of the speed of light in relativistic particle dynamics. There are actually two species of fermions in this model, corresponding to excitations about the two distinct Dirac points in the Brillouin zone of graphene (a distinct pseudospin is associated). There also is a (real) spin degeneracy factor 2. We consider an infinite flat graphene sample on which a uniform electric field is applied, directed along the x axis on the plane of the sample. We assume that the applied field is the *T*-constant electric field that exists during a macroscopic large time period T comparing to the characteristic time scale $\Delta t_{\text{st}} = \left(e|E|v_F/\hbar\right)^{-1/2} \gg 0.24\ \text{fs}$, $10^{-12}\,\text{s} \gtrsim T > \Delta t_{\text{st}}$. This field turns on to E at $-T/2 = t_{\text{in}}$ and turns off to 0 at $T/2 = t_{\text{out}}$.

The electromagnetic field is not confined to the graphene surface, $z = 0$, but rather propagates (with the speed of light c) in the ambient $3+1$ dimensional space-time, where z is the coordinate of axis normal to the graphene plane. Thus, we have the so-called reduced $\text{QED}_{3,2}$ with distinct velocities for relativistic dynamics of charged particles

[1]National Research Tomsk State University, Tomsk, Russia, [2]Herzen State Pedagogical University of Russia, Saint Petersburg, Russia, e-mail: gavrilovsergeyp@yahoo.com; gavrilovsp@herzen.spb.ru; [3]P. N. Lebedev Physical Institute of the Russian Acedemy of Sciences, Moscow, Russia; [4]Instituto de Fisica, Universidade de São Paulo, São Paulo, Brazil, e-mail: gitman@@if.usp.br. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 11, pp. 123–126, November, 2016. Original article submitted July 7, 2016.

and clasical and quantum electromagnetic fields. Low-frequency ($\omega \lesssim T^{-1}$) crossed electromagnetic field is radiated in the direction orthogonal to the graphene plane by a mean current of pairs created from vacuum (see [1] for details). High-frequency ($\omega \gg T^{-1}$) emission (absorption) of a photon occures due to a particle state transition. For example, this can be (1) emission by an electron in initial state or (2) emission with pair creation from vacuum.

Methods of strong-field QED are relevant for the Dirac model, since any low-frequency electric field requires a nonperturbative treatment of massless carriers in case it remains unchanged for a sufficiently long time interval, $T > \Delta t_{\rm st}$ In particular, the effect of particle creation is crucial for understanding the conductivity of graphene, especially in the so-called nonlinear regime. In this regime, the effects of creation and annihilation of electron-hole pairs produced from vacuum by a slowly varying and small-gradient electric field are relevant, thereby substantially affecting the radiation pattern. For this reason, the standard QED text-book theory of photon emission (relevant assuming that vacuum is stable) cannot be of help.

EFFECTIVE PERTURBATION THEORY OF THE PHOTON EMISSION

We construct the Fock-space representation of the Dirac model, which takes exact accounts of the effects of vacuum instability caused by external electric fields, and in which the interaction between electrons and photons is taken into account perturbatively, following the general theory (the generalized Furry representation) [2]. We use boldface symbols for three-dimensional vectors and symbols with arrows for in-plane components, for example, $\vec{r} = (x, y)$. In the usual dipole approximation, z-dependence of the QED Hamiltonian can be integrated out and we obtain the Hamiltonian of the electron-photon interaction as

$$\widehat{\mathcal{H}}_{\rm int} \approx \int \vec{j}_{\rm in}(t,\vec{r})\,\hat{\vec{A}}(t,\boldsymbol{r})\Big|_{z=0} d\vec{r}\,,\ \vec{j}_{\rm in}\left(t,\vec{r}\right) = -\frac{ev_{\rm F}}{2c}\left[\hat{\Psi}^{\dagger}(t,\vec{r}), \gamma^0\vec{\gamma}\hat{\Psi}(t,\vec{r})\right]_{-}, \tag{1}$$

where quantum fields $\hat{\Psi}(t,\vec{r})$ and $\hat{\Psi}^{\dagger}(t,\vec{r})$ obey both the Dirac equation with the potential $\vec{A}^{\rm ext}(t,\vec{r})$ and the standard equal time anticommutation relations. We decomposed quantum electromagnetic field in the interaction representation into terms of the annihilation and creation operators of photons, $C_{\boldsymbol{k}\vartheta}$ and $C^{\dagger}_{\boldsymbol{k}\vartheta}$:

$$\hat{\boldsymbol{A}}(t,\boldsymbol{r}) = c\sum_{\boldsymbol{k},\vartheta}\sqrt{\frac{2\pi\hbar}{\varepsilon V\omega}}\,\boldsymbol{\epsilon}_{\boldsymbol{k}\vartheta}\left[C_{\boldsymbol{k}\vartheta}e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} + C^{\dagger}_{\boldsymbol{k}\vartheta}e^{-i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\right], \tag{2}$$

where $\vartheta = 1, 2$ is a polarization index, $\boldsymbol{\epsilon}_{\boldsymbol{k}\vartheta}$ are unit polarization vectors transversal to each other and to the wavevector $\boldsymbol{k}$, $\omega = ck\ \ k = |\boldsymbol{k}|$, V is the volume of the box regularization, and ε is the relative permittivity ($\varepsilon = 1$ for graphene suspended in vacuum).

The in - and out -operators of creation and annihilation of electrons ($a^{\dagger}_n$, a_n) and holes ($b^{\dagger}_n$, b_n) are defined by the two representations of the quantum Dirac field $\hat{\Psi}(t,\vec{r})$ as

$$\hat{\Psi}(t,\vec{r}) = \sum_n\left[a_n(\text{in})\,{}_{+}\psi_n(t,\vec{r}) + b^{\dagger}_n(\text{in})\,{}_{-}\psi_n(t,\vec{r})\right] = \sum_n\left[a_n(\text{out})\,{}^{+}\psi_n(t,\vec{r}) + b^{\dagger}_n(\text{out})\,{}^{-}\psi_n(t,\vec{r})\right], \tag{3}$$

where ${}_{\zeta}\psi_n(t,\vec{r})$ and ${}^{\zeta}\psi_n(t,\vec{r})$ are in - and out -solutions of the Dirac equation with the potential $\vec{A}^{\rm ext}(t,\vec{r})$ for given quantum numbers n and well-defined sign of frequency ζ either before turning on or after turning off of a field, respectively. They are related by a linear transformation of the form:

$$ {}^{\zeta}\psi_n(t,\vec{r}) = g_n({}_+|^{\zeta})_+\psi_n(t,\vec{r}) + g_n({}_-|^{\zeta})_-\psi_n(t,\vec{r}),\ {}_{\zeta}\psi_n(t,\vec{r}) = g_n\left({}^+|_{\zeta}\right){}^+\psi_n(t,\vec{r}) + g_n\left({}^-|_{\zeta}\right){}^-\psi_n(t,\vec{r}), \tag{4} $$

where g are some complex coefficients. Here the notation $g\,({}^{\zeta'}|_{\zeta}) = g\,({}_{\zeta}|^{\zeta'})^*$ is used. These coefficients obey the unitarity relations which follow from the orthonormalization and completness relations for the corresponding solutions. It is known that all g can be expressed in terms of two of them, e.g. of $g\,({}_+|^+)$ and $g\,({}_-|^+)$. However, even the latter coefficients are not completely independent,

$$ \left|g_n({}_-|^+)\right|^2 + \left|g_n({}_+|^+)\right|^2 = 1. \tag{5} $$

Then a linear canonical transformation (the Bogoliubov transformation) between in - and out - operators which follows from Eq. (3) is defined by these coefficients.

The initial and final states with definite numbers of charged particles and photons can be generally written in the following way:

$$ |\,\mathrm{in}>= C^\dagger \ldots b^\dagger(\mathrm{in})\ldots a^\dagger(\mathrm{in})\ldots|\,0,\mathrm{in}\rangle,\ |\,\mathrm{out}>= C^\dagger \ldots b^\dagger(\mathrm{out})\ldots a^\dagger(\mathrm{out})\ldots|\,0,\mathrm{out}\rangle. $$

The $\mathcal{S}$ matrix or the scattering operator in the first-order approximation with respect to electron-photon interaction (it is exact with respect to an interaction with an external field) is

$$ \mathcal{S} \approx 1 + i\Upsilon^{(1)},\ \Upsilon^{(1)} = -\hbar^{-1}\int_{-\infty}^{\infty}\widehat{\mathcal{H}}_{\mathrm{int}}dt. \tag{6} $$

In general, the emission of a single photon by an electron is accompanied by the creation of $M \geq 0$ electron-hole pairs from the vacuum by the quasiconstant electric field:

$$ \mathcal{P}_M\left(\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right) = \sum_{\{m\}\{n\}}\left[M!(M+1)!\right]^{-1}\left|\langle 0,\mathrm{out}|b_{n_M}(\mathrm{out})\ldots b_{n_1}(\mathrm{out})a_{m_{M+1}}(\mathrm{out})\ldots a_{m_1}(\mathrm{out})C_{\boldsymbol{k}\vartheta}i\Upsilon^{(1)}a_l^\dagger(\mathrm{in})\,|\,0,\mathrm{in}\rangle\right|^2. $$

The probability of transition from the single-electron state characterized by the quantum numbers l with the emission of one photon with given $\boldsymbol{k}$ and ϑ and production of arbitrary number of pairs from the vacuum, that is, the total probability of the emission of the given photon from the single-electron state, is

$$ \mathcal{P}\left(\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right) = \sum_{M=0}^{\infty}\mathcal{P}_M\left(\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right). \tag{7} $$

The probability of the process with the emission of one photon with given $\boldsymbol{k}$, ϑ and the production of $M \geq 1$ arbitrary pairs from the vacuum is

$$ \mathcal{P}_M(\boldsymbol{k},\vartheta) = \sum_{\{m\}\{n\}}(M!)^{-2}\left|\langle 0,\mathrm{out}|b_{n_M}(\mathrm{out})\ldots b_{n_1}(\mathrm{out})a_{m_M}(\mathrm{out})\ldots a_{m_1}(\mathrm{out})c_{\boldsymbol{k}\vartheta}i\Upsilon^{(1)}\,|\,0,\mathrm{in}\rangle\right|^2. \tag{8} $$

The total probability of emission of the given photon from the vacuum and the production of an arbitrary number of pairs from the vacuum is

$$\mathcal{P}(\boldsymbol{k},\vartheta) = \sum_{M=1}^{\infty} \mathcal{P}_M(\boldsymbol{k},\vartheta). \tag{9}$$

The unitary transformation V relates the in- and out- Fock spaces, $|\,\mathrm{in}\rangle = V\,|\,\mathrm{out}\rangle$. This means that we can pass from the basis of the final Fock space to the basis of the initial Fock space and, for example, represent the total probabilty (7) as

$$\mathcal{P}\left(\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right) = \sum_n \left| w_{in}^{(1)}\left(\overset{+}{n};\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right)\right|^2, \; w_{in}^{(1)}\left(\overset{+}{n};\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right) = \langle 0,\mathrm{in}|a_n(\mathrm{in})C_{\boldsymbol{k}\vartheta} i\Upsilon^{(1)} a_l^{\dagger}(\mathrm{in})\,|\,0,\mathrm{in}\rangle. \tag{10}$$

Note that if the number of created pairs is not small, then the matrix element $w_{\mathrm{in}}^{(1)}(\overset{+}{n};\boldsymbol{k}\vartheta|\overset{+}{l})$ is quite distinct from the amlitude of the relative probability for a one-particle transition with emission of a photon,

$$w^{(1)}\left(\overset{+}{n};\boldsymbol{k}\vartheta\middle|\overset{+}{l}\right) = \frac{\langle 0,\mathrm{out}|a_n(\mathrm{out})C_{\boldsymbol{k}\vartheta} i\Upsilon^{(1)} a_l^{\dagger}(\mathrm{in})\,|\,0,\mathrm{in}\rangle}{\langle 0,\mathrm{out}\,|\,0,\mathrm{in}\rangle}.$$

CHARACTERISTICS FOR EMISSION OF A PHOTON BY AN ELECTRON

We apply this theory to calculation of the total probability for emission of a photon by an electron in a constant electric field. We define an orthonormal triple

$$\boldsymbol{k}/k = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \; \boldsymbol{\epsilon}_{\boldsymbol{k}1} = \mathrm{e}_z \times \boldsymbol{k}/\left|\mathrm{e}_z \times \boldsymbol{k}\right|, \quad \boldsymbol{\epsilon}_{\boldsymbol{k}2} = \boldsymbol{k}\times\boldsymbol{\epsilon}_{\boldsymbol{k}1}/\left|\boldsymbol{k}\times\boldsymbol{\epsilon}_{\boldsymbol{k}1}\right|, \tag{11}$$

then $\boldsymbol{\epsilon}_{\boldsymbol{k}1} = (-\sin\phi, \cos\phi, 0)$, $\boldsymbol{\epsilon}_{\boldsymbol{k}2} = (-\cos\theta\cos\phi, -\cos\theta\sin\phi, \sin\theta)$ for $\boldsymbol{k}$ in the upper spatial region, and $k_z \geq 0$. Using the parametrization, $d\boldsymbol{k} = c^{-3}\omega^2 d\omega d\Omega$, we find that the probabilty of emission per unit frequency and solid angle $d\Omega$ is

$$\frac{d\mathcal{P}(\boldsymbol{k}\vartheta|\overset{+}{\vec{p}})}{d\omega d\Omega} = \frac{\alpha}{\varepsilon}\left(\frac{v_{\mathrm{F}}}{c}\right)^2 \frac{\omega\Delta t_{\mathrm{st}}^2}{(2\pi)^2}\left|M_{\overset{+}{\vec{p}'}\vec{p}}\right|^2\Bigg|_{\vec{p}'=\vec{p}-\hbar\vec{k}}, \quad M_{\overset{+}{\vec{p}'}\vec{p}} = v_F^2 S C' C \exp\left(-i\omega\frac{p_x + p_x'}{2eE}\right)$$

$$\times(eE\hbar/v_{\mathrm{F}})^{1/2}\left[(1-i)\zeta p_y' \chi_\vartheta^{1,0} Y_{10} + (1+i)\zeta p_y \chi_\vartheta^{0,1} Y_{01} + p_y' p_y \chi_\vartheta^{1,1} Y_{11} + 2(eE\hbar/v_{\mathrm{F}})^{1/2}\chi_\vartheta^{0,0} Y_{00}\right], \tag{12}$$

$$C = \left(2eE\hbar v_{\mathrm{F}} S\right)^{-1/2}\exp(-\pi\lambda/8), \;\; \lambda = v_{\mathrm{F}} p_y^2/(eE\hbar), \;\; C' = C\big|_{p_y\to p_y'},$$

where $\alpha = e^2/c\hbar$ is the fine structure constant, S is the graphene area, $\chi_\vartheta^{(1+s')/2,(1+s)/2} = U_{s'}^{\dagger}\gamma^0\vec{\gamma}\cdot\vec{e}_{\boldsymbol{k}\vartheta}U_s$, and $\chi_1^{0,0} = -\chi_1^{1,1} = \sin\phi$, $\chi_1^{1,0} = -\chi_1^{0,1} = i\zeta\cos\phi$, $\chi_2^{0,0} = -\chi_2^{1,1} = \cos\theta\cos\phi$, and $\chi_2^{1,0} = -\chi_2^{0,1} = -i\zeta\cos\theta\sin\phi$. Here $Y_{j'j}(\rho)$ is the Fourier transformation of the product of the Weber parabolic cylinder functions,

$$Y_{j'j}(\rho) \simeq \int_{-\infty}^{+\infty} D_{-\nu'-j'}[-(1+i)u]D_{\nu-j}[-(1-i)u]\mathrm{e}^{i\rho u}du, \tag{13}$$

where $\nu = \frac{i\lambda}{2}, \nu' = \frac{i\lambda'}{2}, \lambda' = \lambda|_{p_y \to p_y'}$, and $\rho \approx \Delta t_{st}\omega$. Applying the saddle-point method to integral (13), we establish the law of conservation of kinetic energy,

$$v_F\left(2eEt + p_x + p_x'\right) = \hbar\omega, \tag{14}$$

at the saddle-point, $u = \rho/2$. The wide high-frequency range follows as $2\Delta t_{st}^{-1} < \omega < 2\Delta t_{st}^{-2}T,\ t_{st}^{-1}T \gg 1$. We find the formation interval for emission of a photon with given $\boldsymbol{k}$. The center of formation of the interval for the given initial momentum p_x is $t_c = -(p_x / eE) + (\omega\Delta t_{st}^2 / 2)$ The width of the formation interval Δt is determined only by the electric field: $= \left(|eE|v_F / \hbar\right)^{-1/2} \approx 2.6a^{-1/2} \cdot 10^{-14}\,\mathrm{s}$, where $E = aE_0,\ E_0 = 1\cdot 10^6\,\mathrm{V/m}$ and $7\times 10^{-4} \ll a \ll 8$. It can be shown that leading contribution to probability (12) is from terms with Y_{00} and Y_{01}.

Taking into account that $\left|\lambda - \lambda'\right| \lesssim 1$, we find the main contribution to Eq. (12) as

$$\left|M^{+}_{\underset{p}{-}'\underset{p}{-}}\right|^2 \approx (2\pi)^2 f(\lambda) e^{-3\pi\lambda'/4}\left|\chi_{\vartheta}^{0,1}\right|^2,\ f(\lambda) = \frac{\mathrm{sh}(\pi\lambda/2)}{2\pi\left[(\lambda/2)^2+1\right]} e^{-\pi\lambda/4} \tag{15}$$

at $\sqrt{\lambda} \sim 1$. This is the Gaussian function of k at fixed $\theta \neq 0$ and $\phi \neq 0$, where $p_y/\hbar$ is the position of the peak center. We see polarized emission to directions $\phi \to 0$ ($k_y \to 0$) and $\phi \to \pm\pi/2$ ($k_x, k_z \to 0$). The probabilty of unpolarized emission per unit frequency and solid angle is

$$\sum_{\vartheta=1,2} \frac{d\mathcal{P}(\boldsymbol{k}\vartheta|\overset{\pm}{\vec{p}})}{d\omega d\Omega} = \frac{\alpha}{\varepsilon}\left(\frac{v_F}{c}\right)^2 \omega\Delta t_{st}^2 f(\lambda) e^{-3\pi\lambda'/4}\left(1 - \frac{k_y^2}{k^2}\right). \tag{16}$$

For any given p_y and k_y/k, the maximum probabilty is realised with $\lambda' \to 0$ ($k_y \sim p_y/\hbar$). The angular distribution is maximal at $k_y \to 0$ ($\boldsymbol{k}$ is in plane that is orthogonal to graphene and parallel to the electric field $\boldsymbol{E}$. We suggest the emission of a photon by an electron in graphene in the presence of a constant electric field for experimental observations.

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