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LIGHT-BY-LIGHT HADRONIC CORRECTIONS TO THE MUON *G*-2 PROBLEM WITHIN THE NONLOCAL CHIRAL QUARK MODEL

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Results of calculation of the light-by-light contribution from the lightest neutral pseudoscalar and scalar mesons and the dynamical quark loop to the muon anomalous magnetic moment are discussed in the framework of the nonlocal SU(3) × SU(3) chiral quark model. The model is based on four-quark interaction of the Nambu–Jona–Lasinio type and Kobayashi–Maskawa–'t Hooft six-quark interaction. The full kinematic dependence of vertices with off-shell mesons and photons in intermediate states in the light-by-light scattering amplitude is taken into account. All calculations are elaborated in explicitly gauge-invariant manner. These results complete calculations of all hadronic light-by-light scattering contributions to a_μ in the leading order in the $1/N_c$ expansion. The final result does not allow the discrepancy between the experiment and the Standard Model to be explained.

Keywords: anomalous magnetic moment, nonlocal quark model, Standard Model.

Quantum mechanics predicts the gyromagnetic ratio g for the charged point-like fermions with spin $1/2$ equals to 2. In relativistic quantum theory this fact is direct consequence of the Dirac equation. From the quantum field theory (quantum electrodynamics in that time) formulated by R. Feynman, J. Schwinger, and S. Tomonaga the existence follows of virtual particles, leading to the so-called vacuum polarization effects. The most famous examples of these effects are the Lamb shift in the hydrogen atom levels and the appearance of the anomalous magnetic moment of the electron. These effects were theoretically predicted and confirmed experimentally almost at the same time.

In QED the general form of the interaction vertex of fermions (with incoming and outgoing momenta p and $p^{'}$, correspondingly) with photon of momentum $q = p^{'} - p$ reads as (e.g., see [1])

$$\Gamma^\mu(p,p^{'}) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2), \quad p^{'2} = p^{'2} = m^2, \tag{1}$$

where F_1 and F_2 are the Dirac and Pauli form factors, respectively, and $\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$. At tree level for the charged point-like fermions, one has $F_1 = 1$ and $F_2 = 0$. In QED it is possible to get the relation between the form factors $F_1(0) = 1$, $F_2(0)$, and the gyromagnetic ratio g:

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$$g = 2\left[F_1(0) + F_2(0)\right] = 2 + 2F_2(0)\,. \tag{2}$$

Thus, the new quantity – the anomalous magnetic moment (AMM) $a = F_2(0) = (g-2)/2$ – appears. In quantum field theory, $a \neq 0$ due to the internal structure of fermions emergent from the virtual radiative corrections.

The AMM of the leptons (electron and muon) is one of the most accurately measured and theoretically studied quantities in the elementary particle physics. Interest in this problem is motivated by our wish to understand the most delicate features of our microword at its boundary and extension beyond of the modern knowledge. The simple rule [2, 3] is that the effect of the second-order contribution to the AMM of the lepton a_l with mass m_l due to a possible particle exchange of mass M is proportional to $a_l \propto (m_l / M)^2$. Thus sensitivity of the muon to hypothetical interaction with the scale M is 40 000 times higher than of electron. This fact compensates for a lower experimental accuracy of measurements of the muon AMM and makes this study more perspective from the point of view of search of new physics.

Recent experiment E821 at the Brookhaven National Laboratory (BNL, USA) got the muon AMM with very high precision [4]: $a_\mu^{\rm exp} = 659208.0(6.3)\cdot 10^{-10}$. In near future it is planed to increase the experimental accuracy by a factor of 4 in new experiments at FermiLab (USA) [5] and JPARC (Japan) [6]. In the Standard Model, it appears from the radiative corrections to the tree fermion-photon vertex due to the coupling of the lepton spin to virtual fields, which in the SM are induced by QED, weak and strong (hadronic) interactions

$$a^{\rm SM} = a^{\rm QED} + a^{\rm weak} + a^{\rm hadr}\,. \tag{3}$$

The electroweak corrections are known precisely. The strong interaction piece, in the orders leading in α, can be separated into three terms

$$a_\mu^{\rm hadr} = a_\mu^{\rm HVP} + a_\mu^{\rm ho} + a_\mu^{\rm HLbL}\,. \tag{4}$$

Here $a_\mu^{\rm HVP}$ is the contribution leading in α due to the hadron vacuum polarization (HVP) effect in the internal photon propagator of the one-loop diagram, $a_\mu^{\rm ho}$ is the next-to-leading order contribution related to iteration of HVP. These terms are estimated with good accuracy by using the dispersion relations and the data on hadron inclusive total cross sections. The last term in Eq. (4) is not reduced to HVP iteration and it is due to the hadronic light-by-light (HLbL) scattering mechanism.

The difference between the experimental value of the AMM and the Standard Model prediction, excluding HLbL contribution, is

$$a_\mu^{\rm BNL} - a_\mu^{\rm QED} - a_\mu^{\rm weak} - a_\mu^{\rm HVP} - a_\mu^{\rm ho} = 37.95\left(7.64\right)\cdot 10^{-10}\,, \tag{5}$$

where the error is due to experimental and $a_\mu^{\rm HVP}$ uncertainties. We calculate $a_\mu^{\rm HLbL}$ within the nonlocal chiral quark model (NχQM). The partially bosonized action of the *SU*(2) × *SU*(2) nonlocal chiral quark model is written as [7]

$$S = \int d^4x \Big\{ \bar{q}(x)\left(i\hat{\partial} - m_c + \hat{V}(x) + \hat{A}(x)\gamma_5\right)q(x) - \frac{1}{2G_1}\left(\pi^a(x)^2 + \sigma(x)^2\right) + \frac{1}{2G_2}\left(\rho^{\mu a}(x)^2 + a_1^{\mu a}(x)^2\right)$$

$$+ \sum_{\Phi_i = \sigma,\pi,\rho,\omega,a_1} \Phi_i(x)\int d^4x_1 d^4x_2\, f(x_1) f(x_2)\bar{Q}(x - x_1, x)\Gamma_i Q(x, x + x_2)\Big\}\,, \tag{6}$$

where $q(x)=\{u(x),d(x)\}$ are the fermion fields of the u and d quarks, $Q(x,y)$ are the corresponding guage quark fields:

$$Q(x,y)=P\exp\left\{-i\int_x^y dz^\mu (V_\mu^a(z)+A_\mu^a(z)\gamma_5)T^a\right\}q(y),$$

$$\bar{Q}(x,y)=\bar{q}(x)P\exp\left\{-i\int_x^y dz^\mu (V_\mu^a(z)-A_\mu^a(z)\gamma_5)T^a\right\},$$

$V_\mu^a(z)$ and $A_\mu^a(z)$ are external vector and axial–vector fields, respectively (the notation is $\hat{V}=V^\mu\gamma_\mu=V_\mu^a\gamma^\mu T^a$), T^a are the generators of the flavor group, P is the ordering operator of T^a along the integration path in every term of the Taylor series of the exponent, m_c is the current quark mass, σ, π, ρ, ω, and a_1 are the boson fields for the mesons; G_1 and G_2 are the coupling constants determined by experimental input for the masses and other low energy properties of the light mesons; $f(x)$ is the form factor of the nonlocal interaction with the characteristic nonlocality parameter Λ. The spin-flavor matrices Γ_i for mesons are given by $\Gamma_\sigma=1$, $\Gamma_\pi^a=i\gamma_5\tau^a$, $\Gamma_\rho^{\mu a}=\gamma^\mu\tau^a$, $\Gamma_\omega^\mu=\gamma^\mu$, $\Gamma_{a_1}^{\mu a}=\gamma_5\gamma^\mu\tau^a$, where τ^a are the Pauli matrices.

Action (6) contains the gauge invariant interaction of quarks and mesons with external fields. At small momenta, the action takes into account the nonperturbative structure of the strong interaction, as it follows from the instanton liquid model or the Schwinger–Dyson approach. The nonlocal action interpolates physics of low momenta to the region of large momenta, where the nonlocality disappears and one has only free current quarks. Note that incorporation of gauge fields with the help of *P*-ordered exponent into action (6) generates the contact interaction of quarks and mesons with any number of photons.

The $SU(2)$ model contains five parameters: the current quark mass m_c, the dynamical quark mass m_d, the nonlocality parameter Λ, and the scalar, G_1, and vector, G_2, coupling constants. The gap equation relates the parameters m_c, m_d, Λ, and G_1 with each other. The couplings G_1 and G_2 are fitted by physical masses of the pion and ρ–meson, respectively. One more parameter is fixed, e.g., by the pion decay $\pi^0\to\gamma\gamma$. Thus one parameter remains free and may be varied.

In [8–12] we estimated the partial contributions to $a_\mu^{\rm HLbL}$ due to the pseudoscalar mesons (π^0, η, and η'): $a_\mu^{\rm HLbL,PS}=5.85(0.87)\cdot 10^{-10}$; the scalar mesons ($\sigma$, $a_0(980)$, and $f_0(980)$): $a_\mu^{\rm HLbL,S}=0.34(0.48)\cdot 10^{-10}$; and the loop of dynamical quarks $a_\mu^{\rm HLbL,QLoop}=11.0(0.9)\cdot 10^{-10}$. The total contribution obtained in the leading order in the $1/N_c$ expansion is

$$a_\mu^{\rm HLbL,N\chi QM}=16.8(1.25)\cdot 10^{-10}. \tag{7}$$

The error bar accounts for the spread of the results depending on the model parameterization. Comparing with other model calculations, we conclude that our results are quite close to the recent results obtained in [13, 14]. If we add result (7) to all other known contributions of the standard model, we get that the difference between the experiment and theory is $a_\mu^{\rm BNL}-a_\mu^{\rm SM}=18.73\cdot 10^{-10}$ which corresponds to 2.43σ. If one uses the hadronic vacuum polarization contribution from the τ hadronic decays instead of e^+e^- data, then $a_\mu^{\rm HVP,LO-\tau}=701.5(4.7)\cdot 10^{-10}$ [15], the difference decreases to $a_\mu^{\rm BNL}-a_\mu^{\rm SM}=12.14\cdot 10^{-10}$, which is a 1.53σ deviation.

Clearly, further reduction of both the experimental and theoretical uncertainties is necessary. On the theoretical side, the calculation of the still badly known hadronic light-by-light contributions in the next-to-leading order in the

$1/N_c$ expansion (the pion and kaon loops) and extension of the model by including heavier vector and axial-vector mesons is the next goal. New experiments at FNAL and J-PARC have to resolve the muon $g-2$ problem, increasing the effect or leading to its disappearance.

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