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STUDY OF THE DYNAMICS OF ASTEROIDS – COMPANIONS TO VENUS

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The present work is devoted to a study of motion of near-Earth asteroids (NEA) 2002 VE68 and 2013 ND15 moving in the vicinity of the 1:1 resonance with Venus. To construct the probability domain of these NEA, 10 thousand clones covering the initial probability domain of the object were used. The investigation time intervals chosen individually are 4500 and 1520 years, respectively. The orbit of asteroid 2013 ND15 is ill-defined that allows no conclusion to be made on its capture in a resonance. The given object regularly approaches to Mercury, Venus, and the Earth thereby causing a substantial growth of the probability domain. Our study shows that new observations of the asteroid from the Earth surface are impossible till 2021. An analysis of the evolution of the average MEGNO parameter demonstrates that the predictability time of motion of the given object is about 250 years. Asteroid 2002 VE68 behaves differently. A study of perturbation structure demonstrates that for this object it is necessary to take into account the influence of major planets, the Moon, the Sun oblateness, and relativistic effects of the Sun. On the entire investigation time interval the asteroid moves in the vicinity of stable resonance and its critical argument librates. Asteroid 2002 VE68 approaches to Mercury and Venus, but not very close. The predictability time of motion is about 800 years.

Keywords: Asteroids, dynamics, approach, orbital resonance, perturbation structure, predictability time, visibility conditions.

INTRODUCTION

A study of motion of asteroids in the vicinity of orbital resonances with planets of the terrestrial group is an urgent problem for some reasons, one of which is the problem of potentially hazardous asteroid. As is well known, stable resonances serve as a protective mechanism against approaches, while unstable resonances and overlap of various resonances can lead to randomness and unpredictability of motion. Previously we revealed asteroids approaching to the Earth and moving in the vicinity of resonances of low orders with planets of the terrestrial group [1, 2]. The purpose of the present work is a study of the probabilistic orbital evolution of near-Earth asteroids (NEA) in the vicinity of the 1:1 resonance with Venus.

The 1:1 resonance with major planets is of special interest from the viewpoint of celestial mechanics; the orbital period of such objects differs slightly from the orbital period of a planet. For the asteroids moving in the 1:1 resonance (the so-called *companions*), three different types of motion in the system of coordinates rotating with the angular velocity of the planet can be identified: classical *tadpole*, *horseshoe*, and *quasi-satellite* [3] that correspond to oscillations of the critical argument about $\pm 60°$, $180°$, and $0°$, respectively. It should be noted separately that for objects moving in the vicinity of the 1:1 resonance with Venus, problems with observations may arise since the most

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TABLE 1. NEA Moving in the Vicinity of the 1:1 Resonance with Venus

Object	α, ″/day	Object	α, ″/day
322756 2001 CK32	(–30, 27)	2007 AG	(–16, 65)
2002 LT24	(–72, 60)	2013 ND15	(–30, 31)
2002 VE68	(–15, 15)		

part of time they are within the Earth orbit. It seems likely that the small number of the known objects is caused by this fact [4].

The present work is devoted to a study of the probabilistic orbital evolution of NEA moving in the vicinity of the 1:1 resonance with Venus on time interval of the order of thousand years. In Section 1 the procedure of revealing such asteroids based on the E. Bowell catalogue [5] is described. In Section 2 data on objects 2002VE68 and 2013 ND15 are presented. In Section 3 results of investigation of the perturbation structure for these asteroids are given. Section 4 is devoted to the improvement of their orbits and construction of initial probability domains for the chosen objects. Section 5 describes results of studying the probabilistic orbital evolution of the examined NEA, and Section 6 is devoted to the possibility of carrying out new positional observations of the examined asteroids from the Earth surface.

1. REVEALING NEA MOVING IN THE VICINITY OF THE 1:1 RESONANCE WITH VENUS

To reveal asteroids moving in the vicinity of the 1:1 resonance with Venus, the motion equations of all NEA were integrated on the time interval (1000, 3000) years. Initial data were taken from the E. Bowell catalogue for February 2015 [5]. The equations of motion were integrated numerically by the Everhart method [6]. The model of forces included the influence of major planets, Pluto, the Moon, Ceres, Pallas, and Vesta. All investigations were performed using specially developed software [7] that allowed high-precision prediction of asteroid motion to be made.

As resonant characteristics, the critical (resonant) argument β defining the connection longitude of the asteroid and the planet

$$\beta = \lambda_1 - \lambda_2, \tag{1}$$

and its time derivative α (called *resonant band* [8])

$$\alpha = n_1 - n_2, \tag{2}$$

were used, where n_1 and n_2 are mean motions and λ_1 and λ_2 are mean longitudes of the asteroid and Venus, respectively. For $\alpha = 0$, the asteroid is in exact resonance with the planet caused by the 1:1 commensurability of their mean motions. The asteroid moves in the vicinity of resonance if α and β oscillate about the exact commensurability so that $|\beta - \beta_{av}| \le 180°$ and $|\alpha| \le \alpha_{max}$ (here β_{av} is the libration center of the critical argument). Here α_{max} characterizes boundaries of resonant motion and is defined by the maximal amplitude of oscillations of the critical argument β.

Five asteroids moving in the vicinity of the 1:1 resonance with Venus on the time interval (1000, 3000) years were identified. Table 1 presents boundaries of variation of resonant band (2) for the examined NEA. Only one of the revealed objects is numbered – 322756 2001 CK32. Over the next 1000 years the given asteroid will move along the orbit of the Venus quasi-companion, regularly passing through the exact commensurability that cannot be said about its past – during the previous one thousand years it was on one side of the resonance. Object 2002 LT24 also moved most of the time on one side of the exact commensurability; only occasionally the resonant band changed its sign, and the critical argument almost always circulated. Object 2007 AG, unlike other examined asteroids belonging to the Aton class, is Atira, i.e., its orbit lies entirely within the Earth orbit. However, the behavior of the resonant characteristics of 2007 AG is similar to that of 2002 LT24.

Only two objects, 2002 VE68 and 2013 ND15, from those presented in Table 1 demonstrate regular oscillations of the resonant band and of the critical argument about the exact commensurability on the time interval (1000, 3000)

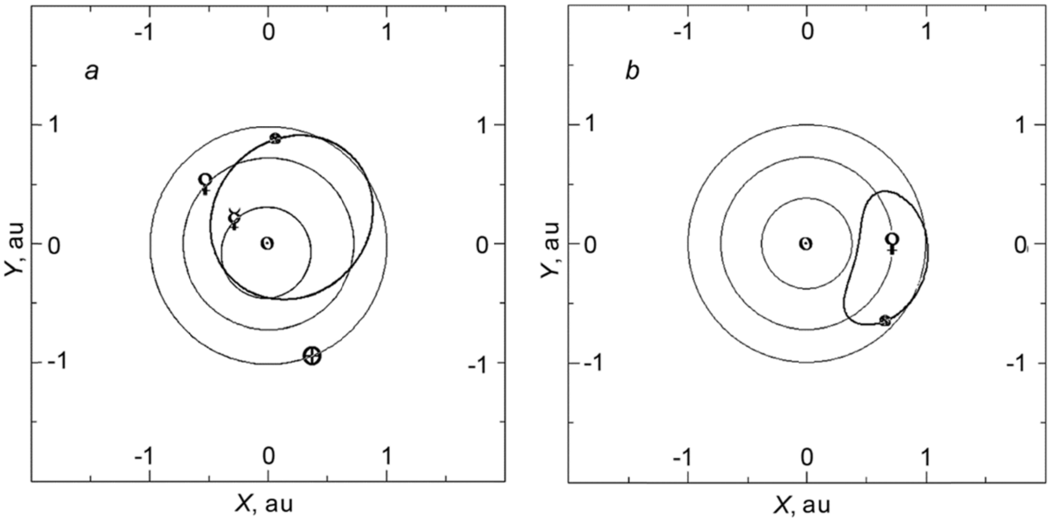

Fig. 1. Projections of the orbit of near-Earth asteroid 2002 VE68 onto the ecliptic plane in the fixed system of coordinates (*a*) and the system of coordinates rotating with the angular velocity of Venus (*b*).

years. Asteroid 2002 VE68 moves along the orbit characteristic for quasi-companions, and asteroid 2013 ND15 moves along a horseshoe in the system of coordinates rotating with the angular velocity of Venus. Exactly these objects are investigated below.

2. DATA ON ASTEROIDS 2002 VE68 AND 2013 ND15

Asteroid 2002 VE68 [9, 10] was discovered on November 11, 2002 at Lowell Observatory. As of September 2015, 457 observations of this object on time interval of about 8 years have been included in the database of the Minor Planet Center (http://www.minorplanetcenter.net/). Figure 1 shows the projection of the orbit of this asteroid onto the ecliptic plane in the fixed system of coordinates (*a*) and in system of coordinates rotating with the angular velocity of Venus (*b*). Figure 1*a* shows projections of the Mercury, Venus, and Earth orbits, and Fig. 1*b* shows the average distances to the planets. From Fig. 1*a* it can be seen the asteroid slightly enters into the orbit of Mercury in perihelion, and in aphelion it orbits beyond the Earth orbit, i.e., belongs to the Aton class. In the system of coordinates rotating with the angular velocity of Venus, the object orbit describes a horseshoe (Fig. 1*b*) displaced in time and describes a torus around the Earth orbit. The minimum orbit intersection distance (MOID) between the 2002 VE68 and Earth orbits, obtained by the Kholshevnikov and Vasiliev method [11] on the basis of elements from the E. Bowell catalogue for September 2015, was 0.0262 au that together with the absolute magnitude H = 20.5 allows it to be referred to potentially hazardous asteroids (http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2002%20VE68;orb=1).

Asteroid 2013 ND15 was discovered on July 13, 2013 at Haleakala Observatory under the Pan-STARRS Project. In the database of the Minor Planets Center, 22 observations of the object were reported for 26 days till September 2015. Sufficiently large orbital eccentricity (e = 0.611) allowed the asteroid to cross the Mercury, Venus, and Earth orbits in the projection onto the ecliptic plane. The fact that it belongs to a flat subsystem (i = 4.796°) allows it to approach to the above-listed planets. Despite the small MOID (0.0078 au), the given object does not belong to the class of potentially hazardous asteroids due to the low absolute magnitude (H = 24.1). However, if we consider that the albedo of asteroid 2013 ND15 is within the limits 0.20–0.04, its diameter will change from 40 to 100 m [4]. It should be noted that the hazardous objects of such sizes must not be ignored [12].

TABLE 2. Data on NEA Observations

Object	N	Δt, days (years)	t_0	σ, sec. of arc	$\sigma(\boldsymbol{X}_0)$, au	$\mu(A)$
2002 VE68	457	2947 (2002–2010)	31.03.2007	0.549	$7.4 \cdot 10^{-8}$	$3.3 \cdot 10^{8}$
2013 ND15	22	26 (2013)	18.07.2013	0.366	$1.3 \cdot 10^{-4}$	$1.4 \cdot 10^{8}$

3. INVESTIGATION OF THE PERTURBATION STRUCTURE FOR ASTEROIDS 2002 VE68 AND 2013 ND15. THE FORCES MODEL

The first step in the study of dynamics of near-Earth asteroids is estimation of perturbing factors acting on the object and choice of the model of forces that have been taken into account. The NEA motion is traditionally considered within the framework of the perturbed two body problem in the heliocentric system of coordinates reffered to the equator and the equinox 2000.0. To choose the perturbing factors, the technique described in [13, 14] was used. The given technique allows the perturbing factors to be estimated based not only on the evolution of the nominal orbit, but also on the sizes of the initial probability domain. In addition, the application of five different methods allows the most objective conclusion to be made on the action of different forces on the asteroid motion.

The perturbations to be estimated included the influence of major planets Pluto, the Moon, Ceres, Pallas, Vesta, the Sun oblateness, the Earth, and Jupiter, and relativistic effects of the Sun, major planets, Pluto, and the Moon. The coordinates of the major planets, Pluto, and the Moon were determined from fundamental ephemeris DE431 [15]. The differential equations of motion for Ceres, Pallas, and Vesta were integrated together with equations of motion for the examined asteroid. The relativistic effects were considered by addition of the Schwarzschild terms to the equations of motion.

The study of the perturbation structure of asteroid 2002 VE68 demonstrated that the influence of major planets, the Moon, the Sun oblateness, and relativistic effects of the Sun should be taken into account. Other forces can be neglected without loss of precision, since their action is much less than errors in the initial data and has no essential influence on the orbital evolution.

Another situation was observed for 2013 ND15. As already mentioned above, its orbit is ill-defined, thereby leading to large initial probability domain. The domain is so large that the initial data obtained disregarding almost all perturbing factors (except for the Earth) do not fall outside of the limits of the domain obtained for the full model. This fact makes it impossible to predict reliably the influence of various perturbations on the object motion. In this case, it seems expedient to take advantage of the full force model.

4. IMPROVEMENT OF ORBITS AND CONSTRUCTION OF INITIAL PROBABILITY DOMAINS FOR ASTEROIDS 2002 VE68 AND 2013 ND15

In the next stage of investigation of the examined objects, the orbit was improved by the least squares method (LSM) based on the available optical observations, and the probabilistic orbital evolution was studied. Results of orbit improvement are presented in Table 2, where N is the number of observations used for the improvement; Δt is the observation period in days and years; t_0 is the arithmetic mean of the observation moments; σ is the root-mean-square error of observation representation, in seconds of arc; $\sigma(\boldsymbol{X}_0)$ is the root-mean-square error of LSM for the position vector $\boldsymbol{X}_0$, in au; and $\mu(A)$ is the Todd condition number of the matrix of normal equations for the epoch t_0. For all objects improvement was made at the time moment t_0 using the model chosen above.

From Table 2 it can be seen that the orbit of 2002 VE68 is well defined, which is not the case for asteroid 2013 ND15 – it was observed only in one appearance and its initial probability domain was very large. The estimation of the nonlinearity coefficient demonstrated that for the examined NEA at the time moment t_0 it did not exceed a critical value of 0.1 [16, 17]. This allowed us to use the linear method to construct the initial probability domain. The domain was constructed in six-dimensional phase space of coordinates and velocity components based on the full covariance matrix

TABLE 3. Data on Close Approaches

Object	Time interval, years	d, au	
		Nominal orbit	Clones
2002 VE68	(–1000, 3500)	$d_1 = 0.00922$ $d_3 = 0.02314$	$d_1 = 0.00832$ $d_3 = 0.02314$
2013 ND15	(763, 2283)	$d_1 = 0.00451$ $d_2 = 0.018455$ $d_3 = 0.003185$	$d_1 = 0.001155$ $d_2 = 0.000935$ $d_3 = 0.000158$

TABLE 4. Extreme Values of the Resonant Bands and Orbit Elements

Object	α, ″/day	a, au	e	i, deg
2002 VE68	(–14.6, 14.8) (–15.1, 15.4)	(0.722099, 0.724548) (0.722065, 0.724589)	(0.402004, 0.413314) (0.400536, 0.413314)	(8.420689, 9.949749) (8.420650, 9.956253)
2013 ND15	(–61.7, 17.1) (–539, 383)	(0.721939, 0.730050) (0.692973, 0.772349)	(0.603725, 0.612116) (0.569158, 0.637304)	(4.567958, 4.953630) (0.955047, 7.472963)

shaped as an ellipsoid whose center was determined by improved coordinates and velocity components of the nominal orbit. In the initial probability domain, 10 thousand clones were chosen whose coordinates and velocity components were distributed by a normal law.

5. STUDY OF THE PROBABILISTIC ORBITAL EVOLUTION

The probabilistic orbital evolution was studied by the nonlinear method of numerical integration of the differential equations of motion for each clone. The time interval was limited by the integration precision. Results of the evolution study are presented in Tables 3 and 4 and are shown in Figs. 2 and 3. Table 3 gives the time interval and the closest approach on the time interval (d_i here is the minimal distance to the center of the ith planet), and Table 4 presents extreme values of the resonant band α and oscillating orbit elements (the major semiaxis a, eccentricity e, and inclination i of the orbit plane to the ecliptic). For each object, the upper row corresponds to the nominal orbit and the lower row corresponds to the clones.

The approaches to the major planets and the evolution of the resonant band, critical argument, major semiaxis, eccentricity, inclination of the orbit plane to the ecliptic, and average MEGNO parameter are shown in Figs. 2 and 3. The evolution for the clones is illustrated by grey color, and the nominal orbit is indicated by black color. The average MEGNO (Mean Exponential Growth of Nearby Orbit) parameter $\overline{Y}(t)$ allows the chaotic and regular motions to be separated and their predictability time to be estimated [18, 19]. For the exponential divergence of close orbits, $\overline{Y}(t)$ first linearly grows, passes through value 2, and then the motion becomes unpredictable.

Asteroid 2002 VE68 approaches to Mercury and the Earth; however, the approaches are not very close – outside of the corresponding Hill spheres. Nevertheless, a great number of approaches cause irregular oscillations of the resonant band in the vicinity of the exact commensurability from –15 to 15″/day. Since the beginning of the examined time interval (–1000 year) till the middle of the current millennium the critical argument librates about 0° with the amplitude of the order of 40°, which implies that the asteroid is a quasi-companion of Venus. Approximately when the resonant argument changes its behavior, the average MEGNO parameter starts to grow and passes through value 2, which allows the conclusion about the exponential growth of the probability domain to be made. The predictability time of motion of the given asteroid is about 800 years. Within this time interval, the evolution of the clones slightly differs from the evolution of the nominal orbit.

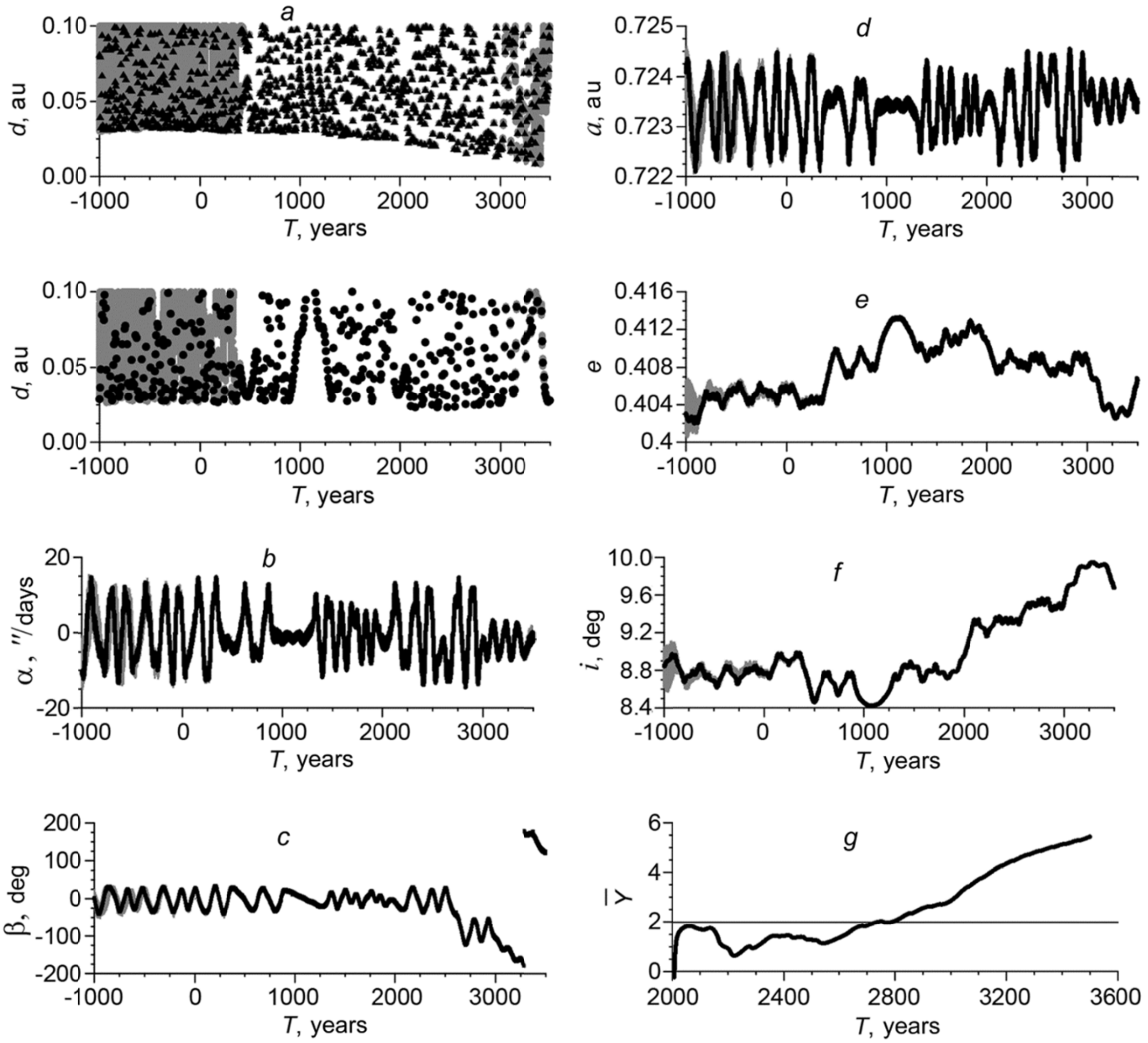

Fig. 2. Asteroid 2002 VE68: approach to Mercury (Δ) and the Earth (•) (*a*) and evolution of the resonant band (*b*), critical argument (*c*), major semiaxis (*d*), eccentricity (*e*), inclination of the orbit plane to the ecliptic (*f*), and average MEGNO parameter (*g*).

Asteroid 2013 ND15 behaves differently. It approaches many times to Mercury, Venus, and the Earth; moreover, even a particle in the nominal orbit traverses twice the sphere of action of the Earth (in 2216 and 2276) and four times the Hill sphere (in 2160, 2192, 2216, and 2276). Moreover, on June 27, 2216 the nonzero probability of collision was revealed. The critical argument on the time interval since 763 (the initial moment of the study) approximately till 1600 undergoes regular librations about 180°, that is, the asteroid moves along a horseshoe orbit. In this case, the resonant band oscillates about the exact commensurability with the small amplitude not exceeding 18″/day. Then under the influence of numerous approaches to the Earth, which becomes closer and closer, the libration center of the resonant argument is displaced. As a result, after the asteroid passage through the sphere of action of the Earth in 2216, libration is replaced by circulation. Approximately at the same time, the average MEGNO parameter starts to grow and passes through value 2, which testifies to an exponential growth of the probability domain. This means that the time of motion predictability of the given object is about 250 years.

However, it should be noted that all said above about 2013 ND15 concerns only the nominal orbit. In particular, the plot in Fig. 3*b* is limited for clarity by values $|\alpha| < 100''$/day, while the resonant band for the clones after

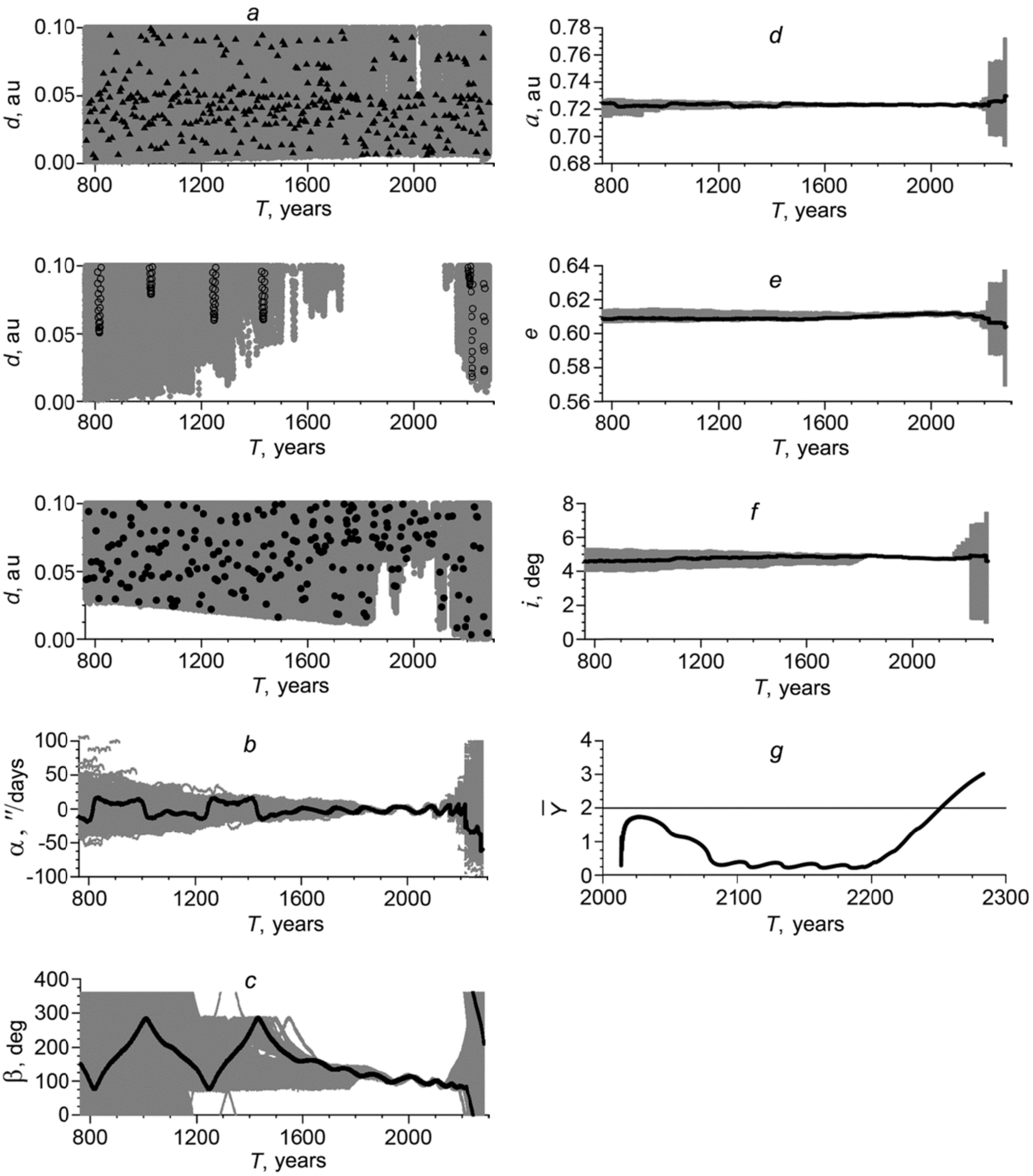

Fig. 3. Asteroid 2013 ND15: approaches to Mercury (Δ), Venus (°), and the Earth (•) *(a)* and evolutions of the resonant band (*b*), critical argument (*c*), major semiaxis (*d*), eccentricity (*e*), inclination of the orbit plane to the ecliptic (*f*), and average MEGNO parameter (*g*).

asteroid passage through the Hill sphere of the Earth in 2216 reaches several angular minutes a day. Even at the initial moment of time, the probability domain was large ($1.3 \cdot 10^{-4}$ au), and under the influence of close approaches it increased many times. A significant part of the clones left the resonance that did not allow a conclusion about stability or instability of resonant relations to be made. Thus, to judge objectively about the dynamic evolution of asteroid 2013ND15, new observations are required. Section 6 is devoted to this problem.

6. POSSIBILITY OF NEW OBSERVATIONS

As already demonstrated above, the orbit of asteroid 2013 ND15 is ill defined that does not allow its dynamic evolution to be studied reliably. In this regard, the problem of performing new observations arises. To elucidate such possibility, conditions under which the examined objects can be observed from the Earth should be revealed. The magnitude and the angular distance of the object from the Sun (elongation) were chosen as the parameters to be estimated. Observations of all objects moving in the vicinity of the 1:1 resonance with Venus (see Table 1) presented for September, 2015 in the database of the Minor Planet Center (http://www.minorplanetcenter.net/db_search) were analyzed. As a result, it was revealed that the overwhelming majority of positional observations from the Earth surface were performed for the magnitudes not exceeding 22 and the elongations greater than 60°. To perform investigations, we used program complexes EROS [20] and IDA [7].

In the next stage, it was necessary to determine the time moment when the object position on the celestial sphere satisfied the conditions obtained. As a result, it was determined that asteroid 2002VE68 could be observed already at the moment of investigations (in October 2015). The correctness of the prediction was confirmed by new observations on October 24 at Pan-STARRS 1 Observatory. The new observations influence only slightly the probabilistic orbital evolution; therefore, they were disregarded in this study.

Another situation was observed for asteroid 2013 ND15 – it could be observed from the Earth surface only in July 2021. In addition, one more problem arose, namely, the large probability domain on the celestial sphere that by our estimates was about one degree for the right ascension and several degrees for the declination. For the given object, surveys are necessary, which considering the small magnitude of the object (about 22 in 2021) is not so simple. It should be noted that the employed visibility criteria are in essence estimates from above and cannot guarantee new observations. In this regard, asteroid 2013 ND15 risks being among the lost objects.

CONCLUSIONS

Thus, results of our study of long-term orbital evolution of asteroids 2002 VE68 and 2013 ND15 moving in the vicinity of the 1:1 resonance with Venus have been presented. The improvement of the orbits from the available optical positional observations allow us to conclude that the orbit of asteroid 2013 ND15 is ill defined that does not allow us to make reliable conclusion about its capture in resonance. In addition, the object regularly approached to Mercury, Venus, and the Earth thereby leading to essential growth of the probability domain. Unfortunately, as demonstrated our studies, new observations of the examined asteroid from the Earth surface are inaccessible till 2021. The analysis of the evolution of the average MEGNO parameter demonstrates that the predictability time of motion of the object is about 250 years.

Asteroid 2002 VE68 behaves differently. Study of the perturbation structure demonstrates that for this object it is necessary to consider the influence of major planets, the Moon, of the Sun obalteness, and relativistic effects of the Sun. On the entire examined time interval, the asteroid moves in the vicinity of the stable resonance and its critical argument librates. Asteroid 2002 VE68 approaches to Mercury and Venus, but not very close. The predictability time is about 800 years.

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