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THE COSMOLOGICAL CONSTANT AS A CONSEQUENCE OF THE EVOLUTION OF SPACE

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Conditions are considered in various approaches, determining the dimensionality of a space in which specific physical interactions are described. The dimensionality of the Universe does not necessarily have a fixed value. The cosmological constant is interpreted as the energy density being released in the remaining dimensions when the dimensionality of space is decreased.

Keywords: cosmological constant, quintessence, dark matter, dark energy.

INTRODUCTION

The development of philosophical ideas about the structure of the world surrounding us has led to the concept of space as a category characterized by extension in length, width, and depth. It seemed natural that such a space enters in the role of only a *storage place of things* and that it is three-dimensional. The experimental development of electrodynamics at the turn of the nineteenth century (the late 1800's through the early 1900's) led H. Lorentz in a treatment of the electron to some convenient transformations of the spatial coordinates and time. A. Poincaré generalized these transformations to the point of absolute accuracy and considered them as elements of transformations of a four-dimensional manifold, where 4-vectors consisting of the spatial coordinates and time preserving an invariant interval of the point coordinates of this manifold.

However, the four-dimensional character of space did not last long. It did not take long before the first efforts to unify gravitational and electromagnetic interactions, undertaken within the framework of an extension of the general theory of relativity (GTR), showed that the dimensionality of the Universe should be greater than four. A possible treatment of the nature of the cosmological constant (CC) and its connection with the dimensionality of spacetime with reference to current approaches to its description is presented in the paper.

LOCAL PHYSICAL THEORIES

Significant progress in physics has been associated with the use of the Lagrange functions with local density. The Lagrange function (Lagrangian) or its density is defined on spatial structures, the simplest of which is a point in space. Such Lagrange functions are called local. It is thought that the interactions that are well known at the present time – gravitational, electromagnetic, weak, and strong – are a manifestation of some unified interaction that splits into the indicated components as one climbs down the scale of the characteristic energy of the interaction of their representatives. The success of A. Salam and S. Weinberg in unifying the electromagnetic and weak interactions strengthened this conviction. According to the available estimates, unification of the electroweak and strong interactions in such a case should take place at energies on the order of 10^{16} GeV (Grand Unification), and of all four interactions – at energies on the order of 10^{19} GeV.

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One of the approaches to unification consists in a consideration of multidimensional spaces, the structure of the metric tensor of which enables a description of not only the gravitational field, but also the other fields that we seek to unify. The first of these approaches was that of T. Kaluza [1] in an attempt to unify gravitation and electromagnetism. The fifteen 5-dimensional Hilbert–Einstein equations (HEEs) split into ten ordinary 4-dimensional HEEs, the four Maxwell equations, and an additional scalar equation. It is assumed that the fourth spatial dimension is curled up into a very small-sized loop, not accessible to direct perception (it is compactified).

The low-lying spectra of systems in the presence of the compact curled-up dimension and without it coincide. The difference in the spectra begins with terms of order a/R, where a is the characteristic size of the system and R is the radius of the compact dimension into which one of the extra coordinates is curled up [2]. As R tends to zero, the corrections tend to infinity. As of yet, no contribution from such corrections has been seen in experiments.

A similar approach to the unification of the gravitational-electroweak interactions leads to a 6-dimensional space for one generation and a 7-dimensional space for three generations of leptons. In the case of unification of the gravitational-electrostrong interactions, space should be 7-dimensional, taking the three generations of quarks into account and 8-dimensional if the color-triplet nature of quarks is also taken into account [1, 3]. But progress in the description of the physical picture of the world by the local Lagrangians has been accompanied by conceptual difficulties in attempts to wring out such a description, which have still not been resolved. Let us consider a few of them.

The equality of inertial and gravitational masses. The equality of gravitational and inertial masses in GRT is considered to be an exact law of nature. It is assumed that this fact is proven in the theory, its *proofs* are given, for example, in [4, 5]. But in actual fact, the equality of these two types of masses in the theory is valid only in systems of special form, in particular, Cartesian. In the remaining systems, inertial mass can take an arbitrary value – negative as well as positive [6].

Conservation laws in GRT. There is also the more essential, indeed key problem of determining the gravitational energy and specifying the laws of conservation of energy-momentum in GRT. Great efforts to resolve it have not led to success. It turned out that the introduction of the Einstein pseudotensor pointed the way to its solution. But in this case, in the spacetime transformations, the energy of the gravitational field varies (for example, the Boyer paradox). C. Møller formulated conditions on the pseudotensor of the gravitational field that should preclude the possibility of obtaining incongruous results, but he himself proves a theorem to the effect that in principle these conditions cannot be satisfied [4].

Gravitational waves. A wave transports energy and momentum, but there are serious problems with their definition (see above). It is difficult to indicate a generally covariant criterion for the wave nature of an effect associated with solutions of the HEEs, there are difficulties associated with the use of reference systems (the reference criterion), with the determination of the dynamical degrees of freedom (orientational, polarizational), it is even unclear specifically how the curvature of spacetime should be affected by a gravitational wave. Many problems fall away in the linearized version of a description of gravitational waves. But the HEEs are nonlinear, and the superposition principle does not work for them. For this reason, it is not clear in this context how to distinguish wave solutions from nonwave solutions. The quantum theory of the linearized gravitational field is not renormalizable, and efforts to adapt it to this purpose lead to the appearance of infinities which cannot be eliminated by introducing a finite number of counter-terms [1].

Many difficulties in the description of gravitation are apparently associated with the fact that it cannot be quantized [1, 7]. Subsequent to the lack of success in quantizing gravitation, the idea was advanced of a secondary character of the curvature of spacetime [8]. It is assumed that gravitation is not a fundamental interaction, but represents the macroscopic (long-wavelength) limit of a more general theory due to a quantized field [5]. Thus, in theories with the local Lagrangians the dimensionality of spacetime is obtained as a consequence of an extension of the metric tensor to include components describing other, nongravitational interactions. The difficulties mentioned above must be borne in mind in any question pertaining to the completeness of such a description.

NONLOCAL PHYSICAL THEORIES

The next object up in complication after a point is a one-dimensional structure – a line. The use of theories with the local Lagrange functions greatly simplifies the treatment, but nowhere does it follow that these are the only possible physical theories. Lines on which a Lagrange function is defined are called strings; correspondingly, we are dealing with string theories. For the most part, optimism in regard to string theories is due to the fact that this approach has the potential to lead to quantization of gravity. The gravitational interaction has not yet been quantized, and gravitational waves also have not yet been discovered. However, the gravitational interaction should, it seems, in a positive solution of the given problems, be realized by a particle whose spin is equal to 2 [1]. In the spectrum of the oscillations of a string, particles have been detected with spin 2 [2], and this is the main argument that physics is nonlocal and that further development of the theory must be sought in a string approach. Note that so far all of the advantages of string theory remain without experimental confirmation.

What does the string approach give in regard to the question pertaining to the dimensionality of our space? Quantization of strings for the boson sector leads to the transverse Virasoro operators of various modes. The generators of the Lorentz transformations of the string coordinates (the Lorentz charges) are expressed in terms of the Virasoro operators, the mass of the string is expressed in terms of the zero-mode Virasoro operator, etc. Calculation of various characteristics containing the given operators is quite specific and makes use, for example, of an analytic continuation of the Riemann zeta function. The commutative relations for generators of the Lorentz charges depend both on the Virasoro operators and on the dimensionality of the space under consideration. Valid commutation relations are obtained for the dimensionality of the space in which the strings are considered, equal to 26. Including the fermion sector in the treatment leads to the theory of superstrings. A similar calculation in the theory of superstrings leads to 10-dimensional spacetime.

There are five types of nontrivial 10-dimensional supersymmetric theories of superstrings: types I, IIA, IIB, and two heterodyne types: $E_8 \times E_8$ and $SO(32)$. It turned out, for example, that for a certain dimensional reduction of 11-dimensional membrane (n = 2) theory, superstrings of type IIA are obtained [9]. From an 11-dimensional theory containing membranes with n = 2 and 5 (the M5-brane is magnetically dual to the M2-brane in 11 dimensions), in various limits it is possible to obtain all 5 types of 10-dimensional superstring theories. This theory is called M-theory. Note that the 11-dimensional theory of supergravitation is also obtained from M-theory in the low-energy limit. The meaning of M-theory has still not been elucidated. One thing is clear, however, namely that the five superstring theories and M-theory are different sides or limits of one theory [2].

DESCRIPTION OF GEOMETRY ON THE BASIS OF PHYSICAL STRUCTURES

There is also an approach to the description of geometry on the basis of physical structures, considered by Yu. I. Kulakov and coworkers. The given approach is not based on a search for *primordial matter*, i.e., on the traditional historical path of development, but on a search for *primordial structures*. In particular, neither fields nor space are contained in it. The form of the relations between elements of a structure allow us to become aware of the prototype of the emerging concepts of space and interaction [1].

Unary physical structures. At the basis of the approach lies a set of elements, the number of which is the rank of the structure r, and between which pairwise relations a_{ik} are established. We seek a general form of the function

$$\Phi = 0, \tag{1}$$

establishing a connection between these relations. All possible laws (1) are found which the elements of the structures obey. These laws are found from the requirement that r arbitrary elements not coincident with the originally chosen elements lead to the same identity satisfied by the initial elements, i.e., identity (1). The obtained solutions of Eq. (1) have the form of connections between $n = r - 2$ elements of the structure, and the number n is associated with the dimensionality of the space. These very connections have the form of a quadratic relation between $r - 2$ elements of the structure and are directly interpreted as the square of the length between n *coordinates* of the elements. Such an approach for a set of 5 elements leads to three-dimensional spaces with different geometries: Euclidean, pseudo-Euclidiean, the first non-Eucliean geometry (Lobachevskian), the second non-Euclidean geometry (Riemannian), a peculiar symplectic geometry, etc. Three practically unknown exotic geometries are also obtained, mentions of which were later found in forgotten works of past geometers.

Binary physical structures, or binary systems of complex relations (BSCRs). In this case, binary connections or relations between r elements of one set and s elements of another set are established, i.e., binary structures of rank (r, s) are considered. Similarly as in the unary case, a general form of functions establishing a connection between these relations is sought. It turns out that only structures of rank (4, 2), (2, 4), (r – 1, r), (r, r), and (r, r + 1) are nontrivial. In contrast to the case of unary structures, for systems of binary structures, the problem of the form of functions Φ is solved in general form. Unary systems of relations can be obtained from binary systems by way of a certain gluing of elements of different sets, where the relations between them are constructed from the primordial binary relations. The obtained unary relations lead to the prototype of space (see above).

WHAT IN FACT IS THE DIMENSIONALITY OF SPACE?

Local theories. In the descriptions of our world by local theories the dimensionality of space grows proportionately as one takes an increasing number of interactions into account. Thus, the presence of the electromagnetic interaction along with the gravitational interaction increases the dimensionality of space from 4 to 5, etc., and taking all of the interactions known to us into account leads to 11-dimensional space. It is now thought that the evolution of our world takes place in accordance with the concept of the Big Bang [10]. If there are interactions we don't know about, then the dimensionality of space should be greater than eleven. Such a situation was entirely possible at times, for example, from 10^{-32} to 10^{-12} s, when, according to the Big Bang, separation of interactions took place. At the present time, the representatives of these unknown interactions could have disappeared (for example, as a result of annihilation processes) or they cannot be detected for whatever reasons. It is also possible that some representatives of these interactions have remained in vanishingly small quantities, but, by virtue of their high-energy character, may be inaccessible to present-day measurement techniques. But even at the present time (13.7 billion years) it is not possible to completely rule out the presence of interactions unknown to us. It is well known that the baryonic component of matter by itself is not capable of explaining the emergence of galaxies [10]. The difference between the observed mass and the dynamical mass in the Universe also speaks on behalf of the assertion that there should be a mass, called dark mass, which exceeds the baryonic mass by at least a factor of three and does not interact with radiation, but does interact with ordinary matter (and with itself) only gravitationally. Neutrinos have been advanced as the most probable candidate for the role of dark matter since they interact with one another, with ordinary baryonic matter, and with radiation very weakly. However, their excessive velocity stands in the way of any explanation of the growth of small-scale inhomogeneities and it has become necessary to reject such an explanation. At the present time, there are no candidates for the role of a carrier of dark mass [11]. Thus, we do not know in what interactions, besides the gravitational interaction, dark matter participates. The same thing can be said about dark energy (see below) – it is not known what it is made of.

Nonlocal theories. In attempts to describe our world by nonlocal theories, the dimensionality of space (eleven) is based on the existence of an M-theory of the same dimension, the corresponding limits of which lead to well-known 10-dimensional string theories. In particular, certain reductions of the Lagrangian of membrane theory (n = 2) lead to a Lagrangian of string theory [9]. The interconnections between the Lagrangians of the various membrane theories have not yet been completely explored. The existence of found reductions does not exclude the existence of other reductions. Thus, the dimensionality of space in such an approach can also grow larger. Moreover, the spectra of string theories should reproduce the particles known to us. If interactions that are not known to us will be revealed, then this will lead to the necessity of reproducing their representatives in the spectra of string theories, i.e., once more to an increase in the dimensionality of space.

Theory of structures. Classical spacetime can be described within the framework of a BSCR of rank (3, 3). Correspondingly, we arrive at 4-dimensional space with allowable signatures (+, +, +, +), (+, +, +, –), (+, +, –, –) and equivalent signatures with the plus signs replaced by minus signs. To construct multidimensional Kaluza–Klein theories, it is necessary to step up to a BSCR of rank (4, 4) [1, 7], and correspondingly the dimensionality of spatial structures will not exceed nine. A description of leptons and quarks is already possible within the framework of a BSCR of rank (6, 6) [1], and the emerging dimensionalities can go as high as 25. With the growth of the number of interactions, the rank of the structure also grows, and correspondingly also the dimensionality of space. Thus, as in the case of a description with the help of the local and nonlocal Lagrangians, taking a continually increasing number of interactions into account leads to a growth of the dimensionality of space.

Multidimensional time. The constructions considered above pertain to one-dimensional time because they have a direct connection with the Universe in which we live. We cannot reject the possible existence of worlds in which time is multidimensional. The possibility of the existence of spaces with the signatures (+, +, –, –) or (–, –, +, +), remarked upon above, is already testimony to the possibility of the existence of two-dimensional time.

Remark. In the monad method, descriptions of the reference system in which the congruence lines are parallel to the time coordinate of the considered manifold (i.e., these are chronometric lines) are a particular case [1]. In the early works on SRT, the time coordinate was chosen to be proportional to the imaginary unit to ensure the pseudo-Euclidean space. In nonchronometric systems, the generalization of the monadic description in the real-number field to a description in the complex-number field can be connected with how one goes about introducing a time coordinate. Multidimensional time structures can be associated with a further generalization of the monadic description to a description in the field of quaternions.

The fundamental possibility of the existence of various Universes is contained in the inflationary theory proposed by A. Guth and modified by A. Linde, P. Steinhardt, and A. Albrecht [12]. Fluctuations of the primordial scalar field within the limits of the Planck scale lead to conditions for transition to the inflationary regime, a result of which is rapid growth of the spatial dimensions of the three-dimensional part of space, leading to the conditions necessary for commencement of the evolution of the hot Big Bang. These fluctuations lead to a continuous generation of Universes, each with a different nature. Transitions between Universes with different dimensionalities of time are still unknown; therefore, we will assume that the dimensionality of the time continuum of our Universe does not change and is equal to 1.

As we have seen, the dimensionality of space in all three approaches to its description is determined by the existence of different kinds of interactions. The more there are of them, the greater is the dimensionality of space. Thus, the dimensionality of our space can be greater than 11. This depends on the existence of interactions as yet unknown to us.

Varying dimensionality of space. The possibility of the existence of a still greater and even a substantially greater dimensionality of space in comparison with the dimensionality discussed above flows out of other considerations. String theory, by virtue of its nonlocal character, contains additional symmetries not contained in local theories. One such symmetry is the T-duality of free strings, from which the impossibility of the electric field strength taking infinitely large values follows [2]. In thermodynamic equilibrium, the same energy is assigned on average to each degree of freedom (the equidistribution law). If this law is still valid at superhigh energy densities, this will lead to the result that a finite number of fields will contain only a finite energy density and under the conditions of infinite energy density of the Big Bang in its initial moments they will not be able to accumulate the released energy. This, in turn, should lead to an infinite or very large (if the energy density of the Big Bang in its initial moments was not infinite) dimensionality of space during the initial moments of the Big Bang.

The above considerations does not exclude the possible existence of a dimensionality of space that is greater than eleven, which with the expansion of the Universe will gradually decrease. W. Heisenberg remarked that modern physics is similar to the teaching of Heraclitus. If we replace the word *fire* by the word *energy*, then it is possible to take it as the primal cause of all changes in the world. We may add here: and possibly the reason for the dimensionality of space as well.

THE COSMOLOGICAL CONSTANT

Current status. The Hilbert–Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi k T_{\mu\nu}$$

are obtained by setting the variational derivative $\delta S_g/\delta g_{\mu\nu}$ of the action equal to zero, where the action

$$S_g = S_{\text{gr}} + S_{\text{m}} = \int d^4x \sqrt{-g}(L_{\text{gr}} + L_{\text{m}}), \tag{2}$$

is constructed using the metric tensor $g_{\mu\nu}$ (μ, ν = 1–n) and its first derivatives with respect to the coordinates of spacetime, g is the determinant of the metric tensor,

$$L_{\text{gr}} = \frac{1}{16\pi k} R = \frac{1}{16\pi k} g^{\mu\nu} R_{\mu\nu}$$

is the gravitational scalar density of the Lagrangian, k is Newton's gravitational constant, R is the curvature of space, i.e., the convolution of the metric tensor with the Ricci tensor $R_{\mu\nu}$: $R = g^{\mu\nu}R_{\mu\nu}$, L_{m} is the density of the Lagrangian of matter, $T_{\mu\nu}$ is the energy-momentum tensor of matter,

$$\frac{1}{2}\sqrt{-g}T_{\mu\nu} = \frac{\partial\sqrt{-g}L_{\text{m}}}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^{\alpha}} \frac{\partial\sqrt{-g}L_{\text{m}}}{\partial \frac{\partial g^{\mu\nu}}{\partial x^{\alpha}}},$$

and the integration is carried out over four-dimensional spacetime. The HEEs are generally covariant and do not depend on the choice of local coordinates. Since the quantity $\sqrt{-g}d^4x$ is invariant, this allows us to introduce the term

$$S_{\Lambda} = -\int d^4x \sqrt{-g}\Lambda,$$

$$\tilde{S}_g = S_g + S_{\Lambda} = S_{\text{gr}} + S_{\text{m}} + S_{\Lambda} = \int d^4x\sqrt{-g}(L_{\text{gr}} + L_{\text{m}} - \Lambda), \tag{3}$$

in Eq. (2) without violating the general covariance of the HEEs. The new equations take the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi k (T_{\mu\nu} + \Lambda). \tag{4}$$

Einstein introduced the Cosmological Constant (CC) (Λ) in 1917 in order to achieve a static Universe. Without introducing this term, either the energy density or the pressure of matter would have to be negative. The introduction of Λ or the energy density $\rho_{\text{vac}} = \frac{\Lambda}{8\pi k}$ makes it possible to avoid such an unphysical result. Since the right-hand side of the given relation does not depend on the characteristics of matter, it is interpreted as the energy density of empty space or the vacuum. After the appearance of the Friedman models, interest in a static Universe disappears and in an overwhelming number of works the CC is taken to be equal to zero.

Models with $\Lambda \neq 0$. Nevertheless, the possibility ($\Lambda \neq 0$) has not been completely excluded. The dynamics of the expansion of the Universe is investigated in such models, and the consequences of the existence of the CC in various astrophysical questions are addressed [10, 11, 13].

The induced theory of gravitation. In this theory, the CC arises as the zeroth term of the expansion of the density of the Lagrange function in a power series over the curvature of space [8].

Propagation of gravitational waves. In a consideration of a gravitational wave far from matter, the only source supporting its propagation can be the wave itself. R. Feynman concludes that nonlinear corrections should be added to the action. After this, his analysis became quite generally accepted and led to the conclusion that a sufficiently general, consistent field equation that includes not more than two derivatives is the HEE with the CC. A similar approach was developed by S. Gupt and R. Kraichnan. At the end of the 1990s it became clear that a nonzero value of the CC is in many ways fundamental.

Expansion of the Universe. Results of analyses of the red shift of emission lines, performed by different international astrophysical groups, provide evidence of the accelerated expansion of the Universe. Such behavior necessitates a large energy density in the Universe, which has been called *dark energy* or quintessence (other names for it are vacuum-like matter, the cosmological Λ-term, and the Cosmological Constant) [11–13]. In contrast to *dark matter*, the given component is distributed uniformly and is not subject to clustering. It is thought that this consists of physical fields of an unknown nature.

The density of matter in the Universe. A useful parameter in cosmology is the ratio of the density of various structural forms of material components to its critical density $\Omega = \frac{\rho}{\rho_{cr}}$, $\rho_{cr} = \frac{3H_0^2}{8\pi k}$, where H_0 is the Hubble constant. The contribution of the baryonic component Ω_b does not exceed 0.023 ($\Omega_b < 0.023$). The contribution of all matter of a nonfield character, including *dark matter*, Ω_m , does not exceed 0.3 ($\Omega_m < 0.3$). This was a stumbling block for a long time since the Universe having a flat geometry necessitates that $\Omega_m = 1$. In a number of works from 1980–1990 it was considered as a statement having a fundamental character. The need to introduce *dark energy* made it possible to remove this contradiction, as it is assumed that the remaining part of the density rests on it: $\Omega_\Lambda \approx 0.7$ [11–13].

Modification of Newtonian dynamics. A modification of the theory was developed at first to describe the rotation curves of galaxies [14] and to reject the necessity of introducing dark matter. The first approach is associated with a modification of the gravitational interaction, and the second – with a modification of Newton's second law. In both cases, results at great distances are modified (for accelerations greater than some value a_0). The question arises of the choice of the parameter a_0. It would be natural to associate it with the Hubble constant ($\propto H_0$), with the curvature of space ($\propto 1/R$), or with the CC ($\propto \Lambda^{1/2}$) [14]. We also note that the existence of the CC affects the growth of the Universe, the anisotropy of the relict radiation, etc. [11–13], and a consideration of various questions associated with its value being different from zero leads to the value $\Omega_\Lambda = 10^{-0.1\pm0.1}$ [12].

On the nature of the cosmological constant. Let us consider the action S_G in a space of a greater number of dimensions (N), where G_{AB} (A, B = 1–N) is the metric tensor in this space. If the dimensionality of space is lowered ($N \to n$), then this will lead to the result that the metric tensor of the space of a greater number of dimensions will be replaced by the metric tensor of a space of a lower number of dimensions $G_{\mu\nu} \to g_{\mu\nu}$, i.e.,

$$G_{\mu\nu} = g_{\mu\nu},\ \mu,\ \nu = 1\text{–}n. \tag{5}$$

All the remaining quantities, including the Ricci tensor, and the curvature will undergo a similar change, thus $R_G \to R_g$, where R_G and R_g are the curvatures of spaces calculated using G_{AB} and $g_{\mu\nu}$, etc. Now, integrating the action over the space of lower dimensionality, we go from S_G to S_g.

But we can also do this another way, working as before with the original action S_G and metric tensor G_{AB}. The transition to the space of lower dimension is a restriction of the initial problem by a specified condition; therefore, we can seek the extremum of the action (i.e., the HEEs) in the initial space under this condition, i.e., we can seek the conditional extremum in the initial problem. We are not imposing any conditions on any of the elements of the matrix of the metric tensor G_{AB} (A, B = 1–n), i.e., as before we have Eqs. (5) since they should follow as the solution of the variational problem. For $A, B > n$ we can choose

$$G_{AB} = 0,\ A \neq B,\ G_{A\nu} = 0,\ G_{\nu A} = 0,\ \nu = 1\text{–}n,\ G_{AA} = 1/V^2{}_A \tag{6}$$

(A is not being summed over, and V_A is the *volume* of the space characterized by the coordinate A) since as a result of the disappearance of interactions underlying the existence of a greater number of dimensions, the nondiagonal elements vanish. Only the choice of components G_{AA}, $A > n$, is not unique. We note that in the Kaluza theory and its generalizations, in the description of 5-dimensional spaces G_{55} can be chosen to be constant, a result of which will be invariance of the ratio of the charge of a particle to its mass, or it will be a function of the spacetime coordinates, which is equivalent to introducing a scalar field. So far, effects of a scalar field on variation of the ratios of the charges of particles to their masses have not been detected [1]. By virtue of this, we will assume that V_A, $A > n$, does not depend on x_μ, $\mu = 1–n$.

A condition is in fact imposed only on the determinant of this matrix, wherefore

$$\sqrt{-G} = \frac{\sqrt{-g}}{V_{n+1}...V_N} . \tag{7}$$

The general Lagrange method of searching for an extremum of some function f, in the given case the functional S_G, under the condition $F(y_1, …, y_i, …, y_M) = 0$ for the variables $y_1, …, y_i, …, y_M$ consists in introducing the term $\lambda F(y_1, …, y_i, …, y_M)$ into this function and searching for an unconditional extremum for it for the variables $y_1, …, y_i, …, y_M$, λ [15]. Thus, according to the general algorithm of searching for the conditional extremum employing the Lagrange multipliers, we should introduce the term

$$-\lambda \int \left(\sqrt{-G} - \frac{\sqrt{-g}}{V_{n+1}...V_N} \right) dx_1 dx_2 ... dx_N$$

into the initial action S_G and search for the no-longer-conditional, but total (unconditional) extremum of the new action over the variables G_{AB}, λ. In this case, setting the variation of the new action

$$\tilde{S}_G = S_G - \lambda \int \left(\sqrt{-G} - \frac{\sqrt{-g}}{V_{n+1}...V_N} \right) dx_1 dx_2 ... dx_N$$

with respect to λ equal to zero will give us the same condition (condition (7)), under which we will seek the conditional extremum of the initial action. Setting the variational derivative with respect to G_{AB} equal to zero then leads us to the equations

$$\frac{\delta \tilde{S}_G}{\delta G_{AB}} = \frac{\delta \left(S_G - \lambda \int \sqrt{-G} dx_1 dx_2 ... dx_N \right)}{\delta G_{AB}} = 0 .$$

In this case, condition (5) in the sector of lowered dimensionality will now be written as

$$\frac{\delta \left(S_g - \lambda \int \sqrt{-G} dx_1 dx_2 ... dx_N \right)}{\delta g_{\mu\nu}} = 0 ,$$

since V_A, $A, B > n$, do not depend on x_μ, $\mu = 1–n$, and, by virtue of Eqs. (6), calculation of the Christoffel symbols, the curvature tensor, and the Ricci tensor in this sector will not lead to the appearance of additional contributions from the components G_{AB}, $G_{A\nu} = 0$ and $G_{\nu A} = 0$ ($A, B = (n + 1) – N$, $\nu = 1–n$). Condition (7) now leads to the result that the integration will be carried out over a volume of a space of dimensionality n since $\frac{\int dx_{n+1}...dx_N}{V_{n+1}...V_N} = 1$:

$$\int \sqrt{-G}dx_1...dx_N = \frac{\int \sqrt{-g}dx_1...dx_N}{V_{n+1}...V_N} = \int \sqrt{-g}dx_1...dx_n\ ,$$

i.e., we arrive (see Eqs. (3)) at the equations $\frac{\delta \tilde{S}_g}{\delta g_{\mu\nu}} = 0$ and correspondingly at Eqs. (4).

Thus, the CC can be interpreted as a Lagrange multiplier introduced to search for a conditional extremum of action in the transition from a space of higher dimensionality to a space of lower dimensionality as a result of the evolution of the Universe. From a physical point of view, in the transition to a space of lower dimensionality the energy contained in the disappearing degrees of freedom should be released in the remaining space. And it will be released in the form of the CC or in the form of quintessence. Until this transition to the space of lower dimensionality has been completed, this energy will most likely grow in time although its concrete behavior will depend on the dynamics of the transition, about which we still do not know anything. We still don't have any data on the nature of dark energy, but there have been efforts to describe it in some scalar field models. In these models, a growth of quintessence with time is needed to explain the growth of the observed red shift. [13].

CONCLUSIONS

Among various Universes, our Universe is characterized by one-dimensional time. Current thinking about the nature of space maintains that its dimensionality is closely connected with the existence of interactions and grows with increasing number of interactions of different nature. According to current thinking, the dimensionality of our space is equal to eleven. This position is based on taking the existence of gravitational, and also electromagnetic, weak, and strong interactions into account (in the standard $SU(3) \times SU(2) \times U(1)$ model). The nature of dark matter and dark energy is unknown to us. What kinds of interactions do they represent? If we ultimately are not able to reject them in a description of the Universe, then the dimensionality of our space in the current epoch should be greater than eleven.

As we move to the beginning of the Big Bang, other interactions, unknown to us, in addition to known ones, could be manifested, which entails a growth of the dimensionality of space. Moreover, if the equidistribution law is valid during the initial moments of the Big Bang, then the existence of T-duality in string theory can lead to an infinite (or very large) dimensionality of space. Thus, a scenario of the evolution of the Universe is possible, in which as it cools off we transit to an increasingly low-energy sector of interactions, which leads to a gradual decrease in the dimensionality of space.

From a mathematical point of view, the CC can be interpreted as a Lagrange multiplier describing the transition of the action in a space of higher dimensionality to the action in a space of lower dimensionality. From a physical point of view, in the transition to a space of lower dimensionality the energy associated with disappearing degrees of freedom should be released in the remaining space. And it is released in the form of the CC or quintessence. Such a way of looking at the nature of dark energy can explain its emergence, but not its composition. The time interval over which the dimensionality of our space transitioned to lower values will be accompanied by a change in the quintessence over time.

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