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SPECIAL FEATURES OF THE STRUCTURE OF SECULAR RESONANCES IN THE DYNAMICS OF NEAR-EARTH SPACE OBJECTS

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The special features of the structure of secular resonances in the near-earth orbital space bounded by the following range of orbital parameters: semimajor axis from 8000 to 55 000 km, inclination from 0 to 90°, and eccentricity equal to 0.01, 0.6, and 0.8 are analyzed. The influence of stable and unstable secular resonances on the long-term orbital evolution of near-earth space objects is also considered. It is demonstrated that the joint effect of the stable secular resonances of different spectral classes does not violate the regularity of motion. The chaoticity arises when stable secular resonances of one spectral class are imposed.

Keywords: artificial Earth satellites, secular resonances, dynamic evolution.

INTRODUCTION

The present article is a continuation of a large cycle of works [1–6] devoted to investigation of the influence of secular resonances on the dynamic evolution of near-earth artificial space objects. In [1] the technique of revealing the secular resonances was presented in detail and it was demonstrated that for orbits with inclination angles chosen for constellations of navigating systems, deviations from the secular lunar-solar resonances led to an increase in the orbit eccentricities that changes significantly the position of these orbits in space and led to falling of defunct objects within the region of orbiting of functioning objects. In [2] the prevalence in the near-earth space of almost circular orbits ($e = 0.01$) of regions where the chaoticity of motion arose due to the cumulative effect of various secular resonances was investigated. The influence of the secular resonances on the dynamic evolution of objects moving in elongated orbits ($e = 0.6$ and 0.8) was investigated in [3] for a wide range of inclinations and semimajor axes from 8000 to 55000 km, and in [4] it was investigated for objects moving in circumpolar orbits with eccentricity $e = 0.01$, 0.6, and 0.8. Results of analysis of the mean exponential growth factor of nearby orbits (MEGNO) for long-term orbital evolution of uncontrollable objects of satellite radio navigating systems were presented in [5]. It was shown that the given objects are subject to the secular resonances that can cause dynamic chaoticity in their long-term orbital evolution. In [6] time variations of the critical argument whose behavior allowed the stability of a resonant configuration to be judged was considered for objects located in the expanded super-geo zone and moving along almost circular orbits together with the evolution of the resonant relation describing the secular resonance.

In the present work a refined version of the employed technique is given and results of analysis of some interesting peculiarities of the examined object dynamics under conditions of the imposition of the secular resonances are presented.

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1. RESEARCH TECHNIQUE

Let us briefly describe the research technique. The analytical technique of revealing secular resonances consists in calculation of conditions for resonance occurrence.

Let us represent the arguments of perturbing functions for singly and doubly averaged bounded three-body problem of the form

$$\underline{\psi} = (l-2p'+q')M' - (l-2p)\omega + (l-2p')\omega' - \overline{m}(\Omega - \Omega'),$$
$$\underline{\underline{\psi}} = (l-2p')\omega' - (l-2p)\omega - \overline{m}(\Omega - \Omega'), \tag{1}$$

where $M' = M_0' + \overline{n}'(t-t_0)$, $\omega' = \omega_0' + \dot{\omega}'(t-t_0)$, $\Omega' = \Omega_0' + \dot{\Omega}'(t-t_0)$, $\omega = \omega_0 + \dot{\omega}(t-t_0)$, and $\Omega = \Omega_0 + \dot{\Omega}(t-t_0)$. Then the resonance condition can be formulated as follows:

$$\dot{\underline{\psi}} \approx 0, \quad \dot{\underline{\underline{\psi}}} \approx 0. \tag{2}$$

Let us call expressions (2) resonant relations. Secular frequencies in satellite motion

$$\dot{\Omega} = \dot{\Omega}_{J_2} + \dot{\Omega}_L + \dot{\Omega}_S, \; \dot{\omega} = \dot{\omega}_J + \dot{\omega}_L + \dot{\omega}_S \tag{3}$$

are determined by the influence of the second zonal harmonic [7]

$$\dot{\Omega}_{J_2} = -\frac{3}{2} J_2 \overline{n} \left(\frac{r_0}{a}\right)^2 \cos i (1-e^2)^{-2}, \;\; \dot{\omega}_{J_2} = \frac{3}{4} J_2 \overline{n} \left(\frac{r_0}{a}\right)^2 \frac{5\cos^2 i - 1}{(1-e^2)^2}, \tag{4}$$

and of the third bodies – the Moon (L) and the Sun (S) [8]:

$$\dot{\Omega}_{L,S} = -\frac{3}{16} \overline{n} \frac{m'_{L,S}}{m_\oplus} \left(\frac{a}{a'}\right)^3 \frac{2+3e^2}{\sqrt{1-e^2}} (2 - 3\sin^2 i')\cos i,$$
$$\dot{\omega}_{L,S} = \frac{3}{16} \overline{n} \frac{m'_{L,S}}{m_\oplus} \left(\frac{a}{a'}\right)^3 \frac{4 - 5\sin^2 i + e^2}{\sqrt{1-e^2}} (2 - 3\sin^2 i'). \tag{5}$$

Here e, i, and $\overline{n}$ denote the eccentricity, inclination, and average motion of the satellite, e', i', and $\overline{n}'$ denote the eccentricity, inclination, and average motion of the third body, $m'_{L,S}/m_\oplus$ is the ratio of the mass of the third body $m'_{L,S}$ to the mass of the Earth $m_\oplus$. In these formulas the dependence of the frequencies on the satellite orbit inclination is distinctly traced; therefore, these resonances are called inclination-dependent.

The procedure of revealing of this or that resonance in the orbital dynamics of an object is reduced to investigation of the degree of smallness of Eqs. (2) for various sets of subscripts l, p, p', q, q', and $\overline{m}$. Then the time evolution of relations (1), the so-called critical arguments, is considered for the same values of the subscripts. This is necessary [9, 10] to establish what character: stable with libration variations of Eqs. (1) or unstable with circulation variations of resonant configurations have the resonant configurations. To investigate the long-term time evolution of Eqs. (1) and (2), the satellite orbit elements were determined by numerical modeling [11].

TABLE 1. Types of Resonant Relations of Low Orders

No.	Resonant relation type	No.	Resonant relation type	No.	Resonant relation type
1	$\dot{M}'_S - \dot{\omega} + \left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega}'_S$	11	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) - \dot{\omega} + \dot{\omega}'_S$	21	$\dot{M}'_L - \left(\dot{\Omega} - \dot{\Omega}'_L\right)$
2	$\dot{M}'_S - \dot{\omega} - \left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega}'_S$	12	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) + 2\dot{\omega} - 2\dot{\omega}'_S$	22	$\dot{M}'_L - 2\dot{\omega} + 2\dot{\omega}'_L$
3	$\dot{M}'_S - 2\dot{\omega} + \left(\dot{\Omega} - \dot{\Omega}'_S\right) - 2\dot{\omega}'_S$	13	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) - 2\dot{\omega} + 2\dot{\omega}'_S$	23	$\dot{M}'_L - \dot{\omega} + \dot{\omega}'_L$
4	$\dot{M}'_S + 2\left(\dot{\Omega} - \dot{\Omega}'_S\right)$	14	$\left(\dot{\Omega} - \dot{\Omega}'_S\right)$	24	$\left(\dot{\Omega} - \dot{\Omega}'_L\right) + \omega - \dot{\omega}'_L$
5	$\dot{M}'_S - 2\left(\dot{\Omega} - \dot{\Omega}'_S\right)$	15	$\dot{M}'_L - \dot{\omega} + \left(\Omega - \Omega'_L\right) + \dot{\omega}'_L$	25	$\left(\dot{\Omega} - \dot{\Omega}'_L\right) - \dot{\omega} + \dot{\omega}'_L$
6	$\dot{M}'_S + \left(\dot{\Omega} - \dot{\Omega}'_S\right)$	16	$\dot{M}'_L - \dot{\omega} - \left(\dot{\Omega} - \dot{\Omega}'_L\right) + \dot{\omega}'_L$	26	$\left(\dot{\Omega} - \dot{\Omega}'_L\right) + 2\dot{\omega} - 2\dot{\omega}'_L$
7	$\dot{M}'_S - \left(\dot{\Omega} - \dot{\Omega}'_S\right)$	17	$\dot{M}'_L - 2\dot{\omega} + \left(\dot{\Omega} - \dot{\Omega}'_L\right) - 2\dot{\omega}'_L$	27	$\left(\dot{\Omega} - \dot{\Omega}'_L\right) - 2\dot{\omega} + 2\dot{\omega}'_L$
8	$\dot{M}'_S - 2\dot{\omega} + 2\dot{\omega}'_S$	18	$\dot{M}'_L + 2\left(\dot{\Omega} - \dot{\Omega}'_L\right)$	28	$\left(\dot{\Omega} - \dot{\Omega}'_L\right)$
9	$\dot{M}'_S - \dot{\omega} + \dot{\omega}'_S$	19	$\dot{M}'_L - 2\left(\dot{\Omega} - \dot{\Omega}'_L\right)$	29	$\dot{\omega}$
10	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega} - \dot{\omega}'_S$	20	$\dot{M}'_L + \left(\dot{\Omega} - \dot{\Omega}'_L\right)$		

Setting the subscripts $l = 2$, $p, p', \overline{m} = 0, 1, 2,$ and $q, q' = -1, 0, 1$, we obtain all resonant relations of low orders; following [12], they can be subdivided into 4 groups or classes: the resonant relations with the average motion of the Sun, the resonant relations with the average motion of the Moon, the apsidal-nodal resonances with the Sun, and the similar resonances with the Moon; moreover, the Lidov–Kozai geometric resonance is a special case of the apsidal-nodal resonances. The basic types of resonant relations of low orders are given in Table 1.

Relations 1–9 and 15–23 correspond to resonances with average motion of the Sun and the Moon, respectively; relations 1–3 and 15–17 describe the mixed apsidal-nodal resonance; relations 4–7 and 18–21 describe the nodal resonance; and relations 8–9 and 22–23 characterize the apsidal resonance with average motion of the Moon and the Sun. Relations 10–13 and 24–27 describe the mixed secular resonance, and relations 14 and 28 characterize the pure nodal resonance. Relation 29 represents the Lidov–Kozai resonance which by its nature is a geometric resonance since it depends only on the mutual arrangement of the objects and is independent of the frequencies of motion of orbiting bodies.

Rosengren *et al*. [12] and Daqun *et al*. [13] recommended that all the spectrum of resonant relations of one class should be considered not to miss the phenomenon of the imposition of stable resonances of one class that can result in chaoticity of object motion, as shown by B. V. Chirikov [14].

Full spectra of resonances of each class are sufficiently extensive; therefore, in Table 2 we present the full spectrum of apsidal-nodal resonances. Other groups of resonances can be represented analogously.

If by analogy with the data of Rosengren *et al*. [12] and Daqun *et al*. [13] and of some other authors we consider that $\dot{\omega}'_S$ and $\dot{\Omega}'_S$ related with the precession of the Earth orbit are negligibly small, the formulas for the apsidal-nodal resonances with the Sun, presented in Table 2, will be simplified significantly.

The dynamic portraits of the secular resonances are constructed in the phase plane [9, 10]

$$x = e\cos\psi, \quad y = e\sin\psi,$$

where e is the eccentricity of the satellite orbit, and ψ is the critical argument in the form of Eq. (1). They are used in the given technique to determine the stability limits of the resonance.

The long-term orbital evolution was modeled numerically using the program complex *Numerical Model of Motion of Systems of Artificial Satellites* [11] and the MEGNO-analysis [15] of the orbital evolution of objects [16]. Estimates of the accuracy of predicting the motion of artificial satellites for long time periods can be found in [1].

TABLE 2. Apsidal-Nodal Resonances

Types of Resonant Relations		
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)+\dot\omega-\dot\omega'_{S,L}$	$\left(\dot\Omega-\dot\Omega'_{S,L}\right)-2\dot\omega-2\dot\omega'_{S,L}$	$\left(\Omega-\Omega'_{S,L}\right)-2\dot\omega'_{S,L}$
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)-\dot\omega+\dot\omega'_{S,L}$	$\left(\dot\Omega-\dot\Omega'_{S,L}\right)+2\dot\omega$	$\left(\Omega-\Omega'_{S,L}\right)+2\dot\omega'_{S,L}$
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)+2\dot\omega-2\dot\omega'_{S,L}$	$\left(\dot\Omega-\dot\Omega'_{S,L}\right)-2\dot\omega$	$\left(\dot\Omega-\dot\Omega'_{S,L}\right)$
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)-2\dot\omega+2\dot\omega'_{S,L}$	$\left(\dot\Omega-\dot\Omega'_{S,L}\right)+\dot\omega$	$\dot\omega-\dot\omega'_{S,L}$
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)+\dot\omega+\dot\omega'_{S,L}$	$\left(\dot\Omega-\dot\Omega'_{S,L}\right)-\dot\omega$	$\dot\omega+\dot\omega'_{S,L}$
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)-\dot\omega-\dot\omega'_{S,L}$	$\left(\Omega-\Omega'_{S,L}\right)+\dot\omega'_{S}$	$\dot\omega$
$\left(\dot\Omega-\dot\Omega'_{S,L}\right)+2\dot\omega+2\dot\omega'_{S,L}$	$\left(\Omega-\Omega'_{S,L}\right)-\dot\omega'_{S,L}$	

2. STRUCTURE OF THE NUMERICAL EXPERIMENT

The numerical experiment described in the present work encompassed the region of the near-earth space with the following ranges of variations of the orbital parameters:

$$a = \{8000\text{–}55000\ \text{km}\},\ i = \{0\text{–}90°\},\ e = \{0.01, 0.6, 0.8\}.$$

From the given ranges of the parameters, 200 models were chosen for which the long-term orbital evolution was constructed during a 100-year time period using the program complex *Numerical Model of Motion of Systems of Artificial Satellites* and the MEGNO parameters were estimated. This allowed the presence of chaoticity in the object motion to be judged. For each model orbit, plots of time dependences of all basic resonant relations, critical arguments corresponding to them, and dynamic portraits of the secular resonances were constructed. In addition, the resonant relations passing through the zeros during the examined time period and the secular resonances whose critical arguments either librated during the examined time period or change over from the libration to the circulation and back were identified.

3. ANALYSIS OF THE DISTRIBUTION OF STABLE SECULAR RESONANCES IN THE NEAR-EARTH ORBITAL SPACE

Let us consider first of all the distribution of the stable secular resonances in the orbital space of almost circular orbits with inclinations $i = \{0\text{–}90°\}$ and semimajor axes a from 8000 to 55000 km. Our analysis is based on a study of the phase portraits of the secular resonances and of the evolution of the corresponding critical arguments for 200 model objects with the above-specified orbits.

The results obtained demonstrated that stable secular resonant configurations are practically absent for low almost circular orbits with semimajor axes up to 20 000 km and inclinations up to 85°. An exception is the nodal resonance with the average motion of the Sun $\dot M'_S + 2\left(\dot\Omega-\dot\Omega'_S\right)\approx 0$ and the secular resonances described by the relations $\dot M'_L-\dot\omega+\left(\Omega-\Omega'_L\right)+\dot\omega'_L\approx 0$ and $\left(\dot\Omega-\dot\Omega'_L\right)-\dot\omega+\dot\omega'_L\approx 0$.

The stable resonant configurations in almost circular orbits with inclinations up to 70° for different models gave the following secular resonant relations: $\left(\dot\Omega-\dot\Omega'_S\right)+\dot\omega-\dot\omega'_S\approx 0$, $\left(\dot\Omega-\dot\Omega'_S\right)+2\dot\omega-2\dot\omega'_S\approx 0$, $\dot M'_L-\dot\omega+\left(\Omega-\Omega'_L\right)+\dot\omega'_L\approx 0$, $\left(\dot\Omega-\dot\Omega'_L\right)-\dot\omega+\dot\omega'_L\approx 0$, and $\dot\omega\approx 0$. The corresponding critical arguments librated during the entire 100-year period. For the model objects with the same ranges of variation of the orbital parameters, the critical arguments of the resonant relations $\dot M'_S-\dot\omega+\left(\dot\Omega-\dot\Omega'_S\right)+\dot\omega'_S\approx 0$, $\dot M'_S-2\dot\omega+\left(\dot\Omega-\dot\Omega'_S\right)-2\dot\omega'_S\approx 0$,

$\dot{M}'_S + \left(\dot{\Omega} - \dot{\Omega}'_S\right) \approx 0$, $\dot{M}'_S - 2\dot{\omega} + 2\dot{\omega}'_S \approx 0$, $\dot{M}'_S - \dot{\omega} + \dot{\omega}'_S \approx 0$, $\left(\dot{\Omega} - \dot{\Omega}'_S\right) - \dot{\omega} + \dot{\omega}'_S \approx 0$, $\left(\dot{\Omega} - \dot{\Omega}'_S\right) - 2\dot{\omega} + 2\dot{\omega}'_S \approx 0$, $\left(\dot{\Omega} - \dot{\Omega}'_S\right) \approx 0$, and $\left(\dot{\Omega} - \dot{\Omega}'_L\right) \approx 0$ librated piecewise changing the libration variations of the critical argument into the circulation ones and back. And the concrete set of the resonant relations depended on the length of the semimajor axis of the model object.

In orbits with inclinations from 75 to 90° in the entire range of variations of the semimajor axes, the stable configurations of the secular nodal resonances $\left(\dot{\Omega} - \dot{\Omega}'_S\right)$ and $\left(\dot{\Omega} - \dot{\Omega}'_L\right)$ took place. Then depending on the semimajor axis, the apsidal-nodal resonances with the Sun joined to them.

The largest number of stable configurations was given by the Lidov–Kozai geometric secular resonance $\dot{\omega} \approx 0$, the apsidal resonances with average motion of the Sun, the mixed apsidal-nodal resonances with the Sun, and the nodal secular resonances with the Sun and the Moon.

Let us dwell in more detail on the Lidov–Kozai secular resonance $\dot{\omega} \approx 0$. An analysis of the phase portraits of this resonance in the plane $x = e\cos\psi_{29}$, $y = e\sin\psi_{29}$ demonstrated that the stable Lidov–Kozai secular resonance arose in the space of almost circular orbits with inclination of 45° and semimajor axes 40000–55000 km. For inclinations from 55 to 70°, the region of the stable Lidov–Kozai resonance was stretched over the semimajor axes from 25000–30000 to 55000 km. For the circumpolar orbits with inclinations from 75 to 80°, the region of the stable action of the Lidov–Kozai resonance was limited by the semimajor axes 40000–50000 km. For the semimajor axes close to 55000 km, the resonant argument ψ_{29} goes over to the libration–circulation mode, and for inclinations close to 90°, the action of the Lidov–Kozai resonance becomes stable again.

It should be noted that an analysis of the full data of the entire numerical experiment allowed us to conclude that the increase of the eccentricity did not change essentially the ranges of the secular resonance stability in the near-earth orbital space.

Thus, our analysis of the numerical experiment showed that the secular resonances with stable configurations in the examined orbital space region were concentrated at inclinations from 45 to 90° and semimajor axes from 20000 to 55000 km.

4. INFLUENCE OF THE SECULAR RESONANCES ON THE LONG-TERM ORBITAL EVOLUTION OF NEAR-EARTH SPACE OBJECTS

For 200 examined model objects we considered the evolution of all resonant relations listed in Table 1 and of the critical arguments corresponding to them. The secular resonances with stable configurations were identified during 100-year period. The results obtained were compared with the special features of the orbital evolution of various model objects, which allowed a number of special features to be established.

It was shown that the motion of the model objects that were not subject to the action of the secular resonances and had one or several stable secular resonances of different spectral classes was regular, and the average MEGNO parameter was $\overline{Y}(t) \approx 2$ irrespective of the type of the examined orbits. Figure 1 shows examples of two objects from the intermediate orbit zones with inclinations of 45 and 60°. The resonance $\dot{M}'_S - \dot{\omega} - \left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega}'_S \approx 0$ with average motion of the Sun acts on the object *a* together with the apsidal-nodal resonance $\left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega} - \dot{\omega}'_S \approx 0$ with the Sun. The resonance $\dot{M}'_L - \dot{\omega} + \left(\Omega - \Omega'_L\right) + \dot{\omega}'_L \approx 0$ with average motion of the Moon and the apsidal-nodal resonance $\left(\dot{\Omega} - \dot{\Omega}'_L\right) - \dot{\omega} + \dot{\omega}'_L \approx 0$ with the Moon acted on the object *b*. The resonances acting on each object belong to different spectral classes and, judging by the behavior of the critical arguments librating during the entire examined time period, are stable. The motion of the objects is regular during the entire time period. The fluctuation amplitudes of all three basic orbital parameters are limited. Both MEGNO parameters did not exceed 2 anywhere. The numbers of resonant relations and critical angles corresponding to them are shown for the numbering accepted in Table 1.

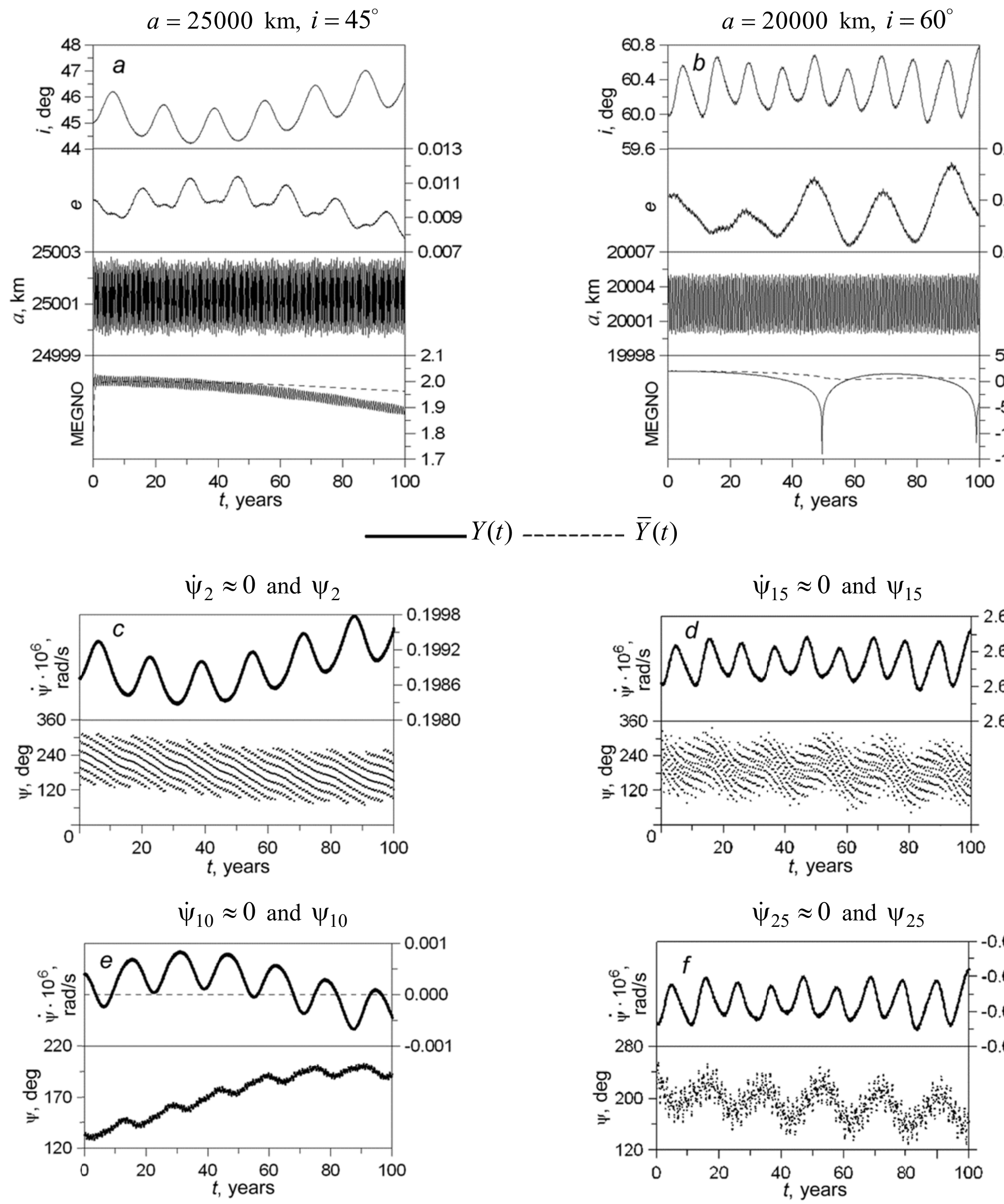

Fig. 1. Orbital dynamics of the model objects in the presence of two stable secular resonances of different spectral classes: *a* and *b* show the long-term orbital evolution of the orbit elements and of the MEGNO parameters, and *c*–*f* show the time variations of the resonant relations and their critical arguments.

The characteristic feature of the action of each of the two secular apsidal-nodal resonances $\left(\dot\Omega - \dot\Omega'_S\right) + 2\dot\omega - 2\dot\omega'_S$ and $\dot\omega \approx 0$ increased the eccentricity of orbits of the objects that retained the regularity of motion. In most cases the apsidal-nodal resonances acted in a group, and in the group, as a rule, in addition to

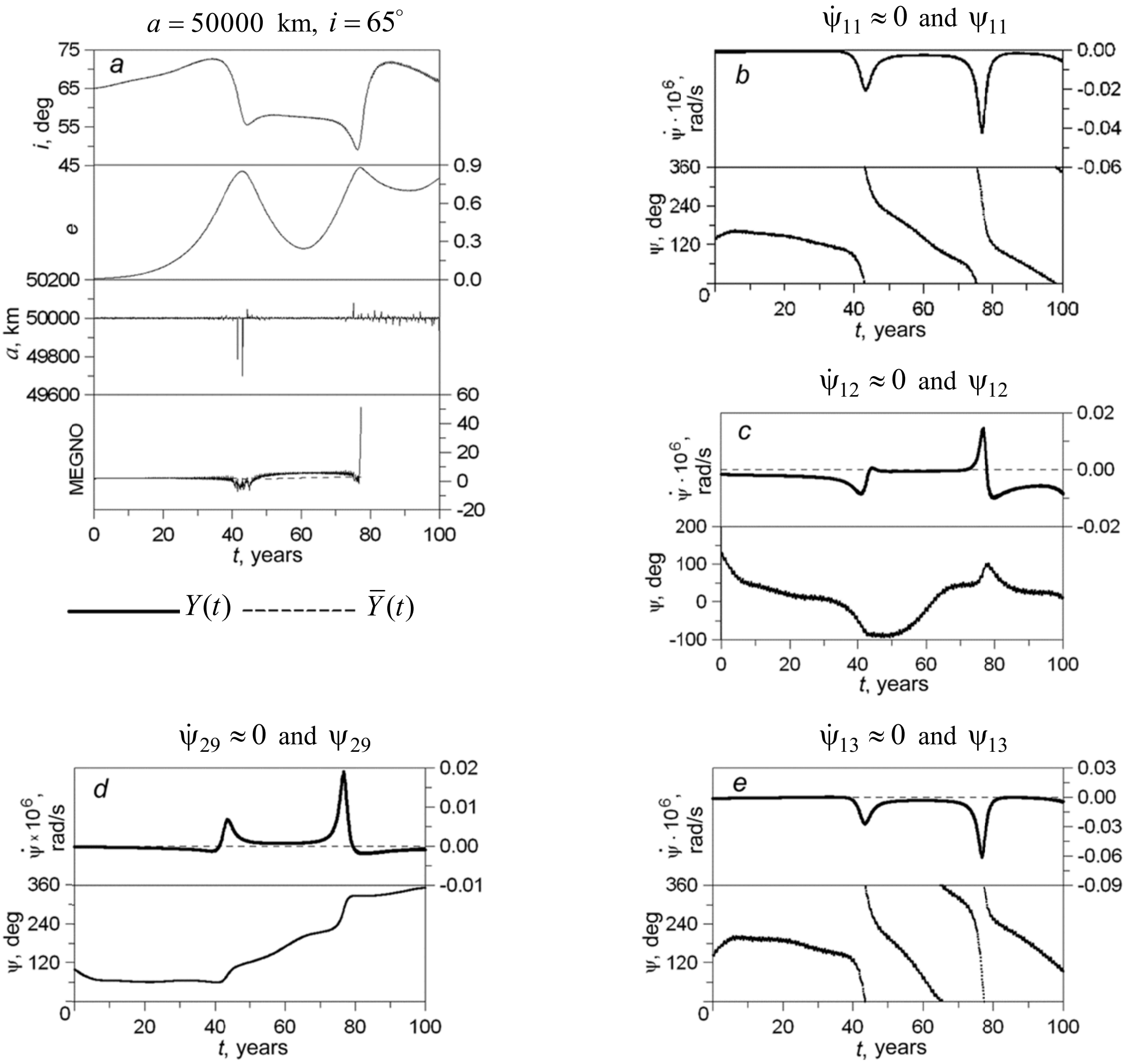

Fig. 2. Orbital dynamics of the model object after the imposition of several secular resonances of one spectral class. Here *a* shows the long-term orbital evolution of the orbit elements and the MEGNO parameters, and *b*–*e* show the time variations of the resonant relations for the secular resonances and their critical arguments.

resonances with the stable librating resonances, the resonances were met with the critical arguments that alternated from libration to circulation and back during the examined 100-year period. The motion of objects with such set of the acting secular resonances, as a rule, was accompanied by chaotization.

Figure 2 shows the orbital evolution of the object subject to the action of four apsidal-nodal secular resonances

$$\left(\dot{\Omega}-\dot{\Omega}'_S\right)-\dot{\omega}+\dot{\omega}'_S \approx 0,\ \left(\dot{\Omega}-\dot{\Omega}'_S\right)+2\dot{\omega}-2\dot{\omega}'_S \approx 0,\ \left(\dot{\Omega}-\dot{\Omega}'_S\right)-2\dot{\omega}+2\dot{\omega}'_S \approx 0,\ \dot{\omega} \approx 0,$$

with the Sun. All four resonances are sharp: the resonant relations not only pass through the zeros, but also have zeros during long time periods.

The evolution of the critical arguments demonstrated that the apsidal-nodal resonance $\left(\dot{\Omega}-\dot{\Omega}'_S\right)+2\dot{\omega}-2\dot{\omega}'_S \approx 0$ and the Lidov–Kozai resonance $\dot{\omega} \approx 0$ had stable configurations during the 80-year period.

At the same time, the apsidal-nodal secular resonances $\left(\dot{\Omega}-\dot{\Omega}'_S\right)-\dot{\omega}+\dot{\omega}'_S\approx 0$ and $\left(\dot{\Omega}-\dot{\Omega}'_S\right)-2\dot{\omega}+2\dot{\omega}'_S\approx 0$ librated piecewise repeatedly going over from libration to circulation and back. The stable action of two secular resonances $\left(\dot{\Omega}-\dot{\Omega}'_S\right)+2\dot{\omega}-2\dot{\omega}'_S\approx 0$ and $\dot{\omega}\approx 0$ led to a sharp increase in the eccentricity to 0.9, and the semimajor axis underwent sharp fluctuations with a large amplitude. In the region of the eccentricity peak, two other resonances alternated from libration to circulation, the eccentricity started to decrease, and the MEGNO parameters demonstrated an increase, though insignificant. In the second peak of eccentricity increase to 0.9, the resonance $\dot{\omega}\approx 0$ also went over from the libration mode to the circulation mode, and after that the MEGNO parameters started to increase linearly and very quickly.

Thus, from our viewpoint, the imposition of secular resonances alone is insufficient; it is required that the resonances intersecting the separatrix (which was exactly observed when going over from the libration mode to the circulation mode) and falling within the region of the phase space where other secular resonances of the same spectral class could act were among them.

CONCLUSIONS

In the present work, results of analysis of the special features in the structure of the secular resonances in the dynamics of the near-earth objects with the semimajor axis from 8000 to 55 000 km, inclination from 0 to 90°, and eccentricity equal to 0.01, 0.6, and 0.8 have been presented. The results obtained are compared with the special features of the dynamic evolution of objects. This allowed us to make a number of interesting conclusions:

1. For all examined eccentricities of the orbits, the secular resonances with stable configurations are concentrated in the range of inclinations from 45 to 90° and semimajor axes from 20000 to 55000 km.

2. In all cases, the stable influence of the apsidal-nodal resonances, including the Lidov–Kozai resonance, leads to long-period variations of the eccentricities of the orbits with large oscillation amplitudes.

3. The motion of the model objects that are not subject to the action of the secular resonances and have one secular resonance is regular.

4. The action of two or several stable secular resonances of different spectral classes is not accompanied by the occurrence of chaoticity.

5. The imposition of several secular resonances of one spectral class, among which there are stable resonances and resonances going over from the stable to unstable state, can lead to chaotization of the object motion.

6. Joint action of a great number of secular resonances whose critical arguments during the examined time period change repeatedly the libration character of motion to the circulation one and back leads to the occurrence of chaotization in the motion of the objects.

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