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# COHERENT LIGHT AT THE INTERFACE BETWEEN TWO MEDIA

## N. D. Kundikova<sup>1,2</sup>

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Reflection and refraction of coherent polarized radiation at the interface between two media are considered. It is shown that deviations from the well-known laws of geometrical optics are possible under certain conditions. The causes of such a deviation are considered.

Keywords: Goos-Hänchen shift, optical Magnus effect, spin and orbital angular momenta, spin-orbit interaction of a photon, optical Hall effect.

## INTRODUCTION

Reflection of light from the interface between two media has been utilized since ancient times. In the Museum of Science in London an ancient Egyptian bronze mirror is preserved which harkens back to the period 800–100 years before our era. In the British Museum (London) there is an entire exposition of Japanese bronze mirrors from the XIII<sup>th</sup> through the XV<sup>th</sup> centuries. These mirrors were considered as symbols of power. Mirrors fabricated from metal and glass by the British astronomer Sir William Herschel (1738–1822) for telescopes are found in the Museum of Science in London.

In all likelihood, the first formulation of the law of reflection of light can be said to harken back to 300 years before our era, when Euclid (ca. 365 - ca. 300 before our era), associating light rays with straight lines, observed that the angle of reflection is equal to the angle of incidence, and that the incident ray and the reflected ray lie in the same plane [1, 2]. Claudius Ptolomeus (ca. 100 - ca. 170), considering the refraction of light at the interface of two transparent media, observed that a beam of light, propagating along a straight line, is deflected from its original direction as it passes through such an interface.

In 1704 Sir Isaac Newton in his treatise *Opticks: Or, a Treatise on Reflections, Refractions, Inflections and Colours of Light* formulated three axioms: 1) The angles of reflection and refraction lie in the same plane as the angle of incidence. 2) The angle of reflection is equal to the angle of incidence. 3) A ray that is refracted from an optically less dense medium into an optically denser medium is deflected toward the perpendicular, that is, the angle of refraction is less than the angle of incidence [3]. The well-known mathematical formulation of the laws of reflection and refraction is rightfully ascribed to Willebrord Snellius (1580–1626) [4]. In 1821 Augustin Fresnel (1788–1827) derived fundamental expressions describing the variation of intensity and phase upon reflection and refraction of light at the interface between two media [5].

Among the achievements of the middle years of the twentieth century, it is necessary to include the creation of lasers and the discovery of two theretofore completely unknown phenomena, namely the possibility of the existence of materials with a negative coefficient of refraction [6] and wave front reversal [7]. At the same time, more subtle effects which are observed at the interface between two media remained practically unnoticed. Interest in these effects arose with the development of nanophysics and nanophotonics and was motivated by the need to take these effects into account in the development of new devices, new technologies, and new instrumentation.

<sup>&</sup>lt;sup>1</sup>Institute of Electrophysics of the Ural Branch of the Russian Academy of Sciences, Ekaterinburg, Russia; <sup>2</sup>South Ural State University, Chelyabinsk, Russia, e-mail: kundikovand@susu.ac.ru. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 10, pp. 5–12, October, 2015. Original article submitted August 25, 2015.



Fig. 1. Speckle patterns of left (a) and right (b) circularly-polarized radiation that has passed through an optical fiber.

The present review is dedicated to a consideration of transverse and longitudinal spatial shifts as well as transverse and longitudinal angular shifts which are observed upon reflection and refraction of radiation at the interface between two media.

### STUDY OF SPATIAL SHIFTS DUE TO EFFECT ACCUMULATION

The laws of reflection and refraction, formulated by Snell, are valid within the framework of geometrical optics or for plane waves. Attention was first drawn to deviations from Snell's laws in [8, 9]. Under total internal reflection, linearly polarized light undergoes a longitudinal shift, the magnitude of which is equal in order of magnitude to the wavelength and depends on the radiation polarization. This shift was first experimentally observed in the propagation of linearly polarized radiation of a different azimuth in a planar waveguide [10, 11] and has come to be known as the Goos–Hänchen shift from the names of the authors of that experimental work.

Under total internal reflection a circularly polarized reflected ray departs from the plane of incidence into a parallel plane [12, 13]. This transverse spatial shift is of the order of magnitude of a wavelength, and the direction of the shift depends on the sign of the circular polarization. This transverse shift was first observed experimentally under multiple total internal reflection upon propagation through a triangular prism [14, 15]. This shift has come to be called the Fedorov–Imbert shift.

An accumulation of the transverse spatial shift can be observed when circularly polarized radiation propagates through a multimode optical fiber (the optical Magnus effect) [16, 17]. The effect is manifested by a rotation of the speckle pattern of circularly polarized radiation that has passed through a multimode optical fiber upon a change in the sign of the circular polarization. Speckle patterns of circularly polarized radiation, rotated with respect to each other, calculated for propagation of radiation with wavelength  $\lambda = 488$  nm in a few-mode optical fiber with core radius  $\rho = 4.5$  µm, core refractive index  $n_{co} = 1.47$ , numerical aperture  $N_A = 0.11$ , and length 10 cm, are shown in Fig. 1.

If linearly polarized radiation undergoes total internal reflection, then a longitudinal shift should take place, accompanied by splitting of the beam into two circularly polarized beams. Such a splitting was first observed upon wave front reversal of a speckle pattern of radiation that had passed through a multimode optical fiber [18]. The reversed beam (or conjugate beam) consisted of two beams with circular polarization of opposite signs. Later, splitting of a linearly polarized beam into two beams with orthogonal circular polarization was observed in the propagation of light in a cylinder for grazing angle of incidence [19].

#### PRINCIPLE OF WEAK MEASUREMENTS

Experimental studies which investigated the accumulation of the effect after multiple reflections made it possible to detect longitudinal and transverse spatial shifts, but did not make it possible to carry out a detailed investigation of the magnitude of the shift due to a single total internal reflection. The possibilities of such an investigation became apparent only with the development of weak measurements. The principle of such measurements in classical optics was demonstrated in the case of the splitting of a Gaussian beam upon passage through a thin crystalline plate [20].

Let us consider the principle of weak measurements in more detail. An incident beam is linearly polarized with polarization azimuth angle  $\alpha$  in the plane of incidence. Upon passing through a crystalline plate the initial beam is split into two beams with orthogonal polarizations. The field of the transmitted wave has the following form:

$$\boldsymbol{E}_{w} = \begin{pmatrix} \left(E_{w}\right)_{x} \\ \left(E_{w}\right)_{y} \end{pmatrix} = E_{0} \left( \exp\left(-\frac{x^{2} + \left(y + a\right)^{2}}{w_{0}^{2}}\right) e^{i\phi} \begin{pmatrix} \cos \alpha \\ 0 \end{pmatrix} + \exp\left(-\frac{x^{2} + y^{2}}{w_{0}^{2}}\right) \begin{pmatrix} 0 \\ \sin \alpha \end{pmatrix} \right).$$

Here  $\begin{pmatrix} (E_w)_x \\ (E_w)_y \end{pmatrix}$ ,  $\begin{pmatrix} \cos \alpha \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ \sin \alpha \end{pmatrix}$  is the Maxwell column vector,  $E_w = (E_w)_x \hat{x} + (E_w)_y \hat{y}$ ,  $\hat{x}$  and  $\hat{y}$  are unit

vectors in the x and y directions, respectively,  $\phi$  is the phase difference between the x-component  $(E_w)_x$  and the y -component  $(E_w)_y$  of the field  $E_w$ ,  $w_0$  is the width of the Gaussian beam, and a is the displacement (spatial shift), where  $a/w_0 \ll 1$ . The split beam passes through an analyzer oriented at an angle  $\beta = \alpha + \pi/2 + \varepsilon$  relative to the plane of incidence, where the angle  $\varepsilon \ll 1$ , which is to say that the polarizer and the analyzer are practically crossed. Let  $\alpha = \pi/4$ , then after passage through the analyzer the field

$$\boldsymbol{E}_{\Sigma} = \begin{pmatrix} (E_{\Sigma})_{x} \\ (E_{\Sigma})_{y} \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} A_{x} (1+\sin 2\varepsilon) - A_{y} \cos 2\varepsilon \\ -A_{x} \cos 2\varepsilon + A_{y} (1-\sin 2\varepsilon) \end{pmatrix},$$

where

$$A_x = E_0 \exp\left(-\frac{x^2 + (y+a)^2}{w_0^2}\right) e^{i\phi}, \qquad A_y = E_0 \exp\left(-\frac{x^2 + y^2}{w_0^2}\right).$$

Along the x axis the intensity distribution remains Gaussian, but along the y axis it acquires the following form:

$$\left| \boldsymbol{E}_{\Sigma} \left( \boldsymbol{x} = 0, \boldsymbol{y} \right) \right|^{2}$$

$$= \frac{\left( \boldsymbol{E}_{0} \right)^{2}}{4} \exp\left( -\frac{2\boldsymbol{y}^{2}}{\boldsymbol{w}_{0}^{2}} \right) \left( \exp\left( -\frac{4\boldsymbol{a}\boldsymbol{y} + 2\boldsymbol{a}^{2}}{\boldsymbol{w}_{0}^{2}} \right) (1 + \sin 2\varepsilon) + (1 - \sin 2\varepsilon) - 2\exp\left( -\frac{2\boldsymbol{a}\boldsymbol{y} + \boldsymbol{a}^{2}}{\boldsymbol{w}_{0}^{2}} \right) \cos \phi \cos 2\varepsilon \right). \tag{1}$$

If we differentiate the intensity distribution  $|E_{\Sigma}(y)|^2$  with respect to the coordinate and set the result equal to zero, then it is possible to see that the position of the maximum  $y_{\text{max}} \neq 0$ . To simplify the calculations, we set  $\cos \phi = 1$ , and in the solution of the quadratic equation we take into account the smallness of the quantities *a* and  $\varepsilon$ . As a result, we obtain the following expression for the position of the coordinate  $y_{\text{max}}$  of the maximum intensity in the intensity distribution of the resulting beam:

$$y_{\max} \approx -\frac{a}{2} \cot \alpha \varepsilon$$
 (2)

It is clear from expression (2) that a change in the sign of the angle  $\varepsilon$  leads to a change in the sign of  $y_{\text{max}}$ . Substituting expression (2) into Eq. (1), it is easy to show that an increase in the observed shift leads to a significant lowering of the intensity of the observed distribution, that is to say, the price one has to pay to increase the spatial resolution is an increase in the sensitivity of the devices recording the intensity.

## SPATIAL AND ANGULAR SHIFTS INCIDENT TO REFRACTION AND REFLECTION

It appears that the longitudinal Goos–Hänchen shift was first measured experimentally inside a laser cavity in the vicinity of the angle of total internal reflection [21]. In [22] a single longitudinal Goos–Hänchen shift was measured using a one-dimensional position-sensitive detector. This method made it possible to determine with an accuracy of hundreds of nanometers the difference in the longitudinal shift for light polarized in the plane of incidence (*p*-polarization) and perpendicular to the plane of incidence (*s*-polarization).

According to the expressions obtained in [23], the longitudinal shift depends on the angle of incidence  $\vartheta$ , the shift for *p*-polarization  $d_p$  is greater than the shift for *s*-polarization  $d_s$ , and the difference between these two shifts  $\Delta = d_p - d_s$  has the following form:

$$\Delta = d_p - d_s = \frac{\lambda}{\pi} \frac{\sin \vartheta}{\sqrt{n^2 \sin^2(\vartheta) - 1}} \left( \frac{1}{\sqrt{(1+n)^2 \sin^2(\vartheta) - 1}} - 1 \right).$$
(3)

It is clear from expression (3) that as the angle of incidence approaches the angle of total internal reflection  $\vartheta_{cr} = \arcsin(1/n)$  the magnitude of  $\Delta$  tends to infinity. In the study reported in [22] measurements were carried out in a prism fabricated from BK7 glass at two wavelengths,  $\lambda = 0.67 \mu m$  (n = 1.511, critical angle 41.4°) and  $\lambda = 1.083 \mu m$  (n = 1.506, critical angle 41.6°). The authors of this work determined the maximum value of the difference  $\Delta$  to be equal to 19  $\mu m$  at the wavelength  $\lambda = 0.67 \mu m$ , and 9  $\mu m$  at the wavelength  $\lambda = 1.083$ .

The use of a two-dimensional position-sensitive detector made it possible for Pillon *et al.* [24] under conditions of a single total internal reflection to measure not only the longitudinal Goos–Hänchen shift, but also the much-smallerin-magnitude transverse Fedorov–Imbert shift. These measurements were performed for linearly, circularly, and elliptically polarized radiation at the wavelength  $\lambda = 1.083$  µm for a fixed angle of incidence  $\vartheta = 41.65^{\circ}$ , close to the angle of total internal reflection for BK7 glass. It turned out that linearly polarized radiation undergoes only a longitudinal shift with a maximum value on the order of 9 µm. A circularly polarized beam was shifted only in the transverse direction with the magnitude of the shift equal to 0.6 µm. As expected, a shift was observed both in the positive and in the negative directions. An elliptically polarized beam was shifted both in the positive and in the negative directions.

In 2004 Onoda *et al.* [25] showed that not only under conditions of total internal reflection but also for partial reflection, and also under conditions of refraction, circularly polarized radiation undergoes a transverse shift. The magnitude of the shift is also on the order of a wavelength, but the direction of the shift is equal for the reflected and the refracted radiation. This effect has come to be known as the spin Hall effect for light in analogy with the spin Hall effect in which a deflection of electrons with antiparallel spins occurs perpendicular to the direction of current toward opposite sides of a conductor in the absence of a magnetic field [26]. In the case of light the analog of the *force* is the gradient of the refractive index at the interface between the two media and photons with spins of different signs deflect in opposite directions, perpendicular to the gradient of the refractive index and to the plane of incidence.

The first experimental study of a transverse shift under conditions of refraction was performed in 2008 by Hosten *et al.* [27]. The measurements were performed at the wavelength  $\lambda = 632.8 \ \mu\text{m}$ . The light was refracted in BK7 glass; in order to eliminate the effect of refraction at the second interface, a prism was fabricated from it consisting of

two parts, the mutual rotation of which, for a given angle of incidence, allowed light to exit the prism along a path perpendicular to the surface. The principle of weak measurements was employed to amplify the shifts. Splitting of a linearly polarized beam into two circularly polarized beams with different signs of circulation was experimentally observed. It was shown first analytically, and then experimentally, that the magnitude of the shift depends on the azimuth of the linear polarization. The dependence of the transverse shift for the *p*-component  $\delta_{\sigma=\pm 1}^{p}$  and the *s*-component  $\delta_{\sigma=\pm 1}^{s}$  on the angle of incidence  $\vartheta$  has the following form:

$$\delta^{p}_{\sigma=\pm 1} = \pm \frac{\lambda}{2\pi} \frac{\cos \Theta_{T} - (t_{s}/t_{p})\cos \Theta}{\cos \Theta_{I}},$$

$$\delta^{s}_{\sigma=\pm 1} = \pm \frac{\lambda}{2\pi} \frac{\cos \Theta_{T} - (t_{p}/t_{s})\cos \Theta}{\cos \Theta_{I}}.$$
(4)

Here  $\vartheta_T$  is the refraction angle,  $t_s$  and  $t_p$  are the Fresnel coefficients for the amplitude, and  $\sigma = +1$  for right circular polarization and  $\sigma = -1$  for left circular polarization. In the experiment good agreement was observed between the measured values of the splitting and expressions (4). The maximum shift, recorded for near-grazing incidence (in the limit  $\vartheta \rightarrow 90^\circ$ ), was approximately  $\pm 80$  nm.

Under conditions of partial reflection of light from the interface between two media, polarization of the incident light has a big effect on the spatial shifts since, as is well known, in the case of reflection at the Brewster angle radiation with polarization lying in the plane of incidence is zeroed out. Such an effect was observed in [28]. For linear polarization perpendicular to the plane of incidence, the magnitude of the splitting increases with increase of the angle of incidence  $\vartheta$  to its maximum value, equal to  $\approx 69.6$  nm at  $\vartheta \approx 48^{\circ}$ , and then decreases to zero as the angle of incidence approaches 90°. In the case of radiation that is linearly polarized in the plane of incidence, for  $\vartheta < \vartheta_B$  the splitting is significantly increased with approach to the Brewster angle  $\vartheta_B \approx 57^{\circ}$ , reaching a value  $\approx 1800$  nm, and for  $\vartheta > \vartheta_B$  the shift changes sign and decreases with further increase of the angle of incidence. For a fixed angle of incidence, as the azimuth of linear polarization is varied from  $\alpha = 0$  (polarization in the plane of incidence) to  $\alpha = 90^{\circ}$  the shift decreases smoothly and for the azimuth of linear polarization equal to  $39^{\circ}$  it changes sign while growing in absolute value.

The longitudinal spatial shift under conditions of partial reflection was investigated experimentally for the Brewster angle of incidence [29]. It turned out that the longitudinal shift is exceptionally sensitive to a change in the azimuth of linear polarization in the vicinity of  $\alpha = 0^{\circ}$ . In the experiment both a longitudinal shift and a transverse shift were observed as the azimuth was varied within the limits  $-0.6^{\circ} \le \alpha \le 0.6^{\circ}$ . The shift changed sign at  $\alpha = 0^{\circ}$ ; in absolute value the longitudinal shift reached a value of 900 µm, and the transverse shift reached a value of 300 µm.

A longitudinal angular shift, specifically nonspecular reflection, was observed experimentally in [30] upon reflection of a Gaussian beam with wavelength  $\lambda = 820$  nm from BK7 glass (n = 1.51,  $\vartheta_B = 56.5^\circ$ ). According to the results of calculations, a longitudinal angular shift should not be observed for radiation with linear polarization perpendicular to the plane of incidence (the *s*-component); for the orthogonal polarization (the *p*-component) the shift grows as the Brewster angle is approached, and at angles greater than the Brewster angle it changes sign while falling off in absolute value. At  $\vartheta = \vartheta_B = 56.5^\circ$  the *s*-component disappears, while the *p*-polarized Gaussian beam undergoes

a deformation. In absolute value the deflection lies within the limits from  $10^{-5}$  to  $10^{-2}$  rad.

Departure of the reflected beam from the plane of incidence at some angle to it (the transverse angular shift, referred to here as an *out-of-plane shift*) was observed upon reflection of light from a metallic surface [31]. Both a spatial shift and an angular shift were observed for linearly polarized radiation; the largest splitting occurred for switching of the azimuth of linear polarization from  $\alpha = 45^{\circ}$  to  $\alpha = -45^{\circ}$ , but for switching between the *s*- and *p*-polarizations it was zero. For circular polarization only a transverse spatial shift was observed.

Under partial reflection a transverse spatial shift is affected by the structure of the field of the light beam. Thus, in [32] it was shown that for a linearly polarized partially reflected beam with orbital angular momentum  $l \neq 0$  the difference in the transverse spatial shifts between beams with *p*- and *s*-polarization  $\Delta = d_p - d_s$  has the following form:

$$\Delta = d_p - d_s = \frac{\lambda}{\pi} l \frac{\sin \vartheta}{\sqrt{n^2 - \sin^2(\vartheta)}} \left( 1 - \frac{n^2}{(n^2 + 1)\sin^2(\vartheta) - n^2} \right).$$
(5)

Here  $\lambda$  is the wavelength,  $\vartheta$  is the angle of incidence, and *n* is the relative refractive index of the medium. It follows from expression (5) that the quantity  $\Delta = d_{TM} - d_{TE}$  is equal to zero for l = 0 and depends linearly on the magnitude of the orbital angular momentum. It changes sign when the orbital angular momentum changes sign, and has a different sign for angles of incidence less than or greater than the Brewster angle.

The experimental results obtained in [33] demonstrate striking agreement between calculated and experimental values of  $\Delta = d_p - d_s$ . The measurements were performed at the wavelength  $\lambda = 632.8$  nm, light was reflected from a rotary prism, and the refractive index of the prism material was n = 1.5. The transverse distribution of the intensity of the reflected beam was recorded with a CCD array for each polarization state and each value of the orbital angular momentum. The cross-correlation method of image processing was employed to determine the distance between the centers of gravity of the recorded beams. The maximum shift  $\Delta = 2 \ \mu m$  was recorded in the vicinity of the Brewster angle for Laguerre–Gaussian beams with l = 3 and 5. For the angle of incidence  $\vartheta = 81^{\circ}$  a linear dependence of the transverse spatial shift  $\Delta = d_{TM} - d_{TE}$  on the magnitude of the orbital angular momentum was experimentally demonstrated in the range l = 1, 2, ..., 5.

In [34] the influence of the orbital angular momentum on the spatial and angular Goos–Hänchen shifts ( $\Delta_{GH}$  is spatial,  $\Theta_{GH}$  is angular) and Fedorov–Imbert shifts ( $\Delta_{FI}$  is spatial,  $\Theta_{FI}$  is angular) was investigated both theoretically and experimentally. It was shown that the influence of the orbital angular momentum leads to an interrelationship between the angular and spatial shifts which can be described by the following matrix equation:

$$\begin{bmatrix} \Delta_{\rm GH}^{l} \\ \Theta_{\rm FI}^{l} \\ \Delta_{\rm FI}^{l} \\ \Theta_{\rm GH}^{l} \end{bmatrix} = \begin{bmatrix} 1 & -2l & 0 & 0 \\ 0 & 1+|2l| & 0 & 0 \\ 0 & 0 & 1 & 2l \\ 0 & 0 & 0 & 1+|2l| \end{bmatrix} \begin{bmatrix} \Delta_{\rm GH} \\ \Theta_{\rm FI} \\ \Delta_{\rm FI} \\ \Theta_{\rm GH} \end{bmatrix}.$$
(6)

Here l is the orbital angular momentum. It follows from expression (6) that the presence of orbital angular momentum can amplify as well as suppress spatial and angular shifts.

Experimental studies were carried out at the wavelength  $\lambda = 632.8$  nm both for total internal reflection and for partial internal reflection. A prism was used that was fabricated from BK7 glass with refractive index n = 1.51. Shifts that arose when the polarization was modulated were recorded with the help of a two-dimensional position-sensitive detector. To separate the angular shifts, the beams were focused, and to separate the spatial shifts, they were collimated. The experimental results showed that under total internal reflection the orbital angular momentum has no influence on the magnitude of the shifts. Under partial reflection the magnitudes of the shifts for beams with orbital angular momenta  $l = 0, \pm 1$  depend on the angle of incidence, and the experimental results proved to match up well with the analytical results.

Upon reflection of radiation from a film deposited on a substrate, it is possible to observe a transverse shift, whose magnitude depends not on the sign of the circular polarization, but on the sign of the orbital angular momentum. In [35] Kundikova and Zaitsev modeled reflection of a Gauss–Bessel beam from a thin sapphire film deposited on silicon. The thickness of the film was varied in the range from 10 to 20 wavelengths. The wavelength was taken to be



Fig. 2. Intensity distribution of the beam before (a) and after (b) reflection.

equal to  $\lambda = 0.63 \ \mu\text{m}$ ; the following values of the relative permittivity were used:  $\varepsilon_s = 13.2$  for silicon and  $\varepsilon_f = 3.2$  for sapphire. The angle of incidence was varied from 0° to 45°.

Figure 2 displays the intensity distribution of a Gauss–Bessel beam before and after reflection from a film on a substrate with dimensions  $400 \times 400 \ \mu\text{m}$ , film thickness 12.3  $\mu\text{m}$  and incident angle  $\vartheta = 15^{\circ}$ . The topological charge was assumed to be l = +1, and the beam had left circular polarization.

In Fig. 2 a deformation of the intensity distribution of the beam after reflection is clearly visible, leading to a longitudinal shift of the center of gravity of the beam. The value of the longitudinal shift is equal to  $-45 \,\mu\text{m}$ , and in the transverse direction the beam is shifted by 0.75  $\mu$ m. It turned out that changing the polarization state does not affect either the longitudinal or the transverse shift; however, a change in sign of the orbital angular momentum, on the contrary, leads to a change in the sign of the shift, i.e., when the sign of the orbital angular momentum is reversed, the center of gravity of the beam is shifted by 1.5  $\mu$ m.

## PHYSICAL REASONS FOR THE APPEARANCE OF SPATIAL AND ANGULAR SHIFTS

A coherent laser beam, incident at an angle  $\vartheta_0$  onto the interface between two media, has some angular width. A spatially bounded beam consists of a set of plane waves, each of which is incident at an angle  $\vartheta_i$  lying in some range of angles  $\vartheta_0 \pm \Delta \vartheta$ . It is possible to obtain a representation of the intensity distribution of the plane waves over angles if the beam is focused and we consider the intensity distribution in the focal plane. For a Gaussian beam, this representation will also be described by a Gaussian function. Each plane wave after reflection or refraction does not keep its original field, but changes the amplitude and phase, depending on the angle of incidence. After reflection or refraction, the plane waves interfere, thereby forming a transverse intensity distribution and defining the direction of propagation of the reflected or refracted beam.

The transverse intensity distribution and direction of propagation of the reflected or refracted beam can be calculated if we assign the field of each plane wave with the help of a Fourier integral (for a continuous set of angles) or Fourier series (for a discrete set of angles). The amplitude and phase of each reflected or refracted plane wave are determined by the Fresnel formulas. The inverse Fourier transform allows us to obtain the intensity distribution I(x, y) over a cross section of the reflected or refracted beam. To determine the shifts, we make use of the concept of the center of gravity of the beam, having the coordinates  $x_0$  and  $y_0$ , which are determined as follows:

$$x_0 = \frac{\int I(x, y) x dx}{\int I(x, y) dx}, \quad y_0 = \frac{\int I(x, y) y dy}{\int I(x, y) dy}.$$

Obviously, the center of gravity of a Gaussian beam coincides with the position of its maximum intensity.

Such an approach does not always allow us to obtain an analytical solution; therefore, to estimate the magnitudes of the shifts, some approaches have been developed in [36].

## CONCLUSIONS

Studies of spatial shifts in the middle of the last century as a manifestation of the influence of polarization of radiation on its trajectory were of an extraordinarily fundamental character and together with studies of the influence of the trajectory on the polarization [37–41] have enabled a consideration of the spin-orbit interaction [16, 42] of a photon and made it possible to predict a number of effects associated with its appearance. The development of modern experimental methods, and also technologies, associated with the transition to the nanoscale range, has led to the need to consider the possibility of bringing fundamental results into the picture. The spatial and angular shifts considered above are very sensitive to a change in the physical state of systems and are promising for application in high-precision metrology. They can be used to determine the spatial distribution of electronic spin states in semiconductors [43], to determine the parameters of films of nanometer thickness [35, 44], to image graphene layers [45], and to investigate topological insulators [46].

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