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INTERACTION OF SOUND VIBRATIONS WITH A DISLOCATION CHAIN IN A PIEZOELECTRIC CRYSTAL

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Sound waves in a piezoelectric medium containing a chain of dislocations parallel to each other are investigated. It is demonstrated that interaction between vibrations localized on individual dislocations engenders waves propagating along the chain in the direction perpendicular to dislocations. The spectrum of allowable vibration frequencies consists of bands separated from each other and from the spectrum of volume vibrations. The total number of different independent wave vectors corresponding to frequencies forming a band is equal to the number of independent states of the vibrating dislocation chain and coincides with the number of dislocations contained in it.

Keywords: piezoelectric materials, dislocations, localized vibrations.

Various aspects of ultrasound interaction with defects of the crystal structure have been investigated in a number of works (for example, see [1–4]). Thus, peculiarities of elastic wave propagation in actual crystals were studied in [1], where mathematical modeling was performed of the dynamic behavior of dislocations under the effect of ultrasound on a crystal. Possible changes in the dislocation structures under the effect of ultrasound were described, and detailed analysis of the influence of these changes on the structure-sensitive properties of crystals was performed. In [2] the joint influence of ultrasound and electric field on the dynamics of dislocations in alkaline halide crystals was analyzed.

In [3] a system consisting of two parallel dislocations with sound waves [4] localized on these dislocations in a piezoelectric crystal was investigated for the first time. It was demonstrated that a general solution of the differential equation describing wave perturbations in this system can be represented as the sum of two solutions. The first solution corresponds to inphase vibrations, and the second solution corresponds to antiphase vibrations with frequencies ω_1 and ω_2. In this regard, it seems natural to consider a more realistic case of the medium containing a significant number of dislocations. Wave perturbations in a piezoelectric material with structural defects in the form of a chain of N dislocations are investigated below. It is demonstrated that the interaction between sound waves localized on individual dislocations [4] leads to the appearance of wave perturbations of new type propagating along the chain (perpendicular to dislocation lines) and the law of their dispersion is found. In this case, frequency bands of localized sound vibrations arise, separated by a finite gap from frequencies of volume vibrations.

We note that waves localized on dislocations exist in crystals having different physical nature. Thus, in [5–9] the possibility of localization of plasma waves, polaritons, Frenkel's excitons, and spin waves on dislocations was pointed out. Because of the similarity between the differential equations describing localized waves in various media, the results obtained below can also be generalized to them.

Let us consider longitudinal waves in a piezoelectric crystal which belongs to the class C_{4v} (tetragonal system) localized on dislocations oriented along the C_4 axis. We choose the coordinate system with the *z* axis coinciding with

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the C_4 axis and x and y axes perpendicular to two of the vertical symmetry planes. The equation for small vibrations $u_z(x,y,z,t) = u_{z0}(x,y,k)\exp i(kz-\omega t)$ in the crystal containing a dislocation chain has the form

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u_{z0}-u_{z0}\left(\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k^2-\frac{\rho_0}{\tilde{\lambda}_1}\omega^2\right)=-a_0^2k^2\frac{\gamma}{\tilde{\lambda}_1}\sum_{s=1}^{N}\delta(x-x_s)\delta(y)u_{z0}(x_s,0,k). \tag{1}$$

The terms of the sum in the right-hand side of Eq. (1), comprising the delta-functions $\gamma\delta(x-x_s)\delta(y)$, describe perturbations in the crystal formed by N dislocations spaced at identical distances d_0 from each other along the x axis parallel to the z axis, u_i is the deformation vector, ρ_0 is the density of the medium, a_0 is the lattice constant,

$$\tilde{\lambda}_1 \equiv \lambda_1+\frac{2\pi\beta_0\beta_2}{\varepsilon_2},\ \ \tilde{\lambda}_0 \equiv \lambda_0+\frac{4\pi\beta_0^2}{\varepsilon_2}, \tag{2}$$

$\lambda_0 \equiv \lambda_{zzzz}$ and $\lambda_1 \equiv \lambda_{zxzx}$ are the components of the stress tensor λ_{iklm}, $\varepsilon_1 = \varepsilon_{xx} = \varepsilon_{yy}$ and $\varepsilon_2 = \varepsilon_{zz}$ are the components of the dielectric permittivity tensor ε_{ik}, and $\beta_0 \equiv \beta_{z,zz}$ and $\beta_2 \equiv \beta_{x,xz} = \beta_{y,yz}$ are the components of the tensor $\beta_{i,kl}$ characterizing the piezoelectric effect [10]:

$$D_i = D_{0i} + \varepsilon_{ik}E_k - 4\pi\beta_{i,kl}u_{kl}. \tag{3}$$

Here D_i and E_i are the components of the electric induction vector and of the electric field strength, respectively. The dislocation spacing d_0 is much greater than the distance r_0 from the dislocation at which the amplitude of localized vibrations decreases e times.

Furthermore, we assume for simplicity that vibrations localized on dislocation number s interact only with vibrations localized on dislocations with numbers $s-1$ and $s+1$ neighboring to it. In this case, Eq. (1) assumes the form

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u_{z0}-u_{z0}\left(\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k^2-\frac{\rho_0}{\tilde{\lambda}_1}\omega^2\right)=-a_0^2k^2\frac{\gamma}{\tilde{\lambda}_1}\delta(y)[\delta(x-x_{s-1})u_{z0}(x_{s-1},0,k)$$

$$+\delta(x-x_s)u_{z0}(x_s,0,k)+\delta(x-x_{s+1})u_{z0}(x_{s+1},0,k)]. \tag{4}$$

The edge dislocations are in special positions, since they have neighbors only on one side. It is clear that for greater number N of dislocations, the influence of the edge effects should be insignificant. To simplify the problem, we limited ourselves to the cyclic boundary conditions:

$$u_{z01} = u_{z0N}. \tag{5}$$

All dislocations are in equivalent conditions now. We seek for a solution of Eq. (4) in the form $u_{z0}(x_s,0,k) \propto \exp iqx_s$, where q is the projection of the perturbation wave vector onto the x axis. Then the vibration amplitudes near the neighboring dislocations are related by the expressions

$$u_{z0}(x_{s-1},0,k)=u_{z0}(x_s,0,k)e^{-iqd_0},\ u_{z0}(x_{s+1},0,k)=u_{z0}(x_s,0,k)e^{iqd_0}. \tag{6}$$

Substituting Eqs. (6) into Eq. (4), we obtain

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u_{z0s}-u_{z0s}\left(\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k^2-\frac{\rho_0}{\tilde{\lambda}_1}\omega^2\right)=-a_0^2k^2\frac{\gamma}{\tilde{\lambda}_1}\delta(y)u_{z0}(x_s,0,k)\Big[\delta(x-x_{s-1})e^{-iqd_0}$$

$$+\delta(x-x_s)+\delta(x-x_{s+1})e^{iqd_0}\Big]. \qquad (7)$$

We further take advantage of the delta-function representation:

$$\delta(x-x_{s-1})\delta(y)=\frac{1}{(2\pi)^2}\int \exp i\left(\kappa_x(x-x_{s-1})+\kappa_y y\right)d^2\boldsymbol{\kappa}\,. \qquad (8)$$

A solution of Eq. (7) we seek in the form

$$u_{z0s}(x,y,k)=\frac{1}{(2\pi)^2}\int\tilde{\chi}(\boldsymbol{\kappa},k)\exp(i\boldsymbol{\kappa}\boldsymbol{\rho}_s)d^2\boldsymbol{\kappa}=\frac{1}{(2\pi)^2}\int\tilde{\chi}(\boldsymbol{\kappa},k)\exp\Big[i\left(\kappa_x(x-x_s)+\kappa_y y\right)\Big]d^2\boldsymbol{\kappa}\,, \qquad (9)$$

where $\boldsymbol{\rho}_s=(x-x_s,y)$ and $\boldsymbol{\kappa}=(\kappa_x,\kappa_y)$. Substituting Eqs. (8) and (9) into Eq. (7), we obtain

$$\left(\kappa^2+\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k^2-\frac{\rho_0}{\tilde{\lambda}_1}\omega^2\right)\tilde{\chi}(\boldsymbol{\kappa},k)\exp(-i\kappa_x x_s)$$

$$=a_0^2k^2\frac{\gamma}{\tilde{\lambda}_1}u_{z0}(x_s,0,k)\Big(\exp(-i\kappa_x x_{s-1})e^{-iqd_0}+\exp(-i\kappa_x x_s)+\exp(-i\kappa_x x_{s+1})e^{iqd_0}\Big). \qquad (10)$$

The coordinates of the neighboring dislocations are related by the expressions

$$x_{s-1}=x_s-d_0\,,\ x_{s+1}=x_s+d_0\,. \qquad (11)$$

From Eqs. (9), (10), and (11) it follows that

$$u_{z0s}(x,y,k)=\frac{a_0^2k^2}{(2\pi)^2}\frac{\gamma}{\tilde{\lambda}_1}u_{z0}(x_s,0,k)\int\frac{1+2\cos(\kappa_x-q)d_0}{\kappa^2+\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k^2-\frac{\rho_0}{\tilde{\lambda}_1}\omega^2}\exp(i\boldsymbol{\kappa}\boldsymbol{\rho}_s)d^2\boldsymbol{\kappa}\,. \qquad (12)$$

Let us represent Eq. (12) in the form of the sum of two components:

$$u_{z0s}=\hat{u}_{z0s}+\breve{u}_{z0s}\,, \qquad (13)$$

where

$$\hat{u}_{z0s}(x,y,k)=\frac{a_0^2k^2}{(2\pi)^2}\frac{\gamma}{\tilde{\lambda}_1}u_{z0}(x_s,0,k)\int\frac{\exp(i\boldsymbol{\kappa}\boldsymbol{\rho}_s)}{\kappa^2+\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k^2-\frac{\rho_0}{\tilde{\lambda}_1}\omega^2}d^2\boldsymbol{\kappa}\,. \qquad (14)$$

Integral (14) is an integral representation of the McDonald function of zero order $K_0(x)$, and $\hat{u}_{z0s}$ is the amplitude of vibrations localized on the isolated dislocation number s. For large values of the argument $x \gg 1$, the function $K_0(x)\approx\sqrt{\pi/2x}\cdot\exp(-x)$. Thus, far from the dislocation, the vibration amplitude exponentially decreases, which

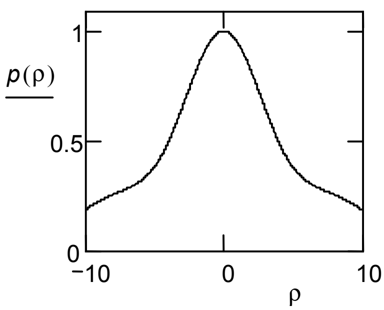

Fig. 1

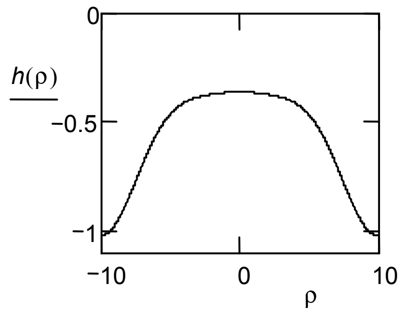

Fig. 2

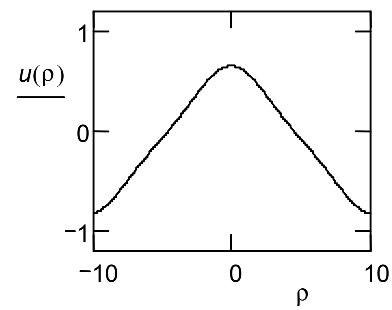

Fig. 3

confirms its localization on the dislocation. Near the dislocation $(x << 1)$, the vibration amplitude $K_0(x) \approx -\ln(x/2)$ has a logarithmic singularity, which is connected with model assumption (1) about the δ-shaped perturbation created by the dislocation in the crystal. Replacing in Eq. (14) the upper infinite integration limits by finite ones $\kappa_0 \sim 1/a_0$, which allows us to exclude from consideration short-wavelength perturbations having no physical sense and thereby to eliminate the logarithmic singularity, we obtain

$$\hat{u}_{z0s}(\rho_s, k) = \frac{a_0^2 k^2 \gamma u_{z0}(x_s, 0, k)}{(2\pi)^2 \tilde{\lambda}_1} \int_0^1 \int_0^{2\pi} \frac{\theta \cdot \cos(\theta \rho_s \cos\varphi / a_0)}{\theta^2 + \chi^2 a_0^2} d\varphi d\theta \, . \tag{15}$$

In Eq. (15) we have used the dimensionless variable $\theta = \kappa a_0$, $\rho_s = \sqrt{(x - x_s)^2 + y^2}$, and $\chi \equiv \sqrt{\left(\tilde{\lambda}_0 k^2 - \rho_0 \omega^2\right)/\tilde{\lambda}_1}$. The plot of the function $p(\rho_s) \equiv \hat{u}_{z0s}(\rho_s, k)/\hat{u}_{z0}(x_s, 0, k)$ for $\chi a_0 = 0.1$, $a_0 k = 0.74$, and $\gamma/\tilde{\lambda}_1 = 5$, drawn in *MATHCAD*, is shown in Fig. 1. Here ρ is given in units of the lattice constant. The term

$$\breve{u}_{z0s}(\rho_s, k) = \frac{a_0^2 k^2}{(2\pi)^2} \frac{\gamma}{\tilde{\lambda}_1} u_{z0}(x_s, 0, k) \int \frac{2\cos(\kappa_x - q) d_0}{\kappa^2 + \frac{\tilde{\lambda}_0}{\tilde{\lambda}_1} k^2 - \frac{\rho_0}{\tilde{\lambda}_1} \omega^2} \exp(i \boldsymbol{\kappa} \boldsymbol{\rho}_s) d^2 \boldsymbol{\kappa} \tag{16}$$

describes the influence on the dislocation with number s of vibrations localized on two neighboring dislocations with numbers $s-1$ and $s+1$.

From cyclic boundary condition (5) we obtain that $1 = \exp(iqNd_0)$; whence

$$\cos(qNd_0) = 1 \Rightarrow q_j N d_0 = 2\pi j \Rightarrow q_j = 2\pi j / N d_0, \; j = 0, \pm 1, \pm 2, ... \tag{17}$$

As an example, Fig. 2 shows $h(\rho_s) \equiv \breve{u}_{z0s}(\rho_s, k)/\breve{u}_{z0}(x_s, 0, k)$ for $N = 100$, $j = 50$, $ak = 0.74$, $\chi a = 0.1$, and $d_0 = 10 a_0$. In this case, the wavelength of the perturbation propagating along the dislocation chain is $\lambda_j = 2 d_0 = 20 a_0$. Figure 3 shows the plot of $u(\rho) = p(\rho) + h(\rho)$ near the dislocation with number s.

Setting $\boldsymbol{\rho}_s = 0$ in Eq. (12), we obtain the dispersion equation for the sound waves interacting with the dislocation chain:

$$1 = \frac{a_0^2 k^2}{(2\pi)^2} \frac{\gamma}{\tilde{\lambda}_1} \left[\int \frac{d^2 \boldsymbol{\kappa}}{\kappa^2 + \frac{\tilde{\lambda}_0}{\tilde{\lambda}_1} k^2 - \frac{\rho_0}{\tilde{\lambda}_1} \omega^2(j)} \right.$$

$$+2\left(\cos q_j d_0 \int \frac{\cos \kappa_x d_0}{\kappa^2 + \frac{\tilde{\lambda}_0}{\tilde{\lambda}_1} k^2 - \frac{\rho_0}{\tilde{\lambda}_1}\omega^2(j)} d^2\boldsymbol{\kappa} + \sin q_j d_0 \int \frac{\sin \kappa_x d_0}{\kappa^2 + \frac{\tilde{\lambda}_0}{\tilde{\lambda}_1} k^2 - \frac{\rho_0}{\tilde{\lambda}_1}\omega^2(j)} d^2\boldsymbol{\kappa}\right)\Bigg]. \tag{18}$$

Taking advantage of the method of stationary phase [3] to calculate the second and third integrals in Eq. (18), we obtain

$$\int \frac{\cos(\kappa_x d_0)\, d^2\boldsymbol{\kappa}}{\left(\kappa^2 + \frac{\tilde{\lambda}_0}{\tilde{\lambda}_1} k^2 - \frac{\rho_0}{\tilde{\lambda}_1}\omega^2(j)\right)} \approx 2\pi \frac{\sin(\bar{\kappa}_0 d_0)}{\kappa_0 d_0} \exp\left(\frac{2\pi\tilde{\lambda}_1}{a_0^2 k^2 \gamma}\right). \tag{19}$$

In calculations of integral (19), the infinite limits of integration over κ_x have been replaced by finite ones, where $\bar{\kappa}_0 \sim 1/d_0$.

Analogous integral containing $\sin(\kappa_x d_0)$ in Eq. (18) is equal to 0. Thus, we have suggested that vibrations localized on dislocations interact mainly by means of long-wavelength perturbations with $|\kappa_x| < \bar{\kappa}_0 \sim 1/d_0$. We also have replaced $\omega(j)$ in Eq. (19) by the frequency ω_0 of localized vibrations observed in the crystal in the presence of only one dislocation:

$$\omega_0^2 \approx \frac{\tilde{\lambda}_0}{\rho_0} k^2 - \frac{\tilde{\lambda}_1}{\rho_0} \kappa_0^2 \exp\left(-\frac{4\pi\tilde{\lambda}_1}{a_0^2 k^2 \gamma}\right), \tag{20}$$

and assumed that the condition $\kappa_0^2 \exp\left(-4\pi\tilde{\lambda}_1 / a_0^2 k^2 \gamma\right) >> \bar{\kappa}_0^2$ was satisfied. From the given inequality for the typical values of the crystal parameters: $\rho_0 = 5$ g/cm^3, $\tilde{\lambda}_0 = 6 \cdot 10^{12}$ dynes/cm^2, $\tilde{\lambda}_1 = 10^{12}$ dynes/m^2, $\gamma = 5 \cdot 10^{12}$ dynes/cm^2, $a_0 = 5 \cdot 10^{-8}$ cm, and $k = 1.26 \cdot 10^7$ cm^{-1} we obtained $d_0 > 24 a_0$, and from Eq. (20), we obtained $\omega_0 \approx 1.38 \cdot 10^{13}$ Hz.

Let us further substitute Eq. (19) into Eq. (18). Taking the remaining integrals assuming that $\left(\tilde{\lambda}_0 k^2 - \rho_0 \omega^2(j)\right) / \tilde{\lambda}_1 << \kappa_0^2$, we find the expression for the possible vibration frequencies:

$$\omega^2(j) \approx \frac{\tilde{\lambda}_0}{\rho_0} k^2 - \frac{\tilde{\lambda}_1}{\rho_0} \kappa_0^2 \exp\left(-\frac{4\pi\tilde{\lambda}_1}{a_0^2 k^2 \gamma}\right) \exp\left(4\cos\left(q_j d_0\right) \frac{\sin(\bar{\kappa}_0 d_0)}{\kappa_0 d_0} \exp\left(\frac{2\pi\tilde{\lambda}_1}{a_0^2 k^2 \gamma}\right)\right), \tag{21}$$

where q_j is given by Eq. (17). By virtue of the periodicity of dependence (21), values $|j| > N/2$ do not cause new states of the vibrating chain. Each of the states represents a wave running along the chain, where $\omega(j)$ is the wave frequency and q_j is the wave vector. However, unlike a continuous medium, the possible independent q values are limited by the condition $|q| \le \pi/d_0$. Therefore, the total number of different independent q values is equal to the number of independent states of the vibrating chain being equal to N .

If the crystal represents a plate bounded by the planes $z = 0$ and $z = L$, the modulus of the wave vector k takes values $k_n = \pi n / L, n = 1, 2, 3...$, and the vibrations localized on dislocations represent standing waves. Then from Eq. (21) we obtain

$$\omega(n, j) \approx \Omega(n) - \frac{1}{2} \frac{\tilde{\lambda}_1}{\sqrt{\rho_0 \tilde{\lambda}_0}} \frac{\kappa_0^2}{k_n} \exp\left(-\frac{4\pi\tilde{\lambda}_1}{a_0^2 k_n^2 \gamma}\right) \exp\left(4\cos\left(\frac{2\pi j}{N}\right) \frac{\sin(\bar{\kappa}_0 d_0)}{\kappa_0 d_0} \exp\left(\frac{2\pi\tilde{\lambda}_1}{a_0^2 k_n^2 \gamma}\right)\right), \tag{22}$$

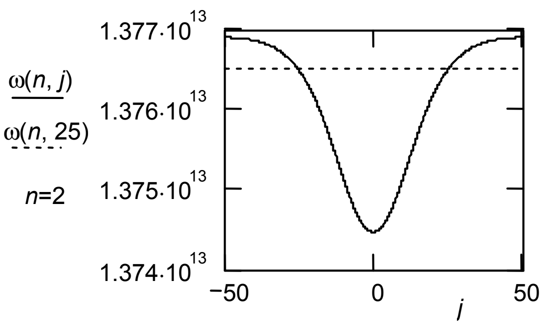

Fig. 4. Results of calculation for $d_0 = 50a_0$.

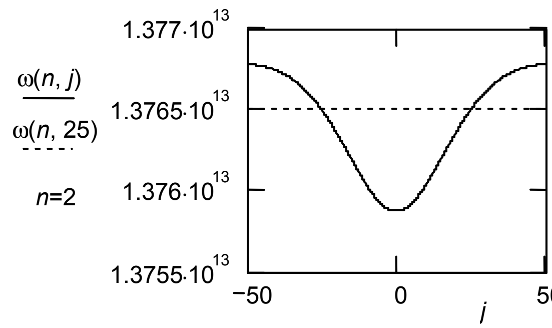

Fig. 5. Results of calculations for $d_0 = 100a_0$.

where $\Omega(n) = \sqrt{\tilde{\lambda}_0/\rho_0} \cdot k_n$ is the frequency of volume vibrations. We note that for $j = N/4$, the frequency $\omega(n, j)$ coincides with the frequency $\omega_0(n)$ of vibrations localized on a single dislocation.

Formula (22) determines the possible values of frequencies of sound vibrations in a piezoelectric crystal containing a dislocation chain. As can be seen from Eq. (22), the interaction between vibrations localized on individual dislocations leads to splitting of each frequency in the discrete spectrum depending on the number n into a series of side frequencies characterized by different values of the number j given by Eq. (17) and to the formation of dislocation zones.

To estimate the width of the range occupied by allowable frequencies, we used the following values of the parameters of piezoelectric material: $\rho_0 = 5$ g/cm^3, $\tilde{\lambda}_0 = 6 \cdot 10^{12}$ dynes/cm^2, $\tilde{\lambda}_1 = 10^{12}$ dynes/cm^2, $\gamma = 5 \cdot 10^{12}$ dynes/cm^2, and $a_0 = 5 \cdot 10^{-8}$ cm. Let us consider a film with the thickness $L = 10a_0$; for this film, the possible values of the number n will lie in the limits from $n_{\min} = 1$ to $n_{\max} = 10$. Wavelengths smaller than $2a_0$ correspond to $n > n_{\max}$, and their consideration has no physical sense. We assume that the chain contains $N = 100$ dislocations. From Eq. (22), for example, for $n = 2$ we obtain $\lambda_2 = 10 \cdot a_0$, $k_2 = 1.257 \cdot 10^7$ cm^{-1}, and the frequency of volume vibrations $\Omega(n) \approx 1.377 \cdot 10^{13}$ Hz. The frequency of vibrations localized on a non-interacting dislocation is $\omega(n, 25) \approx 1.3765 \cdot 10^{13}$ Hz. Results of calculations using *MATHCAD* for the indicated distances d_0 between dislocations are shown in Figs. 4 and 5.

It can be seen that the width of the dislocation zone increases with decreasing distance between dislocations and, hence, strengthening the interaction between the vibrations localized on them. We now estimate the width of the range occupied by the allowable frequencies. From Eq. (22) we obtain

$$\Delta\omega(n) = 4\frac{\tilde{\lambda}_1}{\rho_0}\frac{\kappa_0^2}{\Omega(n)}\frac{\sin(\overline{\kappa}_0 d_0)}{\kappa_0 d_0}\exp\left(-\frac{2\pi\tilde{\lambda}_1}{a_0^2 k_n^2 \gamma}\right). \tag{23}$$

The plot of the dependence $\Delta\omega(n)$ is shown in Fig. 6. With increasing number n, the width of the frequency range first increases, reaches a maximum at $n = 5$, and then slightly decreases. For a crystal plate we obtain

$$u_z(x, y, z, t) = \sum_n \sum_{j=-N/2}^{N/2} \sum_{s=1}^{N} \frac{a_0^2 k_n^2}{2\pi}\frac{\gamma}{\tilde{\lambda}_1} u_{z0nj} K_0\left(\sqrt{\frac{\tilde{\lambda}_0}{\tilde{\lambda}_1}k_n^2 - \frac{\rho_0}{\tilde{\lambda}_1}\omega_0^2}\sqrt{(x - x_s)^2 + y^2}\right)$$

$$\times\cos(k_n z) \cdot \exp i\left(\frac{2\pi}{N} js - \omega(n, j)t\right). \tag{24}$$

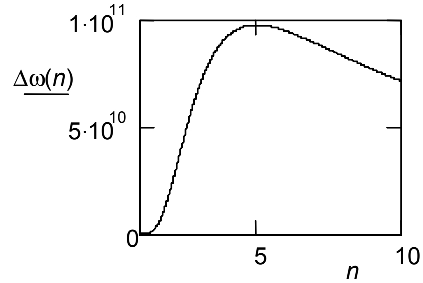

Fig. 6. Results of calculations for $d_0 = 50a_0$.

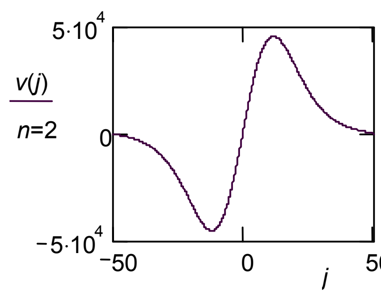

Fig. 7. Results of calculations for $d_0 = 50a_0$.

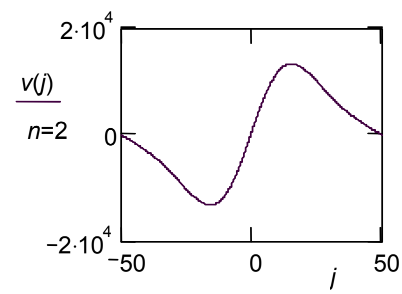

Fig. 8. Results of calculations for $d_0 = 100a_0$.

The wave perturbation propagates along the chain with the group velocity $v(j) = d\omega/dq$. The dependence $v(j)$ is shown in Figs. 7 and 8 for $d_0 = 50a_0$ and $d_0 = 100a_0$. The maximum group velocity of waves propagating along the chain is $\sim 5 \cdot 10^4$ cm/s for $d_0 = 50a_0$ (Fig. 7), which is much less than the velocity of volume vibrations $\sqrt{\tilde{\lambda}_0/\rho_0} \approx 10^6$ cm/s. With increasing distance between dislocations and hence, weakening of the interaction between them, the group velocity decreases down to $1.5 \cdot 10^4$ cm/s for $d_0 = 100a_0$ (Fig. 8).

The same frequency ω corresponds to j values that differ only by their signs for fixed n value (see Figs. 4 and 5); therefore, the standing wave $\sim \cos\left(q_j s d_0\right)$ can exist in the chain along with perturbations $\sim \exp i\left(q_j s d_0\right)$ running in the direction perpendicular to dislocations.

Thus, sound vibrations in the piezoelectric crystal containing a dislocation chain have been investigated in this work. It was demonstrated that the interaction between the vibrations localized on individual dislocations caused the occurrence of waves propagating along the chain with the group velocity much smaller than the velocity of volume vibrations. In this case, the number of different independent values of the wave vector was equal to the number of independent states of the vibrating dislocation chain and coincided with the number of dislocations contained in it. The width of the frequency range occupied by the allowable frequencies of localized vibrations was estimated for different values of the parameters characterizing the piezoelectric medium and the dislocation chain. The ranges of allowable frequencies were separated both from each other and from the spectrum of volume vibrations.

REFERENCES

1. N. A. Tyapunina, E. K. Naimi, and G. M. Zinenkova, Influence of Ultrasound on Crystals with Defects [in Russian], Publishing House of Moscow State University, Moscow (1999).
2. N. A. Tyapunina and É. P. Belozerov, Usp. Fiz. Nauk, **156**, No. 4, 683–717 (1988).
3. S. G. Gestrin and E. B. Shchukina, Russ. Phys. J., **56**, No. 12, 1326–1332 (2013).

4. A. M. Kosevich, Principles of Mechanics of a Crystal Lattice [in Russian], Nauka, Moscow (1972).
5. S. G. Gestrin, Russ. Phys. J., **39**, No. 10, 938–943 (1996).
6. S. G. Gestrin, Russ. Phys. J., **41**, No. 2, 174–177 (1998).
7. S. G. Gestrin, A. N. Sal'nikov, and E. B. Shchukina, Russ. Phys. J., **49**, No. 10, 1112–1116 (2006).
8. S. G. Gestrin and A. N. Sal'nikov, Russ. Phys. J., **48**, No. 7, 690–693 (2005).
9. S. G. Gestrin and E. A. Sal'nikova, Russ. Phys. J., **54**, No. 11, 1177–1184 (2011).
10. L. D. Landau and E. M. Lifshits, Electrodynamics of Continuous Media. Theoretical Physics, Vol. VIII [in Russian], Nauka, Moscow (1982).