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SECULAR RESONANCES IN THE DYNAMIC EVOLUTION OF NEAR-EARTH SPACE OBJECTS ON POLAR ORBITS

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Results are presented of a study of the effect of secular resonances on the dynamic evolution of objects moving along polar orbits with eccentricities changing over a wide range and semi-major axes from 8 000 to 55 000 km. It is shown that motion of polar objects with semi-major axes in the range 50 000–55 000 km is most complicated in the examined altitude range.

Keywords: artificial Earth satellites, secular resonances, dynamic evolution.

INTRODUCTION

This work is the last one from the cycle of papers [1–4] on the abundance of secular resonances in near-Earth orbital space and on the estimate of their effect on the dynamic evolution of orbiting objects. In [1] it was shown that secular resonances are widespread in near-Earth orbital space. An analysis of the results of the extensive numerical experiment presented in [1, 3, 4] allows us to conclude that in the near-Earth space orbits with inclinations from 10 to 65° there exist overlap zones of several secular resonances and orbits with overlap of secular and tesseral resonances. The dynamic evolution of objects subject to the simultaneous action of several resonances is distinguished by its great complexity, which is manifested through long-period variation of their main orbital elements: semi-major axis, eccentricity, and inclination of their orbits as well as through the emergence of chaoticity in motion of such objects. With other things being equal, the presence of large eccentricity causes speed-up of orbital evolution, but the character of this evolution remains independent of the eccentricity value [4]. The present work considers the effect of secular resonances on motion of polar objects

1. METHOD OF INVESTIGATION

The method employed here for identifying and investigating the effect of secular resonances on near-Earth space objects was presented in detail in [5]. It consists of three parts: an analytical method for identifying the secular resonances [4], numerical modeling of the long-time orbital evolution with the help of the software package *Numerical Model of the Motion of Artificial Earth Satellites* (AESs) [6] and a Megno analysis of the orbital evolution of orbiting objects [7]. Estimates of the accuracy of prediction of AES motion over large time intervals can be found in [5].

We discuss only some fragments of the method necessary for an analysis of the results of the numerical-analytical experiment connected with the dynamics of polar orbits

Let us use the arguments of the perturbing function for the single-averaged, $\underline{\psi}$, and double-averaged, $\underline{\underline{\psi}}$, bounded three-body problem, represented in the form

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$$\underline{\psi} = (l - 2p' + q')M' + (l - 2p)\omega - (l - 2p')\omega' + j(\Omega - \Omega'), \tag{1}$$

$$\underline{\underline{\psi}} = (l - 2p)\omega - (k - 2p')\omega' + j(\Omega - \Omega'), \tag{2}$$

where

$$\omega = \omega_0 + \dot{\omega}(t - t_0), \ \ \Omega = \Omega_0 + \dot{\Omega}(t - t_0), \ M' = M'_0 + \overline{n}'(t - t_0),$$

$$\omega' = \omega'_0 + \dot{\omega}'(t - t_0), \ \ \Omega' = \Omega'_0 + \dot{\Omega}'(t - t_0),$$

and write down the condition for the appearance of a resonance in the following way:

$$\dot{\underline{\psi}} = 0, \quad \dot{\underline{\underline{\psi}}} = 0.$$

The secular frequencies in the motion of a satellite

$$\dot{\Omega} = \dot{\Omega}_J + \dot{\Omega}_L + \dot{\Omega}_S, \ \dot{\omega} = \dot{\omega}_J + \dot{\omega}_L + \dot{\omega}_S$$

due to the influence of the second zonal harmonic and a third body are calculated according to well-known formulas (see, for example, [8])

$$\dot{\Omega}_J = -\frac{3}{2}J_2 n\left(\frac{r_0}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}, \ \ \dot{\omega}_J = \frac{3}{4}J_2 n\left(\frac{r_0}{a}\right)^2 \frac{5\cos^2 i - 1}{(1-e^2)^2}, \tag{5}$$

for the second zonal harmonic, and

$$\begin{aligned} \dot{\Omega}_{L,S} &= -\frac{3}{16}\overline{n}\frac{m}{m'}\left(\frac{a}{a'}\right)^3 \frac{2+3e^2}{\sqrt{1-e^2}}(2 - 3\sin^2 i')\cos i, \\ \dot{\omega}_{L,S} &= \frac{3}{16}\overline{n}\frac{m}{m'}\left(\frac{a}{a'}\right)^3 \frac{4 - 5\sin^2 i + e^2}{\sqrt{1-e^2}}(2 - 3\sin^2 i'), \end{aligned} \tag{6}$$

for the third body: Moon (L) or Sun (S).

The full set of resonance relations of lower orders consists of 29 algebraic expressions presented in Table 1. The description of all these resonance relations is presented in [2–4].

2. DESCRIPTION OF RESULTS OF A NUMERICAL-ANALYTICAL EXPERIMENT FOR OBJECTS WITH NEAR-POLAR ORBITS

With the help of the above-indicated method, we carried out an extensive numerical-analytical experiment. For all 29 relations listed in Table 1, we determined their numerical values for the following variations of the parameters:

e = {0.01, 0.1–0.8 with a step of 0.1}, i ={70–90° with step 5°}, a = {8000–55000 km with a step of 1 km}.

An analysis of peculiarities of the orbital evolution of objects moving along almost circular orbits with inclination of 10–65° was performed in [2, 3], where the characteristics of orbital evolution of objects on prolate orbits with the same eccentricities were presented [4].

TABLE 1. Resonance Relations

Serial No.	Resonance relation	Serial No.	Resonance relation	Serial No.	Resonance relation
1	$\dot{M}'_S - \dot{\omega} + \left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega}'_S$	11	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) - \dot{\omega} + \dot{\omega}'_S$	21	$\dot{M}'_M - \left(\dot{\Omega} - \dot{\Omega}'_M\right)$
2	$\dot{M}'_S - \dot{\omega} - \left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega}'_S$	12	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) + 2\dot{\omega} - 2\dot{\omega}'_S$	22	$\dot{M}'_M - 2\dot{\omega} + 2\dot{\omega}'_M$
3	$\dot{M}'_S - 2\dot{\omega} + \left(\dot{\Omega} - \dot{\Omega}'_S\right) - 2\dot{\omega}'_S$	13	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) - 2\dot{\omega} + 2\dot{\omega}'_S$	23	$\dot{M}'_M - \dot{\omega} + \dot{\omega}'_M$
4	$\dot{M}'_S + 2\left(\dot{\Omega} - \dot{\Omega}'_S\right)$	14	$\left(\dot{\Omega} - \dot{\Omega}'_S\right)$	24	$\left(\dot{\Omega} - \dot{\Omega}'_M\right) + \omega - \dot{\omega}'_M$
5	$\dot{M}'_S - 2\left(\dot{\Omega} - \dot{\Omega}'_S\right)$	15	$\dot{M}'_M - \dot{\omega} + \left(\Omega - \Omega'_M\right) + \dot{\omega}'_M$	25	$\left(\dot{\Omega} - \dot{\Omega}'_M\right) - \dot{\omega} + \dot{\omega}'_M$
6	$\dot{M}'_S + \left(\dot{\Omega} - \dot{\Omega}'_S\right)$	16	$\dot{M}'_M - \dot{\omega} - \left(\dot{\Omega} - \dot{\Omega}'_M\right) + \dot{\omega}'_M$	26	$\left(\dot{\Omega} - \dot{\Omega}'_M\right) + 2\dot{\omega} - 2\dot{\omega}'_M$
7	$\dot{M}'_S - \left(\dot{\Omega} - \dot{\Omega}'_S\right)$	17	$\dot{M}'_M - 2\dot{\omega} + \left(\dot{\Omega} - \dot{\Omega}'_M\right) - 2\dot{\omega}'_M$	27	$\left(\dot{\Omega} - \dot{\Omega}'_M\right) - 2\dot{\omega} + 2\dot{\omega}'_M$
8	$\dot{M}'_S - 2\dot{\omega} + 2\dot{\omega}'_S$	18	$\dot{M}'_M + 2\left(\dot{\Omega} - \dot{\Omega}'_M\right)$	28	$\left(\dot{\Omega} - \dot{\Omega}'_M\right)$
9	$\dot{M}'_S - \dot{\omega} + \dot{\omega}'_S$	19	$\dot{M}'_M - 2\left(\dot{\Omega} - \dot{\Omega}'_M\right)$	29	$\dot{\omega}$
10	$\left(\dot{\Omega} - \dot{\Omega}'_S\right) + \dot{\omega} - \dot{\omega}'_S$	20	$\dot{M}'_M + \left(\dot{\Omega} - \dot{\Omega}'_M\right)$		

TABLE 2. Numeration of the Near-Polar Models

a, km	Serial No.	$i°$	Serial No.	$i°$	Serial No.	$i°$	Serial No.	$i°$	Serial No.	$i°$
$e = 0.01$										
8000	00	70	010	75	020	80	030	85	040	90
10000	01	70	011	75	021	80	031	85	041	90
15000	02	70	012	75	022	80	032	85	042	90
20000	03	70	013	75	023	80	033	85	043	90
25000	04	70	014	75	024	80	034	85	044	90
26000	05	70	015	75	025	80	035	85	045	90
30000	06	70	016	75	026	80	036	85	046	90
40000	07	70	017	75	027	80	037	85	047	90
50000	08	70	018	75	028	80	038	85	048	90
55000	09	70	019	75	029	80	039	85	049	90
$e = 0.6$										
20000	135	70	142	75	149	80	156	85	163	90
25000	136	70	143	75	150	80	157	85	164	90
26000	137	70	144	75	151	80	158	85	165	90
30000	138	70	145	75	152	80	159	85	166	90
40000	139	70	146	75	153	80	160	85	167	90
50000	140	70	147	75	154	80	161	85	168	90
55000	141	70	148	75	155	80	162	85	169	90
$e = 0.8$										
40000	215	70	218	75	221	80	224	85	227	90
50000	216	70	219	75	222	80	225	85	228	90
55000	217	70	220	75	223	80	226	85	229	90

We have selected from all the sets of calculated resonance relations whose values pass through zero over a time interval of 100 years. We have introduced numerical notation for the considered models in Table 2.

TABLE 3. Resonance Relations Passing through Zero over a Time Interval of 100 Years for the Objects on Near-Polar and Near-Circular Orbits

Serial numbers of the objects	Serial numbers of the resonance relations
e = 0.01	
00, 01, 03–06, 010–016, 020–027, 030–038, 040, 044–047	–
02	25
07	1, 3, 4, 6, 8, 9, 11 – 13, 15, 17, 18, 20, 22, 23, 25–27, 29
08	1, 3, 4, 6, 8, 9, 12, 13, 15, 17, 18, 20, 22, 23, 25–27, 29
09	1, 3, 4, 6, 8, 9, 13, 15, 17, 18, 20, 22, 23, 25, 27, 29
017	11
018	1, 3, 4, 6, 8, 9, 11, 13, 15, 17, 18, 20, 22, 23, 25, 27, 29
019	1, 4, 6, 11, 14, 18, 20, 25
028, 029	4, 6, 14
039	4, 5, 7, 11, 14, 28
041–043	14
048	14, 28
049	2, 5, 7, 10, 12, 14, 16, 19, 21, 24, 26, 28

TABLE 4. Resonance Relations Passing through Zero over a Time Interval of 100 Years for the Objects on Prolate Near-Polar Orbits

Serial numbers of the objects	Serial numbers of the resonance relations	Serial numbers of the objects	Serial numbers of the resonance relations
e = 0.6			
142, 143, 149–152, 156–158,	–	145	28
135	25	148	3, 4, 11–14, 25, 27, 29
136–138	13, 25, 27, 29	153–155, 159–161, 163–167	14
139, 140	11–13, 25, 27, 29	168	10, 14
141, 147	1, 3, 4, 8, 11–14, 25–27, 29	169	2, 5, 8,10–14, 24, 26–29
144, 146	11		
e = 0.8			
215	3, 4, 11, 13, 25, 27, 29	220	11, 13, 14, 25, 28
216	11–13, 25, 27, 29	221, 222, 224, 227	14
217	1, 3, 4, 8–13, 25–27,29	223, 225	14, 28
218	11, 13, 25, 27, 29	226	11, 13, 14, 25, 27, 29
219	3, 4, 11, 13, 25, 27, 29	228, 229	10, 14, 28

Table 3 lists the serial numbers of the resonance relations that pass through zero over a time interval of 100 years for the models on near-polar and near-circular orbits. The similar estimations for the objects on prolate near-polar orbits are given in Table 4. The absence of such relations is also indicated.

The influence of nodal secular resonances described by resonance relations 14 and 28 is an essential characteristic of the motion on near-polar orbits. As formulas (5) and (6) show the secular frequency $\dot{\Omega}$ in the satellite nodal motion are close to zero for near-polar orbits. The values of the suitable frequencies in the Moon and the Sun motion are small too. This leads to the appearance of zero or near-zero values for resonance relations (Table 5).

It is interesting to discuss the behavior of near-circular models with semi-major axis of 8000 km which move in the neighborhood of tesseral resonance 1:3 with the Earth rotation frequency. The motion is completely regular with the

TABLE 5. Values of Critical Resonance Relations for Near-Circular Near-Polar Orbits in Units of 10^{-6} rad/s

Resonance relations	Serial No.	Min	Max	Serial No.	Min	Max
$14\left(\dot{\Omega}-\dot{\Omega}'_s\right)$	040	–0.011349216	–0.000186970	041	–0.003850604	0.004294064
$28\left(\dot{\Omega}-\dot{\Omega}'_m\right)$		–0.021714179	–0.010551933		–0.014215566	–0.006070898
$14\left(\dot{\Omega}-\dot{\Omega}'_s\right)$	042	–0.002089336	0.001926674	043	–0.001605245	0.000415956
$28\left(\dot{\Omega}-\dot{\Omega}'_m\right)$		–0.050357023	–0.050133202		–0.018521356	–0.018389952
$14\left(\dot{\Omega}-\dot{\Omega}'_s\right)$	044	–0.001292531	–0.000279394	045	–0.001268299	–0.000328401
$28\left(\dot{\Omega}-\dot{\Omega}'_m\right)$		–0.011657493	–0.010644356		–0.011633261	–0.010693363
$14\left(\dot{\Omega}-\dot{\Omega}'_s\right)$	046	–0.001153023	–0.000329211	047	–0.002485707	–0.000330111
$28\left(\dot{\Omega}-\dot{\Omega}'_m\right)$		–0.011517986	–0.010694173		–0.012850670	–0.010695073
$14\left(\dot{\Omega}-\dot{\Omega}'_s\right)$	048	–0.003815845	0.041136707	049	–0.000940915	37.025934450
$28\left(\dot{\Omega}-\dot{\Omega}'_m\right)$		–0.014180807	0.030771744		–0.011305877	37.015569488

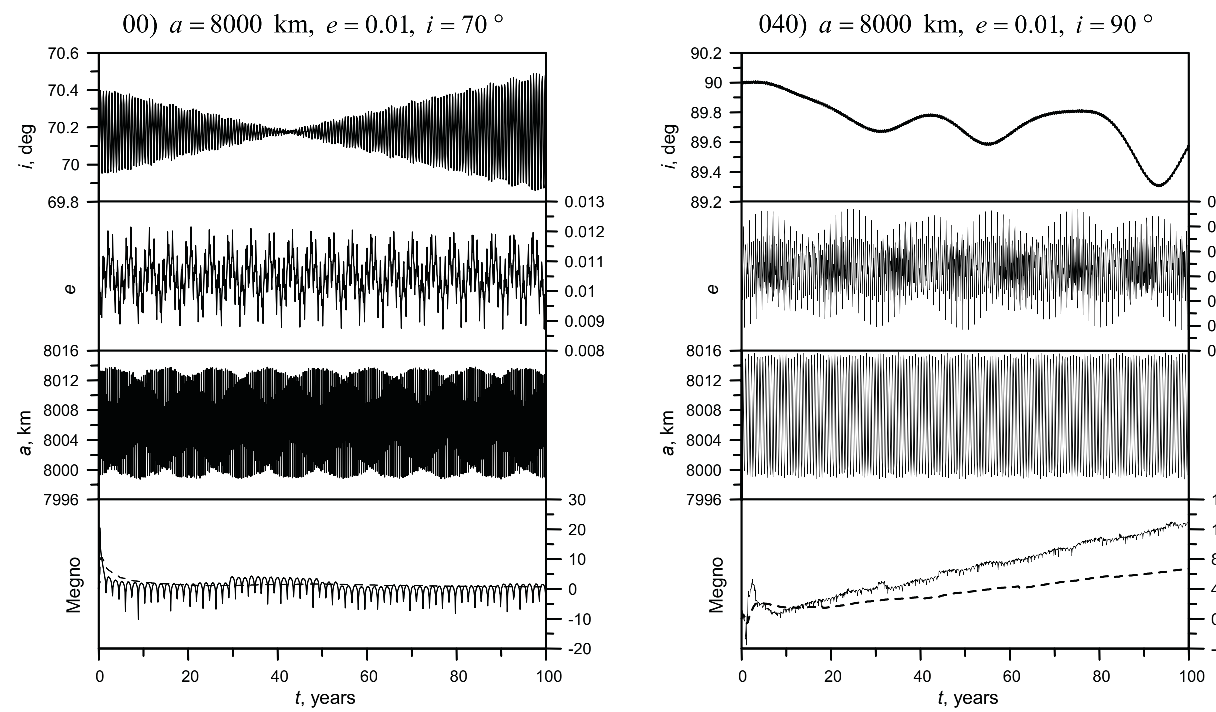

Fig. 1. Orbital evolution of objects on low near-circular and near-polar orbits.

inclination equal to 70° and the Megno parameters are close to zero over the entire 100 years interval of motion. In Figs. 1–5 the average Megno parameter is shown by the dashed curve and the unaveraged parameter is shown by solid curve. The effect of nodal secular resonances reveals itself from the inclination equal 80°. The irregular long-period oscillations appear in the inclination, and the Megno parameter increases. This indicates the appearance of the chaoticity in the object motion.

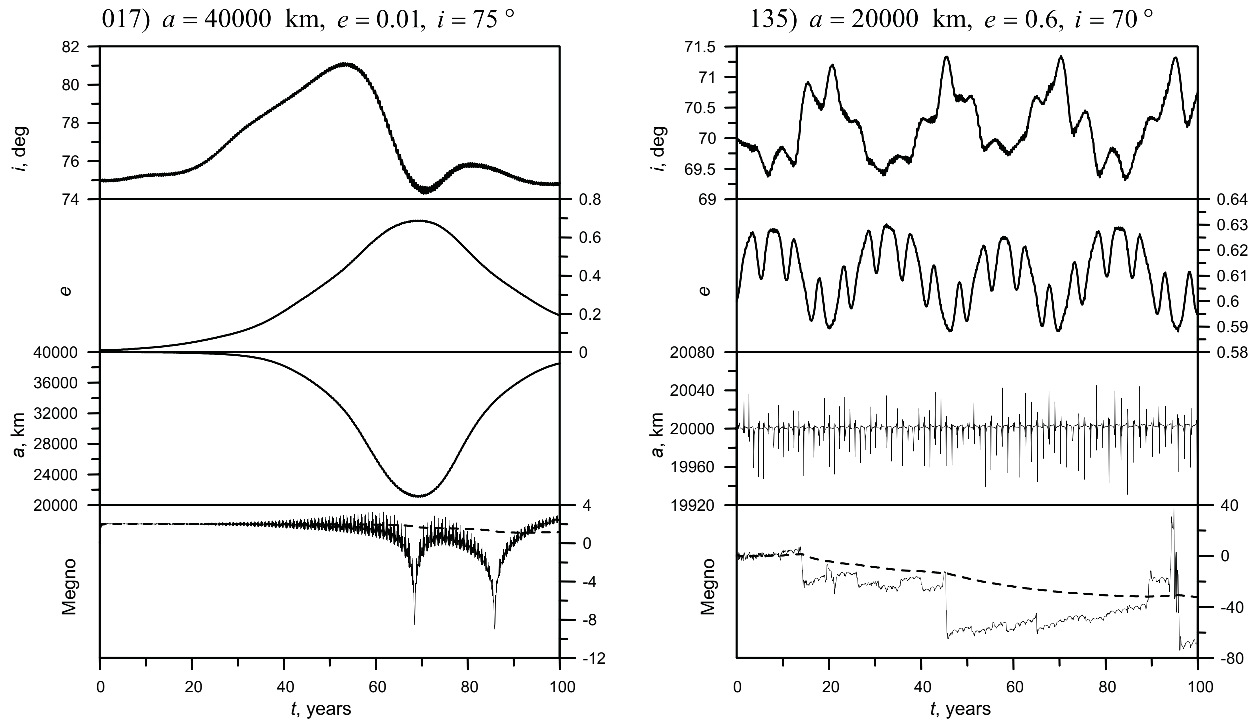

Fig. 2. Orbital evolution of objects on near–polar orbits under the influence of a single secular resonance.

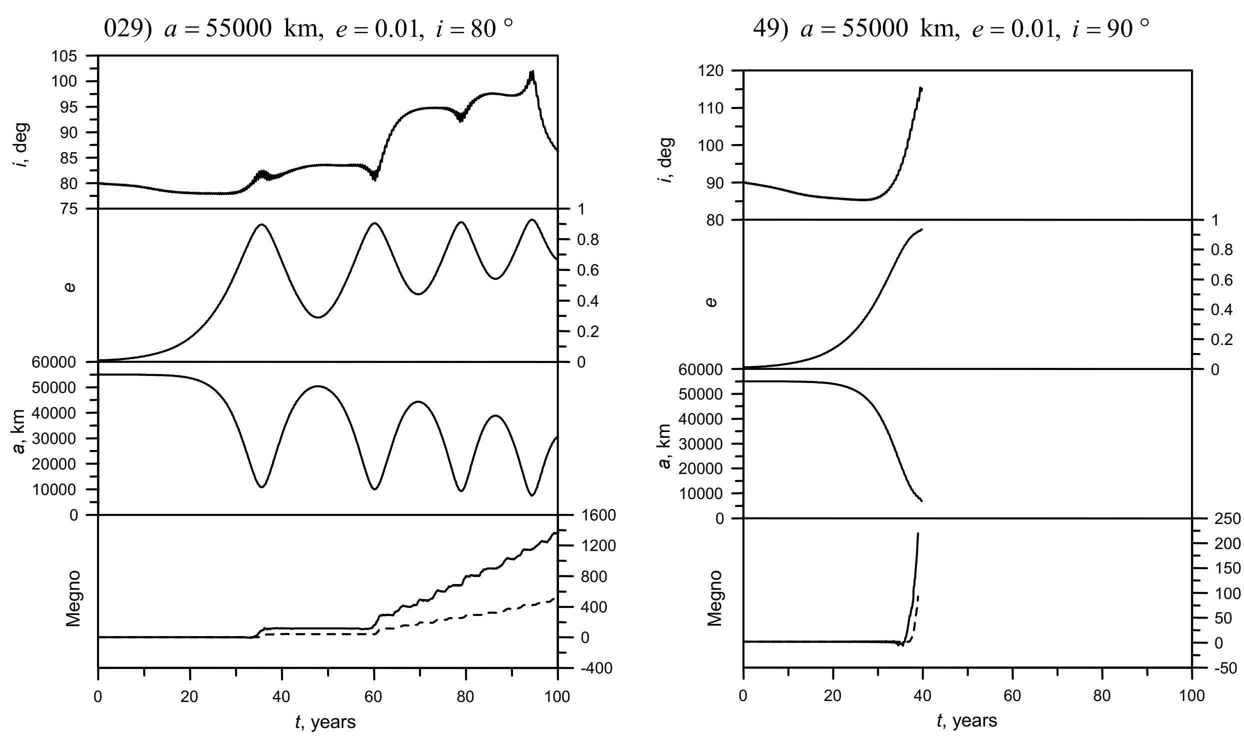

Fig. 3. Peculiarities of orbital evolution of high orbiting objects on near-circular and near-polar orbits.

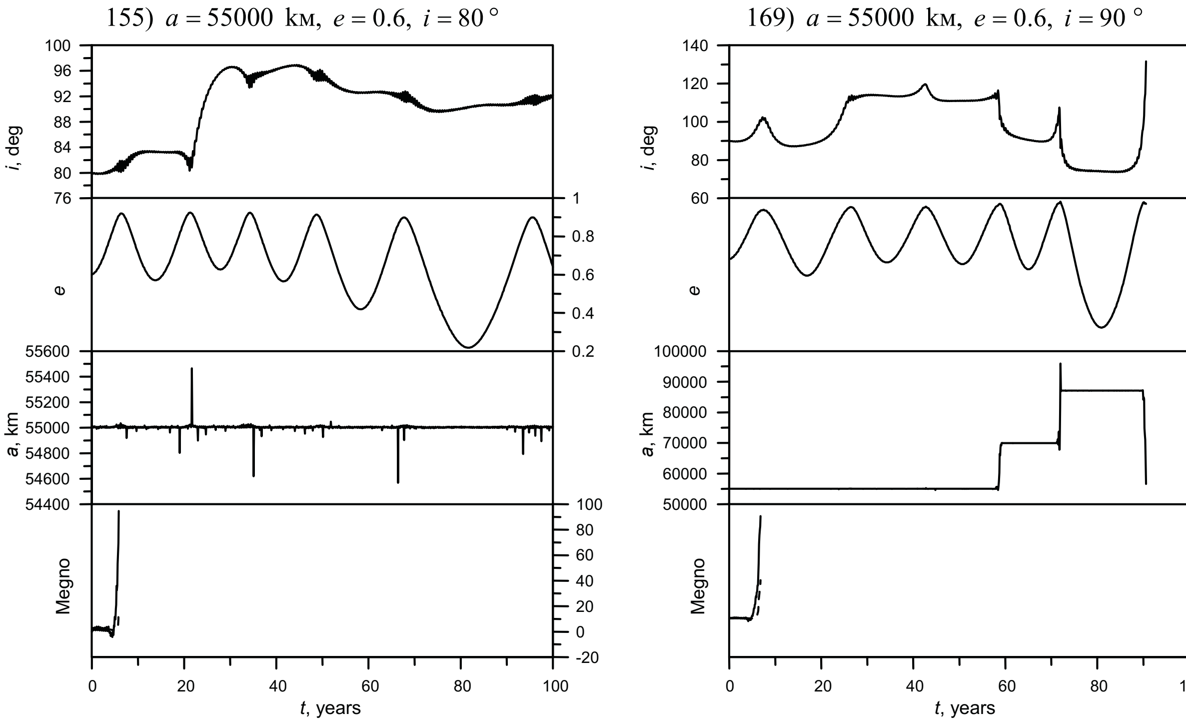

Fig. 4. Peculiarities of orbital evolution of high orbiting objects on prolate and near–polar orbits.

The regular motion with the Megno parameter that does not exceed 2 takes place for objects on near-polar orbits affected by only one secular resonance (Fig. 2) as well as for objects on similar orbits with inclinations smaller than 70°.

The motion of near-polar objects with semi-major axes between 50 000 and 55 000 km is most complicate in the examined altitudes range. The results of the numerical experiment that demonstrates the peculiarities of the orbital evolution for six models with semi-major axes equal to 55 000 km, inclinations of 80 and 90° and eccentricities equal to 0.01, 0.6, and 0.8 are shown in Figs. 3–5.

As the results show, the motion on orbits with inclination of 80° has the complicated but not catastrophic evolution. The presence in the motion of the large number of resonance relations passing through zero or near it leads to the secular variations of all basic orbital parameters of the objects. Even for the near-circular motion with inclination equal to 80° the variations of the inclination leads to the reverse motion on some parts of the trajectory, the eccentricity approaches unity and the semi-major axis decreases. As a result, the orbital space position is changed and the Megno parameter increases. The velocity of chaotization increases with initial eccentricity, but on the whole, the orbital evolution does not look catastrophic. This is due to different actions of the secular resonances in different parts of the trajectory with large eccentricity.

The pattern of long-time evolution has a catastrophic character for orbits with the semi-major axis equal to 55 000 km and inclination of 90°. The motion of the orbital model 49, which goes through 12 secular resonances, practically breaks in 40 years because the orbit eccentricity approaches unity, the change of the inclination translates the object on orbit with reverse motion, and the semi-major axis rapidly decreases. The chaotization of the motion takes place with fast increase of the Megno parameter. The chaotization velocity increases with the eccentricity, but the orbit destruction happens later with the nearly-zero initial eccentricity. As mentioned above, this may be explained by the different actions of secular resonances on different parts of the trajectory.

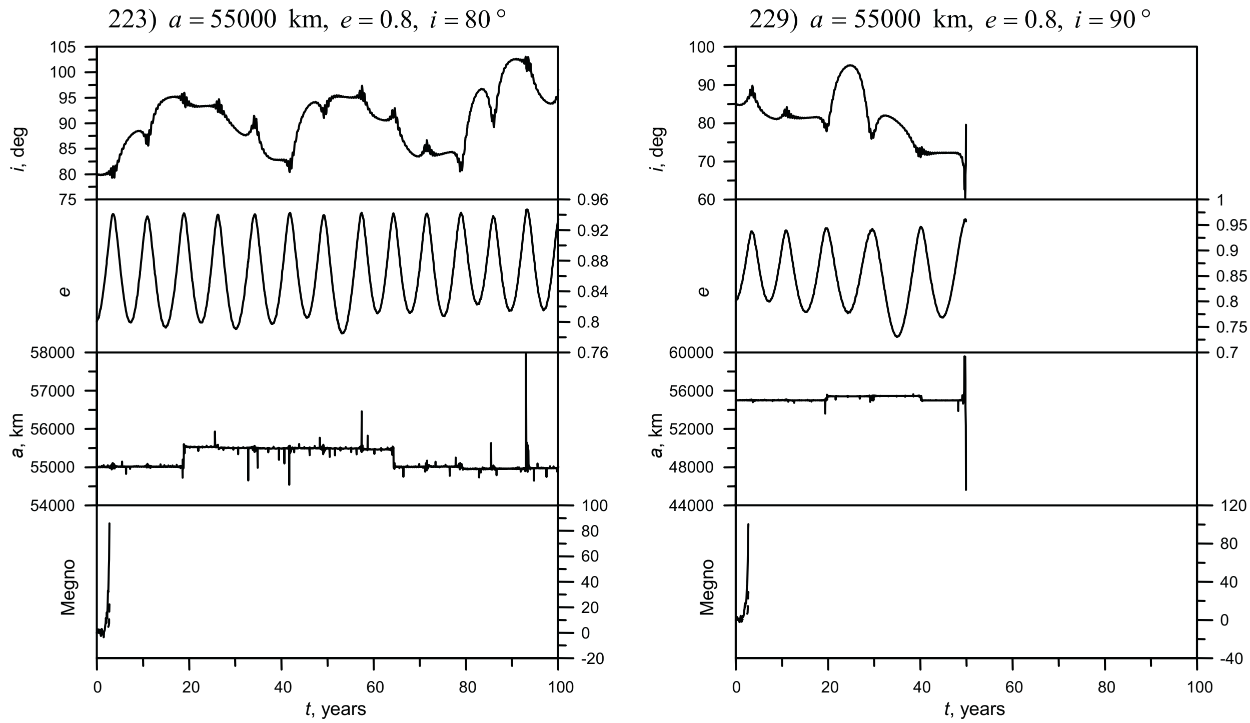

Fig. 5. Peculiarities of orbital evolution of objects with high eccentricity and near-polar orbits.

CONCLUSIONS

As indicated in the Introduction, this paper is the last one from the cycle of papers [1–4] devoted to the description of the extensive numerical-analytical experiment which has been carried out to elucidate the abundance of secular resonances in near-Earth orbital space and to study their influence on the long-time orbital evolution of uncontrollable objects with orbital parameters in the range:

e = {0.01 and 0.1 – 0.9 with a step of 0.1}; { i = 0–90° with a step of 5°}; a = {8000 km – 55000 km with a step of 1 km}.

In this connection, it seems appropriate here to summarize the results of our investigations. The results obtained show the following:

− secular resonances are very common phenomena in the near-Earth orbital space, but then values of semi-major axis and inclination are factors responsible for the appearance of secular resonances in the motion of near–Earth space objects;

− secular resonances with resonance relations smaller than 10^{-8} rad/s affect the orbital evolution of near-Earth objects;

– it is interesting to note that the existence of even one sharp secular resonance in the object motion changes the character of motion so that the object passes in another class of orbits and the dynamical evolution of the object changes essentially with time;

– the so-called semi-secular resonances with the mean motion of the Sun are more significant than the similar resonances with the mean motion of the Moon;

– the superposition of several sharp secular resonances or the proximity of the average motion of the object to the tesseral resonance in the presence of a sharp secular resonance are sources of randomness in the motion of objects;

– objects from the region of the orbital space with semi-major axes of 40 000 km and higher and inclinations between 55 and 70° are subject to the simultaneous influence of a large number of orbital resonances; the motion of such objects is irregular and shows long-period oscillations of the eccentricity and the inclination with large amplitudes; the chaotization of motion may be considerable, the velocity of chaotization increases with the initial eccentricity, and the process of the catastrophic development of the evolution accelerates;

– the evolution of the near-polar orbits with inclinations of 80 and 90° is very complicated, but degrees of the orbital destruction are different;

– for the near–circular motion with inclination equal to 80° the variations of the inclination leads to the motion turned into opposite one on some parts of the trajectory; the eccentricity approaches to unity, and the semi-major axis decreases; as a result, the orbital space position is changed, and the Megno parameter increases; the velocity of chaotization increases with the initial eccentricity, but on the whole, the orbital evolution does not look catastrophic;

– the long–time evolution of orbits with inclination of 90° has a catastrophic character for any eccentricities, and the chaotization of motion with rapidly increasing in the Megno parameter takes place; as in the preceding case, the process of chaotization is accelerated with increasing in the initial eccentricity, but the destruction of orbits comes later than with the near zero value of the initial eccentricity.

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