Russian Physics Journal, Vol. 52, No. 6, 2009

ASYMMETRICAL WAVES IN A HELIX AND A SYSTEM OF HELICES FORMING AN ARTIFICIAL MEDIUM

A. S. Zavyalov

UDC 621.336.674.3

Propagation along a helix of asymmetrical waves excited at the open waveguide end with H_{11} wave in the circular cross section is investigated. A system of helices forming a square array is considered. The effective dielectric constant of the array representing an artificial medium is determined. The calculated results are checked experimentally.

Keywords: helix, waves, artificial medium.

INTRODUCTION

Conductors bent as helices were widely used in the 40s–50s of the last century in retardation systems of running wave lamps [1] and antennas [2]. Helices with the turn length much smaller than the radiation wavelength were used in the running wave lamps and omnidirectional antennas. In these cases, axially symmetric waves propagating along the helix were considered. The approximation used in [1] to study the wave propagation along the helix was based on the helix approximation by a conducting cylinder of the same radius a. Two directions were chosen on the helix surface (Fig. 1*b*). Let the direction s be coincident with the wire direction, and the direction τ be perpendicular to the direction s.

Let us designate the fields outside the helix by superscript 1, and the fields inside the helix by superscript 2. In the approximation of the anisotropic conducting surface, the boundary conditions on the helix surface at $r = a$ are written as follows:

$$E_s^{(1)} = E_\varphi^{(1)} \cos\psi + E_z^{(1)} \sin\psi = E_s^{(2)} = E_\varphi^{(2)} \cos\psi + E_z^{(2)} \sin\psi = 0\,, \quad (1a)$$

$$E_\tau^{(1)} = -E_\varphi^{(1)} \sin\psi + E_z^{(1)} \cos\psi = E_\tau^{(2)} = -E_\varphi^{(2)} \sin\psi + E_z^{(2)} \cos\psi\,, \quad (1b)$$

$$H_s^{(1)} = H_\varphi^{(1)} \cos\psi + H_z^{(1)} \sin\psi = H_s^{(2)} = H_\varphi^{(2)} \cos\psi + H_z^{(2)} \sin\psi\,. \quad (1c)$$

ASYMMETRICAL WAVE PROPAGATION ALONG THE HELIX

The propagation along the helix of symmetric $\left(\frac{\partial}{\partial\varphi} = 0\right)$ and asymmetrical waves whose dependence on the angle φ was described by the function $e^{im\varphi}$ was studied in [3]. Here we consider the propagation along the helix of surface waves excited by wave H_{11} from the open waveguide end, having a circular cross section. To solve the problem, we take advantage of the Hertz vectors. For fields with a harmonic time dependence $e^{-i\omega t}$, the complex field

Tomsk State University, Tomsk, Russia, e-mail: zavyalov@elefot.tsu.ru. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 6, pp. 38–44, June, 2009. Original article submitted May 13, 2008; revision submitted April 16, 2009.

 1064-8887/09/5206-0590

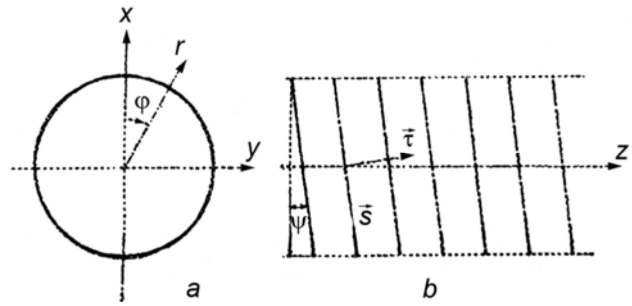

Fig. 1. Helix face (*a*) and side views (*b*).

amplitudes are related to the complex amplitudes of the electric ($\mathbf{\Pi}^e$) and magnetic ($\mathbf{\Pi}^m$) Hertz vectors (for example, see [3]):

$$\boldsymbol{E} = \operatorname{grad}\operatorname{div}\mathbf{\Pi}^e + k_0^2\mathbf{\Pi}^e + ik_0W_0 \operatorname{rot}\mathbf{\Pi}_m,$$
$$\boldsymbol{H} = -i\frac{k_0}{W_0}\operatorname{rot}\mathbf{\Pi}^e + \operatorname{grad}\operatorname{div}\mathbf{\Pi}^m + k_0^2\mathbf{\Pi}^m, \tag{2}$$

where $k_0 = \dfrac{2\pi}{\lambda}$ is the wave number and $W_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$ is the wave resistance of free space in the international system of units (SI), unlike formulas presented in [3] where the absolute system of Gauss units was used.

The magnetic wave field H_{11} in the wave guide with a circular cross section can be expressed through one longitudinal component of the magnetic Hertz vector

$$\Pi_z^m(r,\varphi,z) = \tilde{A}_1 J_1(\alpha r)\sin\varphi e^{i\gamma z}. \tag{3}$$

The electric and magnetic field components of the wave H_{11} have the form

$$E_r = ik_0W_0\frac{\tilde{A}_1}{r}J_1(\alpha r)\cos\varphi\cdot e^{i\gamma z},$$
$$E_\varphi = -ik_0W_0\tilde{A}_1J_1'(\alpha r)\sin\varphi\cdot e^{i\gamma z t},$$
$$E_z = 0,$$
$$H_r = i\alpha\gamma\tilde{A}_1J_1'(\alpha r)\sin\varphi\cdot e^{i\gamma z},$$
$$H_\varphi = i\gamma\frac{\tilde{A}_1}{r}J_1(\alpha r)\cos\varphi\cdot e^{i\gamma z},$$
$$H_z = (k_0^2-\gamma^2)\tilde{A}_1J_1(\alpha r)\sin\varphi\cdot e^{i\gamma z}, \tag{4}$$

where $J_1'(\alpha r)$ is the derivative of the Bessel function with respect to the argument, $\alpha = \dfrac{\mu_{11}}{a}$, and μ_{11} is the root of the derivative of the Bessel function. The fields of waves propagating along the helix can be expressed through the longitudinal components of the electric or magnetic Hertz vectors [3]. The waves with the angular field dependence of the form $\sin\varphi$ or $\cos\varphi$ can be represented as a superposition of partial waves having the field dependence on the angle φ in the form $e^{i\varphi}$ or $e^{-i\varphi}$.

We now express solutions of the Helmholtz equation for the longitudinal components of the electric and magnetic Hertz vectors as

$$\Pi_z^e(r,\varphi,z) = A_j^+ R_j(\alpha_+ r)e^{i\varphi}e^{i\gamma_+ z} + A_j^- R_j(\alpha_- r)e^{-i\varphi}e^{i\gamma_- z},$$
$$\Pi_z^m(r,\varphi,z) = \tilde{A}_j^+ R_j(\alpha_+ r)e^{i\varphi}e^{i\gamma_+ z} - \tilde{A}_j^- R_j(\alpha_- r)e^{-i\varphi}e^{i\gamma_- z}, \tag{5}$$

where $\alpha_\pm = \sqrt{\gamma_\pm^2 - k_0^2}$, subscripts "+" and "–" correspond to the angular dependence $e^{\pm i\varphi}$, $j = 1,2$ indicates the serial number of the region. The functions $R_j(\alpha_\pm r)$ satisfy the following conditions: $R_1(\alpha_\pm r) \to 0$ at $r \to \infty$, and the value of the function $R_2(\alpha_\pm r)$ remains finite when $r \to 0$. Cylindrical functions of imaginary arguments, including the McDonald function $R_1(\alpha_\pm r) = K_1(\alpha_\pm r)$ and the modified Bessel function $R_2(\alpha_\pm r) = I_1(\alpha_\pm r)$, satisfy these conditions. The partial wave field components can be written as follows. For the dependence on the angle φ in the form $e^{i\varphi}$, we obtain

$$E_r = \left[i\alpha_+\gamma_+ A_j^+ R_j'(\alpha_+ r) - k_0 W_0 \frac{\tilde{A}_j^+}{r} R_j(\alpha_+ r) \right] e^{i\varphi} \cdot e^{i\gamma_+ z},$$

$$E_\varphi = -\left[\gamma_+ \frac{A_j^+}{r} R_j(\alpha_+ r) + i k_0 W_0 \alpha_+ \tilde{A}_j^+ R_j'(\alpha_+ r) \right] e^{i\varphi} \cdot e^{i\gamma_+ z},$$

$$E_z = -[\alpha_+^2 A_j^+ R_j(\alpha_+ r)] e^{i\varphi} \cdot e^{i\gamma_+ z},$$

$$H_r = \left[\frac{k_0}{W_0} \frac{A_j^+}{r} R_j(\alpha_+ r) + i\alpha_+\gamma_+ \tilde{A}_j^+ R_j'(\alpha_+ r) \right] e^{i\varphi} \cdot e^{i\gamma_+ z}, \tag{6}$$

$$H_\varphi = \left[i \frac{k_0}{W_0} \alpha_+ A_j^+ R_j'(\alpha_+ r) - \gamma_+ \frac{\tilde{A}_j^+}{r} R_j(\alpha_+ r) \right] e^{i\varphi} \cdot e^{i\gamma_+ z},$$

$$H_z = -[\alpha_+^2 \tilde{A}_j^+ R_j(\alpha_+ r)] e^{i\varphi} \cdot e^{i\gamma_+ z}.$$

For the dependence on the angle φ in the form $e^{-i\varphi}$, we obtain

$$E_r = \left[i\alpha_-\gamma_- A_j^- R_j^-(\alpha_- r) - k_0 W_0 \frac{\tilde{A}_j^-}{r} R_j(\alpha_- r) \right] e^{-i\varphi} \cdot e^{i\gamma_- z},$$

$$E_\varphi = -\left[\gamma_- \frac{A_j^-}{r} R_j(\alpha_- r) + i k_0 W_0 \alpha_- \tilde{A}_j^- R_j'(\alpha_- r) \right] e^{-i\varphi} \cdot e^{i\gamma_- z},$$

$$E_z = -[\alpha_-^2 A_j^- R_j(\alpha_- r)] e^{-i\varphi} \cdot e^{i\gamma_- z}, \tag{7}$$

$$H_r = -\left[\frac{k_0}{W_0} \frac{A_j^-}{r} R_j(\alpha_- r) + i\alpha_-\gamma_- \tilde{A}_j^- R_j'(\alpha_- r) \right] e^{-i\varphi} \cdot e^{i\gamma_- z},$$

$$H_\varphi = \left[i \frac{k_0}{W_0} \alpha_- A_j^- R_j'(\alpha_- r) - \gamma_- \frac{\tilde{A}_j^-}{r} R_j(\alpha_- r) \right] e^{-i\varphi} \cdot e^{i\gamma_- z},$$

$$H_z = -[\alpha_-^2 \tilde{A}_j^- R_j(\alpha_- r)] e^{-i\varphi} \cdot e^{i\gamma_- z}.$$

Let us assume that expressions enclosed in the square brackets in formulas (6) and (7) (we designate them by $[E_r^+]$, $[E_\varphi^+]$, $[E_z^+]$... and $[E_r^-]$, $[E_\varphi^-]$, $[E_z^-]$...) for the corresponding field components are equal, that is, $[E_r^+] = [E_r^-] = [E_r]$. Then summation of the fields at $z = 0$ yields

$$\begin{aligned} E_r &= 2[E_r]\cos\varphi, \\ E_\varphi &= -2i[E_\varphi]\sin\varphi, \\ E_z &= 2[E_z]\cos\varphi, \\ H_r &= 2i[H_r]\sin\varphi, \\ H_\varphi &= 2[H_\varphi]\cos\varphi, \\ H_z &= 2[H_z]\sin\varphi. \end{aligned} \tag{8}$$

As can be seen, the dependences on the angle φ of the field components representing a superposition of two partial waves correspond to the dependences on the angle φ of exciting field (4). We now consider boundary conditions (1) separately for each partial wave. From condition (1a), we can express the coefficients $\tilde{A}_j$ in terms of A_j. Substituting the obtained coefficients into Eqs. (1b) and (1c) and considering the ratio

$$\frac{E_\tau^{(1)}}{H_s^{(1)}} = \frac{E_\tau^{(2)}}{H_s^{(2)}}, \tag{9}$$

we obtain the dispersion equation in the form

$$[n \pm (n^2 - 1)\tan\psi \cdot k_0 a]^2 = -\frac{\alpha a K_1'(\alpha a)}{K_1(\alpha a)} \frac{\alpha a I_1'(\alpha a)}{I_1(\alpha a)}, \tag{10}$$

where $n = \dfrac{\gamma}{k_0}$ is the surface wave retardation. The sign "+" here corresponds to the partial wave with the dependence on the angle φ in the form $e^{i\varphi}$, and the sign "–" corresponds to the wave with the time dependence $e^{-i\varphi}$.

As follows from Eq. (10), the partial waves propagate with different phase velocities, and beats will be observed in the presence of the velocity difference. Results of numerical calculations demonstrated that for helix angles ψ and winding radius a small compared to the wavelength, the difference between n_+ and n_- appears very small, and we can approximately consider

$$n_+ = n_- = n_{\text{av}}. \tag{11}$$

A solution of Eq. (10) in this approximation can easily be obtained. The dependence of the retardation n on the wavelength λ_0 for the helix winded by the wire 1.5 mm in diameter with a pitch of 3 mm and average radius $a_{\text{av}} = 6.75$ mm was calculated. The helix angle was approximately equal to 4°, and $\tan\psi = 0.071$.

For experimental investigations, a 50-cm helix was winded on the tube made from a thin elastic cardboard. The helix was excited with the help of transition from the rectangular 28.5 × 12.6 mm waveguide to the circular waveguide 36 mm in diameter. The helix was fixed in the waveguide transition region with a foam plastic bush. The reflector in the form of a metal washer was placed at the end of the helix. The standing wave pattern was registered with a mobile

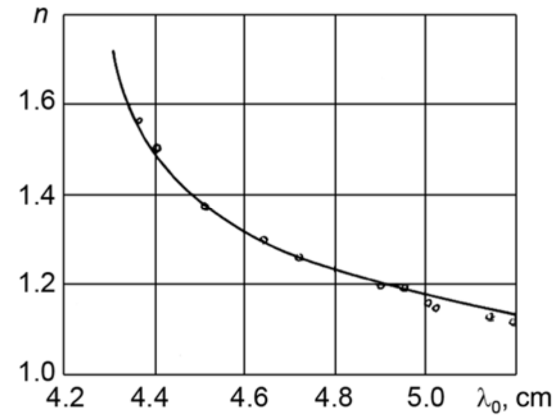

Fig. 2. Dependence of the wave retardation in the helix on the wavelength.

probe fabricated from a cable section whose external sheathing was partly removed from the length approximately equal to the quarter of wavelength. A signal from the cable was fed into a detector and then into a measuring amplifier. The retarded wavelength was averaged over several (five or more) standing waves.

In this case, no wave beats were registered. It was established that when the wavelength approached the helix turn length ($L = 4.35$ cm), the standing wave amplitudes started to decrease as the probe moved away from the waveguide face. This testifies to an increase in the portion of power radiated in the direction of the helix axis. Exactly this effect is used to produce helix antennas with longitudinal radiation excited by the internal conductor of a coaxial line [2]. The calculated and experimental data are shown in Fig. 2.

Good agreement of the experimental data with the calculated results demonstrates that the helix representation by the anisotropic conducting cylinder is justified for the given wire thickness and winding pitch. It was demonstrated experimentally that sections of the helix with length $L_{\text{opt}} = 0.3\dfrac{\lambda_0}{n-1}$ excited by the waveguide face wave H_{11} can be used as directing systems that narrow significantly the directivity pattern in the axial direction.

ARTIFICIAL MEDIUM IN THE FORM OF A SQUARE ARRAY OF HELICES

Short sections of helices were used to create a chiral artificial medium early in the last century [4]. Here we consider rather extended sections of helices assembled in a square array (Fig. 3). The direction of wave propagation in the system is set parallel to the helix axes.

The effective dielectric permittivity of the square array of two-dimensional elements can be calculated by analogy with the cubic array of three-dimensional elements by introducing the Lorentz sphere having the center coinciding with the center of a certain particle outside of which the dielectric is considered uniform. For the square array, the cylindrical void whose axis coincides with the axis of the reference element can be introduced. The effective dielectric permittivity of the square array is determined from the formula (see [5])

$$\varepsilon_{\text{eff}} = \frac{1 + \dfrac{1}{2}\dfrac{\alpha_{\tilde{n}}}{\varepsilon_0}N}{1 - \dfrac{1}{2}\dfrac{\alpha_c}{\varepsilon_0}N}, \tag{12}$$

where α_c is the polarizability per unit length of the array element, and N is the number of elements in unit area.

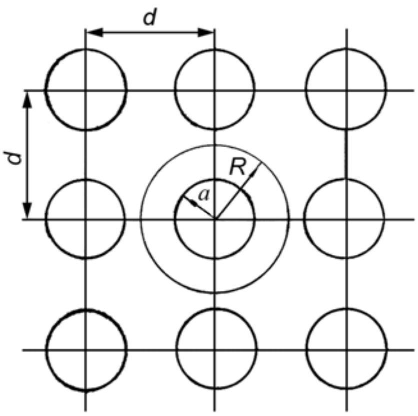

Fig. 3. Square array of helices.

We note that in the Lorentz method, the radius of the sphere must be much greater than the particle sizes. We calculated the effective dielectric permittivity of the square array of helices as follows. We considered the array of helices as an array of anisotropic conducting cylinders. We represented the array itself (except one cylinder and the void surrounding it) as a continuous magnetodielectric with dielectric and magnetic constants ε_{eff} and μ_{eff} in a plane wave field. The void radius was such that the cross sectional void area was equal to the square area, that is,

$$\pi R^2 = d^2 \text{, where } R \cong 0.568\, d. \tag{13}$$

Then we expressed a solution of the Helmholtz equation inside the void in terms of the modified Bessel and McDonald functions. The longitudinal components of the electric and magnetic fields are written as

$$\begin{aligned} E_z &= -\alpha^2 A_1 [I_1(\alpha r) + C_1 K_1(\alpha r)] \cos\varphi\, e^{i\gamma z}, \\ H_z &= -\alpha^2 \tilde{A}_1 [I_1(\alpha r) + \tilde{C}_1 K_1(\alpha r)] \sin\varphi\, e^{i\chi z}. \end{aligned} \tag{14}$$

By virtue of the assumption about the transverse field outside the void, the tangential field components at $r = R$ vanish: $E_z = 0$ and $H_z = 0$

From here we obtain

$$\begin{aligned} R_1(\alpha r) &= I_1(\alpha r) - \frac{I_1(\alpha R)}{K_1(\alpha R)} K_1(\alpha r), \\ R_1'(\alpha r) &= I_1'(\alpha r) - \frac{I_1(\alpha R)}{K_1(\alpha R)} K_1'(\alpha r). \end{aligned} \tag{15}$$

The dispersion helix equation in the void can be written as follows:

$$n^2 = -\frac{\alpha a R_1'(\alpha a)}{R_1(\alpha a)} \frac{\alpha a I_1'(\alpha a)}{I_1(\alpha a)}. \tag{16}$$

For small arguments $(\alpha a << 1 \text{ and } \alpha R << 1)$, the functions $I_1(x)$ and $K_1(x)$ can be substituted by their approximate values $\left(I_1(x) \cong \frac{x}{2},\ \ I_1'(x) \cong \frac{1}{2},\ \ K_1(x) \cong \frac{1}{x},\ \text{ and } K_1'(x) \cong -\frac{1}{x^2} \right)$, and we arrive at the expression

$$n^2 = \varepsilon_{\text{eff}} = \frac{1+v}{1-v}, \tag{17}$$

where $v = \dfrac{a^2}{R^2} = \dfrac{\pi a^2}{d^2}$.

Formula (17) gives the effective dielectric permittivity of the array of continuous metal cylinders. This is easy to verify by substitution of the expression for the polarizability per unit cylinder length $\alpha_c = 2\pi\varepsilon_0 a^2$ and $N = \dfrac{1}{d^2}$ into Eq. (12). Considering that the continuous metal cylinder has the magnetic polarizability $\alpha_m = -2\pi\mu_0 a^2$, the effective magnetic permittivity of the array of cylinders is

$$\mu_{\text{eff}} = \frac{1-v}{1+v} < 1. \tag{18}$$

As a result, we obtain

$$n^2 = \varepsilon_{\text{eff}} \cdot \mu_{\text{eff}} = 1. \tag{19}$$

From formula (16) it follows that $n^2 > 1$ for the array of helices.

The dispersive properties of the array of helices can be easily taken into account by the method of successive approximations with the zero approximation n_0 given by formula (17). Calculations were carried out for the square array of helices winded by a copper wire 1.75 mm in diameter with a pitch of 5 mm, average radius of 9 mm, and the array parameter d = 30 mm. In the zero approximation, $n = n_0 = 1.81$. Retardations were calculated from formula (16) for wavelengths $\lambda_0 = 10, 11$, and 12 cm. The model used for experimental testing of the calculated results represented a series of seven helices 7 cm long screwed into a plexiglass plate. The presence of the plate perpendicular to the electric field is equivalent to the arrangement of helices in the dielectric with the effective dielectric permittivity

$$\varepsilon_{\text{eff}}^0 = \frac{d}{\dfrac{d_1}{\varepsilon} + (d - d_1)}, \tag{20}$$

where d_1 is the plate thickness $(d_1 = 2 \text{ mm})$, $\varepsilon \cong 2.6$, and $d = 30$ mm. As a result, we obtain $\varepsilon_{\text{eff}}^0 = 1.04$.

The retardation calculated from formulas (16) and (17) with allowance for $\varepsilon_{\text{eff}}^0 = 1.04$ is shown in Fig. 4, where straight line *1* shows the retardation in the static approximation ($n^2 = 1.81 \cdot 1.04 = 1.88$), and curve *2* shows the results of calculations by Eq. (16).

The results of calculations were compared with measurements of ε_{eff} by the waveguide method. For this purpose, an attachment to the waveguide measuring line having cross sectional dimensions 72×34 and comprising a waveguide section with cross sectional dimensions 210×30 and smooth transition from the 72×34 to 210×30 waveguide section was used. Measurements of ε_{eff} were performed for several wavelengths λ_0 increasing from 9.8 to 11.9 cm. The experimental data are shown by filled circles in Fig. 4. They are in good agreement with the calculated results, which demonstrates the efficiency of the employed calculation method.

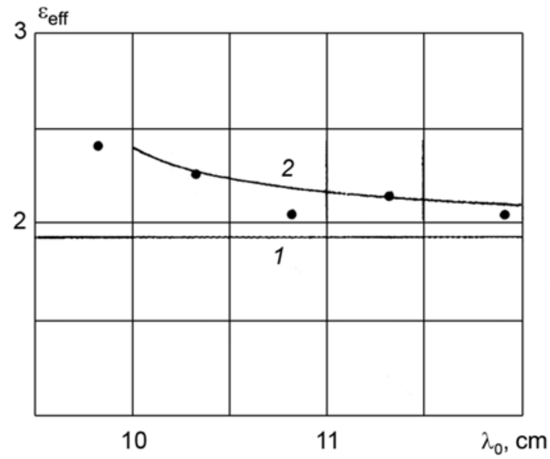

Fig. 4. Dependence of the effective dielectric permittivity of the array of helices on the wavelength.

REFERENCES

1. J. R. Pierce, Proc. IRE, **35**, 114–123 (1947).
2. J. D. Kraus, Electronics, **20**, 109–112 (1947).
3. L. A. Wainshtein, Electromagnetic Waves [in Russian], Radio i Svyaz', Moscow (1988).
4. K. F. Lindman, Ann. Phys., **63**, 621–644 (1920).
5. M. M. Kharadly and W. Jakson, Proc. IEEE, **100**, 119–212 (1953).