Russian Physics Journal, Vol. 48, No. 7, 2005

ELEMENTARY PARTICLE PHYSICS AND FIELD THEORY

POLARIZATION OF TWO IDENTICAL ATOMS IN AN ELECTRIC FIELD

I. I. Khvalchenko

UDC 539.186.097/ 098

The polarization of two atoms which are in a linearly polarized monochromatic field and in a field of optical pulses is calculated. The problem is solved for the 2s→3p transitions taking into account the initial states of the atoms.

INTRODUCTION

Modern technologies allow production of nanostructured objects consisting of a few atoms and molecules. The optical properties of objects of this type have been intensely studied in recent years since they can find applications in quantum computers, in quantum cryptography, in investigations of polymers and biologically important molecules, and in high resolution spectroscopy ([1, 2] and the cited works).

Let us consider a problem of the polarization of two identical atoms in an electric field. Suppose that each of these atoms can be in the 2*s* or 3*p* state. All possible initial states of the system under consideration are described by the wave functions [3]

$$\psi_m^L = \sum_\mu s_{L,\,\mu,m-\mu}^{(l,\bar{l}\,)} \Psi_\mu \bar{\Psi}_{m\text{-}\mu} \,, \tag{1}$$

where $s_{L,\mu,m-\mu}^{(l,\bar{l}\,)}$ are Wigner coefficients. The overbarred symbols refer to the states of the second atom.

Relation (1) determines 16 initial states of the system of atoms:

$$\begin{gathered}
\psi_1 = \Psi_{200}\bar{\Psi}_{200},\ \psi_2 = \Psi_{200}\bar{\Psi}_{31-1},\ \psi_3 = \Psi_{200}\bar{\Psi}_{310},\ \psi_4 = \Psi_{200}\bar{\Psi}_{311},\ \psi_5 = \Psi_{31-1}\bar{\Psi}_{200},\\
\psi_6 = \Psi_{310}\bar{\Psi}_{200},\ \psi_7 = \Psi_{311}\bar{\Psi}_{200},\ \psi_8 = \Psi_{31-1}\bar{\Psi}_{31-1},\ \psi_9 = \frac{1}{\sqrt{2}}\left(\Psi_{31-1}\bar{\Psi}_{310} + \Psi_{310}\bar{\Psi}_{31-1}\right),\\
\psi_{10} = \frac{1}{\sqrt{6}}\left(\Psi_{31-1}\bar{\Psi}_{311} + 2\Psi_{310}\bar{\Psi}_{310} + \Psi_{311}\bar{\Psi}_{31-1}\right),\ \psi_{11} = \frac{1}{\sqrt{2}}\left(\Psi_{310}\bar{\Psi}_{311} + \Psi_{311}\bar{\Psi}_{310}\right),\\
\psi_{12} = \Psi_{311}\bar{\Psi}_{311},\ \psi_{13} = \frac{1}{\sqrt{2}}\left(\Psi_{310}\bar{\Psi}_{31-1} - \Psi_{31-1}\bar{\Psi}_{310}\right),\ \psi_{14} = \frac{1}{\sqrt{2}}\left(\Psi_{311}\bar{\Psi}_{31-1} - \Psi_{31-1}\bar{\Psi}_{311}\right),\\
\psi_{15} = \frac{1}{\sqrt{2}}\left(\Psi_{311}\bar{\Psi}_{310} - \Psi_{310}\bar{\Psi}_{311}\right),\ \psi_{16} = \frac{1}{\sqrt{3}}\left(\Psi_{31-1}\bar{\Psi}_{311} - \Psi_{310}\bar{\Psi}_{310} + \Psi_{311}\bar{\Psi}_{31-1}\right).
\end{gathered} \tag{2}$$

Consider two types of electric field acting on the atoms.

Elabuga State Pedagogical University, e-mail: khvi@mail.ru. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 7, pp. 57–59, July, 2005. Original article submitted December 25, 2004.

LINEARLY POLARIZED FIELD

Let the atoms be under the action of an electric field $\boldsymbol{E}=(E_1\boldsymbol{i}+E_2\boldsymbol{j}+E_3\boldsymbol{k})\cos\omega t$. The quantum states of a system of this type obey the equation

$$i\hbar\frac{\partial\Psi}{\partial t}=\left(\hat{H}_0+\hat{\bar{H}}_0+\hat{V}\right)\Psi, \tag{3}$$

where

$$\Psi=\sum_{k=1}^{16}a_k\psi_k\exp(-i\omega_k t), \tag{4}$$

$\omega_k=\begin{cases}2\omega_1,\ k=1,\\ \omega_1+\omega_2,\ k=\overline{2,7},\\ 2\omega_2,\ k=\overline{8,16};\end{cases}$ $\hbar\omega_1$ and $\hbar\omega_2$ are the energies of the 2*s* and 3*p* states; $\hat{H}_0$ and $\hat{\bar{H}}_0$ are the Hamiltonians of the free atoms, and $\hat{V}=-(\boldsymbol{D},\boldsymbol{E})$, $\boldsymbol{D}=\boldsymbol{d}+\bar{\boldsymbol{d}}$, $\boldsymbol{d},\bar{\boldsymbol{d}}$ are the operators of the dipole moments of the atoms.

Equation (3) yields the system of differential equations

$$i\hbar\dot{a}=Va. \tag{5}$$

To solve system (5), we use the method of successive approximations [4]. This method gives the following set of coefficients as a first approximation:

$$a=I-\frac{i}{\hbar}\int_0^t V(\tau)d\tau. \tag{6}$$

In relation (6), *a* denotes a square matrix of order 16 whose each column defines the wave function $|\Psi_k\rangle$ of the atoms-plus-field system at the corresponding initial state of (2). Let us calculate the polarization of this system using the formula

$$\boldsymbol{P_k}=\mathrm{Sp}\hat{\rho}_k\boldsymbol{D}, \tag{7}$$

where $\hat{\rho}_k=|\Psi_k\rangle\langle\Psi_k|$, and the operator $\boldsymbol{D}$ is constructed on the basis of functions (4). We obtain the following expressions for the corresponding initial states (2):

$$\begin{gathered}\boldsymbol{P}_1=4X(E_1\boldsymbol{i}+E_2\boldsymbol{j}+E_3\boldsymbol{k}),\quad \boldsymbol{P}_2=(XE_1-YE_2)\boldsymbol{i}+(YE_1+XE_2)\boldsymbol{j}+2XE_3\boldsymbol{k},\\ \boldsymbol{P}_3=2X(E_1\boldsymbol{i}+E_2\boldsymbol{j}),\quad \boldsymbol{P}_4=(XE_1+YE_2)\boldsymbol{i}+(-YE_1+XE_2)\boldsymbol{j}+2XE_3\boldsymbol{k},\ \boldsymbol{P}_5=\boldsymbol{P}_2,\\ \boldsymbol{P}_6=\boldsymbol{P}_3,\quad \boldsymbol{P}_7=\boldsymbol{P}_4,\quad \boldsymbol{P}_8=-2[(XE_1+YE_2)\boldsymbol{i}+(-YE_1+XE_2)\boldsymbol{j}],\\ \boldsymbol{P}_9=(-XE_1-YE_2)\boldsymbol{i}+(YE_1-XE_2)\boldsymbol{j}-2XE_3\boldsymbol{k},\quad \boldsymbol{P}_{10}=-2X(E_1\boldsymbol{i}+E_2\boldsymbol{j}+4E_3\boldsymbol{k})/3,\\ \boldsymbol{P}_{11}=(-XE_1+YE_2)\boldsymbol{i}+(-YE_1-XE_2)\boldsymbol{j}-2XE_3\boldsymbol{k},\quad \boldsymbol{P}_{12}=-2[(XE_1-YE_2)\boldsymbol{i}+(YE_1+XE_2)\boldsymbol{j}],\\ \boldsymbol{P}_{13}=(-XE_1-YE_2)\boldsymbol{i}+(YE_1-XE_2)\boldsymbol{j}-2XE_3\boldsymbol{k},\quad \boldsymbol{P}_{14}=-2X(E_1\boldsymbol{i}+E_2\boldsymbol{j}),\\ \boldsymbol{P}_{15}=(-XE_1+YE_2)\boldsymbol{i}+(-YE_1-XE_2)\boldsymbol{j}-2XE_3\boldsymbol{k},\quad \boldsymbol{P}_{16}=-4X(E_1\boldsymbol{i}+E_2\boldsymbol{j}+E_3\boldsymbol{k})/3,\end{gathered} \tag{8}$$

where

$$X = \frac{d^2\omega_0\left[\cos\omega_0 t - \cos\omega t\right]}{3\hbar\left(\omega^2 - \omega_0^2\right)}, \quad Y = \frac{d^2\left[\omega_0\sin\omega_0 t - \omega\sin\omega t\right]}{3\hbar\left(\omega^2 - \omega_0^2\right)}, \quad d = -e\int\limits_0^\infty R_{20}R_{31}r^3 dr,$$

R_{20}, R_{31} are the radial functions, e is the charge of a positron, and $\omega_0 = \omega_2 - \omega_1$.

Note that this solution has been obtained for the case $\omega \neq \omega_0$. The case of precise resonance demands separate consideration.

FIELD OF RECTANGULAR PULSES

Let the atoms be subjected to the action of a rectangular pulse. To consider the pulse, which is directed along an arbitrarily chosen coordinate axis, we write it as the sum of three terms: $\boldsymbol{E} = E_1(t)\boldsymbol{i} + E_2(t)\boldsymbol{j} + E_3(t)\boldsymbol{k}$, where

$$E_1(t) = \begin{cases} A_1, t_{11} < t < t_{12}, \\ 0,\ t < t_{11},\ t > t_{12}, \end{cases} \quad E_2(t) = \begin{cases} A_2,\ t_{21} < t < t_{22}, \\ 0,\ t < t_{21},\ t > t_{22}, \end{cases} \quad E_3(t) = \begin{cases} A_3,\ t_{31} < t < t_{32}, \\ 0,\ t < t_{31},\ t > t_{32}. \end{cases} \tag{9}$$

Calculating the wave functions and polarization for the given field again by formulas (3)–(7), we obtain the following expressions:

$$\boldsymbol{P}_1 = 4(X_1A_1\boldsymbol{i} + X_2A_2\boldsymbol{j} + X_3A_3\boldsymbol{k}), \quad \boldsymbol{P}_2 = (X_1A_1 - Y_2A_2)\boldsymbol{i} + (Y_1A_1 + X_2A_2)\boldsymbol{j} + 2X_3A_3\boldsymbol{k},$$
$$\boldsymbol{P}_3 = 2(X_1A_1\boldsymbol{i} + X_2A_2\boldsymbol{j}), \quad \boldsymbol{P}_4 = (X_1A_1 + Y_2A_2)\boldsymbol{i} + (-Y_1A_1 + X_2A_2)\boldsymbol{j} + 2X_3A_3\boldsymbol{k},$$
$$\boldsymbol{P}_5 = \boldsymbol{P}_2, \quad \boldsymbol{P}_6 = \boldsymbol{P}_3, \quad \boldsymbol{P}_7 = \boldsymbol{P}_4, \quad \boldsymbol{P}_8 = -2[(X_1A_1 + Y_2A_2)\boldsymbol{i} + (-Y_1A_1 + X_2A_2)\boldsymbol{j}],$$
$$\boldsymbol{P}_9 = (-X_1A_1 - Y_2A_2)\boldsymbol{i} + (Y_1A_1 - X_2A_2)\boldsymbol{j} - 2X_3A_3\boldsymbol{k},$$
$$\boldsymbol{P}_{10} = -2(X_1A_1\boldsymbol{i} + X_2A_2\boldsymbol{j} + 4X_3A_3\boldsymbol{k})/3,$$
$$\boldsymbol{P}_{11} = (-X_1A_1 + Y_2A_2)\boldsymbol{i} + (-Y_1A_1 - X_2A_2)\boldsymbol{j} - 2X_3A_3\boldsymbol{k},$$
$$\boldsymbol{P}_{12} = -2[(X_1A_1 - Y_2A_2)\boldsymbol{i} + (Y_1A_1 + X_2A_2)\boldsymbol{j}],$$

$$\boldsymbol{P}_{13} = (-X_1A_1 - Y_2A_2)\boldsymbol{i} + (Y_1A_1 - X_2A_2)\boldsymbol{j} - 2X_3A_3\boldsymbol{k},\ \boldsymbol{P}_{14} = -2(X_1A_1\boldsymbol{i} + X_2A_2\boldsymbol{j}),$$
$$\boldsymbol{P}_{15} = (-X_1A_1 + Y_2A_2)\boldsymbol{\imath} + (-Y_1A_1 - X_2A_2)\boldsymbol{j} - 2X_3A_3\boldsymbol{k}, \tag{10}$$
$$\boldsymbol{P}_{16} = -4(X_1A_1\boldsymbol{i} + X_2A_2\boldsymbol{j} + X_3A_3\boldsymbol{k})/3,$$

where

$$X_k = \frac{d^2[\cos\omega_0(t - t_{k2}) - \cos\omega_0(t - t_{k1})]}{3\hbar\omega_0}, \quad Y_k = \frac{d^2[\sin\omega_0(t - t_{k2}) - \sin\omega_0(t - t_{k1})]}{3\hbar\omega_0}, \tag{11}$$
$$k = 1,2,3.$$

Let us consider the basic results following from this model. Suppose that the atoms are subjected to the action of only the first pulse ($A_2 = A_3 = 0$), which is applied at a time $t_{11} = 0$; then

$$\boldsymbol{P}_1 = 4X_1A_1\boldsymbol{i}, \quad \boldsymbol{P}_2 = (X_1\boldsymbol{i} + Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_3 = 2X_1A_1\boldsymbol{i}, \quad \boldsymbol{P}_4 = (X_1\boldsymbol{i} - Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_5 = \boldsymbol{P}_2,$$

$$\boldsymbol{P}_6 = \boldsymbol{P}_3, \quad \boldsymbol{P}_7 = \boldsymbol{P}_4, \quad \boldsymbol{P}_8 = -2(X_1\boldsymbol{i} - Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_9 = (-X_1\boldsymbol{i} + Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_{10} = -2X_1A_1\boldsymbol{i}/3,$$

$$\boldsymbol{P}_{11} = (-X_1\boldsymbol{i} - Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_{12} = -2(X_1\boldsymbol{i} + Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_{13} = (-X_1\boldsymbol{i} + Y_1\boldsymbol{j})A_1,$$

$$\boldsymbol{P}_{14} = -2X_1A_1\boldsymbol{i}, \quad \boldsymbol{P}_{15} = -(X_1\boldsymbol{i} + Y_1\boldsymbol{j})A_1, \quad \boldsymbol{P}_{16} = -4X_1A_1\boldsymbol{i}/3.$$

We give the expressions for X_1 and Y_1 with four values of t_{12}:

t_{12}	X_1	Y_1
$\pi/2\omega_0$	$-d^2\sqrt{2}\sin(\pi/4 - \omega_0 t)/3\hbar\omega_0$	$-d^2\sqrt{2}\cos(\pi/4 - \omega_0 t)/3\hbar\omega_0$
π/ω_0	$-2d^2\cos\omega_0 t/3\hbar\omega_0$	$-2d^2\sin\omega_0 t/3\hbar\omega_0$
$3\pi/2\omega_0$	$-d^2\sqrt{2}\cos(\pi/4 - \omega_0 t)/3\hbar\omega_0$	$d^2\sqrt{2}\sin(\pi/4 - \omega_0 t)/3\hbar\omega_0$
$2\pi/\omega_0$	0	0

Thus, for the initial states ψ_1, ψ_3, ψ_6, ψ_{10}, ψ_{14}, and ψ_{16} the polarization vector will make oscillations along the x-axis; in all other cases, the motion will occur in a circle. For the pulse duration $2\pi n/\omega_0$, $n = 1, 2, 3\ldots$, the polarization of the medium is equal to zero. Properly choosing the time of application and duration of the second pulse, one can vary the polarization created by the first pulse.

REFERENCES

1. K. N. Koen-Tanudzhi, Usp. Fiz. Nauk, **169**, 292–304 (1999).
2. S. Ya. Kilin, *Ibid*., 507–526.
3. E. Wigner, Group Theory and Its Application to the Quantum Mechanics of Atomic Spectra, Academic Press, NY (1959).
4. F. R. Gantmakher, Matrix Theory [in Russian], Nauka, Moscow (1988).