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REFLECTION OF PLANE BELTRAMI WAVES FROM A PLANE INTERFACE IN AN ISOTROPIC CHIRAL MEDIUM

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The reflection of plane electromagnetic waves of circular polatization from a flat, ideally conducting screen limiting an isotropic chiral-medium half-space is examined. The equations relating the reflection coefficients with the common angle of incidence and material parameters of the medium are discussed.

Many substances found in nature are optically active, whereas their artificial analogs – chiral materials – manifest electromagnetic activity at ultrahigh frequencies [1]. Despite the fact that a chiral medium has a discrete microstructure, in fact, it is homogeneous when chiral inclusions are uniformly distributed over the volume. Phenomenologically, it is considered to be a continuous biisotropic medium and is characterized by three macroscopic material parameters: two scalars (dielectric constant and permeability) and one pseudoscalar – a chiral-order parameter which is a measure of disymmetry of geometrical shape and its structural elements. The medium has a property of birefringence, therefore the magnetic field is a superposition of two self-wave fields of circular polarization with opposite direction of the field vector rotation and different wave numbers. As applied to the set of constitutive Drude – Born – Fedorov equations, we get

$$\boldsymbol{D} = \varepsilon(\boldsymbol{E} + \beta\nabla\times\boldsymbol{E}), \qquad \boldsymbol{B} = \mu(\boldsymbol{H} + \beta\nabla\times\boldsymbol{H}),$$

where $\boldsymbol{D}$ and $\boldsymbol{E}$ are the electric-field induction and strength, $\boldsymbol{B}$ and $\boldsymbol{H}$ are the magnetic-field induction and strength, ε and μ are the dielectric constant and permeability, and β is the chiral-order parameter, and homogeneous vortical Maxwell equations for a monochromatic field with the time factor $\exp(-i\omega t)$ reduce to

$$\nabla\times\boldsymbol{Q}_1 = \gamma_1\boldsymbol{Q}_1, \qquad \nabla\times\boldsymbol{Q}_2 = -\gamma_2\boldsymbol{Q}_2, \tag{1}$$

where $\gamma_1 = k/(1-k\beta)$ and $\gamma_2 = k/(1+k\beta)$ are the wave numbers, $k = \omega(\varepsilon\mu)^{1/2}$, and ω is the angular frequency. The equations of type (1) describe the Beltrami wave fields, and Silberstein was the first to use them in electromagnetism [2]. The electric and magnetic fields in a chiral medium are related to the Beltrami fields by means of the following formulas:

$$\boldsymbol{E} = \boldsymbol{Q}_1 - i\eta\boldsymbol{Q}_2, \qquad \boldsymbol{H} = \boldsymbol{Q}_2 - i\eta^{-1}\boldsymbol{Q}_1, \tag{2}$$

where $\eta = (\mu/\varepsilon)^{1/2}$ is the wave impedance. In an infinite homogeneous chiral medium, the Beltrami fields can be excited one at a time by special source distributions [3] and propagate independently. The fields appear to be coupled at the interface and impermeable boundaries because the boundary conditions are imposed on the electric field (2) rather than on the Beltrami fields $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$.

In the applied electrodynamic investigations, one comes across different boundaries impermeable for the field. Most common are isotropic ideally conducting boundaries, where a tangential electric field (“electric wall”) and a tangential

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magnetic field (“magnetic wall”) vanish. Though the phenomena of electromagnetic-wave reflection from the foregoing boundaries in chiral media are always taken into account in the problems of waveguide propagation, diffraction, and wave scattering by objects of various shape, to our knowledge, they were considered in brief in [4] alone as applied to the concept of images in chiral media. This paper studies wave phenomena occurring in a chiral medium for the case of the plane waves associated with the Beltrami fields, where they fall in combination and one by one, and their reflection from flat, ideally conducting screens.

Let the plane xz be the plane of incidence of the Beltrami waves, and a flat, ideally conducting screen is at $z = 0$. In the range $z > 0$, the incident Beltrami fields $\boldsymbol{Q}_1^{\mathrm{i}}(x,z)$ and $\boldsymbol{Q}_2^{\mathrm{i}}(x,z)$ propagate at the angles φ and ψ to the z axis, respectively, and we have

$$\boldsymbol{Q}_1^{i}(x,z) = A_1\left(\cos\varphi \boldsymbol{x}_0 - \mathrm{i} y_0 + \sin\varphi z_0\right)\exp\left[i\gamma_1\left(x\sin\varphi - z\cos\varphi\right)\right], \tag{3}$$

$$\boldsymbol{Q}_2^{i}(x,z) = A_2\left(\cos\psi \boldsymbol{x}_0 + i y_0 + \sin\psi z_0\right)\exp\left[i\gamma_2\left(x\sin\psi - z\cos\psi\right)\right], \tag{4}$$

where A_1 and A_2 are the amplitudes of the incident plane waves and x_0, y_0, and z_0 are the unit vectors of the Cartesian rectangular coordinate system. The reflected field is described as

$$\boldsymbol{Q}_1^{r}(x,z) = B_1\left(-\cos\varphi x_0 - i y_0 + \sin\varphi z_0\right)\exp\left[i\gamma_1\left(x\sin\varphi + z\cos\varphi\right)\right], \tag{5}$$

$$\boldsymbol{Q}_2^{r}(x,z) = B_2\left(-\cos\psi x_0 + i y_0 + \sin\psi z_0\right)\exp\left[i\gamma_2\left(x\sin\psi + z\cos\psi\right)\right], \tag{6}$$

where B_1 and B_2 are the amplitudes of waves departing from the boundary. The angles of arrival and reflection obey the generalized Snell law

$$\gamma_1 \sin\varphi = \gamma_2 \sin\psi. \tag{7}$$

For the sake of definiteness, let us assume that the chiral-order parameter is positive $(\beta > 0)$. Then $\gamma_1 > \gamma_2 > 0$, since there is a restriction on the maximum chiral-order parameter ($k\beta < 1$) [5]. It follows from Eq. (7) that both angles are real ($\psi > \varphi$), as long as the angle φ is in no excess of the critical value $\varphi_{\text{кр}} = \arcsin\left(\dfrac{\gamma_2}{\gamma_1}\right)$. For $\varphi_{\text{кр}} < \varphi < \pi/2$ the Beltrami fields with the wave number γ_2 take on the structure of nonuniform plane waves.

It follows from Eqs. (2) and (3)–(6) that for the case of electric wall, the boundary condition at $z = 0$ becomes

$$\left(\boldsymbol{Q}_1^{i} - i\eta \boldsymbol{Q}_2^{i}\right) z_0 + \left(\boldsymbol{Q}_1^{r} - i\eta \boldsymbol{Q}_2^{r}\right) z_0 = 0. \tag{8}$$

The amplitudes of reflected waves are related to the amplitudes of incident waves by the following formulas:

$$B_1 = R_{11}A_1 + R_{12}A_2, \quad B_2 = R_{21}A_1 + R_{22}A_2, \tag{9}$$

where R_{ij} (i and j are 1 or 2) form a matrix of reflection coefficients. Upon substitution of Eqs. (3)–(4), (5)–(6), and (9) into the boundary condition (8), we get algebraic relations for determination of the R_{ij} coefficients as

$$\left[(1 - R_{11})\cos\varphi + i\eta R_{21}\cos\psi\right]A_1 - \left[R_{12}\cos\varphi + i\eta(1 - R_{22})\cos\psi\right]A_2 = 0,$$

$$(1 + R_{11} + i\eta R_{21})A_1 + \left[R_{12} + i\eta(1 + R_{22})\right]A_2 = 0,$$

whence it follows that

$$R_{11}=\frac{\cos\varphi-\cos\psi}{\cos\varphi+\cos\psi},\quad R_{12}=\frac{-2\mathrm{i}\eta\cos\psi}{\cos\varphi+\cos\psi}, \tag{10}$$

$$R_{21}=\frac{-2\cos\varphi}{\mathrm{i}\eta(\cos\varphi+\cos\psi)},\quad R_{22}=-\frac{\cos\varphi-\cos\psi}{\cos\varphi+\cos\psi}. \tag{11}$$

For the case of magnetic wall, using the boundary condition

$$\left(\boldsymbol{Q}_2^i-i\eta^{-1}\boldsymbol{Q}_1^i\right)z_0+\left(\boldsymbol{Q}_2^r-i\eta^{-1}\boldsymbol{Q}_1^r\right)z_0=0 \tag{12}$$

we get the same R_{ij}, only cross coefficients R_{12} and R_{21} change their signs. The coefficients R_{ij} are interrelated; of interest are the following relations:

$$R_{12}=-i\eta(1+R_{22}),\quad R_{21}=i\eta^{-1}(1+R_{11}), \tag{13}$$

and the invariant

$$R_{11}R_{22}-R_{12}R_{21}=\det\left\|R_{ij}\right\|=-1.$$

It follows from Eqs. (10) and (11) that $R_{22}=-R_{11}$, and taking in to account Eq. (13), all reflection coefficients can be expressed using R_{11}.

Let us simultaneously consider of two interrelated realizations of reflection of the plane Beltrami waves having a common angle of incidence θ. In the first case, the angles of incidence $\varphi=\theta$ and $\psi=\arcsin\left(\frac{\gamma_1}{\gamma_2}\sin\theta\right)$ have the corresponding reflection coefficient

$$R_{11}^{(1)}=r_1=\frac{\cos\theta-\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\sin^2\theta\right]^{1/2}}{\cos\theta+\left[1-\left(\frac{\gamma_1}{\gamma_2}\right)^2\sin^2\theta\right]^{1/2}}, \tag{14}$$

whereas in the second case, the angles of incidence $\psi=\theta$ and $\varphi=\arcsin\left(\frac{\gamma_2}{\gamma_1}\sin\theta\right)$ have the following reflection coefficient:

$$R_{11}^{(2)}=r_2=\frac{\cos\theta-\left[1-\left(\frac{\gamma_2}{\gamma_1}\right)^2\sin^2\theta\right]^{1/2}}{\cos\theta+\left[1-\left(\frac{\gamma_2}{\gamma_1}\right)^2\sin^2\theta\right]^{1/2}}. \tag{15}$$

Provided the angle θ is in no excess of the critical value, it follows from Eqs. (14) and (15) that $r_1 > 0$ and $r_2 < 0$. Eliminating the radicals, we get a relation invariant with respect to the material parameters of the chiral medium as

$$\sin^2\theta = \frac{4}{\left[(-r_2)^{-1/2} + (-r_2)^{1/2}\right]^2 - \left[(r_1)^{-1/2} + (r_1)^{1/2}\right]^2}, \tag{16}$$

and a relation invariant with respect to the common angle of incidence

$$\frac{\gamma_1}{\gamma_2} = \frac{(-r_2)^{-1/2} - (-r_2)^{1/2}}{(r_1)^{-1/2} + (r_1)^{1/2}},$$

which allows us to write the dependence of the chiral-order parameter on the reflection coefficients in explicit way, and we get

$$k\beta = \frac{(-r_2)^{-1/2} - (-r_2)^{1/2} - (r_1)^{-1/2} - (r_1)^{1/2}}{(-r_2)^{-1/2} - (-r_2)^{1/2} + (r_1)^{-1/2} + (r_1)^{1/2}}. \tag{17}$$

Let us consider now a combination of the reflection coefficients for the case where the plane Beltrami waves fall on the screen at the same angle, but one at a time. For the incident field $\boldsymbol{Q}_1^i$ (3), we must assume that $\varphi = \theta$ and $\psi = \arcsin\left(\frac{\gamma_1}{\gamma_2}\sin\theta\right)$. The reflected waves (5) and (6) do not involve R_{12} and R_{22}, and the coefficient R_{11} is calculated using Eq. (14) so that $R_{11} = r_1$ and $R_{21} = i\eta^{-1}(1 + r_1)$. For the incident field $\boldsymbol{Q}_2^i$ (4), we must assume that $\psi = \theta$ and $\varphi = \arcsin\left(\frac{\gamma_2}{\gamma_1}\sin\theta\right)$. The reflected waves do not involve R_{11} and R_{21}, and the coefficient $R_{22} = r_2$ is calculated by means of Eqs. (11) and (15), $R_{12} = -i\eta(1 + r_2)$. Equations (16) and (17) are preserved, and can be shaped to a more symmetric form

$$\left[(-R_{22})^{-1/2} + (-R_{22})^{1/2} + R_{11}^{-1/2} + R_{11}^{1/2}\right]\left[(-R_{22})^{-1/2} + (-R_{22})^{1/2} - R_{11}^{-1/2} - R_{11}^{1/2}\right] = \frac{4}{\sin^2\theta}, \tag{18}$$

$$\frac{(-R_{22})^{-1/2} - (-R_{22})^{1/2} - R_{11}^{-1/2} - R_{11}^{1/2}}{(-R_{22})^{-1/2} - (-R_{22})^{1/2} + R_{11}^{-1/2} + R_{11}^{1/2}} = k\beta. \tag{19}$$

Equations (18) – (19) give an alternative way of calculating the coefficients of reflection from an ideally conducting screen limiting an isotropic chiral medium half-space.

In the case where the angle θ exceeds the critical value, the coefficient R_{11} takes the following complex values:

$$R_{11} = \frac{\cos\theta - i\left[\left(\frac{\gamma_1}{\gamma_2}\right)^2\sin^2\theta - 1\right]^{-1/2}}{\cos\theta + i\left[\left(\frac{\gamma_1}{\gamma_2}\right)^2\sin^2\theta - 1\right]^{-1/2}} = e^{i\chi},$$

where χ is the real phase of the reflection coefficient. To this end, we take a positive branch of the square root from Eq. (14) so as to provide an exponential decrease in the filed $\boldsymbol{Q}_2^r$ along the coordinate z. Since $|R_{11}| = 1$, we observe a total internal reflection of the Beltrami wave $\boldsymbol{Q}_1^i$ at $\theta > \theta_c$. Equations (18) and (19) are valid provided that $R_{11}^{-1/2} + R_{11}^{1/2} = 2\,\mathrm{Re}\left(R_{11}^{1/2}\right)$ is a real value.

A lateral wave is closely related to the phenomenon of total internal reflection both in this case and in the case of interface of achiral dielectics [6]. Lateral waves in a chiral medium are excited in the presence of a geometric inhomogeneity (e.g., boundary kink). In terms of quasioptical diffraction theory, a lateral wave consists of a ray of the Beltrami field $\boldsymbol{Q}_2$ grazing along the screen and a geometrooptical ray of the Beltrami field $\boldsymbol{Q}_1$ arriving at the point of observation at a critical angle of the total internal reflection [7].

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