Russian Journal of Applied Chemistry, Vol. 78, No. 8, 2005, pp. 1294–1300. Translated from Zhurnal Prikladnoi Khimii, Vol. 78, No. 8, 2005, pp. 1317–1323.

PROCESSES AND EQUIPMENT OF CHEMICAL INDUSTRY

Effect of Rheological Properties of the Dispersion Medium on Separation of Suspensions in Hydrocyclones with Various Working Space Configurations

V. O. Yablonskii

Volgograd State Technical University, Volgograd, Russia

Received March 16, 2005

Abstract—The model of separation of suspensions with a non-Newtonian dispersion medium in a cylindroconical hydrocyclone, which takes into account the effect of the Coriolis force on solid particles, was constructed and applied to analysis of the rheological properties of the dispersion medium on separation of suspensions in hydrocyclones with various working space configurations.

Hydrocyclones are one of multi-purpose types of apparatus for separation of heterogeneous systems in chemical, microbiological, food, and other industries. Use of hydrocyclones yields a pronounced economic effect through intensification of technological processes. The economic feasibility and efficiency of separation processes are determined by the degree of thickening of the dispersed phase and by its loss. In this context, optimizing the design parameters of hydrocyclones and determining the optimal modes of their operation are important problems, which can lead, when solved, to intensification and accelerated development of numerous branches of industry.

Until now, there has been no rigorous theory of motion of two-phase systems in a swirling vortex flow. The overwhelming majority of modern studies [1–6] are based on the equation of a unidimensional motion of a small spherical particle suspended in a viscous incompressible turbulent flow.

In [1], methods for calculation of predictable separation parameters and other important performance characteristics of cylindroconical hydrocyclones were developed in the framework of a unified stochastic approach. As noted by the authors, an exact analytical description of particle motion in a centrifugal field, based on Navier–Stokes and flow continuity equations, requires that a number of not quite correct assumptions should be made, because of the complexity of the hydrodynamic situation in hydrocyclones, which impairs the adequacy of the suggested analytical descriptions of the real hydrodynamic pattern.

In [2, 3], fundamental aspects of a calculation of turbulent particle transport in a hydrocyclone, based on the continuity equation for the solid phase flow, were considered. Such an approach makes it possible to calculate both the amount of the solid phase recovered in a hydrocyclone and its parameters (concentration, granulometric composition) at any point of the apparatus. In [4], a numerical study of the flow structure and separation processes in a hydrocyclone was carried out using the Navier–Stokes equations. The momentum transfer by the dispersed phase was described in terms of the theory of multispeed continuum. By now, the equation of radial motion of particles in hydrocyclones of various designs has been solved with account of the action of inertia and Coriolis forces on a particle [5–7]. This made it possible to calculate the size distribution of particles of the dispersed phase and to determine their content in target separation products.

The difficulty encountered in solving equations that characterize sedimentation of solid particles in a hydrocyclone consists in that, in order to satisfy the requirement that the system of equations should be closed, it is necessary to use those solutions to equations of hydrocyclone hydrodynamics, which rigorously relate the circumferential, radial, and axial components of the fluid velocity over the entire working space of the apparatus. In addition, suspensions separated in chemical industry are, in most cases, non-Newtonian media whose effective viscosity decreases as the intensity of deformation rates becomes higher, which affects the hydrodynamics of apparatus.

1070-4272/05/7808-1294 © 2005 Pleiades Publishing, Inc.

It was concluded in [8, 9] that the rheological equation of state of a non-Newtonian liquid, which is commonly written in the form of the Ostwald–de Weyl power law, is applicable to multiphase heterogeneous systems.

A numerical study of the flow of a film of a non-Newtonian fluid in a cylindroconical hydrocyclone, based on solution of complete equations of rheodynamics, was carried out in [10]. Description of separation processes in a hydrocyclone with a freely forming surface of a non-Newtonian fluid phase by solving complete equations of rheodynamics has not been provided until now, but it is of considerable theoretical and practical interest.

The hydrocyclone comprises a cylindrical chamber, into which a suspension to be separated is fed tangentially through the inlet pipe mounted in its upper part, and a conical section. The height of the cylindrical chamber of the hydrocyclone is equal to its diameter. The suspension delivered into the hydrocyclone flows down, swirling, along its walls, having a radial (V_r), circumferential (V_φ), and axial (V_z) velocity components. Solid particles are thrown by the centrifugal force to apparatus walls and are then removed via an unloading device at the bottom discharge. The clarified suspension is also removed via the bottom discharge.

The efficiency of suspension separation in a hydrocyclone is determined by hydrodynamic parameters of a preliminarily swirled suspension film flowing down along the case walls under the action of the gravity force and pressure created by centrifugal forces.

The problem was mathematically formulated under the following assumptions: particles of the substance being recovered are uniformly distributed in the inlet pipe of the hydrocyclone; their concentration at the hydrocyclone inlet is c_0; for a film-type flow of a suspension with a non-Newtonian dispersion medium with a high effective viscosity, the flow mode is laminar and the sedimentation mode of solid particles is laminar [11].

The mathematical modeling of the concentration field in a flow of a suspension with a non-Newtonian dispersion medium in a cylindroconical hydrocyclone is performed by using the differential equation of convective diffusion in cylindrical coordinates, which can be written at a molecular diffusion coefficient equal to zero as follows [12]:

$$\operatorname{div}(\mathbf{V}_\mathrm{h} c) = 0, \tag{1}$$

where $\mathbf{V}_\mathrm{h}$ is the vector of velocity of solid particles, and c is the concentration of these particles (kg m^{-3}).

Equation (1) can be brought to the form

$$\mathbf{V}_\mathrm{h}\operatorname{grad}c + c\operatorname{div}\mathbf{V}_\mathrm{h} = 0. \tag{2}$$

As the velocity of solid particles differs from the velocity of a continuous flow of the dispersion medium, $\operatorname{div}\mathbf{V}_\mathrm{h} = \dfrac{1}{r}\dfrac{\partial[r(V_{r\mathrm{h}} - V_{r\mathrm{l}})]}{\partial r}$ and Eq. (2) takes the following form:

$$V_{r\mathrm{h}}\frac{\partial c}{\partial r} + V_{z\mathrm{h}}\frac{\partial c}{\partial z} = -c\left\{\frac{1}{r}\frac{\partial[r(V_{r\mathrm{h}} - V_{z\mathrm{h}})]}{\partial r}\right\}, \tag{3}$$

where $V_{r\mathrm{h}}(r, z)$ and $V_{z\mathrm{h}}(r, z)$ are the radial and axial velocity components of solid particles (m s^{-1}); $V_{r\mathrm{l}}(r, z)$, radial velocity component of the dispersion medium (m s^{-1}); r, z, radial and axial coordinates (m); h, solid particle; and l, dispersion medium.

This equation is solved together with the equation of motion of a solid particle.

It is assumed that the solid phase of a suspension being separated is monodisperse, which makes it possible to achieve a required degree of thickening of solid particles in modeling the separation process for the finest fraction. The inertia force acting on a solid particles in the axial direction, gravity force, and Archimedes buoyancy force are neglected by assuming that the axial component of the velocity of a solid particle, $V_{z\mathrm{h}}(r, z)$ is equal to the axial component of the velocity of the dispersion medium, $V_{z\mathrm{l}}(r, z)$. This assumption is well justified in practice because separation in hydrocyclones with a non-Newtonian dispersion medium having high effective viscosity occurs at larger Froude numbers Fr (separation factors).

Taking into account that suspensions with a non-Newtonian dispersion medium are separated in hydrocyclones under the conditions in which the Reynolds number, which characterizes the sedimentation mode of a solid particle, is not large, the Magnus force can be neglected. If a centrifugal Archimedes force, resistance force, and a Coriolis force act upon a solid particle, the equation of motion of a solid particle can be written, in projections onto the r and φ axes, as

$$\left.\begin{aligned}
V_{r\mathrm{h}}\frac{\mathrm{d}V_{r\mathrm{h}}}{\mathrm{d}r} + V_{z\mathrm{h}}\frac{\mathrm{d}V_{r\mathrm{h}}}{\mathrm{d}z} &= \frac{V_{\varphi\mathrm{h}}^2}{r}\left(1 - \frac{\rho_\mathrm{l}}{\rho_\mathrm{h}}\right) - \frac{3}{4}\frac{K_{fr}\,\rho_\mathrm{l}(V_{r\mathrm{h}} - V_{r\mathrm{l}})|V_{r\mathrm{h}} - V_{r\mathrm{l}}|}{\rho_\mathrm{h} d_\mathrm{h}\Phi(B)},\\
V_{r\mathrm{h}}\frac{\mathrm{d}V_{\varphi\mathrm{h}}}{\mathrm{d}r} + V_{z\mathrm{h}}\frac{\mathrm{d}V_{\varphi\mathrm{h}}}{\mathrm{d}z} &= \frac{V_{\varphi\mathrm{h}}V_{r\mathrm{h}}}{r} - \frac{3}{4}\frac{K_{f\varphi}\,\rho_\mathrm{l}(V_{\varphi\mathrm{h}} - V_{\varphi\mathrm{l}})|V_{\varphi\mathrm{h}} - V_{\varphi\mathrm{l}}|}{\rho_\mathrm{h} d_\mathrm{h}\Phi(B)},
\end{aligned}\right\} \tag{4}$$

where K_{fr}, $K_{f\varphi}$ are the coefficients of resistance to particle motion in the radial and circumferential directions; $V_{\varphi h}(r, z)$, circumferential velocity component of solid particles (m s^{-1}); d_h, diameter of solid particles (m); ρ_h, ρ_l, densities of solid particles and dispersion medium, respectively (kg m^{-3}); and φ, circumferential coordinate (deg).

The resistance coefficients for radial motion of a particle in a non-Newtonian fluid, K_{fr}, can be calculated using the empirical formula [13]

$$K_{fr} = \frac{24f_1(n)}{\mathrm{Re}_{nr}} + \frac{f_2(n)}{(\mathrm{Re}_{nr})^{f_3(n)}}, \tag{5}$$

where

$$f_1(n) = 3^{1.5(n-1)}\frac{2 + 29n - 22n^2}{n(n+2)(2n+1)},$$

$$f_2(n) = 10.5n - 3.5, \quad f_3(n) = 0.32n + 0.13.$$

$\mathrm{Re}_{nr} = [\rho_l(d_h)^n(V_{rh} - V_{rl})^{2-n}]/k$, local Reynolds number characterizing the sedimentation mode of solid particles; k, consistency index (Pa s^{n-1}); and n, nonlinearly index of the flow curve.

The resistance coefficient for circumferential motion of a particle, $K_{f\varphi}$, is calculated using similar dependences, but with the corresponding difference of particle and fluid velocities in the circumferential direction taken as a characteristic velocity difference.

The coefficient $\Phi(B)$, which accounts for the hindered conditions of particle sedimentation, is calculated using the formula suggested by Sokolov [14]:

$$\Phi(B) = 10^{-1.82(1-B)}, \tag{6}$$

where $B = 1 - c_0/\rho_h$, is a part of a unit volume of the suspension that is occupied by the dispersion medium, and c_0 is the concentration of solid particles in the inlet pipe of the hydrocyclone (kg m^{-3}).

The initial fraction of the suspension volume occupied by solid particles, equal to $1 - B$, was taken to be 0.05.

In accordance with the method of characteristics [15], the system of differential equations in partial derivatives, (3) and (4), is reduced to an equivalent system of ordinary differential equations, which can be represented in dimensionless form as

$$\frac{dR}{dZ} = \frac{G_h}{H_h},$$

$$\frac{dC}{dZ} = -\frac{C}{H_h}\left[\frac{d(G_h - G_l)}{dR} + \frac{G_h - G_l}{R}\right],$$

$$\frac{dG_h}{dZ} = -\frac{\Theta_h^2}{H_h R}(1 - P) - \frac{3}{4}\frac{K_{fr}P(G_h - G_l)|G_h - G_l|}{D_h\Phi(B)},$$

$$\frac{d\Theta_h}{dZ} = \frac{\Theta_h G_h}{H_h R} - \frac{3}{4}\frac{K_{fr}P(\Theta_h - \Theta_l)|\Theta_h - \Theta_l|}{D_h\Phi(B)}, \tag{7}$$

where $G_h(R, Z) = [V_{rh}(r, z)]/U_0$, $H_h(R, Z) = [V_{zh}(r, z)]/U_0$, $\Theta_h(R, Z) = [V_{\varphi h}(r, z)]/U_0$, are the dimensionless radial, axial, and circumferential velocity components of particles of the solid phase; $\Theta_l(R, Z) = [V_{\varphi l}(r, z)]/U_0$, $G_l(R, Z) = [V_{rl}(r, z)]/U_0$, dimensionless circumferential and radial velocity components of the dispersion medium; $C = c/c_0$, dimensionless concentration of solid particles; $P = \rho_l/\rho_h$, density parameter; $D_h = d_h/r_k$, dimensionless diameter of solid particles; $R = r/r_k$, $Z = z/r_k$, dimensionless radial and axial coordinates; U_0 average suspension velocity in the inlet pipe of the hydrocyclone (m s^{-1}); and r_k, radius of the cylindrical chamber of the hydrocyclone (m).

The boundary conditions for the system of ordinary differential equations (7) has the form

$$Z = 0, \quad 1 - F \le R \le 1, \quad C_0 = 1, \tag{8}$$

where C_0 is the dimensionless concentration of solid particles in the inlet pipe of the hydrocyclone; $F = f/r_k$, dimensionless width of the inlet pipe of the hydrocyclone; and f, width of the initial pipe of the hydrocyclone (m).

The approximating dependences for the radial $[H_l(r, z)]$ and axial $[\Theta_l(r, z)]$ velocity components of the dispersion medium can be represented as

$$\left.\begin{aligned} H_l(R, Z) &= H_{ls}(Z)\left[1 - \left(1 - \frac{1-R}{\Delta(Z)}\right)^{M(Z)}\right], \\ \Theta_l(R, Z) &= \Theta_{ls}(Z)\left[1 - \left(1 - \frac{1-R}{\Delta(Z)}\right)^{M(Z)}\right], \end{aligned}\right\} \tag{9}$$

where $H_l(R, Z) = V_{zl}(r, z)/U_0$ is the dimensionless axial velocity component of the dispersion medium; $H_{ls}(Z)$, $\Theta_{ls}(Z)$, dimensionless axial and circumferential velocity components of the dispersion medium at the film surface; $\Delta(Z) = \delta(z)/r_k$, dimensionless thickness of the suspension film; $L(Z)$, $M(Z)$, completeness coefficients of the radial distributions of the circumferential and axial velocity components; $\delta(z)$ suspension film thickness (m).

The values of $H_{ls}(Z)$, $\Theta_{ls}(Z)$, $M(Z)$, $L(Z)$, and $\Delta(Z)$ were approximated with fourth-order polynomials.

The first equation of the system (7) determines the motion trajectory of a solid particle, which depends on the place of particle entry into the apparatus, i.e., sets the direction of the characteristic. The second equation yields the value of the function, i.e., the dimensionless concentration of solid particles, which indicates the degree of concentration of solid particles, on the characteristic. The remaining equations make it possible to find the radial (G_h) and circumferential (Θ_h) velocity components of the particle.

The system of differential equations (7) was solved by the fourth-order Runge–Kutta method with a fixed step. The rate of variation of the axial coordinate for a trajectory approaching the wall is many times that of the radial coordinate in the case of a small Re_{nr} number. Therefore, the given system of equations is a stiff set [16] and its solution requires a very large number of steps. the integration interval from the lower boundary of the inlet pipe of the hydrocyclone to the outlet orifice of the conical part of the case was divided into $M = 3 \times 10^5$ steps along the Z axis and the system of differential equations (7) was integrated using a software composed in Compaq Visual Fortran language. The error in solving the system, evaluated by doubling the number of steps, did not exceed 1×10^{-5}.

The separation of suspensions with a non-Newtonian dispersion medium in a hydrocyclone was simulated by modeling the velocity and pressure component fields in a flow of a non-Newtonian fluid in a cylindroconical hydrocyclone [10] and then approximating the results obtained. This was done by solving the system of differential equations (7) at a boundary condition (8) for widely varied defining similarity numbers and rheological constants of the dispersion medium. The flow of a non-Newtonian fluid in the hydrocyclone is characterized by a centrifugal Froude number Fr (separation factor), modified Reynolds number Re_n, dimensionless flow rate parameter Q, and nonlinearity index n of the flow curve. Sedimentation of solid particles in the field of centrifugal forces in a non-Newtonian dispersion medium is characterized by a local Reynolds number Re_{nr}, which varies along the trajectory of a particle.

The defining similarity numbers [10] were calculated as follows:

$$\mathrm{Fr} = \frac{U_0^2}{g r_k}, \quad \mathrm{Re}_n = \frac{\rho_l U_0^{2-n} r_k^n}{k}, \quad Q = \frac{q}{\pi r_k^2 U_0} = \frac{ef}{\pi r_k^2}, \qquad (10)$$

where q is the suspension flow rate ($\mathrm{m}^3\ \mathrm{s}^{-1}$); e, height of the inlet pipe of the hydrocyclone (m); and g, acceleration of gravity ($\mathrm{m\ s}^{-2}$).

The width F of the inlet pipe of the hydrocyclone was divided in the radial direction into 20 equal intervals and, for each point at boundaries between the intervals, a calculation of the corresponding trajectory and concentrations of solid particles along this trajectory was commenced. This was done so because the trajectory of a particle at prescribed hydrodynamic parameters of a suspension film is uniquely determined by the place of its entry into the apparatus. The arrival of a solid particle to the wall was determined from the condition $1 - R \le D_h$ with account of the accumulation of a deposit layer on the wall. As a solid particle arrived to the wall, the calculation for the given trajectory was terminated and a calculation of the next trajectory was commenced. The results obtained in the numerical simulation are shown in Figs. 1, 2.

Fig. 1a (*I*) shows the trajectories of solid particles, and Fig. 1b (*I*), the distributions of the concentrations of solid particles along the respective trajectories at $\mathrm{Fr} = 85$, $\mathrm{Re}_n = 4 \times 10^3$, $Q = 3.98 \times 10^{-2}$, $n = 0.6$, and $\alpha = 5°$. Analysis of Fig. 1a (*I*) demonstrates that the most intense sedimentation of solid particles in a centrifugal field occurs in the cylindrical chamber of the hydrocyclone, in which solid particles are in the zone of high values of the circumferential velocity component of the dispersion medium, Θ_l, near the surface of the suspension film. In the bottom zone of the conical part of the case, the film becomes considerably thicker and the radial velocity component of the dispersion medium, G_l, directed toward the apparatus axis, grows, with the result that particles move away from the case wall.

The concentrations of solid particles grow along the trajectories because of the thickening of the solid phase, and this increase is the most pronounced in regions close to the boundary between the thickened and clarified suspension.

Fig. 1 (*II*, *III*) shows the trajectories and concentration distributions for Fr, Re_n, Q, and n corresponding to Fig. 1 (*I*) at $\alpha = 10$ and 15°, respectively. Analysis of Fig. 1 (*I–III*) demonstrates that the maximum dimensionless concentration of solid particles, characterizing the degree of its thickening, is observed for $n = 0.6$ at $\alpha = 5°$. This is due to the low rate of decay of the circumferential velocity component of the dispersion medium, Θ_l, in the direction of the Z axis and to the high value of the completeness coefficient L of the radial distribution of Θ_l at a more pronounced anomaly of the non-Newtonian

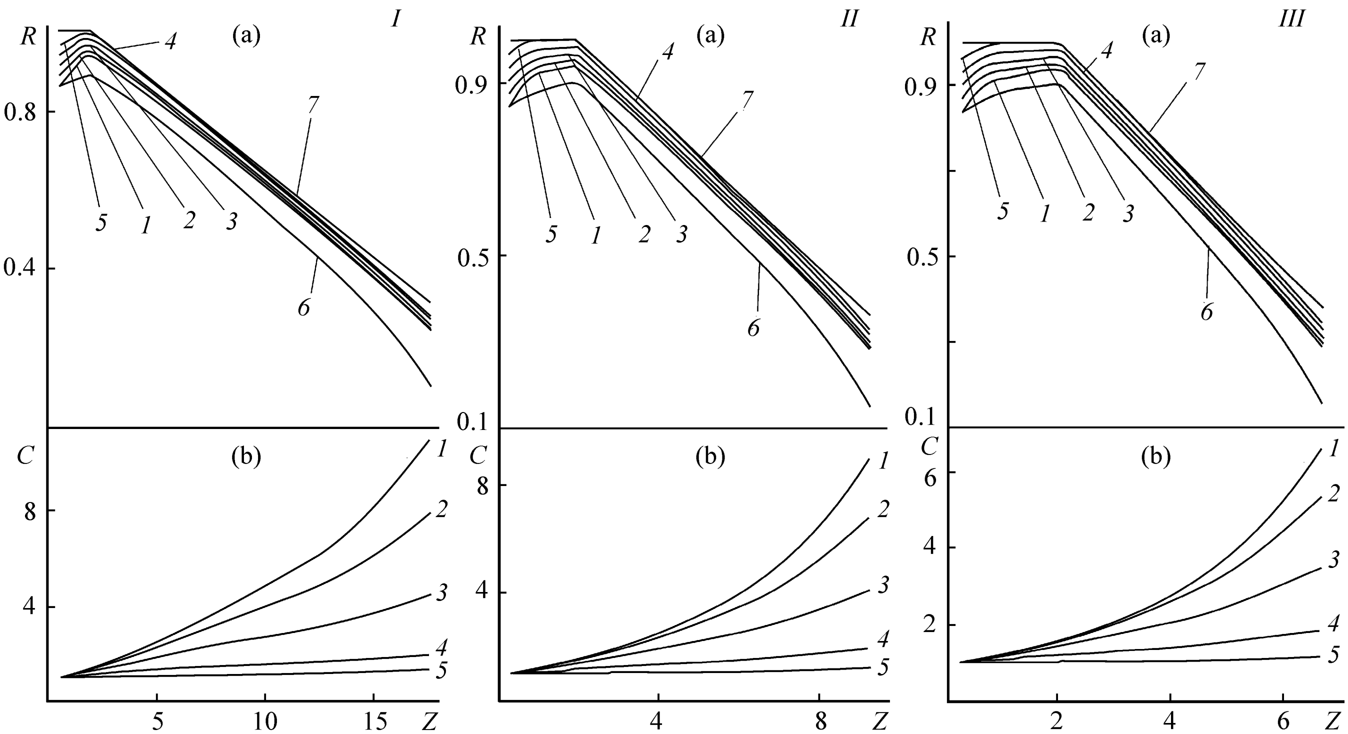

Fig. 1. (a) Trajectories of solid particles and (b) distribution of the concentrations C along the trajectories at Fr = 85, $\mathrm{Re}_n = 4 \times 10^3$, $Q = 3.98 \times 10^{-2}$, $n = 0.6$, $D_\mathrm{h} = 1.75 \times 10^{-3}$, and $P = 0.5$. (R) Dimensionless radial coordinate and (Z) dimensionless axial coordinate; the same for Fig. 2. α (deg): (*I*) 5, (*II*) 10, and (*III*) 15; the same for Fig. 2. (*1*) Trajectory corresponding to the boundary between the zones of clarified and thickened suspension, (*2–5*) trajectories of solid particles, (*6*) boundary of the suspension film, (*7*) wall of the hydrocyclone case; the same for Fig. 2.

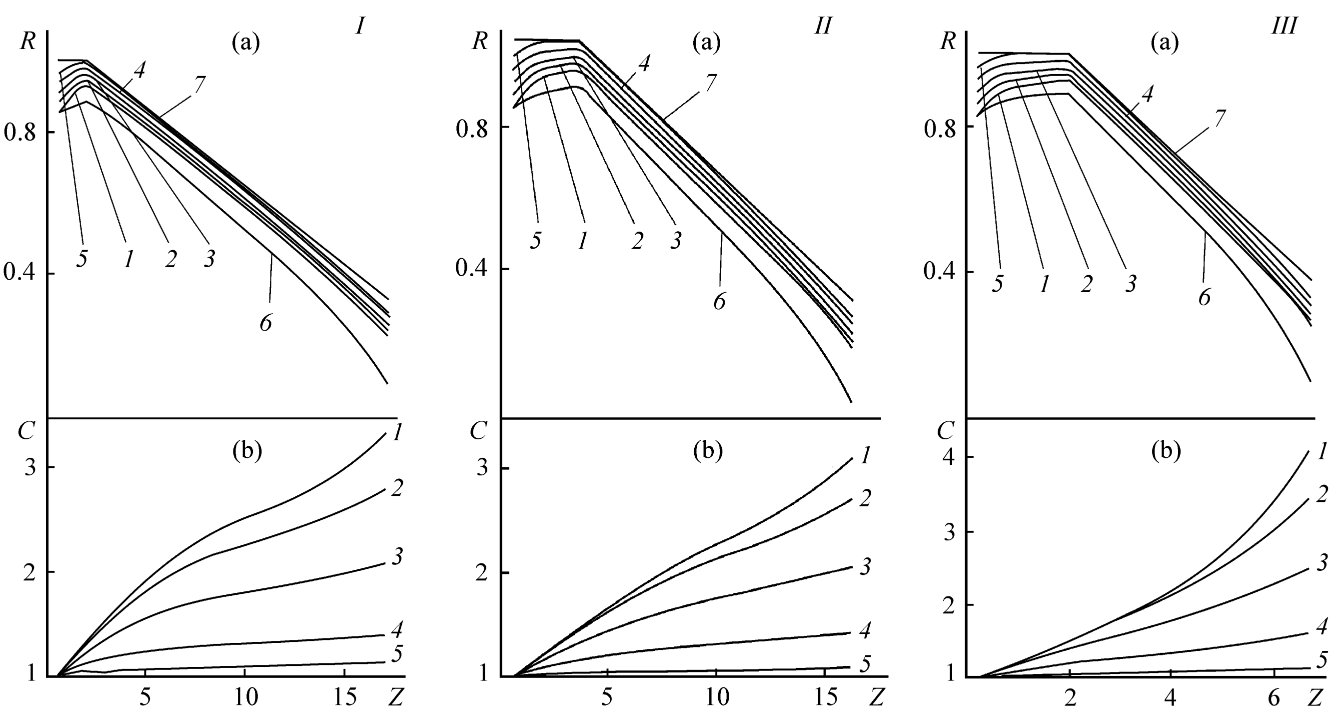

Fig. 2. (a) Trajectories of solid particles and (b) distribution of the concentrations C along the trajectories at Fr = 85, $\mathrm{Re}_n = 4 \times 10^3$, $Q = 3.98 \times 10^{-2}$, $n = 0.8$, $D_\mathrm{h} = 1.75 \times 10^{-3}$, and $P = 0.5$.

properties of the dispersion medium (n = 0.6). This yields a high degree of thickening of solid particles, because the length of the conical part of the hydrocyclone case is larger in the direction of the Z axis at α = 5°, which makes longer the residence time of the suspension in the separation zone.

Fig. 2 (*I–III*) shows the trajectories and concentration distributions for Fr, Re_n, and Q corresponding to Fig. 1 (*I–III*) at n = 0.8 and α = 5°, 10°, and 15°, respectively. Comparison of the concentration distributions at n = 0.8 [Fig. 2 (*I–III*)] and n = 0.6 [Fig. 1 (*I–III*)] shows that the degree of thickening of solid particles is lower at n = 0.8, which can be attributed to a faster decay of the circumferential velocity component Θ_1 in the direction of the Z axis and to a lower completeness coefficient L of the radial distribution of Θ_1 at a less pronounced anomaly of the non-Newtonian properties of the dispersion medium. Comparison of the data in Fig. 2 (*I–III*) suggests that the highest degree of thickening of solid particles at n = 0.8 is observed at α = 15°, because this angle of conicity provides the minimum decay of the circumferential velocity component Θ_1 in the direction of the Z axis. At α = 5°, the degree of thickening of solid particles exceeds that at α = 10°, because the strong decay of the Θ_1 component along the Z axis at 5° is compensated for by the large length of the conical part of the hydrocyclone case in the direction of the apparatus axis.

As follows from the analysis of the calculated data for Fr, Re_n, and Q corresponding to Fig. 1 (*I–III*), even lower degrees of thickening of solid particles are obtained at n = 1.0 (Newtonian fluid), compared with the case of n = 0.8, because the rate of decay of the circumferential velocity component Θ_1 in the direction of the Z axis increases as the anomaly of the non-Newtonian properties of the dispersion medium becomes less pronounced. The lowest degree of thickening of solid particles is observed at α = 5° because of the strong decay of the Θ_1 component along the Z axis. At α = 10 and 15°, the degrees of thickening of solid particles are approximately the same because of the opposite kinds of influence exerted on the degree of thickening by such factors as the rate of decay of the circumferential velocity component Θ_1 of the dispersion medium along the Z axis and the length of the conical part of the hydrocyclone case in the direction of the apparatus axis.

CONCLUSIONS

(1) The degree of thickening of solid particles of the suspension in a cylindroconical hydrocyclone grows as the anomaly of the non-Newtonian properties of the dispersion medium becomes more pronounced because of the decrease in the rate of decay of the circumferential velocity component of the dispersion medium along the hydrocyclone axis and increase in the completeness coefficient of its radial distribution.

(2) The influence exerted by the angle of conicity of the hydrocyclone case on the degree of thickening of solid particles depends on the rheological properties of the dispersion medium, which determine the rate of decay of the circumferential velocity component of the dispersion medium along the hydrocyclone axis, because the rate of decay decreases and, simultaneously, so does the length of the conical part of the case along the apparatus axis, which makes shorter the residence time of the suspension in the separation zone.

(3) The mathematical model of separation of suspensions with a non-Newtonian dispersion medium in a cylindroconical hydrocyclone, constructed in the study, makes it possible to calculate the degree of thickening of solid particles as a function of the defining similarity numbers and rheological properties of the dispersion medium and can be used as a basis for developing a procedure for engineering calculations of hydrocyclones for separation of rheologically complex media.

REFERENCES

1. Ternovskii, I.G. and Kutepov, A.M., *Gidrotsiklonirovanie* (Hydrocyclonation), Moscow: Nauka, 1994.
2. Kudryavtsev, I.A. and Mikhotov, V.V., *Teor. Osn. Khim. Tekhnol.*, 1989, vol. 23, no. 1, pp. 120–121.
3. Kosoi, G.M. and Sapeshko, V.V., *Teor. Osn. Khim. Tekhnol.*, 1983, vol. 17, no. 5, pp. 637–641.
4. Dik, I.G., Matvienko, O.V., and Neesse, T., *Teor. Osn. Khim. Tekhnol.*, 2000, vol. 34, no. 5, pp. 478–488.
5. Baranov, D.A., Kutepov, A.M., and Lagutkin, M.G., *Teor. Osn. Khim. Tekhnol.*, 1996, vol. 30, no. 2, pp. 117–122.
6. Lagutkin, M.G., Baranov, D.A., Bulychev, S.Yu., and Baranova, E.Yu., *Khim. Neftegaz. Mashinostr.*, 2004, no. 5, pp. 3–6.
7. Lagutkin, M.G. and Baranov, D.A., *Teor. Osn. Khim. Tekhnol.*, 2004, vol. 38, no. 1, pp. 9–13.
8. Romankov, P.G., *Teor. Osn. Khim. Tekhnol.*, 1972, vol. 6, no. 6, pp. 855–871.

9. Vainshtein, I.A., *Inzh.–Fiz. Zh.*, 1983, vol. 45, no. 4, pp. 602–608.
10. Yablonskii, V.O., *Zh. Prikl. Khim.*, 2004, vol. 77, no. 6, pp. 971–977.
11. Dyakowski, T., Hornung, G., and Williams, R.A., *Chem. Eng. Res. Des. A.*, 1994, vol. 72, no. 4, pp. 513–520.
12. Bird, R.B., Stewart, V.E., and Lightfoot, E.N., *Transfer Phenomena*, New York: John Wiley and Sons, 1965. Translated under the title *Yavleniya perenosa*, Zhavoronkov, N.M. and Malyusov, V.A., Eds., Moscow: Khimiya, 1974.
13. Acharya, A., Mashelkar, R.A., and Ulbrecht, J., *Rheol. Acta*, 1976, vol. 15, no. 9, pp. 454–463.
14. Sokolov, V.I., *Tsentrifugirovanie* (Centrifugation), Moscow: Khimiya, 1976.
15. Matveev, N.M., *Differentsial'nye uravneniya* (Differential Equations), Minsk: Vysheisha Shkola, 1976.
16. Cahaner, D., Mowler, C., and Nash, S., *Chislennye metody i programmnoe obespechenie* (Numerical Methods and Software), Moscow: Mir, 2001.