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Admissible utility functions for health, longevity, and wealth: integrating monetary and life-year measures

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Abstract The value of reducing health and mortality risks is often measured using value per statistical life (VSL) or one of several life-year measures (e.g., life years, quality-adjusted life years, disability-adjusted life years). I derive the utility function that is admissible when preferences for health and longevity, conditional on wealth, are consistent with any life-year measure (LYM) and examine the implications for marginal willingness to pay (WTP) for increases in health, longevity, and current-period survival probability. I conclude that marginal WTP for any LYM is decreasing and that VSL is increasing in the LYM. These results imply that cost-effectiveness analysis using a fixed monetary value per LYM is not consistent with economic welfare theory and that the benefit of a health improvement cannot be calculated by multiplying the change in a LYM by a constant.

Keywords Value per statistical life · Value per statistical life year · Quality-adjusted life year · Disability-adjusted life year · Healthy years equivalent · Willingness to pay · Risk aversion

JEL Classification D61 · D81 · I10

The value of reducing risk to health or life is conventionally measured using any of several approaches: the value per statistical life (VSL), value per statistical life year (VSLY), quality-adjusted life year (QALY), disability-adjusted life year (DALY), or healthy years equivalent (HYE). VSL and VSLY are monetary measures of the value of mortality risk that can be readily compared with other benefits and costs of an intervention but do not account for changes in non-fatal health risk. QALY, DALY, and HYE are non-monetary measures but incorporate both health and mortality. QALY and DALY are often used in cost-effectiveness analysis where an intervention is said to

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be desirable if the cost per expected QALY or DALY gained is smaller than some threshold monetary value determined outside the analysis; HYE is a generalized form that is rarely used in practice.

This paper investigates the theoretical relationships among these measures, focusing on the monetary value of changes in expected health and longevity. Understanding this relationship is useful for several reasons. First, if VSLY is independent of health and life expectancy, then mortality risks should be valued in proportion to the change in life expectancy rather than the change in current mortality risk, as with VSL. Second, if the monetary value per QALY or DALY is constant, then cost-effectiveness analysis using these measures and benefit-cost analysis using monetary values are essentially equivalent (Johannesson 1995; Garber and Phelps 1997; Dolan and Edlin 2002). Third, because few non-fatal health risks have empirical estimates of monetary value but many have QALY or DALY estimates, a transfer function relating the monetary value of a risk change to the expected change in QALY or DALY would be valuable for estimating monetary benefits of health improvements. The simplest function would be multiplication by a constant monetary value per QALY or DALY (French and Mauskopf 1992; Tolley et al. 1994; Cutler and Richardson 1997); the U.S. Food and Drug Administration and the U.S. Department of Transportation both follow this approach (Adler 2006; Robinson 2007). Alternatively, a non-linear transfer function might be developed (Johnson et al. 1997; Van Houtven et al. 2006; Haninger and Hammitt 2011).

The VSLY, QALY, and DALY approaches have in common that all three value increases in longevity by weighting years lived by some factor. I will call these life-year measures (LYMs). The weighting factor is constant (for VSLY), dependent on health (for QALY and DALY), and can be dependent on age (for DALY). In addition, all three LYMs typically discount future life years (i.e., weight life years for futurity). HYE is also a LYM; it is defined as the number of years of healthy life that are judged indifferent to a specified time path through various health states but does not require a fixed weight for each health state.

If an individual's preferences for his own health, longevity, and wealth are consistent with VSL or a LYM, his preferences must satisfy conditions imposed by the model underlying that approach. The model underlying VSL is quite general; those underlying the LYMs are more restrictive. In this paper, I identify these conditions and present a utility function for health, longevity, and wealth that is consistent with both VSL and any LYM. This utility function characterizes how VSL and the marginal value per LY, QALY, DALY, or HYE vary with health, life expectancy, and wealth. I conclude that it is implausible for an individual's marginal rate of substitution between wealth and a LYM to be constant (e.g., independent of expected longevity). This implies that using a constant monetary value per LYM, as with VSLY, cost-effectiveness analysis using QALY or DALY, or linear transfer from a LYM to a monetary measure, is inconsistent with individual preferences. Hence conventional applications of VSLY, cost-effectiveness analysis, and linear 'monetization' of QALY or DALY are inconsistent with individual welfare.

The paper is organized as follows. The following section provides a brief survey of the diversity of utility functions for health, longevity, and wealth that have been considered in the literature and presents the economic models underlying VSL, VSLY, QALY, DALY, and HYE. Section 2 derives the utility function for health, longevity, and wealth that is admissible under the assumption that preferences for health and longevity are consistent with a LYM. Section 3 characterizes the implications of the admissible utility function for the marginal utility of wealth and for risk postures with respect to longevity and wealth, and imposes restrictions on the admissible utility function so it is consistent with reasonable, standard assumptions about these properties. Marginal willingness to pay (WTP) for a LYM and its dependence on health, longevity, and wealth are examined in Section 4 and marginal WTP to decrease current-period mortality risk (VSL) is examined in Section 5. Section 6 concludes.

1 Utility functions for health, longevity, and wealth

The effects of health, longevity, and wealth (or consumption) on individual utility have been represented by a variety of alternative utility functions (Rey and Rochet 2004). Grossman (1972) assumed a general function,

$$U = U(\varphi_1 h_1, \varphi_2 h_2, \ldots, \varphi_T h_T, \ \ c_1, \ \ c_2, \ldots, c_T), \tag{1.1}$$

where h_τ is the health state, φ_τ the rate of service flow, and c_τ is other consumption in period τ. Subsequent authors have imposed more structure, often adapting Yaari's (1965) model (in which lifetime utility is the expected present value of period utilities that depend on consumption) to include health state,

$$U = \sum_{\tau=0}^{T} \delta^{\tau} s_\tau U(h_\tau, c_\tau), \tag{1.2}$$

where δ is a discount factor,[1] s_τ is the probability of surviving to age τ, and T is a maximum possible age (e.g., Cutler and Richardson 1997; Meltzer 1997). The period-utility function in equation (1.2) is often assumed to be multiplicative in health and consumption,

$$U(h_\tau, c_\tau) = Q(h_\tau) \ V(c_\tau), \tag{1.3}$$

where $Q(\cdot)$ and $V(\cdot)$ are monotone increasing (e.g., Garber and Phelps 1997; Murphy and Topel 2006[2]). Bleichrodt and Quiggin (1999) provide an axiomatic basis for the representation in equations (1.2–1.3) under both expected-utility and rank-dependent expected-utility theories.

Bommier and Villeneuve (2012) propose a generalization of the Yaari (1965) model (1.2) that provides more flexibility in representing risk posture with regard to longevity,

$$U = \int_0^{\infty} s_t u(c_t) \exp\left[-\int_0^t \nu(\tau) d\tau\right] dt, \tag{1.4}$$

but do not consider the effect of health.[3]

[1] Constant exponential discounting is most common, but other discounting functions can be incorporated by replacing δ^τ with δ_τ (Harvey 1994).

[2] Murphy and Topel (2006) define $V(\cdot)$ as a function of both consumption and leisure.

[3] The Yaari model is obtained as a special case of equation (1.4) by setting $\upsilon(\tau) = \frac{1-\delta}{\delta}$, which is the discount rate in model (1.2).

Alternative representations for a single period include the monetary-loss-equivalent model, in which the effect of impaired health is equivalent to the effect of reduced consumption,

$$U(h,c) = U[h^*, c - m(h^* - h)], \tag{1.5}$$

where h^* represents full health and $m(h^* - h)$ is a monetary value (Evans and Viscusi 1991), and an additively separable model,

$$\mathrm{U}(h,c) = Q(h) + V(c), \tag{1.6}$$

in which $Q(\cdot)$ and $V(\cdot)$ are monotone increasing in their arguments (Eeckhoudt et al. 1998). The literature on valuing mortality risk typically uses a health-state-dependent specification,

$$U(h,c) = U_h(c), \tag{1.7}$$

where the health state is restricted to two values, alive and dead (e.g., Drèze 1962; Jones-Lee 1974; Weinstein et al. 1980). Equation (1.7) can be generalized to include multiple alternative health states while living (Evans and Viscusi 1991; Sloan et al. 1998).

1.1 Value per statistical life (VSL)

The conventional single-period model for VSL assumes the individual seeks to maximize expected state-dependent utility of wealth,

$$EU = (1-p)U_a(w) + pU_d(w), \tag{1.8}$$

where p is the probability of dying in the current period and U_a and U_d represent utility conditional on surviving and dying, respectively. Differentiating equation (1.8) with respect to w and p yields

$$VSL = \frac{dw}{dp} = \frac{U_a(w) - U_d(w)}{(1-p)U_a'(w) + pU_d'(w)} = \frac{\Delta U}{EU'}. \tag{1.9}$$

It is conventionally and reasonably assumed that

$$U_a(w) > U_d(w) \tag{1.10a}$$

$$U_a'(w) > U_d'(w) \geq 0 \tag{1.10b}$$

$$U_a''(w) \leq 0, U_d''(w) \leq 0, \tag{1.10c}$$

where primes denote derivatives; i.e., survival is preferred to death, the marginal utility of wealth is non-negative and larger given survival than death, and weak risk aversion with respect to wealth in both states. Under these assumptions, VSL is positive and increases with wealth and current-period mortality risk. The effect of a change in health or longevity conditional on survival is theoretically ambiguous (Hammitt 2002). An increase in health or longevity may be presumed to increase $U_a(w)$, increasing the numerator of equation (1.9). However, it may also increase $U'_a(w)$, increasing the denominator. Hence the effect on VSL cannot be signed without further assumptions.

1.2 Value per statistical life year (VSLY)

VSLY is derived from VSL by dividing VSL by longevity conditional on survival, *t*, or by the present value of a series of *t* years (Moore and Viscusi 1988; Hirth et al. 2000). Implicitly, the function $U_a(w)$ in equation (1.9) is replaced by a function $U_a(w,t)$, which is assumed to satisfy assumptions (1.10a-c) for all positive values of *t* and to be increasing in *t* for relevant values of *w*.[4] This implies VSLY is positive, increasing in wealth and current-period mortality risk.

As for VSL, the effect of an increase in longevity on VSLY cannot be signed without further assumptions. Substituting $U_a(w,t)$ for $U_a(w)$ in equation (1.9) and dividing by *t* yields

$$VSLY = \frac{1}{t}\frac{U_a(w,t) - U_d(w)}{(1-p)U_a'(w,t) + pU_d'(w)} = \frac{1}{t}\frac{N}{D}. \tag{1.11}$$

Differentiating with respect to *t* yields

$$\frac{d}{dt}VSLY = \frac{1}{tD}\left[-\frac{N}{t} + \frac{\partial}{\partial t}U_a(w,t) - \frac{N}{D}(1-p)\frac{\partial^2}{\partial t \partial w}U_a(w,t)\right] \tag{1.12}$$

where *N* and *D* are defined in equation (1.11). The first term in brackets is negative, the second is positive, and the third depends on the sign of the cross-partial derivative of $U_a(w,t)$ with respect to its two arguments. It is clear from equation (1.12) that VSLY cannot be independent of *t* except under restrictive conditions. While the expression in brackets might equal zero for some values of *t*, for it to equal zero for all *t* in a finite range would require that changes in the first term (which is inversely proportional to *t*) would have to be exactly offset by appropriate changes in the first and second-order derivatives in the following terms.

1.3 Quality-adjusted life year (QALY)

The QALY model for constant health state can be expressed as

$$Y_Q = V[q_t(h) \cdot t] \tag{1.13}$$

or

$$Y_Q = q_s(h)V(t). \tag{1.14}$$

It weights longevity *t* by *q*(*h*), the 'health-related quality of life' (HRQL) associated with health state *h*, scaled so that a value of 1 corresponds to full health and 0 to a state of health as bad as dead (health states worse than dead are permitted, for which HRQL<0). The function $V(\cdot)$ accounts for risk posture with respect to longevity (and hence time preference).

[4] For values of *w* sufficiently small, it is possible that utility is decreasing in *t*. Assumption (1.10a) may be interpreted as excluding values of *w* for which life is worse than death.

If V is not the identity function (implying risk neutrality toward longevity), the HRQL associated with health states other than full health and as bad as dead depend on whether QALY is represented using model (1.13) or (1.14). Under model (1.13), the HRQL $q_t(h)$ is the response to a time-tradeoff question in which the respondent states that a fraction $q_t(h)$ of lifespan t lived in full health is as desirable as lifespan t lived in health state h. Under model (1.14), the HRQL $q_s(h)$ is the response to a standard-gamble question in which the respondent states that a lottery with probability $q_s(h)$ of lifespan t lived in full health and complementary probability of immediate death is as desirable as lifespan t lived in health state h.

In their pioneering paper, Pliskin et al. (1980, equations (4a, b)) propose a special case of model (1.13) and derive conditions on preferences under which it represents utility. These include mutual utility independence of h and t and constant proportional tradeoff of t for h. These conditions imply that V exhibits constant proportional risk posture (including risk neutrality as a special case).[5]

Bleichrodt et al. (1997) show that the risk-neutral case can be characterized by the conditions that the individual is risk-neutral with regard to longevity conditional on health for all health states, and indifferent to health state when longevity is zero (the so-called 'zero condition'). Miyamoto and Eraker (1985) propose model (1.14) and Miyamoto et al. (1998) show that it represents preferences that satisfy the zero condition and 'standard-gamble invariance' (a weaker form of the condition that t is utility independent of h). In conventional cost-effectiveness analysis, QALYs are discounted at a constant positive rate, which implies constant absolute risk aversion with respect to longevity (including risk neutrality as a special case).[6]

1.4 Disability-adjusted life year (DALY)

The DALY (Murray 1994) is similar to the QALY in weighting longevity by an index of health. Unlike QALY, DALY is measured as a loss from an ideal health profile (one with full health and a specified survival function), but an inverse measure can be obtained by subtraction from this ideal profile, which has the property that more inverse DALYs are preferred to fewer. Like QALYs, DALYs are often discounted for time preference. In addition, life years are often weighted by a function of age τ (measured in years), $\tau e^{-\beta\tau}$ with $\beta=0.04$. This function gives less weight to years lived when young or old than to years lived at intermediate ages.

1.5 Healthy years equivalent (HYE)

HYE is defined as the number of years of life in full health that are as desirable as a specified time lived in a health state, or to a specified time path through multiple health states (Mehrez and Gafni 1989). Unlike the QALY and DALY models, it does not assume a fixed weight for each health state or that the preference ordering over health states is independent of duration. Because it imposes less structure on preferences than

[5] That is, $V(x)=x^{(1-r)}/(1-r)$, $r\neq1$; $V(x)=ln(x)$, $r=1$, where r is the measure of relative risk aversion (Pratt 1964).

[6] That is, $V(x)=(1/r)\ (1-e^{-rx})$ where r is the discount rate and also the Arrow-Pratt measure of absolute risk aversion (Pratt 1964).

the QALY and DALY models, it requires that the HYE for each time path over health states be elicited independently. It is rarely used in practice.

2 Admissible utility functions for health, longevity, and wealth

Assume that preferences for lotteries on health, longevity, and wealth are consistent with expected utility theory and that lifetime utility $U(h,t,w)$ depends on health h, longevity t, and wealth w (including labor and other income). Lifetime utility may be defined over the entire lifetime or the part remaining at the individual's current age. Longevity and wealth are assumed to be real and longevity to be non-negative. Health may be a scalar or vector, continuous or categorical; it need not be ordered. I limit attention to health states and wealth levels for which life is weakly preferred to death, for simplicity and because other cases are less often of policy interest.

This one-period model contrasts with previous work that assumes lifetime utility can be represented as a discounted sum of period utilities (equation (1.2)), which requires assumptions about intertemporal separability, additivity, and the discounting function (e.g., Shepard and Zeckhauser 1984; Rosen 1988; Ng 1992; Bleichrodt and Quiggin 1999). Moreover, it obviates the need to consider explicitly the extent to which individuals can allocate consumption and health spending over the lifecycle.[7]

Assume that preferences over health and longevity are consistent with some LYM. That is, holding wealth fixed, preferences over health and longevity are consistent with some function $Y(h,t)$ which may represent LY, QALY, DALY, HYE, or some other measure that integrates longevity and health.

Definition 1: Preferences for lotteries on health and longevity are consistent with a life-year measure $Y(h,t)$ iff

$$Y(h',t') > Y(h'',t'') \leftrightarrow U(h',t',w') > U(h'',t'',w') \tag{2.1}$$

for all levels of wealth w' and health h', h'' for which life is preferred to death.

If preferences satisfy Definition 1, then utility functions for health and longevity conditional on different wealth levels are strategically equivalent, i.e., represent the same preference ordering, and hence are related as positive affine transformations. This observation yields Proposition 1 (Keeney and Raiffa 1993, equation (6.5), p. 285):

Proposition 1. If preferences satisfy Definition 1, then

$$U(h,t,w) = Y(h,t)a(w) + b(w) \tag{2.2}$$

where $a(w)>0$.

Proposition 1 describes the utility function for health, longevity, and wealth that is admissible under the assumption that preferences for health and longevity are consistent with a LYM. The condition $a(w)>0$ is necessary for $U(h,t,w)$ to be increasing in $Y(h,t)$.

[7] Using model (1.2) to estimate WTP per QALY, Bleichrodt and Quiggin (1999) assume that individuals allocate medical and other spending to obtain constant health and consumption over their lifetimes. The assumption that health can be made constant over the lifecycle through allocation of spending seems implausible given that health deteriorates with age and not all impairments can be eliminated by treatment.

In addition to assuming preferences are consistent with a LYM, I impose some additional conditions that are motivated by the application to health and longevity. I assume the zero condition (that the individual is indifferent to health when longevity is zero; Bleichrodt et al. 1997) and normalize the LYM to zero when $t=0$ (i.e., for death). As noted above, I restrict attention to health states for which life is weakly preferred to death. These assumptions are formalized as:

$$Y(h,0) = 0 \ \text{ for all } \ h \tag{2.3a}$$

$$Y(h,t) \geq 0 \ \text{ for all } \ t > 0. \tag{2.3b}$$

Note that I need not assume that $Y(h,t)$ is monotone increasing in t or that preferences over health states are independent of t.

3 Properties of the admissible utility function

In this section, I examine properties of the admissible utility function including the marginal utility of wealth, risk posture with respect to longevity and wealth, and their dependence on health and longevity. Requiring these properties to be reasonable leads to additional restrictions on the admissible utility function.

3.1 Marginal utility of wealth

The marginal utility of wealth is given by

$$\frac{\partial}{\partial w} U(h,t,w) = Y(h,t)a'(w) + b'(w). \tag{3.1}$$

Note that $t=0$ implies $U(h,t,w)=b(w)$, so $b(w)$ is the utility for wealth conditional on death (i.e., the utility of a bequest). I adopt the conventional and plausible assumptions of the VSL literature (expression (1.10b)). First, the marginal utility of a bequest is non-negative, hence $b'(w) \geq 0$. Second, the marginal utility of wealth conditional on survival exceeds the marginal utility conditional on death, hence $a'(w)>0$. This in turn implies that the marginal utility of wealth increases with $Y(h,t)$, and hence increases with both health and longevity (except in any cases where $Y(h,t)$ does not increase with t). Limited empirical evidence (Viscusi and Evans 1990; Sloan et al. 1998; Domeij and Johannesson 2006; Finkelstein et al. 2013) and the frequent adoption of the sum of period utilities (1.2) and multiplicative utility function (1.3) also support the notion that the marginal utility of wealth increases with health and longevity.

3.2 Risk postures for longevity and wealth

There is no necessary relationship between the risk postures for longevity and for wealth. The longevity risk posture is characterized by $Y(h,t)$ and is independent of wealth. This follows immediately from the assumption that preferences for health and longevity are independent of wealth. Moreover, a LYM may exhibit risk aversion, risk neutrality, and risk proneness with respect to longevity for different values of t; it need not satisfy constant absolute or constant relative risk aversion as assumed for QALY (e.g., Pliskin et al. 1980).

In contrast, the risk posture with respect to wealth may depend on health and longevity. The Arrow-Pratt measure of absolute risk aversion $\pi(w)$ (Pratt 1964) is given by

$$\pi(w) = -\frac{\frac{\partial^2}{\partial w^2}U(h,t,w)}{\frac{\partial}{\partial w}U(h,t,w)} = -\frac{Y(h,t)a''(w)+b''(w)}{Y(h,t)a'(w)+b'(w)}. \tag{3.2}$$

If the individual is indifferent to the level of his bequest ($b'=0$), then the measure of risk aversion is independent of Y and equal to the Arrow-Pratt measure for the function $a(w)$, i.e., $\pi_a(w)=-a''(w)/a'(w)$. Alternatively, $b'>0$ and differentiating equation (3.2) with respect to Y yields

$$\frac{\partial \pi}{\partial Y} = (\pi_a - \pi_b)\frac{a'b'}{(Ya'+b')^2} \tag{3.3}$$

where $\pi_b(w)$ is the Arrow-Pratt measure for the function $b(w)$. Hence risk aversion with respect to wealth increases with, is independent of, or decreases with Y as the Arrow-Pratt measure for $a(w)$ is respectively greater than, equal to, and less than the Arrow-Pratt measure for $b(w)$. If a'' and b'' are of opposite sign, an individual may be risk averse with respect to wealth for some values of Y and risk seeking for other values. Hammitt et al. (2009) report survey evidence that wealth risk aversion decreases with health and life expectancy (and increases with age, as also found by Barksy et al. 1997). These results suggest (counterintuitively) that individuals are more risk averse with respect to their bequests than their wealth given survival and that $\pi_a<\pi_b$.[8] A sufficient condition for individuals to be risk averse both with respect to wealth conditional on survival and for their bequest (expression 1.10c) is $a''(w)\leq 0$ and $b''(w)\leq 0$ for all w, and hence $\pi(w)\geq 0$ for all $Y(h,t)$.

The effect of a change in aversion to longevity risk on aversion to financial risk is indeterminate. As an example, assume $Y(h,t)=t^{(1-r)}$; i.e. LY with constant relative risk aversion r ($r<1$). Differentiating equation (3.2) with respect to r yields

$$\frac{\partial \pi}{\partial r} = \frac{\partial \pi}{\partial Y}\frac{\partial Y}{\partial r} \tag{3.4}$$

where

$$\frac{\partial Y}{\partial r} = -t^{1-r}\log(t). \tag{3.5}$$

As shown by equation (3.3), the sign of $\frac{\partial \pi}{\partial Y}$ is determined by the sign of $\pi_a - \pi_b$. If $\pi_a = \pi_b$ (or $b'=0$), then risk aversion with respect to wealth is independent of longevity risk aversion. If not, the sign of equation (3.4) depends on the signs of equations (3.3) and (3.5). The sign of (3.5), and hence the effect of a change in longevity risk aversion on wealth risk aversion, depends on whether t is larger or smaller than $t^*=1$. Note that

[8] Empirically, risk aversion may decrease with health and life expectancy because these factors increase expected lifetime wealth (Hammitt et al. 2009). In contrast, equation (3.3) holds wealth fixed.

t^* depends on what is held constant as risk aversion changes. In this example, $Y(h,t)$ is held constant at $t=1$, but the units in which t is measured (and hence the magnitude of t^*) are arbitrary.

3.3 Admissible utility function

To summarize, combining the assumption that preferences for health and longevity are consistent with a life-year measure (Definition 1) with conventional and plausible assumptions about the marginal utility of wealth and aversion to financial risk implies that the admissible utility function for health, longevity and wealth is specified by:

$$U(h,t,w) = Y(h,t)a(w) + b(w) \tag{3.6a}$$

$$a(w) > 0, \tag{3.6b}$$

$$a'(w) > 0, \tag{3.6c}$$

$$b'(w) \geq 0, \tag{3.6d}$$

$$a''(w) \leq 0, \text{ and} \tag{3.6e}$$

$$b''(w) \leq 0. \tag{3.6f}$$

4 Willingness to pay per life-year measure

I examine the implications of the admissible utility function for the marginal rate of substitution between wealth and any life-year measure $Y(h,t)$. Let v denote the individual's marginal rate of substitution between w and Y. It is obtained by totally differentiating equation (3.6a) holding utility constant to obtain

$$v = -\frac{dw}{dY} = \frac{a(w)}{Ya'(w) + b'(w)} + \frac{\partial w}{\partial Y}. \tag{4.1}$$

In general, marginal WTP for Y depends on w and Y. The first term in equation (4.1) represents pure WTP for improvements in health and longevity and is positive under assumptions (3.6b-f). The second term represents the feedback effect of changes in health and longevity on lifetime wealth. The sign and magnitude of $\frac{\partial w}{\partial Y}$ may depend on whether Y is changed by improved health or increased longevity (e.g., fewer sick days during working life or a longer retirement). For example, Meltzer (1997) and Bleichrodt and Quiggin (1999) assume lifetime wealth depends on longevity but not on health. In the remainder of this section, I focus on pure WTP for health and longevity, neglecting any feedback effect,

$$v = -\frac{dw}{dY} = \frac{a(w)}{Ya'(w) + b'(w)}. \tag{4.2}$$

As shown by equation (4.2), WTP for Y decreases with Y. If the individual is indifferent to his bequest, b'=0 and v is inversely proportional to Y. Otherwise, WTP decreases with Y but less than proportionally.

If b'=0, v is proportional to $a(w)/a'(w)$, which is 'fear of ruin' (the reciprocal of 'boldness'), where ruin is defined as the level of w at which $a(w)$=0 and hence the individual is indifferent between life and death. Fear of ruin measures the individual's willingness to risk financial ruin in exchange for a marginal increase in wealth (Aumann and Kurz 1977; Foncel and Treich 2005). Marginal WTP increases with fear of ruin: when the marginal utility of wealth is small, the individual is unwilling to accept a small risk of ruin to increase wealth and is willing to spend more for health and longevity.

Some empirical studies support the result that marginal WTP per LYM decreases with Y, but others do not. Revealed-preference estimates of VSLY find it increases then decreases with age and hence decreases with life expectancy over younger ages and increases with life expectancy over older ages (Aldy and Viscusi 2008). Stated-preference studies (e.g, Tolley et al. 1994; Pinto-Prades et al. 2009; Haninger and Hammitt 2011) and the Johnson et al. (1997) meta-analyses find that WTP is an increasing, concave function of QALYs gained. In contrast, the Van Houtven et al. (2006) meta-analysis finds that average WTP per QALY falls with duration but increases with health-quality gain.

Marginal WTP for Y increases with wealth. Under assumptions (3.6c,e,f), an increase in wealth increases the numerator and decreases the denominator of equation (4.2). This finding is unsurprising and has been anticipated (e.g., Gold et al. 1996; Garber and Phelps 1997).

5 Value per statistical life

Substituting the admissible utility function (3.6a) into the standard VSL model (1.9) yields

$$VSL = \frac{dw}{dp} = \frac{Ya(w)}{(1-p)Ya'(w) + b'(w)}. \tag{5.1}$$

The effects of health and longevity on VSL can be seen by inspection. If the individual is indifferent to the level of his bequest ($b'(w)$=0), then VSL=$(1\text{-}p)^{-1}$ $a(w)/a'(w)$ (i.e., fear of ruin divided by the survival probability) and is independent of health and longevity. If the marginal utility of the bequest is positive ($b'(w)$>0), then the proportionate effect of an increase in Y is larger in the numerator than in the denominator of equation (5.1) and so VSL increases with Y. These results can be verified by differentiating equation (5.1) to obtain

$$\frac{\partial}{\partial Y} VSL = \frac{ab'}{[(1-p)Ya' + b']^2}. \tag{5.2}$$

The right-hand side of equation (5.2) equals zero when $b'(w)$=0 and is positive when $b'(w)$>0. The admissible utility function constrains the relationship between the effects of health and longevity on the utility and the marginal utility of wealth in such a way as to remove the ambiguity about the effects of increased longevity and health on VSL in the standard model (equation (1.9)).

Krupnick et al. (2002) and Smith et al. (2004) report empirical evidence suggesting that reduced health increases VSL in some cases, which is inconsistent with the admissible utility function. In addition, empirical estimates suggest that VSL varies little with age or initially rises then falls with age (Aldy and Viscusi 2007, 2008; Krupnick 2007). Increasing VSL with age suggests decreasing VSL with life expectancy, which is inconsistent with the admissible utility function.[9] A constant VSL is consistent only if the individual is indifferent to his bequest.

The effect of longevity risk aversion on VSL is ambiguous.[10] For example, assume as in Section 3.2 that $Y(h,t)=t^{(1-r)}$, $r<1$. Substituting this specification into equation (5.1) and differentiating with respect to r yields

$$\frac{\partial}{\partial r}VSL = \frac{\partial VSL}{\partial Y}\frac{\partial Y}{\partial r}. \tag{5.3}$$

The first term is given by equation (5.2) and is non-negative. The second term is given by expression (3.5); as discussed in Section 3.2, its sign depends on whether t is larger or smaller than a critical value that depends on the arbitrary units in which t is measured.

6 Conclusion

If an individual's preferences for health and longevity are consistent with the conditions required for a life-year measure (LYM) such as LY, QALY, DALY, HYE, or some alternative, then his utility function for health, longevity, and wealth is tightly constrained: it must be a positive affine transformation of the life-year measure $Y(h,t)$ in which the slope and intercept may depend on wealth, i.e., $U(h,t,w)=Y(h,t)\ a(w)+b(w)$, where $a(w)>0$ (so utility is increasing in Y). Many of the utility functions used in the literature are inconsistent with these assumptions.

Adding conventional and plausible assumptions about the marginal utility of wealth and aversion to financial risk, I obtain the following results:

1. Risk posture with respect to longevity is independent of wealth. Risk aversion with regard to wealth can depend on health and longevity, consistent with survey evidence (Barksy et al. 1997; Hammitt et al. 2009).
2. Marginal willingness to pay (WTP) for $Y(h,t)$ decreases with Y and increases with wealth. Revealed-preference estimates of VSLY (Aldy and Viscusi 2008) are inconsistent with this result, though many stated-preference estimates and meta-analyses of WTP per QALY are consistent with it (Tolley et al. 1994; Johnson et al. 1997; Pinto-Prades et al. 2009; Haninger and Hammitt 2011).
3. The value per statistical life (VSL) or marginal WTP to reduce current mortality risk increases with future health and longevity (consistent with common intuition but contrary to much empirical evidence), except that VSL is independent of health and longevity when the individual is indifferent to the level of bequest.

[9] One reason that VSL increases with age over younger ages is that income rises over these ages and individuals cannot borrow against future increases. Hence VSL and current consumption are correlated over the lifecycle (Kniesner et al. 2006). In contrast, equation (5.2) is conditioned on fixed wealth.

[10] The effect of financial risk aversion on VSL is also ambiguous (Eeckhoudt and Hammitt 2004).

4. The effects of risk posture with respect to longevity on WTP for a LYM and VSL are ambiguous. They depend on the quantity of the LYM and on what is held constant when longevity risk aversion changes.

The result that individual WTP for a LYM depends on future health and longevity (as well as on wealth) implies that WTP to reduce the probability of a non-fatal health condition cannot be accurately estimated by multiplying the expected gain in QALY or some other LYM by a constant WTP per unit. Similarly, VSL is not proportional to life expectancy, and so an individual's VSLY is not constant but depends on life expectancy. WTP to reduce fatal or non-fatal health risk may be estimable as a non-linear function of the expected LYM gain that is decreasing in the baseline LYM and incremental gain. In addition, cost-effectiveness analysis using QALY or DALY is inconsistent with standard welfare economics and benefit-cost analysis. Even if preferences for health and longevity are consistent with one of these LYMs, the use of a fixed threshold value per LYM in cost-effectiveness analysis is inconsistent with individual preferences. Instead, the threshold for an intervention should decrease with the expected gain.

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