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Competition in Research and Development: A Theory for Contradictory Predictions

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Abstract This paper develops reasons for the many seemingly contradictory findings in the literature about competition and research and development (R&D) investment. The theory of R&D competition implies that increases in competitive pressure may increase R&D investment, decrease it, increase it initially but decrease it over greater levels of competitive pressure (an inverted-U relation), decrease it initially but increase it over greater levels (a U relation), or have no effect at all.

Keywords Innovation · Inverted U · Oligopoly · Research and development · R&D investment · Schumpeterian competition

JEL Classifications L10 · L20 · O31 · O32

1 Introduction

Schumpeter (1942) challenged conventional wisdom that competition promotes good economic performance. He predicted that less structural competition—more concentrated markets—would foster technological change.[1] Yet, public policy promotes more competitively structured industries and depends in part on the belief that innovative performance is better with more competitors—the opposite of Schumpeter's view.

[1] Baldwin and Scott (1987, pp. 1–4) provide an overview of Schumpeter's challenge to the conventional wisdom regarding monopoly and competition.

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This paper develops reasons that both views can be correct. The paper contrasts research and development (R&D) behavior for a firm in an industry with many competitors—arguably a "non-Schumpeterian" industry—that perceives competitive R&D pressure as exogenous, with the R&D behavior for a firm in an oligopoly—arguably a "Schumpeterian" industry—that perceives interactive R&D interdependence with its rivals.[2] The oligopoly is "Schumpeterian" in the sense that Schumpeter hypothesized that R&D would be most prevalent and most effective for firms that dominated their industries.[3] Moreover, the empirical literature testing his hypothesis has associated Schumpeterian industry structures with high levels of seller concentration and "tight" oligopoly.

The different perceptions of competitive pressure for firms in competitive, non-Schumpeterian industries versus those in oligopolistic, Schumpeterian industries can imply that in the non-Schumpeterian industries greater competition increases R&D investment, while in the Schumpeterian industries less competition will increase R&D. The theory reconciles Schumpeter's view with the view that he challenged. Will more competitively structured markets stimulate R&D (the broad expectation in antitrust policy)? Or instead will R&D increase with less structural competition (the Schumpeterian expectation)?

This paper explains that because R&D competition is perceived differently in the two types of industries, the answer to both questions can be yes, although that is not necessarily so. In both types of industries greater competitive pressure is associated with a higher anticipated (i.e., statistical expectation) state of the art for the product or process that is the focus of R&D. Consequently, because the relative advancement of any technological outcome from a given R&D investment is less and the marginal effect of R&D is greater, in both non-Schumpeterian and Schumpeterian industries, the marginal value of a firm's investment in R&D is higher. However, in the Schumpeterian industry, increased competitive pressure also causes the oligopolistic firm to anticipate the adjustments of its rivals' R&D investments in response to its own R&D choices, and to recognize that more R&D from those rivals could lower the marginal value of the firm's own R&D because more competing innovations would be anticipated in the post-innovation market.

As long as R&D is profitable, greater competitive pressure implies anticipation that the state of the art technology will be better and, as will be explained in detail, therefore increases R&D in both types of industries, other things being the same. But in the Schumpeterian industry the foregoing effect may be especially likely to be offset because the oligopolist recognizes the impact of rivals' R&D adjustments after the equilibrium is upset by an increase in competitive pressure. The oligopolist

[2] The argument is analogous to Chamberlin's (1929) rationale for a critical concentration ratio determining behavior and performance regarding price and output, with price-taking behavior below the critical concentration ratio in a regime where there was no recognition of oligopolistic interdependence, and with a very different type of behavior above the critical concentration ratio in a regime where firms recognize their interdependence.

[3] "Schumpeterian" firms are large firms with market power that Schumpeter expected to do most of the R&D because (a) there were important economies of scale in R&D (e.g., a research lab) that only a large firm could capture and (b) their market power is necessary to capture the returns from the R&D because Schumpeter was concerned about the appropriability of the results of R&D.

recognizes that a change in its own R&D will trigger a response by its rivals—just as a market's oligopolists recognize their interdependence when choosing outputs and prices. By contrast, the competitor takes the R&D of its rivals as a parameter of its environment, ignoring the impact of a change in its own R&D on the R&D of its rivals—just as a price-taking competitor ignores the impact of its quantity decision on the market's price.

This paper establishes the conditions for which greater competition will increase R&D for a firm with numerous rivals but will decrease R&D for an oligopolist with only a few rivals. More generally, as the paper will show, the possible relations between competitive pressure and R&D investment are many. In a review of the literature about competition in R&D, Gilbert (2006) concludes that theory does not predict a unidirectional link from competition to R&D investment nor does the empirical work demonstrate such a link. This paper develops reasons for competitive pressure's multi-directional effects on R&D. Increases in competitive pressure may increase R&D investment, decrease it, increase it initially but decrease it over higher levels of competitive pressure (an inverted-U relation), decrease it initially but increase it over higher levels (a U relation), or have no effect at all.[4]

The dichotomy between exogenous and endogenous competitive pressure has been important in models of R&D competition. Early models of the effects of competition on R&D investment analyzed exogenous competitive pressure, while subsequent models of R&D rivalry have studied noncooperative equilibrium in markets characterized by recognition of oligopolistic interdependence.[5] Different effects on R&D of competition may result from the differences across samples in the way that competitive pressure is perceived by firms.

With numerous firms, the individual firms may not have sufficient information about their rivals' activities—and the relationships between R&D investment and innovative outcomes for those rivals—to formulate and reach a Nash noncooperative equilibrium set of strategy combinations. As Kreps (1990, p. 31) observes: "Unless a given game has a self-evident way to play, self-evident to the participants, the notion of a Nash equilibrium has no particular claim upon our attention." In circumstances where firms do not perceive a self-evident way to play, they will nonetheless be aware of competitive pressure—pressure that can be more or less intense. Hypothesizing that perception and its effect on the behavior of firms is an alternative to the Nash equilibrium behavior.

Rather than model Nash noncooperative equilibria for the full spectrum of observed competition, this paper emphasizes the old dichotomy: Numerous competitors perceive exogenous competitive pressure; but when competitors are few, each recognizes its

[4] This paper focuses entirely on the relation between competition and the R&D investment of firms. It does not explore the relation between that investment and the socially optimal amount of investment, although the question could be addressed with the paper's model by positing the relation between the private benefits and costs in the model and the associated social benefits and costs. Many of the theoretical articles, beginning with Scherer (1967) and Barzel (1968), do address the issue of the social optimality of private R&D investment; the focus herein is on understanding the theory about the relatively simple, empirically verifiable answer to the question of how competitive pressure affects private R&D spending.

[5] Baldwin and Scott (1987) review the earliest models with exogenous competitive pressure as well as the transition to noncooperative equilibrium models. Gilbert (2006) and Martin (2002) review both the pioneering models and the subsequent models featuring noncooperative equilibrium.

interdependence with the others, and Nash noncooperative equilibria may describe behavior.

This paper proceeds as follows: Sect. 2 introduces the general R&D set-up; Sect. 3 focuses on the case where a firm perceives exogenous pressure; and Sect. 4 examines the case where a firm perceives endogenous determination of a Nash noncooperative equilibrium.

2 A Firm that Invests in Risky R&D

Investment in risky R&D results in innovations—new, commercialized processes or products with better technical performance as indexed by the random variable x. The measure, x, of technical performance is necessarily relative technical performance—that is, performance relative to the anticipated state of the art. For example, x for an innovative computer might represent the ratio of its speed to the anticipated speed of the best practice for alternative computers.[6] Better technical performance reflects higher quality of the firm's completed R&D project. The probability distribution for the measure of performance x is given by the probability density $\mathrm{f}(x;\alpha)$, where greater values of the distribution's parameter α shift the probability distribution rightward over higher levels of performance.[7]

The parameter α is determined by the amount of R&D investment R and an additional set of explanatory variables X. Thus, $\alpha = \alpha(\mathrm{X}, R)$.[8] Greater R&D investment, R, is associated with a greater α. Hence, if a company increases its R&D, its distribution over performance outcomes is shifted rightward over higher values of the index of performance x. The R&D investment R is the present value of the R&D cost schedule chosen by the firm. The details of the R&D cost schedule are described with the partial derivatives and cross-partials for $\alpha(\mathrm{X}, R)$.

A company's innovation has a value that increases at a decreasing rate with its technical performance x. The innovation's value is given by $\mathrm{V}(x;\gamma)$, where V given x increases with the parameter γ and $\partial V/\partial\gamma > 0$ increases with x, and where given the parameter γ, $V(x) > 0$, $V'(x) > 0$, and $V''(x) < 0$.[9]

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6 Details about how computer speed is measured and how a new microprocessor can result in a faster computer are provided below when the effect of competition on relative performance is explained. Familiar examples where relative technological performance is important for innovations include the resolution of video displays, the mileage per unit of fuel for automobiles, and the strength and durability of building materials.

7 In Scott (2003) an explicit functional form—the gamma distribution—was used for the probability density $\mathrm{f}(x;\alpha)$. The present paper uses general functional relations, deriving the results for the general case rather than working with explicit functional forms.

8 For example, in Scott (2003), with α being the shift parameter for the gamma distribution, α was modeled as $\alpha = \alpha(\mathbf{X}, R) = AX_1^{\beta_1}X_2^{\beta_2}\cdots X_n^{\beta_n}e^{\varphi_1 D_1}\cdots e^{\varphi_w D_w}R^{\beta_R}$, with the terms $X_i^{\beta_i}$ for the n positive and continuous variables, and with $e^{\varphi_i D_i}$ for each term where the explanatory variable is a qualitative (0–1 dummy) variable or where theory implies that a continuous variable should be entered in such an exponential form.

9 For example, in Scott (2003), the value of environmental performance was modeled as $V(x) = \gamma - \gamma e^{-x/\theta}$, where θ is a scaling parameter.

The value parameter γ is a function of a set of explanatory variables Z. Thus, $\gamma = \gamma(\mathrm{Z})$.[10] The value-shifting variables in the vector Z would include the company's sales, because larger firms are expected to gain more value from an innovation.[11]

Value reflects the present discounted value of the stream of benefits generated by the innovation; details about the stream of benefits are in the partial derivatives and cross-partials for $\gamma(\mathrm{Z})$. Value increases at a decreasing rate because of diminishing marginal utility for the increased performance measured by x given the parameter γ.

The expected value E of the investment R is:

$$E = \int V(x; \gamma(Z_{-i}, Z_i)) f(x; \alpha(X_{-i}, X_i, R)) dx. \tag{1}$$

The firm chooses R to maximize expected profit (E – R). The first-order condition is:

$$\frac{\partial E}{\partial R} = \int V \frac{\partial f}{\partial \alpha} \frac{\partial \alpha}{\partial R} dx = 1. \tag{2}$$

For expected profit-maximizing equilibrium, the marginal benefit for R&D equals its marginal cost—the marginal dollar of R&D yields expected benefits of a dollar. The marginal benefit of R&D is the result of R&D investment increasing alpha and hence shifting the distribution f over performance x rightward. Marginal benefit is therefore positive because R increases α, shifting the distribution f over x rightward, and V increases with x.

The second-order condition is:

$$\frac{\partial^2 E}{\partial R^2} = \int V \left(\frac{\partial f}{\partial \alpha} \frac{\partial^2 \alpha}{\partial R^2} + \frac{\partial \alpha}{\partial R} \frac{\partial^2 f}{\partial \alpha^2} \frac{\partial \alpha}{\partial R} \right) dx < 0. \tag{3}$$

In the neighborhood of the expected profit-maximizing equilibrium, the marginal benefit of R&D is positive but falling. The marginal benefit of R&D falls with R&D investment at the equilibrium because as R increases, the increase in α, and hence shifting the distribution f over performance x rightward, is decreasing. As well, V increases at a decreasing rate as x increases, so rightward shifts of f generate smaller increases in expected value as R increases. The diminishing effect of R&D on the rightward shift of the distribution f reflects the increasing costs of achieving additional rightward shifts. Increasing cost for improving the probability distribution over

[10] For example, in Scott (2003), the value parameter is modeled as $\gamma = \gamma(\mathbf{Z}) = aZ_1^{b_1} Z_2^{b_2} \cdots Z_m^{b_m} e^{\theta_1 S_1} \cdots e^{\theta_v S_v}$ with the term $Z_i^{b_i}$ for each positive and continuous explanatory variable and with a term $e^{\theta_i S_i}$ if the explanatory variable is a qualitative (0–1 dummy) variable or, again, if theory dictates that exponential expression for a continuous variable.

[11] For development of the theory of how R&D and innovation vary across the size distribution of firms in an industry, see Kohn and Scott (1982). The belief that larger firms gain more value from an innovation rests, of course, on the assumption of imperfect appropriability by the innovator.

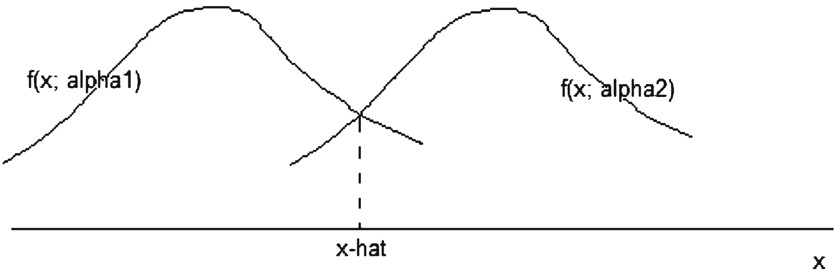

Fig. 1 The density function f

performance is a natural assumption, and the assumption has been a part of the literature from the outset.[12]

The model here describes a firm that invests in risky R&D.[13] There is both technical and market uncertainty.[14] The model captures abstractly the value and cost effects of the R&D technical results (such as the physical features of the product or process and the time of introduction for the innovation), of a firm's size, of competition in multiple-stage games, and so forth.

We will use some properties of the distribution function. The properties are stated in the following lemmas, and the proof for each is given in the appendix. The density function f is assumed to be unimodal, as shown in Fig. 1. For small $\Delta\alpha$, x-hat ($\hat{x}$) separates smaller x for which $\partial f/\partial\alpha < 0$ from larger x for which $\partial f/\partial\alpha > 0$.

Lemma 1 $\int \frac{\partial f}{\partial \alpha} dx = 0$. *The area under the probability distribution function is always* 1*, regardless of the value of* α.

Lemma 2 $\int \frac{\partial^2 f}{\partial \alpha^2} dx = 0$. *The partial derivative, with respect to the value of* α*, for a function that is always equal to zero is zero.*

Lemma 3 *With* $b(x) > 0$*, and* $b'(x) > 0$*, then* $\int b(x) \frac{\partial}{\partial \alpha} f(x;\alpha) dx > 0$*. If a numerical value associated with x is positive and increases with x, its expected value increases as* α *increases and the probability distribution function shifts rightward over higher outcomes for x.*

Lemma 4 $\int V(\frac{\partial f}{\partial \alpha} \frac{\partial^2 \alpha}{\partial X_i \partial R}) dx = \frac{\partial^2 \alpha}{\partial X_i \partial R} \int V(\frac{\partial f}{\partial \alpha}) dx$ *has the sign of* $\partial^2\alpha/\partial X_i \partial R$*. A distribution shifting variable's impact on the marginal value of R&D depends on*

[12] These increasing costs are consonant with Scherer's (1967) cost schedule for achieving a shorter development time.

[13] The model developed in Sects. 2, 3, and 4 is a generalization of the model integrated with the empirical work in Scott (2003) where some of the ideas herein were used, but were studied in the context of specific functional forms rather than general functions.

[14] Technical uncertainty results because, at the time the investments are made, the technical results that will emerge from the R&D are not known with certainty. Market uncertainty results because the market value of known technical results is not known with certainty until after the results have been commercialized in an innovation.

whether the variable is complementary to the firm's R&D investment, increasing the distribution shifting effect of R&D, or instead reduces that effect.[15]

Lemma 5 $\int V(x)\frac{\partial^2 f}{\partial\alpha^2}dx < 0$. *The incremental effect on expected R&D value, of shifting the probability distribution rightward over more favorable outcomes for technical performance, is decreasing.*

We now state the conditions determining the sign for the derivative of R with respect to a value-shifting variable Z_i and for the derivative of R with respect to a probability-shifting variable X_i.

Result 1 If $\partial V/\partial\gamma > 0$ and is increasing with *x*, then $dR/dZ_i > 0$ if $\partial\gamma/\partial Z_i > 0$ and $dR/dZ_i < 0$ if $\partial\gamma/\partial Z_i < 0$.

Proof Displace the equilibrium by changing Z_i and observe:

$$\frac{dR}{dZ_i} = -\frac{\int\left(\frac{\partial f}{\partial\alpha}\frac{\partial\alpha}{\partial R}\right)\left(\frac{\partial V}{\partial\gamma}\frac{\partial\gamma}{\partial Z_i}\right)dx}{\int V\left(\frac{\partial f}{\partial\alpha}\frac{\partial^2\alpha}{\partial R^2}+\frac{\partial\alpha}{\partial R}\frac{\partial^2 f}{\partial\alpha^2}\frac{\partial\alpha}{\partial R}\right)dx}. \tag{4}$$

From Eq. (3), the sign of the denominator of (4) is negative. Together with the negative sign before the right-hand side of (4), we have a positive sign, leaving the sign of the derivative given by Eq. (4) to be determined by the sign of the numerator.

We assume that $\partial V/\partial\gamma > 0$ and increasing with the quality of the investment outcome *x*, and we have $\partial\alpha/\partial R > 0$, since by assumption greater R increases α and thereby shifts the distribution over the investment outcomes to the right. We have:

$$\int\left(\frac{\partial f}{\partial\alpha}\frac{\partial\alpha}{\partial R}\right)\left(\frac{\partial V}{\partial\gamma}\frac{\partial\gamma}{\partial Z_i}\right)dx = \frac{\partial\alpha}{\partial R}\frac{\partial\gamma}{\partial Z_i}\int\frac{\partial f}{\partial\alpha}\frac{\partial V}{\partial\gamma}dx. \tag{5}$$

The integral given by Eq. (5) has the sign of $\partial\gamma/\partial Z_i$ since the integral on the right-hand side is positive by Lemma 3.[16] Hence, the numerator of Eq. (4) and the derivative given by Eq. (4) have the sign of $\partial\gamma/\partial Z_i$. The sign of the derivative of investment, R, with respect to a value shifting variable Z_i is determined by the sign of $\partial\gamma/\partial Z_i$.

Result 2 If $\partial^2\alpha/\partial X_i\partial R$ has a sign that is the opposite of the sign of $\partial\alpha/\partial X_i$, or alternatively is sufficiently small, then $dR/dX_i > 0$ if $\partial\alpha/\partial X_i < 0$ and $dR/dX_i < 0$ if $\partial\alpha/\partial X_i > 0$.

Proof The equilibrium is displaced by changing X_i. Then,

[15] When the probability shifting variable X_i reflects competitive pressure from rivals, the cross-partial derivative here parallels Scherer's (1967) cross-partial derivative of a duopolist's marginal value for R&D with respect to its rival's behavior. The importance of these cross-partial derivatives in Scherer (1967) is mirrored in Result 2 below.

[16] The term $\partial f/\partial\alpha$ is positive or negative for different values of x. But summed in the integral the positive values of $(\partial f/\partial\alpha)dx$, which occur over the larger values of *x*, get greater $\partial V/\partial\gamma$ weights because those weights are positive and increasing with *x*, and the integral is then positive.

$$\frac{dR}{dX_i} = -\frac{\int V(\frac{\partial f}{\partial \alpha}\frac{\partial^2 \alpha}{\partial X_i \partial R} + \frac{\partial \alpha}{\partial R}\frac{\partial^2 f}{\partial \alpha^2}\frac{\partial \alpha}{\partial X_i})dx}{\int V\left(\frac{\partial f}{\partial \alpha}\frac{\partial^2 \alpha}{\partial R^2} + \frac{\partial \alpha}{\partial R}\frac{\partial^2 f}{\partial \alpha^2}\frac{\partial \alpha}{\partial R}\right)dx}. \tag{6}$$

The denominator of (6) is negative from the second order condition. With the negative sign preceding the entire right-hand side expression, the sign of the overall derivative is then determined by the sign of the numerator. The numerator can be separated into two integrals. The first of the two integrals will have the sign of $\partial^2\alpha/\partial X_i \partial R$ by Lemma 4. For the second of the two integrals for the numerator, we know that $\partial\alpha/\partial R > 0$. Further, we know that $\int V(x)\frac{\partial^2 f}{\partial \alpha^2}dx < 0$ by Lemma 5.

Therefore, the sign of the derivative given by Eq. (6) depends on the sign of $\partial^2\alpha/\partial X_i \partial R$ and on the sign of $\partial\alpha/\partial X_i$. The sign of $\partial^2\alpha/\partial X_i \partial R$ is >0 if X_i is complementary to R. If X_i reduces the effectiveness of R, then the sign of $\partial^2\alpha/\partial X_i \partial R$ is <0. The sign of $\partial\alpha/\partial X_i$ is positive or negative depending on whether an increase in X_i shifts the distribution $f(x; \alpha)$ rightward or leftward. If the shift were rightward, X_i would have an effect analogous to the effect of R.

If the cross partial effect of X_i, $\partial^2\alpha/\partial X_i \partial R$, has a sign that is the opposite of the sign of $\partial\alpha/\partial X_i$, or alternatively is sufficiently small, then the effect of X_i on the sign of the derivative given by Eq. (6) is the opposite of the sign of $\partial\alpha/\partial X_i$. In that case, if an exogenous variable shifts the probability distribution over investment outcomes leftward, then an increase in the variable will result in greater investment by the firm. An exogenous variable that shifts the distribution rightward as it increases will thereby cause a decrease in the firm's investment.

The R&D investment is aimed at solving a problem; a tougher problem as measured by the variable X_i implies that the research problem is more challenging and that there will be a lower value for α, other things being the same ($\partial\alpha/\partial X_i < 0$). Moreover, we might expect the cross-partial to have the opposite sign: A more difficult research problem is the type of problem that needs R&D, increasing the shift in the probability distribution that can be achieved by more R&D. If the cross-partial effect reinforces the effect that results because $\partial\alpha/\partial X_i < 0$, a lower value for α will increase the marginal benefit of doing more R&D to shift the probability distribution rightward over better performance outcomes. Hence, with a reinforcing cross-partial, or simply one that is of inconsequential size, a probability-shifting variable X_i that has a negative effect on $\alpha(\partial\alpha/\partial X_i < 0)$ will have a positive effect on R, the investment in R&D ($\partial R/\partial X_i > 0$). Analogously, a variable that has a positive effect on α will have a negative effect on R.

3 A Firm that Responds to Exogenous Competitive Pressure

This section uses the foregoing derivations of the impacts of value-shifting and distribution-shifting variables to describe the R&D behavior of a competitive firm. With a sufficiently large number of competitors—a great amount of structural competition—a firm is a "competitive" firm in the sense that it simply responds to its environment, taking that environment as exogenous and then reacting to it.

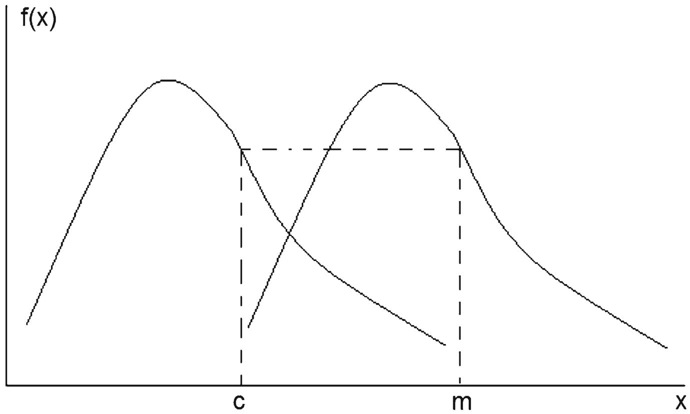

Fig. 2 Greater competitive pressure shifts f(x) leftward

Conjecture 1 *Conjecture about Exogenous Competitive Pressure: Greater competition will increase the representative firm's R&D when competitors perceive exogenous competitive pressure.*

To understand the effect of exogenous competitive pressure, observe that competitive pressure is an "X variable" that shifts leftward the probability distribution over the technical performance evaluation, x, of an innovation. Competitive pressure is also a "Z variable" that affects the value of any technical outcome. Here, in thinking of competitive pressure as an X or a Z variable, it is useful to think of the empirical literature in which competitive pressure has been measured directly by the number of firms or inversely by seller concentration. We now explain the conditions when *Conjecture 1* will hold for a market.

First, consider competitive pressure as an X variable. To develop the argument, let x, the technical performance indicator again be the relative speed of an innovative computer.[17] Given a state of competition, x is a measure of relative speed—relative to the speed for the best alternative computer from the firm's R&D competitors. The measure x is the ratio of the new computer's speed to the speed of the best alternative. A given R&D investment, R, gives the firm its probability distribution of speed, and hence the distribution of its *relative* speed—i.e., the distribution f(x) for performance evaluation x.

As shown in Fig. 2 for the distribution on the right, an outcome for x is not the speed itself, but the assessment of the quality, m, of the technical performance represented by the speed. With more competition, Fig. 2 shows, for the distribution on the left, the same speed is associated with a lower quality, c, of technical performance, because with

[17] A computer's speed will depend on the speed at which a microprocessor executes instructions—the "clock speed". Greater clock speed allows the central processing unit of a computer to execute more instructions per second—measured in millions of instructions per second. Clock speeds are expressed in megahertz (MHz) or gigahertz (GHz). A central processing unit's internal architecture will affect how fast it performs given its clock speed, so a new microprocessor can be much faster than older ones even if clock speed is the same.

more rivals doing R&D the best alternative speed is faster.[18] As illustrated in Fig. 2, the probability distribution over x has shifted leftward as competition increased, and that shift is greater as the competitive pressure increases.[19] At the most abstract level, more "competitive pressure" means firms anticipate a more advanced state of the art resulting in a leftward shift of the probability distribution over performance outcomes for any given amount of R&D. Such greater competitive pressure can come because of more R&D competitors, but it could also come because of other things such as environmental changes (e.g., better information technology) that speed the development process for the other firms or speed the entry of new innovative firms. Even new government policy that made the introduction of new technology more likely could increase "competitive pressure."

Competitive pressure, as an X variable, lowers α and translates the distribution f(x) leftward. Thus, competition and the lower value for α will increase the marginal benefit of doing more R&D to shift the probability distribution rightward over better technical performance outcomes. If the cross-partial $\partial^2\alpha/\partial X_i \partial R$ has a positive sign—greater competitive pressure is accompanied by positive R&D spillovers[20] that increase the shift in the probability distribution achieved with more R&D, or simply that R&D is more effective at shifting the distribution when the R&D problem is more substantial—or is small, then, as a probability-shifting variable X_i that has a negative effect on $\alpha(\partial\alpha/\partial X_i < 0)$, competition will have a positive effect on $R(\partial R/\partial X_i > 0)$, ceteris paribus.

R&D has greater marginal value as competitive pressure increases because any given technological outcome from a given R&D investment has lower relative quality—relative to the anticipated state of the art technology. Hence, more R&D (which increases the quality of the technical outcomes by shifting the distribution over technical outcomes rightward over higher quality) to solve the problem of bettering the anticipated state of the art has higher marginal value. With greater competitive pressure, any given investment in R&D will yield a less favorable distribution over the relative quality of technical outcomes and the marginal value of R&D to shift that distribution rightward will be greater. With greater competitive pressure, there is a tougher challenge to be met, a challenge that R&D (as contrasted with routine engineering work) is uniquely suited to meet.

Thus, when competitive pressure shifts the distribution f(x) leftward for any given R, R&D's expected benefit $\int V(x) f(x) dx$ falls, but marginal benefit increases. Both

[18] The idea here is from the statistics of extreme values. With more trials from the same distribution, an extreme value from the *set* of outcomes has a higher probability.

[19] Consider the often told story that competitive pressure causes firms to invest more in order to beat rivals to the punch. With more rivals, the representative firm invests more to win a patent before its rivals do or to win a first-mover advantage when there are competing innovations, patented or not, and there is not one sure winner. Such a story is a subset of the general story here. In the general story, given R&D investment, with more competition the relative quality of any given outcome is less—whether that quality is measured by the relative time of introduction, by the innovation's relative physical characteristics such as the speed of a computer, or a combination of such traits. With greater competitive pressure, the distribution over relative quality for any given amount of R&D investment is less good; and hence, as we observe next, the marginal value of R&D is greater.

[20] Observe that the model encompasses R&D spillovers.

changes result because more probability weight is given to $V(x)$ and to $V'(x)$ at lower values of relative performance x. Given R, if exogenous competitive pressure increases and R&D remains profitable, R will be increased from the original equilibrium level (before the increase in competitive pressure), because given the old equilibrium R, the marginal benefit of the last R&D dollar will exceed a dollar.

Competitive pressure will also typically be a "Z variable" that lowers value, V(x), of relative performance because greater competition is expected in the post-innovation market.[21] If a positive distribution-shifting effect outweighs that value-reducing effect, greater competitive pressure will increase the representative firm's R&D. *Conjecture 1* corresponds to a prominent result—for example, in Scherer (1967, pp. 389–392)—in the literature: Greater competitive pressure increases the representative firm's R&D efforts as long as the R&D remains profitable.

4 Rivalry and the Noncooperative Nash Equilibrium

In the foregoing description, the R&D behavior is "competitive" in the sense that the firm simply responds to its environment, taking that environment as exogenous and then reacting to it. When the competing firms are few, however, there will be interdependence among them in the sense that once the firm in question has adjusted its investment in response to increased competitive pressure from its rivals, the other firms—the rivals—will find that they too desire to make adjustments to their investment plans. The combination of investment plans for which each rival is doing the best that it can do given the investments of its rivals is a Nash noncooperative equilibrium.

The expected value of the investment R is:

$$E_i = \int V(x; \gamma(Z_{-i}, Z_i, Z_R)) f(x; \alpha(X_{-i}, X_i, X_R, R_i)) dx. \tag{7}$$

The variables Z_R and X_R replace the Z and X variables that denoted the extent of competitive pressure in the case of a "competitive" firm. Those variables will be $\sum_{j \neq i}^{n} R_j$ where R_j denotes the investment of the jth firm among n firms. The ith firm maximizes expected profit, $(E_i - R_i)$ given the R&D investments of its rivals. The first-order condition is:

$$\frac{\partial E_i}{\partial R_i} = \int V \frac{\partial f}{\partial \alpha} \frac{\partial \alpha}{\partial R_i} dx = 1. \tag{8}$$

The second-order condition at the Nash equilibrium is:

$$\frac{\partial^2 E_i}{\partial R_i^2} = \int V \left(\frac{\partial f}{\partial \alpha} \frac{\partial^2 \alpha}{\partial R_i^2} + \frac{\partial \alpha}{\partial R_i} \frac{\partial^2 f}{\partial \alpha^2} \frac{\partial \alpha}{\partial R_i} \right) dx < 0. \tag{9}$$

[21] Observe that the model encompasses appropriability conditions.

For ease of exposition subsequently, define:

$$\Psi \equiv \int V\left(\frac{\partial f}{\partial \alpha}\frac{\partial^2 \alpha}{\partial R_i^2} + \frac{\partial \alpha}{\partial R_i}\frac{\partial^2 f}{\partial \alpha^2}\frac{\partial \alpha}{\partial R_i}\right)dx. \tag{D-1}$$

By the second order condition (9), Ψ is negative. It can be written as two integrals, with the second being negative by Lemma 5, and the first will be negative if the distribution shifting effect of R&D is diminishing, but in any case for the second order condition to be satisfied it cannot be too positive.

Again we delineate the conditions determining the sign for the derivative of R_i with respect to a value shifting variable Z_i and for the derivative of R_i with respect to a probability shifting variable X_i. With rivalrous competition, the new equilibrium after the displacement from a change in a Z or an X variable will require adjustments in the other firms' investments as well as adjustment in the investment of the ith firm.

Result 3 With noncooperative Nash equilibrium, the sign for the derivative of R_i with respect to a value shifting variable Z_i will be the same sign as given in *Result 1* if the equilibrium is stable in the special sense that a cooperative, *R&D-increasing* solution does not dominate the Nash equilibrium. Thus, a cooperative venture that would increase the effectiveness of research and *increase* the amount of R&D investment (as contrasted with a venture that would increase expected profits by reducing competitive pressures and reducing R&D investment) is not possible.

Proof Displace the equilibrium by changing Z_i. With symmetry in the equilibrium, $R_i = R_j = R$, for all i and j, and $\delta Z_R = \delta X_R = (n-1)\delta R$.

For expositional ease, define:

$$\Phi \equiv \int\left(\frac{\partial f}{\partial \alpha}\frac{\partial \alpha}{\partial R}\right)\left(\frac{\partial V}{\partial \gamma}\frac{\partial \gamma}{\partial Z_R}\right)dx + \int V\frac{\partial f}{\partial \alpha}\frac{\partial^2 \alpha}{\partial X_R \partial R}dx + \int V\frac{\partial \alpha}{\partial R}\frac{\partial^2 f}{\partial \alpha^2}\frac{\partial \alpha}{\partial X_R}dx. \tag{D-2}$$

The first of the three integrals comprising Φ is negative by Lemma 3 and because α increases with R and because γ decreases with Z_R.[22] By Lemma 4, the second of the three integrals has the sign of the cross partial derivative. For example, if rivals' R&D lessened the probability-shifting impact of a firm's own R&D, the second integral is negative; it is positive when the rivals' R&D is complementary.[23] The third of the three integrals is positive by Lemma 5 and because the distribution-shifting impact of own R&D is positive, but it is negative for rival R&D.

Consider the following stability condition:

$$\Psi + (n-1)\Phi < 0. \tag{S-1}$$

[22] Again, we see in this last effect that the model encompasses the appropriability conditions.

[23] Here the model encompasses spillovers in R&D.

The left-hand side of the inequality is the rate of change in the ith firm's marginal benefit of R&D per unit change in its R&D given that all firms expand their R&D symmetrically from the symmetric Nash equilibrium.[24] The stability condition (S-1) states that the rate of change must be negative. If it were not, the n firms could form a cooperative research joint venture—not to reduce competition and reduce R&D investment, but to increase the effectiveness of R&D and to increase R&D investment. Such a venture would increase the net benefit of R&D for the participating firms, and it would be encouraged by public policy (Scott 2008).

Displacing the equilibrium condition (8),

$$\frac{dR}{dZ_i} = -\frac{\int \left(\frac{\partial f}{\partial \alpha}\frac{\partial \alpha}{\partial R}\right)\left(\frac{\partial V}{\partial \gamma}\frac{\partial \gamma}{\partial Z_i}\right)dx}{\Psi + (n-1)\Phi}. \tag{10}$$

For the numerator, we have:

$$\int \left(\frac{\partial f}{\partial \alpha}\frac{\partial \alpha}{\partial R}\right)\left(\frac{\partial V}{\partial \gamma}\frac{\partial \gamma}{\partial Z_i}\right)dx = \frac{\partial \alpha}{\partial R}\frac{\partial \gamma}{\partial Z_i}\int \frac{\partial f}{\partial \alpha}\frac{\partial V}{\partial \gamma}dx. \tag{11}$$

We have $\partial V/\partial \gamma > 0$ and increasing with x, and we have $\partial \alpha/\partial R > 0$ because greater R increases α and shifts the distribution over the investment outcomes to the right. The integral given by Eq. (11) has the sign of $\partial \gamma/\partial Z_i$ since the integral on the right-hand side is positive by *Lemma 3*. Hence, the numerator of Eq. (10) has the sign of $\partial \gamma/\partial Z_i$. The denominator of (10) is negative given the stability condition (S-1).

Then the sign for dR_i/dZ_i is the same as for the case of exogenous competitive pressure in *Result 1* because the negative denominator of (10) combined with the minus sign preceding the expression leaves the sign of the derivative to be determined by the sign of the numerator, which has the sign of $\partial \gamma/\partial Z_i$.

Result 4 With Nash equilibrium, the sign for the derivative of R_i with respect to a probability shifting variable X_i will be the same sign as given in *Result 2* given the stability condition (S-1).

Proof The equilibrium is displaced by changing X_i. With symmetry in the Nash equilibrium, $R_i = R_j = R$, for all i and j, and $\delta Z_R = \delta X_R = (n-1)\delta R$. Therefore,

$$\frac{dR}{dX_i} = -\frac{\int V\left(\frac{\partial f}{\partial \alpha}\frac{\partial^2 \alpha}{\partial X_i \partial R} + \frac{\partial \alpha}{\partial R}\frac{\partial^2 f}{\partial \alpha^2}\frac{\partial \alpha}{\partial X_i}\right)dx}{\Psi + (n-1)\Phi}. \tag{12}$$

The leading minus sign and the negative denominator given (S-1) leave the sign to be determined by the numerator. The numerator is the same as for Eq. (6) that underlies *Result 2*. Thus, in response to a change in a firm's environment because of a change in a distribution-shifting variable, the direction of the change in the Nash equilibrium

[24] At this point in the discussion, the symmetric expansion is describing a property of the function and is not an assertion about the behavior of the firms.

investment for the firm facing rivalrous interdependence with other firms will be the same as if it is "competitive" and simply reacts to its environment without making adjustments for the strategies of the other investing firms.

Before developing the effect on Nash equilibrium R&D of a greater number of competitors, consider the following "Schumpeterian" condition:

$$\Phi < 0. \tag{S-2}$$

This condition says that the marginal-value-reducing effect of rivals' R&D, reflected in the negative first integral of Φ as presented in (D-2), outweighs the positive effect of the third integral, which reflects the positive impact of rivals' R&D on the marginal value of a firm's R&D resulting from the leftward shift in the probability distribution, and the possibly positive impact from complementary rival R&D that would be reflected in the second integral.

Result 5 The sign of the change in Nash equilibrium R&D investment for each firm as the number of competitors increases is negative given the stability condition (S-1) and the Schumpeterian condition (S-2), and the effect becomes smaller as the number of firms in the equilibrium increases.

Proof Displace the equilibrium condition (8) by changing n, thereby changing $Z_R = X_R = (n-1)R$. As n changes, $\delta Z_R = \delta X_R = R\delta n + (n-1)\delta R$. Thus, we consider the displacement from equilibrium when the number of firms is changed from n to $n + \delta n$ symmetric firms. The competing firms readjust their R&D investments.

With symmetry in the Nash equilibrium, the change in R per unit change in n is:

$$\frac{\delta R}{\delta n} = -\frac{R\Phi}{\Psi + (n-1)\Phi}. \tag{13}$$

The numerator is negative given (S-2) and the denominator is negative given (S-1); thus, with the leading minus sign, the derivative here is negative. The derivative is smaller in absolute value as n increases.

Although fairly general, the model does build in assumptions that while reasonable may not always hold; moreover, the Schumpeterian condition (S-2) need not hold. Hence, *Result 5* is now restated as a conjecture about *some* markets.

Conjecture 2 *Conjecture about Endogenous Competitive Pressure: Greater competition will reduce the representative firm's R&D when competitors perceive interactive R&D and reach noncooperative equilibrium strategy combinations.*

The conjecture follows directly from *Result 5* if the Schumpeterian condition (S-2) obtains.

Conjecture 2 and *Conjecture 1* together posit opposite effects of competitive pressure in Schumpeterian and non-Schumpeterian industries. When competitive pressure increases, *Conjecture 2* posits a fall in the oligopolistic firm's R&D while, in contrast, *Conjecture 1* posits an increase in the competitive firm's R&D. For *Conjecture 1*, with a large amount of structural competition, a firm perceives R&D competition as exogenous competitive pressure—that is, it perceives the pressure parametrically. The R&D competitive pressure increases with larger numbers of rivals. As long as R&D is profitable, the first conjecture is based on the expectation that the effect of an increase in such exogenous competitive pressure is to increase the firm's marginal value of doing R&D to improve on the anticipated state of the art for its product or process.

In contrast, for *Conjecture 2* with oligopoly, a firm perceives competitive R&D pressure as endogenous. In the displacement from the Nash equilibrium as the number of rivals increases, the firm knows its rivals will be adjusting their R&D. With oligopoly, greater competitive pressure again increases with the number of rivals. But in addition to increasing the marginal value of the firm's own R&D (because of the distribution-shifting impact of the increased competition), there is recognition of an additional effect (because of the value-shifting impact of the increased competition) as all rivals adjust their R&D. This additional effect reduces the marginal value of the firm's R&D and reduces its investment, other things being the same.

The difference between *Conjecture 2* and *Conjecture 1* comes because *Conjecture 2* places more importance on the R&D-marginal-value dampening effect of competitive pressure. Greater competitive pressure stimulates R&D when it increases the marginal value of the firm's R&D. The stimulating effect results primarily because competitive pressure shifts the distribution over R&D's technical performance leftward. Also, greater competitive pressure has an R&D stimulating effect when there are positive spillovers from the R&D of rivals that increase the effectiveness of the firm's own R&D. *Conjectures 1* and *2* have the representative firms in both types of markets—structural competition and oligopoly—perceiving the stimulating effects of competitive pressure.

However, greater competitive pressure also dampens R&D when it decreases the marginal value of the firm's R&D. Such a dampening effect occurs primarily because greater R&D competitive pressure from rivals brings the expectation of greater competition in the post-innovation market from competing innovations that will lower the marginal value of the firm's R&D investment. *Conjecture 2* has the representative firm in an oligopoly perceiving the dampening effect of more competition and has that effect outweigh the stimulating effect. *Conjecture 1* assumes that the stimulating effect from increased marginal value of R&D will outweigh the dampening effect from decreased marginal value.

The different perceptions of competitors in a market with structural competition as contrasted with the perceptions of rivals in an oligopoly play a role in the different importance placed on the stimulating and dampening effects of competitive pressure. In the oligopoly, a firm recognizes that *all rivals* react to reach a new equilibrium after an original equilibrium is disturbed by more competition. The distribution-shifting role of greater competition is clearly perceived with both structural competition and with oligopoly. But with oligopoly, there is the recognition that in response to greater competitive pressure *all rivals* will adjust their R&D and that forces attention on the dampening effect from the value-shifting role of competitive pressure.

Result 6 Total R&D investment could rise or fall as the number of competitors increases. *Result 5* implies that the equilibrium R&D investment for each firm will fall as the number of firms increases; however, the total investment in the market could rise given the set of conditions in *Result 5*. Even given that the firm's R&D falls, the total R&D will rise if a firm's own R&D investments have effects on the marginal benefit of its R&D that are larger in absolute value than the marginal-benefit-dampening effects that result from the R&D investments of its rivals.

The change in total R&D investment as the number of firms increases will be:

$$\frac{\Delta\,(nR)}{\Delta n} = \frac{n\Delta R + R\Delta n}{\Delta n} = n\frac{\Delta R}{\Delta n} + R. \tag{14}$$

Then, using (13) and (14), the sign for the change in total investment as the number of firms increases will be determined by the sign of:

$$n\frac{\delta R}{\delta n} + R = R\left(1 - \frac{n\Phi}{\Psi + (n-1)(\Phi)}\right). \tag{15}$$

Given *Result 5*, the sign of the ratio within the parentheses is positive, since we have seen that both its numerator and its denominator are negative given the circumstances for *Result 5* where each firm's R&D falls as the number of rivals increases. Then, the sign for the expression in Eq. (15) will be positive if:

$$|\Psi| > |\Phi|. \tag{16}$$

Condition (16) will hold if a firm's marginal benefit from its own R&D investments falls faster with the increase in those investments than with the R&D investments of its rivals. Intuitively, the individual firm's equilibrium investment is then driven more by the effects of its own R&D than by the value-eroding effects of its rivals' R&D. If those value-eroding effects of rivals' investments dominate, then as the number of competitors expands, each individual firm's marginal benefit schedule shifts downward sufficiently to reduce the sum of the optimal investments in the equilibrium.

All of the foregoing results hold given the number of R&D rivals in the Nash equilibrium; they hold whether or not there is a free-entry Nash equilibrium. Additionally, if there is a free-entry Nash noncooperative equilibrium in R&D investment, the number of firms n will increase to n^* where for the ith firm in Nash equilibrium the expected profit is greater than or equal to zero and would be less than zero if another firm entered (Loury 1979; Lee and Wilde 1980):

$$(E_i - R_i) \geq 0 | n = n^* \quad \text{and} \quad (E_i - R_i) < 0 | n = n^* + 1 \tag{17}$$

5 Discussion

The theory shows that greater competition can have a different effect on a firm's R&D investment when the firm perceives exogenous R&D competitive pressure as contrasted with when the firm perceives an equilibrium strategy combination given interactive rivalry. With exogenous competitive pressure, the firm takes the amount of competitive pressure as a parameter of its environment. With equilibrium strategy combinations, when the firm reacts to the competitive pressure, we do not have equilibrium unless—given the firm's response—the other firms creating the competitive pressure do not want to change their behavior.

Conjecture 1 hypothesizes that with exogenous competitive pressure, greater competition will cause firms to increase their R&D given that R&D is profitable at all. That is the classic prediction of many models beginning with the work of Scherer (1967) and Barzel (1968). However, as *Result 5* shows, for firms in an interactive equilibrium, given the Schumpeterian condition (S-2), greater competitive pressure will cause firms to decrease their R&D investments. *Conjecture 2* hypothesizes that such a result might actually occur in some markets. The intuition is that an oligopolist, unlike a firm in an industry with greater structural competition, will recognize that its rivals react to its R&D investment, and their R&D will lessen the marginal value of its investment.

At the Nash equilibrium, each firm is maximizing given the R&D of rivals, but each firm will recognize that if the equilibrium is displaced by more competition, its rivals will react and adjust their R&D, and it is the negative effect of rivals' R&D on the marginal value of the firm's own R&D (because of the anticipation of more competition in the post-innovation market) than can outweigh the positive effect (because of the distribution-shifting effect of rivals' R&D). That, however, need not be so. Importantly, not only may the conditions for *Conjecture 2* not hold, but the conditions for *Conjecture 1* may not hold. In the case of exogenous competitive pressure for firms in more competitively structured markets, competition's role as a value-shifting "Z" variable could outweigh its role as a distribution-shifting "X" variable, a possibility that is overlooked in the numerous contributions that reach the conclusion that the effect of competition on R&D is captured by *Conjecture 1*. The predictions encapsulated in *Conjecture 1* and *Conjecture 2* are indeed just conjectures.

Conjectures 1 and 2 together do not imply the often anticipated inverted-U relation (Gilbert (2006)) between competition and R&D investment. With the inverted U, greater competition increases R&D up to a point, and then R&D falls with additional competition. In contrast, *Conjectures 1* and 2 imply the predicted effect is a U relation between competition and R&D for the representative firm. From low levels of structural competition, greater competition *decreases* R&D given that firms perceive the interactive rivalry and Nash noncooperative strategy combinations occur. Schumpeter's prediction that larger more dominant firms with market power are the engines of progress (Schumpeter 1942, p. 106) obtains. But, at higher levels of structural competition, firms perceive exogenous competitive pressure, and greater competition *increases* R&D. Moreover, *Result 5* predicts the convexity of the U relation because the competition-induced reduction in R&D for the representative firm facing interactive rivalry decreases as the number of firms in the noncooperative equilibrium increases.[25]

[25] The U relation, rather than the conventional inverted-U relation, is found in Scott (2003) for U.S. industrial firms' investments in environmental R&D, where environmental R&D is any R&D with the goal of introducing innovations to reduce pollution.

As Gilbert (2006) emphasizes, researchers have found evidence pointing in many different directions. Sects. 2, 3, and 4 predict the great variety of empirical results in the large literature testing the Schumpeterian hypothesis about industrial organization and innovative activity. Gathering the differing empirical results into groups of similar results might be possible by using the simple dichotomy between exogenous competitive pressure versus interactive rivalry with endogenously determined non-cooperative equilibrium strategy combinations. Samples grouped by that dichotomy might show similar results—and results that do not vanish when other controls are added (Gilbert 2006, pp. 190–191)—for the effect of competitive pressure on R&D once other important variables have been controlled.[26]

Appendix: Proofs for Lemmas 1 through 5

Throughout the paper, f is assumed to have continuous first and second derivatives.

Lemma 1 $\int \frac{\partial f}{\partial \alpha} dx = 0.$

Proof $\int \frac{\partial}{\partial \alpha} f(x;\alpha)dx = \frac{\partial}{\partial \alpha} \int f(x;\alpha)dx = 0$ since $\int f(x;\alpha)dx = 1$ both before and after the change in α.

Lemma 2 $\int \frac{\partial^2 f}{\partial \alpha^2} dx = 0.$

Proof $\int \frac{\partial^2 f(x;\alpha)}{\partial \alpha^2} dx = \frac{\partial}{\partial \alpha} \int \frac{\partial}{\partial \alpha} f(x;\alpha)dx = 0$ by the definition of a derivative, since by Lemma 1 the function $\int \frac{\partial}{\partial \alpha} f(x;\alpha)dx$ is always equal to zero regardless of α's value.

Lemma 3 *With* $b(x) > 0$, *and* $b'(x) > 0$, *then* $\int b(x) \frac{\partial}{\partial \alpha} f(x;\alpha)dx > 0.$

Proof As illustrated in Fig. 1, for small $\Delta\alpha$ with the distribution-shifting effect shown in the figure, $x < \hat{x} \Rightarrow \partial f/\partial \alpha < 0$; $x > \hat{x} \Rightarrow \partial f/\partial \alpha > 0$. As $\Delta\alpha$ shrinks to the limiting case of the partial derivative, $\hat{x}$ approaches the modal value of x. Then, $\int \frac{\partial}{\partial \alpha} f(x;\alpha)dx = 0 = \int_{x<\hat{x}} \frac{\partial}{\partial \alpha} f(x;\alpha)dx + \int_{x>\hat{x}} \frac{\partial}{\partial \alpha} f(x;\alpha)dx$. Further,

$$\left| \int_{x<\hat{x}} b(x) \frac{\partial}{\partial \alpha} f(x;\alpha)dx \right| < \int_{x>\hat{x}} b(x) \frac{\partial}{\partial \alpha} f(x;\alpha)dx \Rightarrow \int b(x) \frac{\partial}{\partial \alpha} f(x;\alpha)dx > 0.$$

Lemma 4 $\int V(\frac{\partial f}{\partial \alpha} \frac{\partial^2 \alpha}{\partial X_i \partial R})dx = \frac{\partial^2 \alpha}{\partial X_i \partial R} \int V(\frac{\partial f}{\partial \alpha})dx$ *has the sign of* $\partial^2 \alpha / \partial X_i \partial R$.

Proof The result follows from Lemma 3 and $V(x) > 0,\ V'(x) > 0.$

Lemma 5 $\int V(x) \frac{\partial^2 f}{\partial \alpha^2} dx < 0.$

[26] The examples—the relative speed of a computer, resolution of a video display, mileage per unit of fuel for an automobile, or strength and durability of a building material—given earlier could be expanded and juxtaposed with the different kinds of industries and observations about the hypothesized differences in R&D behavior.

Proof For small $\Delta\alpha$, with $\doteq$ meaning "is approximately equal to" and observing that if the derivative exists the approximation can be made as fine as desired by shrinking $\Delta\alpha$,

$$\int V(x)\frac{\partial^2 f}{\partial\alpha^2}dx \doteq (1/(\alpha_2-\alpha_1))\left(\int V(x)\frac{\partial}{\partial\alpha}f(x;\alpha_2)dx - \int V(x)\frac{\partial}{\partial\alpha}f(x;\alpha_1)dx\right).$$

Further, $\int V(x)\frac{\partial}{\partial\alpha}f(x;\alpha_1)dx > \int V(x)\frac{\partial}{\partial\alpha}f(x;\alpha_2)dx > 0$ by $V(x) > 0,\ V'(x) > 0$, Lemma 3, and $V''(x) < 0$. (Note that in evaluating the integrals, $\hat{x}|\alpha_2 > \hat{x}|\alpha_1$.)

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References

Baldwin, W. L., & Scott, J. T. (1987). *Market structure and technological change*. Chur; London; Paris; New York: Harwood Academic Publishers.

Barzel, Y. (1968). Optimal timing of innovations. *Review of Economics and Statistics, 50*, 348–355.

Chamberlin, E. H. (1929). Duopoly: Value where sellers are few. *Quarterly Journal of Economics, 44*, 63–100.

Gilbert, R. (2006). Looking for Mr. Schumpeter: Where are we in the competition-innovation debate? In A. B. Jaffe, J. Lerner, & S. Stern (Eds.), *Innovation policy and the economy* (Vol. 6, pp. 159–215). Cambridge, Massachusetts: MIT Press.

Kohn, M., & Scott, J. T. (1982). Scale economies in research and development: The schumpeterian hypothesis. *Journal of Industrial Economics, 30*, 239–249.

Kreps, D. M. (1990). *Game theory and economic modelling*. Oxford; New York: Oxford University Press.

Lee, T., & Wilde, L. L. (1980). Market structure and innovation: A reformulation. *Quarterly Journal of Economics, 94*, 429–436.

Loury, G. C. (1979). Market structure and innovation. *Quarterly Journal of Economics, 93*, 395–410.

Martin, S. (2002). *Advanced industrial economics* (2nd ed.). Oxford: Blackwell Publishers.

Scherer, F. M. (1967). Research and development resource allocation under rivalry. *Quarterly Journal of Economics, 81*, 359–394.

Schumpeter, J. A. (1942). *Capitalism, socialism, and democracy*. New York: Harper.

Scott, J. T. (2003). *Environmental research and development: US industrial research, the clean air act and environmental damage*. Cheltenham, UK; Northampton, Massachusetts: Edward Elgar Publishing.

Scott, J. T. (2008). The national cooperative research and production act. In W. D. Collins (Ed.), *Issues in competition law and policy* (Vol. 2, pp. 1297–1317). Chicago: American Bar Association.