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Collective labor supply with many consumption goods

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Abstract We extend Chiappori's (J Polit Econ 100:437–467, 1992) standard, 'collective' model of labor supply to the case of several consumption goods. We show that more robust estimates obtain. Moreover, individual demands for each commodity, although unobservable, can be recovered up to an additive constant. In particular, the impact of changes in wages, non labor income or distribution factors on *individual* consumption patterns can be identified even though no individual consumption is observed.

Keywords Household · Labor supply · Collective model · Identification

1 Introduction

1.1 Collective models of labor supply

An important literature has been devoted recently to empirical analysis of household labor supplies. Many of these works adopt the so-called 'collective' framework, whereby household member each have their own preferences and reach Pareto efficient agreements. The standard version of the model, introduced by Chiappori (1988, 1992) and adopted by most studies, consider a two-person household in a three-commodity setting (each member's leisure and an Hicksian composite good); the consumption good is privately consumed, and leisure is exclusive, in the sense that leisure of each member only enters the person's own utility function.[1]

[1] See also Grossbard-Shechtman (2003).

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In such a framework, efficiency has a simple translation. Specifically, efficient decisions can always be considered as stemming from a two-stage process. In stage one, agents agree on (or bargain over) a *sharing rule* that defines the transfers between members. At stage two, each agent chooses her consumption and labor supply subject to the budget constraint defined by the sharing rule. Such a process is always efficient, whatever the particular sharing rule at stake. This result provides a nice dichotomy between efficiency requirements (which are satisfied by the mere existence of a sharing rule) and bargaining issues (which are paramount in the choice of a particular sharing rule).

1.2 Identifiability and distribution factors

A natural concern, in this context, relates to what is called the *identifiability* problem[2], which can be stated as follows: Is it possible, from the sole observation of household behavior (i.e., here, individual labor supplies as functions of wages and non labor income), to recover the underlying structure, i.e. preferences and the sharing rule? Chiappori (1992) shows that the answer is positive. The sharing rule is identifiable from labor supplies up to an additive constant; for each choice of the constant, preferences are exactly (ordinally) identified. Moreover, the constant is welfare irrelevant: any welfare judgment formulated for a particular choice of the constant will remain relevant for any other choice.

This result has been extended by Chiappori et al. (2002; from now on CFL) to allow for *distribution factors*. These are defined as variables that influence group behavior, but only through their impact on the decision process (i.e., they affect neither the shape of preferences nor the budget constraint). Think, for instance, of the choices as resulting from a bargaining process. Typically, the outcomes will depend on the members' respective bargaining positions; hence, any factor of the group's environment that may influence these positions (EEPs in McElroy's (1990) terminology) potentially affects the outcome. Such effects are of course paramount, and their relevance is not restricted to bargaining in any particular sense. In general, group behavior depends not only on preferences and budget constraint, but also on the members' respective 'power' in the decision process. Any variable that changes the powers may have an impact on observed household behavior.

Individual non labor incomes provide a typical example. According to standard models of household behavior, based on a 'unitary' representation (in which a single utility function is maximized under budget constraint), household demand must satisfy income pooling; i.e., only total (non labor) income may matter, not its individual components. Empirically, however, this prediction has been proved to be largely counterfactual. Early empirical studies of labor supply had already recognized that the individual non labor income or each spouse had different impact on labor supply (see for instance Cain and Dooley 1976). More recently, an influential paper by Thomas (1990), based on Brazilian cross-sectional data, finds that the relative share of non labor income coming from the wife has a very significant impact on the health status of children within the household. From a

[2] See Chiappori and Ekeland (2009) for a precise discussion.

collective viewpoint, such a ratio is a typical distribution factor: it does not alter the budget constraint (which only depends on total income), but it may (and typically does) affect the members' respective bargaining positions. The literature on 'targeting' provides related and very convincing examples of distribution factors: their common finding is that the *identity* of the recipient of a benefit has a considerable importance on the outcome, which is totally incompatible with the income pooling property. Lundberg et al. (1997) present quasi-experimental evidence based on a reform of the UK child public support system in April 1977, which resulted in the benefit being paid to the mother; they show that this shift 'from wallet to purse' significantly increased the ratio of expenditures on children's clothing and women's clothing (both relative to men's clothing). Duflo (2000) has derived related conclusions from the analysis of a reform of the South African social pension program that extended the benefits to a large, previously not covered black population. She finds that the recipient's gender is of considerable importance for the consequences of the transfers on children's health. All in all, people working in the field of public policy have long recognized that paying a benefit to the wife makes a difference—but only recently has this intuition been introduced in models of household behavior. From a collective perspective, the technical translation of these intuitions is straightforward. On the one hand, total (non labor) income enters the budget constraint (just as in a unitary model), and possibly the sharing rule. On the other hand, the respective role of *individual* contributions can be taken into account by constructing a corresponding distribution factor (generally the fraction of total non labor income coming from one of the spouses), and allow the sharing rule to *also* depend on it. Then an increase in her non labor income (say, because a benefit is now being paid to her) affects *both* total non labor income and the distribution factor. The first impact is the same as for any other increase in income; the second, on the contrary, is totally specific.

Other distribution factors have been used in the empirical literature. CFL use the state of the marriage market, as proxied by the sex ratio by age, race and state, and the legislation on divorce as particular distribution factors affecting intrahousehold decision process and thereby its outcome, i.e. labor supplies.[3] On the theoretical side, they show how the use of distribution factors allows a simpler and more robust identification of collective models of labor supply. Empirically, they find, indeed, that factors more favorable to women significantly decrease (resp. increase) female (resp. male) labor supply, and they compute the implicit impact on intrahousehold allocation of resources. Using similar tools, Oreffice (2007) concludes that the legalization of abortion had a significant impact on intrahousehold allocation of power. Grossbard-Shechtman and Neuman (1988) refer to ethnicity, while Rubalcava and Thomas (2000) use the generosity of single parent benefits and reach identical conclusions. Thomas, Contreras, and Frankenberg(1997), using an Indonesian survey, show that the distribution of wealth by gender at marriage—another candidate distribution factor—has a significant impact on children health in those areas where wealth remains under the contributor's control.[4]

3 See also Grossbard-Shechtman and Neideffer (1997) and Grossbard and Amuedo-Dorantes (2007).

4 See also Galasso (1999) for a similar investigation.

1.3 Collective labor supply with many commodities

A clear limitation of these works is the Hicksian composite good assumption. Quite often, more detailed information is available; household consumption of various commodities may be recorded. In many data sets, for instance, household expenditures on food, clothing, services, transportation, alcohol, tobacco and many other goods are available. Most of the time, however (with the possible exception of clothing), *individual* consumptions are unknown; only the household's total demand for each commodity is recorded.

This raises two questions. First, is it possible to use this additional information to get more robust estimates of preferences and the sharing rule? For instance, if data tell us that an increase in male wage tends to correlate with a surge in the household's demand for alcoholic beverages, can this fact shed more light on how the additional income is divided between spouses? Second, can the theory help assessing individual consumptions of the various commodities, especially those for which only aggregate demand is observed? Is it possible, for instance, to infer from such data the respective income elasticities of the husband's and the wife's demand for food, even though the spouses' individual food consumptions are not observed?

The goal of the present paper is to investigate these questions. We show that in a framework in which consumptions are all private,[5] both answers are positive. The observation of household consumption in several commodities generate additional, overidentifying restrictions that increase the robustness of the estimations. Moreover, and perhaps more surprisingly, individual demands for each commodity can be recovered (up to an additive constant). It is thus possible to analyze how given changes in wages, non labor supply and distribution factors affect each person's consumption patterns, even though these individual consumptions are not directly observable.

1.4 Existing literature

The identifiability issue in a collective framework has recently attracted renewed attention. Several perspectives have been adopted in the literature. Browning and Chiappori (1998) and Chiappori and Ekeland (2009) consider a general setting in which all prices vary, so that demand functions can be observed. They show that while the most general setting is not identifiable, simple restrictions are sufficient to guarantee (generic) identifiability of the collective indirect utilities (which are needed to generate welfare assessments). Namely, it is sufficient that (1) each commodity is either purely private or public (which excludes consumption externalities for private goods), and (2) for each agent, there exists one commodity (at least) that the agent does not consume. Bourguignon, Browning and Chiappori (2008) consider an alternative polar case, in which prices do not vary; the econometrician only observes aggregate household consumption as a function of total household income and one distribution factor at least. Assuming that

[5] The case of public consumptions is considered in a companion paper, Chiappori and Ekeland 2009, and a revealed preferences perspective is provided by Cherchye, De Rock and Vermeulen (2010).

commodities are privately consumed, they show that, surprisingly enough, individual consumptions are generically identifiable up to an additive constant even without exclusion restrictions, provided that the number of commodities is at least three. Blundell et al. (2007) extend this identification result to a collective model of labor supply in which leisures are private, but the (unique) consumption good is public. Donni (2007) shows that when only one labor supply (say, the wife's) exhibits significant variations, structural elements of the decision process, such as individual preferences or the rule that determines the intra-household distribution of welfare, can still be identified provided that household demand for at least one commodity, together with the wife's labour supply, is observed. Finally, Cherchye, De Rock and Vermeulen (forthcoming) show that in a revealed preferences framework (where demand is assumed to be observed for only a finite set of price-income combinations) one can derive bounds on the sharing rule without any exclusive goods restrictions.

The present paper, which considers private consumption of multiple goods, is a clear complement of the previous results. It considers a multi-commodities labor supply framework in which some prices (individual wages) and the corresponding labor supplies vary, while others (commodity prices) do not. It focuses on models with private goods only.[6] Finally, it considers the extreme case in which *no* individual consumption is observed, which differentiates it from Donni. The paper thus fills a gap in the existing literature.

Finally, several articles have been devoted to empirical estimations of the collective model of labor supply.[7] Using a collective approach to analyze intra household allocation patterns in the UK over two decades, Lise and Seitz (forthcoming) argue that the standard approach largely underestimates the level of inequality between individuals, but overestimates its increase during the 80s. Recently, Kapan (2009) has investigated the consequences of a drastic change of the common laws governing divorce in the UK; he shows that labor supply of women who directly benefit from the reform decreases, suggesting larger transfers from the husband. Our results suggests that these works could be extended to look more carefully at the structural shifts in individual consumptions patterns arising from these evolutions.

2 The model

We consider a two-person household in a $(n + 2)$-good economy. For $s = A, B$, let L^s denote member s's leisure (with $0 \leq L^s \leq 1$), $h^s = 1 - L^s$ her labor supply, and

[6] It is interesting to note that the vast majority of empirical papers devoted to household labor supply actually use a private good version of the model; in that sense, this paper is in line with a common practice in the literature. Note, however, that the results derived below could be extended to the case in which public goods are present, through the notion of *conditional sharing rule*(see for instance Chiappori and Ekeland 2009).

[7] Among earlier contributions, one can mention Fortin and Lacroix (1997), Dauphin and Fortin (2001), Vermeulen (2005), Dauphin (2003), Donni (2003, 2007, 2009), Couprie (2004), Bargain et al. (2006), Blundell et al. (2007) among many others.

C_i^s her consumption of commodity i. The framework is cross sectional, in the sense that prices of the consumption goods are constant over the sample; therefore we normalize these prices to be equal to 1. However, different households face different wages (w_A, w_B) and non labor income y. Let $\mathbf{z}$ denote a vector of K distribution factors, with $K \geq 1$. We assume that these factors are behavior-relevant, in the sense that $\partial L^s / \partial z_k \neq 0$ almost everywhere ($s = A, B; k = 1, ..., K$). Finally, we assume that preferences are either egoistic (with utilities of the form $u^s\left(L^s, C_1^s, \ldots, C_n^s\right)$, $s = A, B$) or 'caring' (then the form is $W^s\left[u^A\left(L^A, C_1^A, \ldots, C_n^A\right), u^B(L^B, C_1^B, \ldots, C_n^B)\right]$, $s = A, B$).

The following result is an immediate generalization of the notion of sharing rule:

Proposition 1 Assume that an allocation $\left(L^A, C_1^A, \ldots, C_n^A, L^B, C_1^B, \ldots, C_n^B\right)$ is efficient. Then there exists a sharing rule ρ, and efficiency is equivalent the two individual programs:[8]

$$\begin{gathered} \max_{\left(L^A, C_1^A, \ldots, C_n^A\right)} u^A\left(L^A, C_1^A, \ldots, C_n^A\right) \\ subject/:to \\ \sum_i C_i^A + w_A L^A \leq w_A + \rho, \\ 0 \leq L^A \leq 1, \end{gathered} \tag{1}$$

and

$$\begin{gathered} \max_{\{L^B, C^B\}} u^B\left(L^B, C_1^B, \ldots, C_n^B\right) \\ subject/:to \\ \sum_i C_i^B + w_B L^B \leq w_B + (y - \rho), \\ 0 \leq L^B \leq 1, \end{gathered} \tag{2}$$

Proof This is a standard consequence of the second welfare theorem (see Chiappori 1992).

Note that now ρ may be negative or larger than one, since one member may receive all non labor income plus part of the spouse's labor income.

Three remarks can be made at this point:

- The sharing rule ρ may (and in general will) be an arbitrary function $\rho(w_A, w_B, y, \mathbf{z})$ of wages, non labor income and distribution factors. However, our assumptions imply that ρ *cannot* depend on the agents' total labor income, $w_i(1 - L^i)$. Indeed, efficiency precludes a person's allocation to depend on an

[8] In what follows, we shall assume for simplicity that only one distribution factor is available; if not, the argument is similar but additional, proportionality conditions must be introduced.

endogenous variable such as the labor supply of this person (or of her spouse, for that matter). The intuition is that such a link would act as a tax (or a subsidy) that would distort the price of leisure faced by the agents, thus violating the efficiency assumption.

- Also, ρ is the part of total non labor income *allocated to* member A as an outcome of the decision process. This should be carefully distinguished from A's *contribution* to household non labor income (although the two are obviosuly related). If A's contribution (say, as a share of total non labor income) matter, it is as a distribution factor that may influence the allocation process. In other words, if non labor income comes either from A (denoted y^A, and representing for instance return on A's capital) or from B (denoted y^B and representing, say, a benefit paid exclusively to B), so that $y = y^A + y^B$, then A's part of total expenditures, denoted ρ, may depend (among other things) on y^A or on the ratio y^A/y—just as it may depend on any relevant distribution factor. But it needs *not*-and in general will not—be equal to y^A.
- Finally, it should be mentioned that the result can be extended to the presence of public goods, and actually even to household production; then the sharing rule is defined conditionally on the level of public good. Such an extension is outside the scope of the present paper; the interested reader is referred to Blundell, Chiappori and Meghir (2005), Chiappori and Ekeland (2009) and Browning et al. (2010, Chapter 5) for a detailed presentation.

In what follows, we assume moreover that the function ρ is smooth (i.e., continuously differentiable).

3 The basic result

We first state a regularity condition:

Condition R (regularity): *there exists at least one distribution factor z_k such that*

$$\frac{\partial L^A/\partial z_k}{\partial L^A/\partial y} \neq \frac{\partial L^B/\partial z_k}{\partial L^B/\partial y} \tag{3}$$

almost everywhere.

From a mathematical viewpoint, Condition R is 'generically' satisfied. From a more economic perspective, we can expect it to always hold for all distribution factors. In general, an increase in non labor income benefits both members; assuming leisure is a normal good, we therefore expect that $\partial L^A/\partial y > 0$ and $\partial L^B/\partial y > 0$. On the contrary, any distribution factor that is favorable to one member (in the sense that it increase this member's 'power' within the decision process) must, by definition, harm the other, so that $\partial L^A/\partial z_k$ and $\partial L^B/\partial z_k$ must have opposite signs. As long as the distribution factor is behavior-relevant, we therefore expect the ratios in (3) to be of opposite signs. Think, for instance, of z_k as being the ratio of female to total non labor income; then clearly a larger z_k favors the wife, so that its impact on her and his labor supply should be opposite.

Our main result is the following:

Proposition 2 Under condition R, it is possible:

1. to identify the sharing rule up to an additive constant, and
2. to identify individual demands for each commodity $i = 1, ..., n$ as functions of wages and non-labor income, again up to an additive constant.

Although this identifiability result does not allow to recover individual preferences (price variations would be needed for that), it still has an important (and somewhat surprising) implication: in a collective model of consumption and labor supply estimated on cross sectional data, it is possible to recover each person's consumption of each commodity (up to an additive constant); in particular, the income and wage elasticities of individual demands for each good are exactly identifiable.[9]

The proof is in two steps

Step 1: This is a direct extension of CFL. From (1), we know that

$$L^A = \Lambda^A(w_A, \rho(w_A, w_B, y, \mathbf{z})) \tag{4}$$

$$L^B = \Lambda^B(w_B, y - \rho(w_A, w_B, y, \mathbf{z})) \tag{5}$$

where Λ^s denotes the Marshallian demand for leisure corresponding to u^s. A simple derivation gives

$$\begin{aligned} \frac{\partial L^A}{\partial w_B} &= \frac{\partial \Lambda^A}{\partial \rho}\frac{\partial \rho}{\partial w_B}, \\ \frac{\partial L^A}{\partial y} &= \frac{\partial \Lambda^A}{\partial \rho}\frac{\partial \rho}{\partial y} \\ \text{and } \frac{\partial L^A}{\partial z_k} &= \frac{\partial \Lambda^A}{\partial \rho}\frac{\partial \rho}{\partial z_k} \text{ for all } k \end{aligned} \tag{6}$$

hence, assuming $\partial L^A / \partial z_k \neq 0$:

$$\frac{\partial L^A/\partial y}{\partial L^A/\partial z_k} = \frac{\partial \rho/\partial y}{\partial \rho/\partial z_k} \tag{7}$$

Clearly, similar conditions can be derived for B (with obvious notations):

$$\begin{aligned} \frac{\partial L^B}{\partial w_A} &= -\frac{\partial \Lambda^B}{\partial \rho^B}\frac{\partial \rho}{\partial w_A}, \\ \frac{\partial L^B}{\partial y} &= \frac{\partial \Lambda^B}{\partial \rho^B}\left(1 - \frac{\partial \rho}{\partial y}\right) \\ \text{and } \frac{\partial L^B}{\partial z_k} &= -\frac{\partial \Lambda^B}{\partial \rho^B}\frac{\partial \rho}{\partial z_k} \end{aligned} \tag{8}$$

[9] In particular, this conclusion generalizes the results derived by Browning et al. (1994) under the assumption of fully constrained labor supply.

so that

$$\frac{\partial L^B/\partial y}{\partial L^B/\partial z_k} = -\frac{1-\partial\rho/\partial y}{\partial\rho/\partial z_k} \tag{9}$$

For notational simplicity, let F_k^s denote the fraction $\frac{\partial L^s/\partial y}{\partial L^s/\partial z_k}$, $s = A, B$; note that F_k^s can be estimated as a function of (w_A, w_B, y, $\mathbf{z}$). Now, (7) and (9) can be solved in $\partial\rho/\partial z_k$ and $\partial\rho\ /\partial y$:

$$\frac{\partial\rho}{\partial y} = \frac{F_k^A}{F_k^A - F_k^B}$$
$$\frac{\partial\rho}{\partial z_k} = \frac{1}{F_k^A - F_k^B}$$

where $F_k^B - F_k^A \neq 0$ by condition R

We thus conclude that the partials of ρ with respect to income and distribution factor are identifiable. Moreover, a testable condition is generated by the cross differential restrictions; namely:

$$\frac{\partial}{\partial z_k}\left(\frac{F_k^A}{F_k^A - F_k^B}\right) = \frac{\partial}{\partial y}\left(\frac{1}{F_k^A - F_k^B}\right)$$

Finally, from Eqs. (6) and (8), we get:

$$\frac{\partial\rho}{\partial w_B} = \frac{\partial L^A/\partial w_B}{\partial L^A/\partial z_k}\frac{\partial\rho}{\partial z_k} = \frac{\partial L^A/\partial w_B}{\partial L^A/\partial z_k}\frac{1}{F_k^A - F_k^B} \quad \text{and}$$
$$\frac{\partial\rho}{\partial w_A} = \frac{\partial L^B/\partial w_A}{\partial L^B/\partial z_k}\frac{\partial\rho}{\partial z_k} = \frac{\partial L^B/\partial w_A}{\partial L^B/\partial z_k}\frac{1}{F_k^A - F_k^B}$$

Again, testable restrictions are generated by the cross differential restrictions:

$$\frac{\partial}{\partial z_k}\left(\frac{\partial L^A/\partial w_B}{\partial L^A/\partial z_k}\frac{1}{F_k^A - F_k^B}\right) = \frac{\partial}{\partial w_B}\left(\frac{1}{F_k^A - F_k^B}\right)$$
$$\frac{\partial}{\partial y}\left(\frac{\partial L^A/\partial w_B}{\partial L^A/\partial z_k}\frac{1}{F_k^A - F_k^B}\right) = \frac{\partial}{\partial w_B}\left(\frac{F_k^A}{F_k^A - F_k^B}\right)$$
$$\frac{\partial}{\partial z_k}\left(-\frac{\partial L^B/\partial w_A}{\partial L^B/\partial z_k}\frac{1}{F_k^A - F_k^B}\right) = \frac{\partial}{\partial w_A}\left(\frac{1}{F_k^A - F_k^B}\right)$$
$$\frac{\partial}{\partial y}\left(-\frac{\partial L^B/\partial w_A}{\partial L^B/\partial z_k}\frac{1}{F_k^A - F_k^B}\right) = \frac{\partial}{\partial w_A}\left(\frac{F_k^A}{F_k^A - F_k^B}\right)$$
$$\frac{\partial}{\partial w_B}\left(-\frac{\partial L^B/\partial w_A}{\partial L^B/\partial z_k}\frac{1}{F_k^A - F_k^B}\right) = \frac{\partial}{\partial w_A}\left(\frac{\partial L^A/\partial w_B}{\partial L^A/\partial z_k}\frac{1}{F_k^B - F_k^A}\right)$$

Step 2: Now consider the demand for commodity i. We have that $C_i = C_i^A + C_i^B$ where

$$C_i^A(w_A, w_B, y, \mathbf{z}) = \xi_i^A(w_A, \rho(w_A, w_B, y, \mathbf{z}))$$
$$C_i^B(w_A, w_B, y, \mathbf{z}) = \xi_i^B(w_B, y - \rho(w_A, w_B, y, \mathbf{z}))$$

where $\xi_i^s(w_s, \rho)$ is member s's Marshallian demand for good i. It follows that

$$\frac{\partial C_i}{\partial z_k} = \frac{\partial C_i^A}{\partial z_k} + \frac{\partial C_i^B}{\partial z_k} = \frac{\partial \xi_i^A}{\partial \rho}\frac{\partial \rho}{\partial z_k} - \frac{\partial \xi_i^B}{\partial \rho}\frac{\partial \rho}{\partial z_k}$$
$$\frac{\partial C_i}{\partial y} = \frac{\partial C_i^A}{\partial y} + \frac{\partial C_i^B}{\partial y} = \frac{\partial \xi_i^A}{\partial \rho}\frac{\partial \rho}{\partial y} + \frac{\partial \xi_i^B}{\partial \rho}\left(1 - \frac{\partial \rho}{\partial y}\right)$$

where $\frac{\partial \xi_s^i}{\partial \rho}$ denotes s's marginal propensity to consume. These two equations allow to identify the $\frac{\partial \xi_s^i}{\partial \rho}$, i.e. member s's marginal propensity to consume commodity i:

$$\frac{\partial \xi_i^B}{\partial \rho} = \frac{\partial C_i}{\partial y} - \frac{\partial \rho/\partial y}{\partial \rho/\partial z_k}\frac{\partial C_i}{\partial z_k}$$
$$= \frac{\partial C_i}{\partial y} - F_k^A \frac{\partial C_i}{\partial z_k}$$

and by the same token:

$$\frac{\partial \xi_i^A}{\partial \rho} = \frac{\partial C_i}{\partial y} - F_k^B \frac{\partial C_i}{\partial z_k}$$

Finally:

$$\frac{\partial C_i}{\partial w_A} = \frac{\partial C_i^A}{\partial w_A} + \frac{\partial C_i^B}{\partial w_A} = \left(\frac{\partial \xi_i^A}{\partial \rho} - \frac{\partial \xi_i^B}{\partial \rho}\right)\frac{\partial \rho}{\partial w_A} + \frac{\partial \xi_i^A}{\partial w_A}$$

gives

$$\frac{\partial \xi_i^A}{\partial w_A} = \frac{\partial C_i}{\partial w_A} - \left(\frac{\partial \xi_i^A}{\partial \rho} - \frac{\partial \xi_i^B}{\partial \rho}\right)\frac{\partial \rho}{\partial w_A}$$
$$= \frac{\partial C_i}{\partial w_A} - \frac{\partial C_i/\partial z_k}{\partial L^B/\partial z_k}\frac{\partial L^B}{\partial w_A}$$

and, similarly:

$$\frac{\partial \xi_i^B}{\partial w_B} = \frac{\partial C_i}{\partial w_B} - \frac{\partial C_i/\partial z_k}{\partial L^A/\partial z_k}\frac{\partial L^A}{\partial w_B}$$

which identifies the partials of ξ_i^A and ξ_i^B, hence the functions themselves up to an additive constant. Testable restrictions, here, come from the fact that $\frac{\partial \xi_i^A}{\partial \rho}$ and $\frac{\partial \xi_i^A}{\partial w_A}$ (resp. $\frac{\partial \xi_i^B}{\partial \rho}$ and $\frac{\partial \xi_i^B}{\partial w_B}$) are functions of ($w_A$, ρ) (resp. (w_B, $y - \rho$)). For instance, $\frac{\partial \xi_i^A}{\partial w_A}$ must be such that:

$$\frac{\partial\left(\frac{\partial \xi_i^A}{\partial w_A}\right)/\partial w_B}{\partial\left(\frac{\partial \xi_i^A}{\partial w_A}\right)/\partial z_k} = \frac{\partial\left(\frac{\partial C_i}{\partial w_A} - \frac{\partial C_i/\partial z_k}{\partial L^B/\partial z_k}\frac{\partial L^B}{\partial w_A}\right)/\partial w_B}{\partial\left(\frac{\partial C_i}{\partial w_A} - \frac{\partial C_i/\partial z_k}{\partial L^B/\partial z_k}\frac{\partial L^B}{\partial w_A}\right)/\partial z_k} = \frac{\partial \rho/\partial w_B}{\partial \rho/\partial z_k}$$
$$\frac{\partial\left(\frac{\partial \xi_i^A}{\partial w_A}\right)/\partial y}{\partial\left(\frac{\partial \xi_i^A}{\partial w_A}\right)/\partial z_k} = \frac{\partial\left(\frac{\partial C_i}{\partial w_A} - \frac{\partial C_i/\partial z_k}{\partial L^B/\partial z_k}\frac{\partial L^B}{\partial w_A}\right)/\partial y}{\partial\left(\frac{\partial C_i}{\partial w_A} - \frac{\partial C_i/\partial z_k}{\partial L^B/\partial z_k}\frac{\partial L^B}{\partial w_A}\right)/\partial z_k} = \frac{\partial \rho/\partial y}{\partial \rho/\partial z_k}$$

and similarly for $\frac{\partial \xi_i^B}{\partial w_B}$

$$\frac{\partial\left(\frac{\partial \xi_i^B}{\partial w_B}\right)/\partial w_A}{\partial\left(\frac{\partial \xi_i^B}{\partial w_B}\right)/\partial z_k} = \frac{\partial\left(\frac{\partial C_i}{\partial w_B} - \frac{\partial C_i/\partial z_k}{\partial L^A/\partial z_k}\frac{\partial L^A}{\partial w_B}\right)/\partial w_A}{\partial\left(\frac{\partial C_i}{\partial w_B} - \frac{\partial C_i/\partial z_k}{\partial L^A/\partial z_k}\frac{\partial L^A}{\partial w_B}\right)/\partial z_k} = \frac{\partial\rho/\partial w_A}{\partial\rho/\partial z_k}$$

$$\frac{\partial\left(\frac{\partial \xi_i^B}{\partial w_B}\right)/\partial y}{\partial\left(\frac{\partial \xi_i^B}{\partial w_B}\right)/\partial z_k} = \frac{\partial\left(\frac{\partial C_i}{\partial w_B} - \frac{\partial C_i/\partial z_k}{\partial L^A/\partial z_k}\frac{\partial L^A}{\partial w_B}\right)/\partial y}{\partial\left(\frac{\partial C_i}{\partial w_B} - \frac{\partial C_i/\partial z_k}{\partial L^A/\partial z_k}\frac{\partial L^A}{\partial w_B}\right)/\partial z_k} = \frac{\partial\rho/\partial y}{\partial\rho/\partial z_k}$$

Similar conditions apply to $\frac{\partial \xi_i^A}{\partial \rho}$ and $\frac{\partial \xi_i^B}{\partial \rho}$.

4 A parametric application

How can these restrictions be taken to data? Clearly, parametric approaches are convenient here. Consider, for instance, the functional form used in CFL:

$$\begin{aligned} h^A = & f_0 + f_1 \log w_A + f_2 \log w_B + f_3 y \\ & + f_4 \log w_A \log w_B + f_5 z_1 + f_6 z_2 + + f_7' a + \varepsilon_A \end{aligned} \tag{hA}$$

$$\begin{aligned} h^B = & m_0 + m_1 \log w_A + m_2 \log w_B + m_3 y \\ & + m_4 \log w_A \log w_B + m_5 z_1 + m_6 z_2 + m_7' a + \varepsilon_B \end{aligned} \tag{hB}$$

where: h^s denotes labor supply of member s (hence $h^s + L^s = 1$); z_1 and z_2 are two distribution factors; a is a $K-$ vector of preference factors, such as age and education of the two agents; the f_i 's and the m_i's, for $i = 1, \dots, 6$, are scalars; f_7' and m_7' are $K-$ vectors of parameters, and ε_A and ε_B are random shocks reflecting measurement errors, unobserved heterogeneity, etc. In addition, we take a similar form for commodity demands, namely:

$$\begin{aligned} C_i = & d_0^i + d_1^i \log w_A + d_2^i \log w_B + d_3^i y \\ & + d_4^i \log w_A \log w_B + d_5^i z_1 + d_6^i z_2 + + d_7'^i a + \varepsilon^i \end{aligned} \tag{Ci}$$

CFL show that the following conditions are necessary regarding labor supply:

$$\frac{m_4}{f_4} = \frac{m_5}{f_5} = \frac{m_6}{f_6}. \tag{10}$$

When they are satisfied, one obtains the following sharing rule:

$$\rho = \kappa(a) + \frac{m_5}{f_3 m_5 - m_3 f_5}\begin{pmatrix} f_3 y + f_5 z + m_1 \log w_A \\ + f_2 \log w_B + f_4 \log w_A \log w_B \end{pmatrix} \tag{11}$$

where $\kappa(a)$ is a constant.

Considering, now, commodity demand, we first have the standard distribution factors proportionality conditions:

$$\frac{m_6}{m_5} = \frac{d_6^i}{d_5^i} = \frac{f_6}{f_5} \tag{12}$$

If they are satisfied, then

$$\frac{\partial \xi_i^A}{\partial \rho} = d_3^i - \frac{m_3}{m_5} d_5^i$$

and

$$\frac{\partial \xi_i^A}{\partial w_A} = \frac{1}{w_A}\left(d_1^i - \frac{m_1}{m_5} d_5^i + \left(d_4^i - \frac{m_4}{m_5} d_5^i\right) \log w_B\right)$$

which cannot depend on w_B, hence an additional restriction:

$$\frac{m_4}{m_5} = \frac{d_4^i}{d_5^i} = \frac{f_4}{f_5} \tag{13}$$

If this condition is fulfilled, then simple identification gives

$$\xi_i^A(w_A, \rho) = \left(d_1^i - \frac{d_4^i}{f_4} m_1\right) \log w_A + \left(d_3^i - \frac{m_3}{m_4} d_4^i\right) \rho + d_0^i + \kappa^i(a)$$

and

$$\xi_i^B(w_B, y - \rho) = \left(d_2^i - \frac{d_4^i}{f_4} f_2\right) \log w_B + \left(d_3^i - \frac{d_4^i}{f_4} f_3\right)(y - \rho) - \kappa^i(a)$$

where $\kappa^i(a)$ is a constant.

To summarize:

- one can *simultaneously* estimate the system (hA), (hB), (Ci) under the constraints (10), (12), (13).
- then the sharing rule and individual consumptions of each commodities are identifiable (up to additive, welfare-irrelevant constants).

5 Conclusion

Most models of household labor supply use a three-commodity setting, in which consumption goods are aggregated into a single Hicksian composite. In a standard, unitary setting, this restriction of scope makes perfect sense. Within a collective setting, however, one can do much better. We show that individual demands for each commodity, although unobservable, can be recovered up to an additive constant. In particular, the impact of changes in wages, non labor income or distribution factors on *individual* consumption patterns can be identified even though no individual consumption is observed. This is a crucial property in view of policy applications. A change in policy generates changes in non labor incomes and (possibly) wages, and the impact of these changes on consumptions, labor supplies and welfare are typically the policy maker's main object of interest. The main message of this paper is that these changes can be exactly identified from existing data. This result should open the way for more precise and more general empirical applications.

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