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Incentives to cooperate and the discretionary power of courts in divorce law

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Abstract In this paper, we study how the uncertainty in the behavior of judges provides parents going to separate with incentives to cooperate. We introduce a model of cooperative bargaining to describe the behavior of parents whose preferences satisfy the characterization of risk averse/pessimistic types proposed by Yaari (1987, *Econometrica*, *55*, 95–116) in his Dual Decision Theory under Risk. The behavior of the judge is modeled in a simple manner: he is either supposed to follow a strict rule (we will say that he uses an imperative scale of alimony), or he may use discretion (he uses an indicative scale of alimony). The point is that for both parents the judgment represents an external opportunity to divorce—the disagreement point in negotiation. We show that the effective decision of parents (cooperation *versus* trial) depends on the specific structure of the costs and risks associated with divorce procedures, such that more uncertainty at trial increases the incentives to cooperate for risk averse parents. Finally, we give a characterization of the optimal degree of the judges' discretionary power required to maximize the parents' gains from negotiation.

Keywords Incentives to cooperate in divorce · Bargaining in divorce litigation with risk averse parties · Rule versus discretion in the settlement of divorces · Scale of child support

JEL Classification K41–J12

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1 Introduction

This paper studies the effects of discretionary power of judges in divorce litigations. The main question is to know if the probability of cooperative bargaining increases when judges makes decisions which appear as random for the parents. This problem is particularly acute in most common law and civil law countries concerned by the introduction of scales of child support. Considering that child support is the object of disputes in a great number of divorce cases, legislators are interested in evaluating whether alimonies have to be fixed by the imperium of the judge or if it could be efficient to impose precise rules of calculus. Some countries have already introduced scales of child support. Some of them have chosen indicative systems (Canada, United States...), but others like England have already decided to impose strict rules, so that the establishment of child support appears to be an administrative task more than a true court decision. Of course, the determination of child support is generally based on allocative (costs of children) and/or redistributive (resources of parents) grounds, but it seems important to consider the problem of cooperation between the two parents. It is surprising to note that this problem has not received much attention in the literature devoted to economics of the family even though it is an important question from a law and economics perspective.[1] Exceptions concern the effects of the content of divorce rules on the behavior of parents (e.g. Mechoulan, 2005 considering no-fault divorce). But the way the rules are enforced by the courts is not considered.

One can consider that the introduction of scales of child support could generate more transparency and more predictability. This means an improvement in parents' information in divorce cases. In other words, precise rules reduce information costs of litigants and adjudicators during the trial. The intuition is that when matters are more difficult to predict, parties' expectations over the outcome of the adjudication are more likely to diverge. As a result, parties to a dispute will tend to settle less often and the litigation rate will increase. In contrast, precise rules should facilitate cooperative settlement. Moreover, considering divorce law, it is important to note that parental cooperation is important not only during litigation, but also after the courts' decision because it could increase collection of child support payments and the participation of non-custodial parents in the education of children (Oldham & Melli, 2000). Empirical research confirms that divorce law negatively impacts children's welfare as a consequence of the reduction in monetary and time contributions of the non-custodial parent (Del Boca & Ribero, 2003). With visitation and direct expenditures, child support transfers fixed by the courts

[1] Surprisingly enough, many judges are not favorable to the introduction of such scales of alimony, arguing the existence of high auditing costs: respecting those scale would imply more work for them, more time for each case of divorce, in order to verify the adequacy between the case and the scale and so on... Thus, they would be prone to accept only indicative scales, keeping some degree of freedom (thus, some discretionary power) in the determination of the alimony, case by case.

have to be considered. The first intuition is that it could be important to limit the discretionary power of judges to facilitate cooperation between parties. However, from an analytical point of view, this is not so easy to demonstrate.

The economic analysis of legal dispute resolution generally explains the choice of the parties between cooperative (agreement) and non cooperative (judgement) solutions via strategic behaviors (Shavell (1982), Cooter, Marks, and Mnookin (1982), Bebchuck (1984)...). Informational asymmetries can explain why a great number of conflicts end up in trial even though a cooperative surplus exists. Using an approach in which models determine the probability that the conflict is resolved through trial rather than negotiation, it is possible to show that when informational asymmetries are reduced, cooperation increases. Therefore, in the case of divorce law, one could argue that the determination of precise guidelines for child support could reduce the asymmetries of information between parents.

We aim to challenge this argument. If informational asymmetries are of great importance in many circumstances, it is sometimes doubtful that informational problems uniquely occur (if at all) between the parties at trial. In divorce cases, for instance, it appears that neither party has an informational advantage with respect to the issue of a dispute settlement at trial. However, both are in a situation of uncertainty in the sense that the decisions of the parties depend on their subjective expectations regarding the decision of the judge. Such situations may occur for example when the legal rules are complex or ambiguous (Kaplow, 1995). Another example is provided by the cases where courts change their way of thinking, leading to an evolution of the *jurisprudence* that individuals have not yet understood because they hold on to expectations not fully updated (Viscusi 1995).

In this spirit, we suggest a new way of thinking about cooperative behavior in litigation games like divorce. While ignoring informational asymmetries between parties, we explicitly deal with another kind of informational problem resulting from the non observability of the judge's type or action by both parties, who can do nothing but assign an expectation regarding the judge's behavior. Moreover, in contrast with the assumption made in the literature, we assume that the parties at trial are no longer risk-neutral decision makers, but that their preferences are of a more general nature, an asumption in line with the axioms of the Dual Theory (DT thereafter) of Yaari (1987). Introducing this DT assumption is both meaningful and powerful for several reasons. In contrast to the usual purely subjectivist optimistic approach, it enables us to consider a context in which both parties have some objective information (probabilistic) with regard to the behavior of the judge that they may use in assigning their own individual beliefs. At the same time, while being in a strategic context in which the common knowledge assumption is required, the DT of Yaari provides us with sound, axiomatically founded but simple arguments to rationalize the disagreement in individuals' beliefs: these are not pure probabilistic decision weights, but they reflect the preferences of the individuals and specifically their risk attitude.

Section 2 presents a basic model of the discretionary power of judges. We show that with Yaari decision makers, the same information about the judge's behavior generates different beliefs with regard to the court's decision. Section 3 analyzes divorce as a cooperative game between parents. We compare strict rules versus discretion in the determination of child support. Demonstrating that the more uncertain the judge's behavior, the higher the incentives to cooperate, we prove that the uncertainty of the outcome at trial cannot *per se* explain the failure of cooperation between parents. On the contrary, it should *increase* the incentives to cooperate. Of course, this result is not inconsistent with redistributive goals in the sense that more uncertainty could imply higher amounts of child support. Finally, Section 4 discusses the optimal degree of discretion which may maximize the gains of cooperation for the parents. We show that the question of the optimal level of judge's discretion is a classical problem of risk sharing : probabilities have to be set so that the less risk averse parent bears a larger share of the risk. Section 5 concludes and suggests possible extensions of the paper.

2 A simple model of judges' discretionary power

2.1 The basic setup

In the following, we assume that the litigation between parents uniquely comes from the monetary aspects of the divorce (amount of child support paid by the non custodial parent—NG thereafter—to the custodial one—G).[2] Hence, we take as reference the case in which parents agree both on discontuing the marriage (they agree to divorce) and on child custody: one of the parents voluntarily gives up child custody. While at a first glance this assumption may appear troublesome, it is in fact both empirically reasonable and theoretically relevant. On the one hand, more than 50% of recent divorces in France are obtained through amicable settlements (joint demand and accepted demand), while in 85% of divorces, one of the parents gives up child custody at the beginning of the procedure. On the other hand, should the divorce legislation provide adverse incentives to cooperation even if the parents would like to reach an amicable divorce, then there is no reason to believe that this would not also be the case when they are at strife and agree neither on the custody of the children nor on the monetary settlement associated with the divorce.

The issue of settlement in divorce may be seen as a negotiation game with two possible outcomes for parents: either the negotiation is successful, and parents reach an agreement corresponding to the value of the monetary transfer between the NG parent and the G parent; or, it is not, and the

[2] In order to avoid the confusion with costs, we do not use the index C and NC (custodial and non custodial). Instead, we use G and NG for the presentation of the model (though inapropriate, the letters for "guardian" and "non guardian").

disagreement point is represented by the utility pair obtained by parents when they go to trial.

We focus here on two polar cases depending on the behavior of the judge. First, the parents may know that the judge will apply an imperative (i.e. legaly binding) scale: he can by no means deviate, and he is bound to strictly apply it. In such a case, the household dissolution, independently of the way it is obtained, is a non risky prospect for the parents: the amount of child support, whether being fixed by the parents themselves or by the judge, is known with a probability equal to 1. Moreover, we assume that it will always be paid, since there is no default of payment from the non custodial parent. In contrast, the law may no longer commit a judge to strictly follow a scale of child support, but gives him discretionary power in the choice of the allowance. Now the judge has the opportunity to ignore the scale, which is only seen as indicative.

More generally, discretion may be the result of at least four different features characterizing the decision context for the judge and parents, which are conceptually different but formally equivalent here:

1. The judge may sometimes be mistaken when facing some cases of divorce, and implicitly use the indicative scale of child support in an irrelevant manner. Nevertheless, his mistakes are independently distributed in time (uncorrelated).
2. The judge always strictly applies a scale of child support (he always uses a deterministic rule), but conditionaly on a private signal that he observes.[3] Thus, the behavior of the judge is perceived as perfectly discretionary (random) for the parents who do not observe the private signal, whereas it is purely deterministic for the judge.
3. Law is not complete (there exist unforeseen contingencies) and/or rules are complex to apply since they are tailored very precisely to acts, requiring from the judge a quality of information on the behavior of the parents more refined than is realistically feasible.
4. When deciding to divorce, the parents do not know which judge they will face; the latter may either be more favorable to the G parent or to the NG parent. To put things differently, the choice of the type of the judge is the result of an initial move of Nature, which is not observable by the parents. As a consequence, divorce settlement at trial appears as a risky procedure in contrast to cooperation.

[3] This view may help us in explaining that there exist at the same time both a kind of *intercohort* (between Courts) heterogeneity as well as an *intracohort* (specific to each Court) homogeneity in courts' decisions. The apparent heterogeneity in family judges's decisions as perceived by individuals or exhibited in panel or survey data would be explained by the existence of a commitment or discipline device between judges belonging to the same Tribunal. Such a commitment makes that the decision of a judge must be conform to a common rule internal to the Tribunal, while the law gives a great degree of freedom in the design of this discipline: as a result, there would exist a great homogeneity in the decisions of the judges operating in the same Tribunal, compatible with more heterogeneity with regards to those taken in different Tribunals; for studies concluding to the existence of a great homogeneity in the behavior of judges attached to a same Tribunal, see Ray (2003) and Jeandidier (2003).

To keep things simple, assume that the judge may choose between three levels of child support (corresponding to three actions: B, A, R): he follows the indicative schedule and sets b, or he sets a higher amount $a > b$, or finally he reduces the allowance with regards to the scale to $r < b$. The probabilities associated with each of these actions are respectively: p_b, p_a and p_r. Hence, a legaly binding scale of child support is a particular case of an indicative one where the judge chooses b in a deterministic manner (*i.e.* $p_b = 1$ and $p_a = p_r = 0$). We also introduce the following notations: c is the cost of a trial for both parents NG and G; δ is the amount of child support resulting from the direct bargaining between the parents. We assume that the costs incurred by the parents when they negotiate are nil.

Since for the parents everything goes as if the judge may randomly choose between three actions A, B or R, we have to deal with uncertainty in the representation of their preferences: we will assume that the parents' preferences satisfy the axioms of the Dual Theory of Yaari (1987),[4] which is a generalization of the more common place expected utility approach *à la* Von Neumann-Morgenstern.

2.2 Preferences: the dual theory of Yaari

The main advantage of Yaari's approach is that it allows us to maintain the technically tractable assumption of constant marginal utility of wealth, while at the same time capturing non neutrality to risk.

Definition 1 Assume that individual preferences satisfy the axioms of the Dual Theory of Yaari; then there exists a probability transformation function φ (continuous, increasing, and unique), with $\varphi(0) = 0$ and $\varphi(1) = 1$, such that for any feasible decision X corresponding to the random variable $(x_1, p_1; \ldots x_n, p_n)$, with $\sum_{i=1}^{n} p_i = 1$, and assuming without loss of generality $x_1 < \cdots < x_i < \cdots < x_n$, the functional representation of the individual's preferences is given by the following real-valued function:

$$V(X) = \sum_{i=1}^{n} \left(\varphi\left(\sum_{j=i}^{n} p_j \right) - \varphi\left(\sum_{j=i+1}^{n} p_j \right) \right) x_i$$

where $\varphi\left(\sum_{j=i}^{n} p_j\right) - \varphi\left(\sum_{j=i+1}^{n} p_j\right)$ is the individual's subjective likelihood of the outcome x_i; by definition, this is the difference between the transformation of two cumulative probabilities: the first one is associated with the event "the outcome is at least x_i" and the second one is associated with the event "the outcome is strictly larger than x_i".

[4] This axiomatic-based decision model is a particular case both of "Rank-Dependent-Expected-Utility" theories (Quiggin, 1982)) and of "Cumulative Prospect Theory" (Wakker & Tversky, 1993))—this last being a generalization of Kahneman and Tversky (1979)'s approach.

The basic meaning of the approach proposed by Yaari may be easily understood. To begin with, first notice that the value-function representing the preferences is nothing else but a weighted sum of outcomes $V(X) = \sum_{i=1}^{n} h_i x_i = E_{\varphi}(X)$, where the decision weights h_i are no more individual probabilities but subjective likelihoods. Secondly, observe that:

$$V(X) = x_1 + \sum_{i=2}^{n-1} \varphi\left(\sum_{j=i}^{n} p_j\right)(x_{i+1} - x_i)$$

i.e. the individual proceeds to a pessimistic evaluation of his prospects. To evaluate a decision, he considers the most unfavorable outcome x_1, and then evaluates his chances to be better off, weighting each additional increase in his outcome $(x_{i+1} - x_i)$ with a decision weight $\varphi\left(\sum_{j=i}^{n} p_j\right)$ defined as the subjective transformation of the probability to be better off ($\varphi\left(\sum_{j=i}^{n} p_j\right) = P(X \geq x_i)$).

Interestingly enough, despite a constant marginal utility of wealth, the decision maker is no longer risk neutral. It can be shown that risk aversion (respectively risk seeking) in the usual sense of aversion to any Mean Preserving Spread *à la* Rothschild & Stiglitz, is equivalent to the concept of pessimism (optimism) which may be characterized as follows:[5]

Theorem 2 (Yaari (1987))

(i) *The decision maker is pessimistic iff* $\varphi''>0, \forall p \in [0,1]$. *Moreover*, $\varphi''>0 \Rightarrow \varphi(p)<p, \forall p \in [0,1]$ *and* $E_{\varphi}(X) < E(X)$.

(ii) *In contrast, the decision maker is optimistic iff* $\varphi''<0, \forall p \in [0,1]$. *Moreover*, $\varphi''<0 \Rightarrow \varphi(p)>p, \forall p \in [0,1]$ *and* $E_{\varphi}(X) > E(X)$.

This means that when a decision maker is pessimistic/risk-averse (respectively optimistic/risk-seeking), he underestimates (overestimates) any probability value, and assesses a value to the prospect which is smaller (larger) than its mathematical expectation.

2.3 Individual assessments of the outcome at trial

Assume that parents are Yaari-decision makers, with parent NG having a probability distortion function denoted $f(p)$, while parent G has a probability distortion function $g(p)$, where f and g are supposed to be twice differentiable.

It is easy to see how the parents assess the likelihoods associated with the outcomes corresponding to the decisions "go to trial" and "negotiate". Negotiation is not a risky decision (the value of the transfer is known with a probability equal to 1). In contrast, trial is risky, since the judge may choose between three actions A, B or R, such that the child support is a random

[5] In decision theory, this concept of pessimism is also called "probabilistic risk aversion": word by word, it means that individuals do not like probabilities mixtures. This alternative view highlights the interest and relevance of our approach, since probabilities mixtures may be produced by the existence of noise in the observable information of individuals.

variable $X = (p_b, b;\ p_a, a;\ p_r, r)$ whose mathematical expectation is $E(X) = p_b \cdot b + p_a \cdot a + p_r \cdot r$.

The following table shows the outcome and the associated likelihood for the NG parent (left table; parent NG face the prospect $-X$, since he pays the child support) and the G parent (right table; parent G faces the prospect X), depending on the decision of the judge to apply A, B or R, with the assumption $r < b < a$:

Outcome	Likelihood
$-a-c$	$1-f(p_b+p_r)$
$-b-c$	$f(p_b+p_r)-f(p_r)$
$-r-c$	$f(p_r)$

Outcome	Likelihood
$r-c$	$1-g(p_b+p_a)$
$b-c$	$g(p_b+p_a)-g(p_a)$
$a-c$	$g(p_a)$

- The most unfavorable outcome for NG would be to pay a: the likelihood weight associated with this event is $1-f(p_b+p_r)$; the intermediate result (pay b) obtains the weight $f(p_b+p_r)-f(p_r)$; finally, the most favorable outcome is coupled with the weight $f(p_r)$.
- For parent G, the most unfavorable outcome is when he receives r: he gives this event the weight $1-g(p_a+p_b)$; the intermediate result (receive b) obtains the likelihood $g(p_a+p_b)-g(p_a)$, while the weight of the most favorable event (obtain a) is $g(p_a)$.

This reveals that parents do not have the same beliefs over the consequences of the judge's decisions:

Judge plays	Belief for NG	Belief for G
A	$1-f(p_b+p_r)$	$g(p_a)$
B	$f(p_b+p_r)-f(p_r)$	$g(p_b+p_a)-g(p_a)$
R	$f(p_r)$	$1-g(p_b+p_a)$

and everything goes as if each parent assesses his own subjective expectation of the outcome at trial, respectively for the NG parent[6] and the G parent:

$$E_{ng}(X) = (1-f(p_b+p_r))a + (f(p_b+p_r)-f(p_r))b + f(p_r)r$$
$$E_g(X) = g(p_a)a + (g(p_b+p_a)-g(p_a))b + (1-g(p_b+p_a))r$$

Note: It is easily seen that $E_{ng}(X) \neq E_g(X) \neq E(X)$; but when $f = g = Id$, we are back to the usual risk neutral-Bayesian world, where both NG and G have the same beliefs about the judge's decision, and evaluate the outcome at trial as its mathematical expectation.

[6] Since parent NG faces the risk $-X$, his true subjective expectation of this risk is $E_{\text{fop}}(-X)$; hence, to be more rigourous, we should have written $E_{\text{ng}}(X) = -E_{\text{fop}}(-X)$. For parent G, we have $E_g(X) = E_{\text{gop}}(X)$. However, our notation rules out secondary difficulties.

Notice that in contrast to the usual *optimistic approach* in litigation settlement, where individual biases are purely subjective, here we suggest a case where biases are rational. The behavior of the judge is common knowledge, hence probabilities lead to common priors for both parents, while they have different preferences over the ranking of the outcomes induced by the judge's decisions: as a result, they have different posterior beliefs concerning the outcomes of the judge's behavior.

It is easy to evaluate the consequences of this bias when parents are pessimistic/risk averse: using theorem 2, we have: $f''>0 \Rightarrow E_{ng}(X)>E(X)$, i.e. NG overestimates the child support to be paid at trial, and $g''>0 \Rightarrow E_g(X)<E(X)$, i.e. G underestimates the child support to be obtained at trial. The opposite inequalities apply in case of optimistic parents.

3 Cooperation under exogenous uncertainty

3.1 Pareto efficient agreements

We first describe the set of agreements that may be obtained by parents such that there exists no other (feasible) agreement leading to additional gains of welfare (a Pareto improvement) for them. For ease of exposition, we denote $a - b = \Delta_1$ and $b - r = \Delta_2$. The basic result is the following:

Proposition 3 *Assume that*:

$$c + \frac{E_{ng}(X) - E_g(X)}{2} \geq 0 \tag{1}$$

Then, there exist two real numbers $WA_g \neq WP_{ng}$, *such that any* $\delta \in [WA_g, WP_{ng}]$ *is a Pareto optimal agreement.*

Proof For parent NG, we have: $V_{ng}(J) = -c - E_{ng}(X)$ and $V_{ng}(N) = -\delta$; thus N is better than J if $\delta<E_{ng}(X) + c \equiv WP_{ng}$. Symmetrically, for parent G: $V_g(J) = -c + E_g(X)$ and $V_g(N) = \delta$; thus N is better than J if $\delta > E_g(X) - c \equiv WA_g$. It is straightforward to verify that $WA_g \leq WP_{ng}$ if condition (1) holds. Moreover, there exists no value of δ outside of $[WA_g, WP_{ng}]$ being welfare improving for parents. □

It makes sense to call the higher bound of the negotiation interval the willingness to pay of the NG parent: WP_{ng} is the maximum amount that the NG parent may accept to pay, while being indifferent between cooperating and going to trial. By the same token, the lower bound will be called the parent G's willingness to accept: WA_g is the minimum amount he accepts from the NG parent, and makes him indifferent between cooperation and going to trial. The willingness to pay of NG and the willingness to accept of G are the sum of two terms; the first one is the associated willingness to pay or accept when the judge is supposed to strictly apply the child support (*i.e.* $b + c$ and

$b - c$ respectively, when $p_b = 1$ and $p_a = p_r = 0$); the second one is a correction in order to take into account the risk of trial.

3.2 Solution to the cooperative divorce

A long and important stream of literature is devoted to the bargaining problem.[7] In order to focus on the main issue of our paper, we simply assume that parents are engaged in a cooperative divorce—and we use a basic concept in cooperative games, known as the (symmetric) Nash solution.

According to this view, the agreement δ^*reached by the parents is defined as the maximand of the Nash product $(V_{ng}(N) - V_{ng}(J))(V_g(N) - V_g(J))$, and the gains of cooperation for parents are defined by:

$$\gamma^* = \max_{\delta}(WP_{ng} - \delta)(\delta - WA_g)$$

Proposition 4 *Assume that* (1) *holds. Then*:

(i) δ^* *is the weighted sum of* WP_{ng} *and* WA_g;
(ii) γ^* *has two components: 1/ the transaction costs at trial, and 2/ the difference between parents' subjective expectation of the outcome at trial.*

Proof

(i) Under condition (1), the first order qualification condition gives the value of δ^*, which may be written as:

$$\delta^* \equiv \frac{1}{2}\left(WP_{ng} + WA_g\right) = \frac{E_{ng}(X) + E_g(X)}{2}$$

with
$E_{ng}(X) + E_g(X) = 2b + [1 - f(1 - p_a) + g(p_a)]\Delta_1 - [1 - g(1 - p_r) + f(p_r)]\Delta_2$.
(ii) Substituting in the value-function of the problem, we obtain the cooperative gains as:

$$\gamma^* \equiv \frac{1}{4}(WP_{ng} - WA_g)^2 = \left(c + \frac{E_{ng}(X) - E_g(X)}{2}\right)^2$$

where
$E_{ng}(X) - E_g(X) = [1 - f(1 - p_a) - g(p_a)]\Delta_1 + [1 - g(1 - p_r) - f(p_r)]\Delta_2$. □

[7] See the illuminating synthesis proposed by Muthoo (1999); it is well known since Binmore, Rubinstein, and Wolinski (1986) that the solution of any sequential bargaining problem *à la Rubinstein*, where parties alternate in making offers/counter-offers to the other party in order to reach an agreement may be as well understood as the solution of a cooperative bargaining game *à la Nash* if appropriately reformulated.

The complete comparative static analysis for δ^* and γ^* obviously comes from the properties of WP_{ng} and WA_g (left to the reader) and heavily depends on the assumption on the risk sensibility of parents. In this spirit, the following paragraph investigates more deeply the impact of uncertainty at trial on the size of the negotiation gains.

3.3 Imperative *versus* indicative scales of child support

Let us discuss the central result of the paper according to which uncertainty created by judges enhance parents' incentives to cooperate in the shadow of the law, beginning with the following note:

Note: When parents are risk neutral (f = g = Id), or simply have the same probability transformation, we obtain: $\delta^* \equiv E(X)$ and $\gamma^* \equiv c^2$.

Then, it is possible to evaluate whether rules are better than discretion in giving parents incentives to cooperate. Everything else equal, we will say that an indicative scale of child support is equivalent to an imperative (i.e. legaly binding) scale of child support if both yield the same cooperative gains. Conversely, we will say that the indicative scale is better (worse) than the imperative scale, that is if it is associated with higher (smaller) cooperative gains.

Corollary 5

(i) *Both scales are equivalent when the parents are risk neutral.*
(ii) *The indicative scale is better when parents are both risk-averse.*
(iii) *The imperative scale is better when parents are both risk-seeking.*

Proof In the case of an imperative scale of child support (*i.e.* with $p_b = 1$ and $p_a = p_r = 0$), the gains associated with cooperation amount to c^2. Then:

(i) In case of an indicative scale of child support, risk neutral parents have the same assessment about the outcome at trial, *i.e.* $E_{ng}(X) = E_g(X) = E(X)$. Hence the result.
(ii) is also straightforward, since when both parents are risk averse: $E_{ng}(X) > E(X) > E_g(X)$. iii) is left to the reader. □

In other words when parents are not risk sensitive, discretion provides no more advantages than a fixed rule, in the sense that it gives no additional incentives to cooperate. In contrast, when both parents are pessimistic/risk-averse, the indicative scale of child support gives more incentives to cooperate than the imperative one: a settlement at trial implies excessive risk for both parents since each overestimates his own subjective likelihood that the settlement at trial leads to the most unfavorable result for himself (paying *a* for

NG, receiving r for G). The opposite is obtained when both parents are optimistic/risk-seeking.[8]

Second, we prove that increasing uncertainty is favorable to the cooperation for risk averse parents. In the following, we assume that the judge uses a randomization with $p_a, p_b, p_r > 0$.

Proposition 6 *Assume that parents are risk averse and face a pure increase in risk at trial, in the sense of a Mean Preserving Spread (MPS); then*:

(i) WP_{ng} *increases and* WA_g *decreases ;*
(ii) γ^* *increases;*
(iii) *Moreover, assume that* $p_a \leq p_r$, *and that* f' *and* g' *are convex. If* g' *is more (less) convex than* f', *then* δ^* *increases (respectively decreases).*

Proof By definition, if parents are risk averse, any MPS at trial decreases their expected utility level. Thus, (i) and (ii) are straightforward since by definition: $WT_{ng} \equiv -V_{ng}(J)$ and $WA_g \equiv V_g(J)$.

In order to prove (iii), consider a simultaneous shift in p_a and p_r such that $E(X)$ is kept constant[9] and $p_a \leq p_r$; it is easy to check that we have:

$$\text{sign}\frac{\partial\delta^*}{\partial p_a} = \text{sign}((f'(1-p_a) - f'(p_r)) - (g'(1-p_r) - g'(p_a)))$$

Assume that g' is more convex than f': given that $p_a + p_r < 1$, and $1 - p_a > 1 - p_r$, then applying a result by Roëll (1987, proposition II.4)[10], we obtain:

$$\frac{f'(1-p_a) - f'(p_r)}{1-p_a-p_r} \geq \frac{g'(1-p_a) - g'(p_r)}{1-p_a-p_r}$$

On the other hand, defining $\alpha = p_r - p_a \geq 0$ and using once more that $p_a + p_r < 1$, by the convexity assumption of g' we have:

$$g'(1-p_r) - g'(p_a) \leq g'(1-p_r+\alpha) - g'(p_a+\alpha) = g'(1-p_a) - g'(p_r)$$

Hence $f'(1-p_a) - f'(p_r) \geq g'(1-p_r) - g'(p_a)$, and as a result $\frac{\partial\gamma^*}{\partial p_a} \geq 0$. The case where g' is less convex than f' may be proven by the same argument, and is left to the reader. □

The analogue applies to optimistic/risk seeking parents.

8 Notice that we obtain an ambiguous result when the parents display the opposite risk sensibility.

9 In order to generate a Mean-Preserving Spread, we require that the simultaneous shifts of p_a and p_r satisfy the following condition: $\Delta_1 \cdot dp_a - \Delta_2 \cdot dp_r = 0$.

10 See also condition (A) thereafter.

Hence, more uncertainty at trial 1/ facilitates cooperation between risk averse parents because it increases the range of the bargaining set $[WA_g, WP_{ng}]$, and 2/ it enhances the incentives of risk averse parents to cooperate since it increases the gains of cooperation $(WA_g - WP_{ng})$. On the other hand, it should be argued that uncertainty may also have perverse effects. As the risk at trial increases, the advantage of the relatively less risk averse party to the bargaining may be strengthened, while the position of the more risk averse parent is weakened. As a result, it may be the case that the agreement on child support is reached for a smaller value of the transfer. In fact, as shown in part (iii) of proposition 8, even when the shift in risk at trial is *a priori* more unfavorable to parent G than to parent NG (in the sense that $p_r > p_a$), it may be the case that more uncertainty ends up in an increase in the transfer between risk averse parents. The proof explicitly shows that the risk aversion assumption is not sufficient in order to unambiguously sign this effect, but more conditions about parents' preferences are required, specifically here on their probability transformation (we need more than f,g convex and g more convex than f).[11]

4 Optimal discretion and cooperation

In the previous section we worked with a purely exogenous uncertainty, since the probabilities (p_a,p_b,p_r) are set on *a priori* ground and may take any value. We consider now a kind of endogenous uncertainty, specifically through an explicit choice of randomization for the judge. Our model may be understood[12] as the reduced form of a more general one, where a population of judges are randomly dealing with a population of divorce cases (couples going to divorce). Knowing the characteristics of parents, judges (who have no specific preferences for their own) may seek to maximize their cooperative gains. Notice also that in such a more general model, it would be possible to justify that this social goal comprises (implies) the minimization of the social costs associated with divorces settlement[13].

Formally, we focus on the socially optimal degree of discretion, corresponding to the socially efficient values of the steady state frequencies (p_b, p_a, p_r) with which the three actions (B, A, R) are used by judges, leading to the maximization of the negotiation gains for the population of parents:

[11] Although this finding may appear counterintuitive, it is not surprising in a context of rational decision making. For example, risk averse consumers need not necessarily increase their savings when they face more uncertainty: they must also be *prudent* (*i.e.* $u''' > 0$ for expected utility consumers, that is u' must be convex; see Kimball (1990)).

[12] See Osborne and Rubinstein (1994) about the debate on the play of mixed strategies.

[13] In France, one of the justifications, at least for the legislator, in favor of scales of alimony is that they may help in reducing the congestion effects of courts specialized in family law. More than half of the decisions taken by the "Tribunaux de Grande Instance" are in fact taken by judges specialized in family law: a great majority of those decisions concern the issue of child support; see for example Munoz-Perez and Ancel (2000).

$$\gamma(p_a,\ p_r) \equiv \left(c + [1 - f(1 - p_a) - g(p_a)]\frac{\Delta_1}{2} + [1 - g(1 - p_r) - f(p_r)]\frac{\Delta_2}{2}\right)^2$$

We have the following result:

Proposition 7

(i) *Assume that both parents are pessimistic; then the judge should choose* (p_a^*, p_r^*)*, the probabilities associated with the pure strategies A and R , in such a way that for each of those strategies, the difference between the likelihood that the judge plays this strategy as evaluated by parent NG and the one evaluated by parent G, be maximum.*

(ii) *When both parents are optimistic, the judge should never deviate from the scale of child support.*

Proof

(i) When both parents are pessimistic, $\gamma\ (p_a, p_r)$ is a concave function. The first order conditions:

$$f'(1 - p_a^*) - g'(p_a^*) = 0 \tag{2}$$

$$g'(1 - p_r^*) - f'(p_r^*) = 0 \tag{3}$$

thus fully characterize the choice of (p_a^*, p_r^*), with $p_b^* = 1 - p_a^* - p_r^*$.

By definition, $1 - f(1 - p_a^*)$ is nothing else but the likelihood for parent NG that the judge plays A and $g(p_a^*)$ is the likelihood for parent G that the judge plays A; then, condition (2) explains that in order to maximize the gains from negotiation, the judge has to play A with probability p_a^* such that the difference between the likelihood of the parents is also maximal. By the same token, $1 - g(1 - p_r^*)$ is the likelihood for parent G that the judge plays R and $f(p_r^*)$ is the likelihood for parent NG that the judge plays R; hence, condition (3) says that R must be played with probability p_r^*, such that the difference between those likelihoods is also maximal.

(ii) Now, when both parents are optimistic, it is easy to see that $\gamma(p_a, p_r)$ is a convex function; moreover, $1 - f(p) - g(p) < 0$ for any p: thus the maximum of $\gamma(p_a, p_r)$ is obtained for $p_b^* = 1$ and $p_a^* = p_r^* = 0$. □

In words, if the social objective is to give parents efficient incentives to cooperate during divorce, then the judge's best strategy is to take into account the parents' beliefs about own decisions, making these beliefs the most divergent possible. However, this objective cannot be reached when parents are risk-neutral or risk-seeking individuals.

Finally, we prove a last useful result concerning the relationship between the mixed strategy played by the judge and the comparative pessimism/risk-aversion index of the parents.

Proposition 8 *Assume that both parents are pessimistic; then:*

(i) $f(p) = g(p), \forall p \in [0, 1] \Rightarrow p_a^* = p_r^* = \frac{1}{2}$ *and* $p_b^* = 0$.
(ii) *if parent G is more risk-averse than parent NG, then* $p_a^* \geq \frac{1}{2}$ *and* $p_r^* \leq \frac{1}{2}$, *with* $p_b^* = 1 - p_a^* - p_r^* \geq 0$.
(iii) *if parent NG is more risk-averse than parent G, then* $p_a^* \leq \frac{1}{2}$ *and* $p_r^* \geq \frac{1}{2}$, *with* $p_b^* = 1 - p_a^* - p_r^* \geq 0$.

Proof

(i) Consider the values (p_a^*, p_r^*) which satisfy conditions (2) and (3). Since both functions f and g are monotone and unique, the result is straightforward.
(ii) Let us first introduce the characterization of the notion of "more risk-averse than" in the context of Yaari's model, which parallels the famous characterization obtained by Pratt for expected utility individuals: Yaari (1987) shows that for two decision makers with probability transformation functions φ_1, φ_2 respectively, φ_2 displays more risk-aversion/pessimism than φ_1, iff φ_2 is a convex transformation φ_1. Roëll (1987, proposition II.4) shows that for all (p,q) satisfying $0 \leq q < p < 1$, this implies:

$$(A) : \frac{\varphi_1(p) - \varphi_1(q)}{p - q} \geq \frac{\varphi_2(p) - \varphi_2(q)}{p - q}$$

Now, assume by contradiction that $p_a^* \leq \frac{1}{2} \Rightarrow p_a^* \leq 1 - p_a^*$; then, using (A) and the convexity assumption of f and g, we obtain:

$$f'(1 - p_a^*) \geq \frac{f(p_a^*) - f(1 - p_a^*)}{p_a^* - (1 - p_a^*)} \geq \frac{g(p_a^*) - g(1 - p_a^*)}{p_a^* - (1 - p_a^*)} \geq g'(p_a^*)$$

hence a contradiction with condition (2). Similarly, assume $p_r^* \geq \frac{1}{2} \Rightarrow p_r^* \geq 1 - p_r^*$; as a result using (A) and the convexity assumption of f and g, we have: $f'(p_r^*) \geq g'(1 - p_r^*)$, hence a contradiction now with condition (3).
(iii) may be obtained using the same argument. □

The intuitive meaning of this last proposition is quite simple. The problem amounts to the optimal sharing of risk between parents. When both parents are equally risk-averse, the judge sets probabilities so as to give the same incentives to both parents (they bear the same risk). But in the case of one of the parents having more risk-aversion, the judge should set the probabilities associated to the play of A and R in such a way that this more risk-averse parent bears less risk than the other: as a result, the latter obtains more incentives to cooperate.

5 Conclusion

In this paper we have analyzed Nash bargaining in divorce cases. In this context, we show that the way parents solve their dispute about child support (settlement versus court decisions) depends on transaction costs and on the behavior of the judge. Our main conclusion is that the incentives to cooperate increase for risk-averse parents when the judge uses a random strategy. The originality of the paper is that risk aversion is not modeled in the standard expected utility framework, but with a particular rank dependent expected utility model with linear utility function: we want to insist on the uncertainty generated by the courts' decisions rather than on the asymmetries of information between parents. Economics of litigation generally insists on this last kind of asymmetries to explain the probability of settlement (i.e. the cooperative solution). But in divorce law, informational asymmetries between parents are not so important and the argument of unpredictability of courts outcomes also needs to be explored. One can observe that the same kind of argument could be developed in liability law and personal injury settings.

From a normative perspective, our analysis suggests some directions for divorce law. In practice, a great number of countries are currently engaged in policy reforms introducing scales of child support. So, it is possible to consider that the introduction of strict guidelines for child support—limitation of the discretionary power of judges—does not represent a good way to encourage cooperative behavior with risk averse people. In this case, if improving cooperation is socially valuable, during and after the litigation, our recommendation should be to promote indicative guidelines and not to impose restrictive calculus. Moreover, if we consider redistributive aspects, it is important to note that the discretionary power of judges could also contribute to an increase in resources available for child support.

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